Siddharth Bahekar – 002417718 – Assignment 1 – INFO 6105

Q1)

## Linear Algebra [50pt]

1. [10pt] Create a matrix array A and vector array in Numpy with **random** integers using random method. For example

$A = \begin{bmatrix} 5 & -2 \\ -3 & -1 \\ 7 & 9 \end{bmatrix}, x = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$. Your A matrix is **5x2** and your x vector is **2x1**. Show your code and result.

Create a diagonal matrix B, with 1,2,3,4,5 in the diagonal. The order doesn't matter. You can have

$B = \begin{bmatrix} 3 & \square & \square \\ \square & 4 & \square \\ \square & \square & 1 \end{bmatrix}$. Note your diagonal matrix is 5x5.

Calculate Ax, and BA. Show your code and result.

Importing necessary libraries and initializing matrix & vectors

[1]:
```python
# import the numpy library
import numpy as np

# create a 5x2 matrix A with random integers
A = np.random.randint(-10, 10, size=(5, 2))
print("Matrix A:")
print(A)

# create a 2x1 vector x with random integers
x = np.random.randint(-10, 10, size=(2, 1))
print("\nVector x:")
print(x)

# create a diagonal matrix B with 1, 2, 3, 4, 5 on the diagonal
B = np.diag([1, 2, 3, 4, 5])
print("\nDiagonal Matrix B:")
print(B)
```

```
Matrix A:
[[-9 -1]
 [ 8  8]
 [-9  8]
 [-4 -9]
 [-4 -6]]

Vector x:
[[  2]
 [-10]]

Diagonal Matrix B:
[[1 0 0 0 0]
 [0 2 0 0 0]
 [0 0 3 0 0]
 [0 0 0 4 0]
 [0 0 0 0 5]]
```

Performing matrix-vector product (Ax) & matrix-matrix product (BA)

Code | JupyterLab | Python 3 (ipykernel)

[3]:
```python
# calculate Ax - matrix vector product operation
Ax = np.dot(A, x)
print("\nAx:")
print(Ax)

# calculate BA - matrix matrix product operation
BA = np.dot(B, A)
print("\nBA:")
print(BA)
```

```
Ax:
[[ -8]
 [-64]
 [-98]
 [ 82]
 [ 52]]

BA:
[[ -9  -1]
 [ 16  16]
 [-27  24]
 [-16 -36]
 [-20 -30]]
```

Q2)

2. [10pt] Calculate the rank of your matrix A and the rank of your matrix B. What about the rank of BA? Are they different? Why?

Matrix rank calculation

Code JupyterLab Python 3 (ipykernel)

```
[5]: # calculating the rank of matrix A
rank_A = np.linalg.matrix_rank(A)
print("\nRank of Matrix A:", rank_A)

# calculating the rank of matrix B
rank_B = np.linalg.matrix_rank(B)
print("Rank of Matrix B:", rank_B)

# calculating the rank of matrix BA
rank_BA = np.linalg.matrix_rank(BA)
print("Rank of Matrix BA:", rank_BA)
```

```
Rank of Matrix A: 2
Rank of Matrix B: 5
Rank of Matrix BA: 2
```

The rank is determined by the following relationship - rank(BA)<=minimum(rank(B), rank(A))

This is the reason for the rank of matrixBA is not equal to rank of matrix B but to the rank of matrix A. The rank of matrix BA will not be 5 because the rank of A is 2, and as per above relationship the minimum rank between matrix B and matrix A is 2.

Q3)

For question 3, 4 and 5, consider the following 3 x 3 real matrices:

$$A = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{bmatrix}.$$

3. [10pt] Find the following expressions by hand. Show your steps.
   (a) AB
   (b) BA
   (c) AB – BA
   (d) ABC

Q-3)

A) Calculate:

$$AB = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (-2)(5)+(1)(6)+(8)(-2) & (-2)(0)+(1)(3)+(8)(-2) & (-2)(-7)+(1)(-9)+(8)(0) \\ (-1)(5)+(-1)(6)+(7)(-2) & (-1)(0)+(-1)(3)+(7)(-2) & (-1)(-7)+(-1)(-9)+(7)(0) \\ (3)(5)+(0)(6)+(4)(-2) & (3)(0)+(0)(3)+(4)(-2) & (3)(-7)+(0)(-9)+(4)(0) \end{bmatrix}$$

$$= \begin{bmatrix} -10+6+(-16) & 0+3+(-16) & +14+(-9)+0 \\ -5+(-6)+(-14) & 0+(-3)+(-14) & 7+9+0 \\ 15+0+(-8) & 0+0+(-8) & -21+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{bmatrix}$$

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B) Calculate

$$BA = \begin{bmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{bmatrix} \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (5)(-2)+(0)(-1)+(-7)(3) & (5)(1)+(0)(-1)+(-7)(0) & (5)(8)+(0)(7)+(-7)(4) \\ (6)(-2)+(3)(-1)+(-9)(3) & (6)(1)+(3)(-1)+(-9)(0) & (6)(8)+(3)(7)+(-9)(4) \\ (-2)(-2)+(-2)(-1)+(0)(3) & (-2)(1)+(-2)(-1)+(0)(0) & (-2)(8)+(-2)(7)+(0)(4) \end{bmatrix}$$

$$= \begin{bmatrix} -10+0+(-21) & 5+0+0 & 40+0+(-28) \\ -12+(-3)+(-27) & 6+(-3)+0 & 48+21+(-36) \\ 4+2+0 & -2+(2)+0 & -16+(-14)+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -31 & 5 & 12 \\ -42 & 3 & 33 \\ 6 & 0 & -30 \end{bmatrix}$$

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c) AB − BA $= \begin{bmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{bmatrix} - \begin{bmatrix} -31 & 5 & 12 \\ -42 & 3 & 33 \\ 6 & 0 & -30 \end{bmatrix}$

$$= \begin{bmatrix} -20-(-31) & -13-5 & 5-12 \\ -25-(-42) & -17-3 & 16-33 \\ 7-6 & -8-0 & -21-(-30) \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -18 & -7 \\ 17 & -20 & -17 \\ 1 & -8 & 9 \end{bmatrix}$$

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d) Calculate : ABC

using matrix association : ABC = AB(C)

$$\begin{bmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{bmatrix} \begin{bmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} (-20)(6)+(-13)(2)+(5)(-1) & (-20)(3)+(-13)(4)+(5)(-1) & (-20)(-1)+(-13)(5)+(5)(8) \\ (-25)(6)+(-17)(2)+(16)(-1) & (-25)(3)+(-17)(4)+(16)(-1) & (-25)(-1)+(-17)(5)+(16)(8) \\ (7)(6)+(-8)(2)+(-21)(-1) & (7)(3)+(-8)(4)+(-21)(-1) & (7)(-1)+(-8)(5)+(-21)(8) \end{bmatrix}$$

$$= \begin{bmatrix} -120+(-20)+(-5) & -60+(-52)+(-5) & 20+(-65)+40 \\ -150+(-34)+(-10) & -75+(-68)+(-16) & 25+(-85)+128 \\ 42+(-16)+(21) & 21+(-32+21 & -7+(-40)+(-168 \end{bmatrix}$$

$$ABC = \begin{bmatrix} -151 & -117 & -5 \\ -200 & -159 & 68 \\ 47 & 10 & -215 \end{bmatrix}$$

Q4)

4. [10pt] Calculate the eigenvalues and eigenvectors of matrix A above by hand.
    a. Show your steps below. You can use external tools to solve for a polynomial equation only.
    b. Show the trace of matrix A.

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Q-4) Calculate eigen values & eigenvectors of matrix A and show trace of matrix A.

a)

$$A = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix}$$

$\therefore Ax = \lambda x$

$Ax - \lambda I x = 0$

$(A - \lambda I)x = 0$

$\det(A - \lambda I) = 0$

$$\begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} -2-\lambda & 1 & 8 \\ -1 & -1-\lambda & 7 \\ 3 & 0 & 4-\lambda \end{bmatrix}$$

Find the determinant →

$$(-2-\lambda)[(-1-\lambda)(4-\lambda)-(7\times 0)] - 1[(-1)(4-\lambda)-(7)(3)] + 8[(-1)(0)-(-1-\lambda)(3)] = 0$$

$$\lambda^3 + \lambda^2 + 33\lambda + 57 = 0$$

$\lambda = -2.192$, $\lambda = -3.747$, $\lambda = 6.939$

Eigenvalues are → $\lambda = -2.192$, $\lambda = -3.747$, $\lambda = 6.939$

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Find eigenvector with eigenvalue $\lambda = -2.192$

$(A - \lambda I)x = 0$

$$x\left[\begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} - (-2.192)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right] = 0$$

$$x\left[\begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} - \begin{bmatrix} -2.192 & 0 & 0 \\ 0 & -2.192 & 0 \\ 0 & 0 & -2.192 \end{bmatrix}\right] = 0$$

$$\begin{bmatrix} 0.192 & 1 & 8 \\ -1 & 1.192 & 7 \\ 3 & 0 & 6.192 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left[\begin{array}{l} 0.192x_1 + x_2 + 8x_3 = 0 \\ -1x_1 + 1.192x_2 + 7x_3 = 0 \\ 3x_1 + 0 + 6.192x_3 = 0 \end{array}\right.$$

Using Cramers rule:

$$\frac{x_1}{\begin{matrix} 1.192 & 7 \\ 0 & 6.192 \end{matrix}} \qquad \frac{-x_2}{\begin{matrix} -1 & 7 \\ 3 & 6.192 \end{matrix}} \qquad \frac{x_3}{\begin{matrix} -1 & 1.192 \\ 3 & 0 \end{matrix}}$$

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$(1.192 \times 6.192) - (7 \times 0) = 7.38$ $\quad (-1 \times 6.192) - (3 \times 7) = 27.192$ $\quad (-1 \times 0) - (1.192 \times 3) = -3.576$

when eigenvalue is $-2.192$, the eigenvectors $\rightarrow \begin{bmatrix} 7.38 \\ 27.192 \\ -3.576 \end{bmatrix}$

$\rightarrow$ Eigenvector with Eigenvalue $\lambda = -3.747$ $\quad (A - \lambda I)x = 0$

$$\left( \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} - \begin{bmatrix} -3.747 & 0 & 0 \\ 0 & -3.747 & 0 \\ 0 & 0 & -3.747 \end{bmatrix} \right) x = 0$$

$$\begin{bmatrix} 1.747 & 1 & 8 \\ -1 & 2.747 & 7 \\ 3 & 0 & 7.474 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$1.747 x_1 + x_2 + 8x_3 = 0$

$-1 x_1 + 2.747 x_2 + 7x_3 = 0$

$3x_1 + 0 \cdot x_2 + 7.474 x_3 = 0$

using Cramers rule $\rightarrow$

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$\dfrac{x_1}{\begin{vmatrix} 2.747 & 7 \\ 0 & 7.474 \end{vmatrix}} \quad \dfrac{-x_2}{\begin{vmatrix} -1 & 7 \\ 3 & 7.747 \end{vmatrix}} \quad \dfrac{x_3}{\begin{vmatrix} -1 & 2.747 \\ 3 & 0 \end{vmatrix}}$

$= 21.28 \quad = 28.747 \quad = -8.241$

when Eigenvalue is $-3.747$, the eigenvectors $\rightarrow \begin{bmatrix} 21.28 \\ 28.747 \\ -8.241 \end{bmatrix}$

$\rightarrow$ Eigenvector with Eigenvalue $\lambda = 6.939$

$(A - \lambda I)x = 0$

$$\left[ \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 6.939 & 0 & 0 \\ 0 & 6.939 & 0 \\ 0 & 0 & 6.939 \end{bmatrix} \right] x = 0$$

$$\begin{bmatrix} -8.939 & 1 & 8 \\ -1 & -7.939 & 7 \\ 3 & 0 & -2.939 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-8.939 x_1 + x_2 + 8x_3 = 0$

$-x_1 - 7.939 x_2 + 7x_3 = 0$

$3x_1 + 0x_2 + (-2.939)x_3 = 0$

using Cramers rule $\rightarrow$

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| $x_1$ | | $-x_2$ | | $x_3$ | |
|---|---|---|---|---|---|
| -7.939 | 7 | -1 | 7 | -1 | -7.939 |
| 0 | -2.939 | 3 | -2.939 | 3 | 0 |
| = 23.33 | | = 18.061 | | = 23.817 | |

When Eigenvalue is 6.939, Eigenvector $\rightarrow \begin{bmatrix} 23.33 \\ 18.061 \\ 23.817 \end{bmatrix}$

b) To find trace of matrix A:

Trace is the sum of all eigenvalues of matrix.
Our eigenvalues are $\lambda = -2.192$, $\lambda = -3.747$, $\lambda = 6.939$

$$(-2.192) + (-3.747) + (6.939) = 1$$

Hence, the trace of matrix A is 1.

Q5)

5. [10pt] Use Numpy to concatenate A, B, C to a 9 x 3 matrix. The order doesn't matter. Let's call the new matrix D. Create b=[3, -10, 2]$^T$. Find the least squares solution x that minimize $||D^T x - b||^2$. Show your code and results.

Initializing the A,B,C matrices, concatenating the 3 matrices and storing it in matrix D, initializing vector b

```
[9]: #intializing matrices A,B & C as given in the question
import numpy as np
matrix_A = np.array([[-2 , 1, 8],
                     [-1, -1, 7],
                     [ 3,  0, 4]])

matrix_B = np.array([[ 5 , 0, -7],
                     [ 6,  3, -9],
                     [ -2, -2, 0]])

matrix_C = np.array([[ 6 , 3, -1],
                     [ 2,  4,  5],
                     [ -1, -1, 8]])
```

```
[17]: #concantenate matrix matrix_A, matrix_B, matrix_C
matrix_D = np.concatenate((matrix_A, matrix_B, matrix_C))
print(f"Concatenated matrix D: \n{matrix_D}")

#initializing vector b and transposing it
vector_b = np.array([[ 3, -10, 2]]).T
print(f"Vector b after transposing it: \n{vector_b}")
```

```
Concatenated matrix D:
[[-2  1  8]
 [-1 -1  7]
 [ 3  0  4]
 [ 5  0 -7]
 [ 6  3 -9]
 [-2 -2  0]
 [ 6  3 -1]
 [ 2  4  5]
 [-1 -1  8]]
Vector b after transposing it:
[[  3]
 [-10]
 [  2]]
```

Least square solution

```
[25]: #finding the least square solution
least_square = np.linalg.lstsq(matrix_D.T,vector_b, rcond=None)[0]
print(f"Least Square Solution is: \n{least_square}")
```

```
Least Square Solution is:
[[-0.6588419 ]
 [ 0.79125205]
 [ 1.2848211 ]
 [ 1.12195654]
 [-0.49220611]
 [ 0.53822844]
 [ 0.10452284]
 [-1.36113905]
 [ 0.86584317]]
```

1. [10pt] Roll a six-sided die 5 times. What is the probability of rolling a six in all 5 rolls? If rolling the die 5 times is considered one trial, perform 500 trials. What is the probability of rolling a six in all 5 rolls in exactly one of these 500 trials? What about rolling a six in all 5 rolls in at least one of the 500 trials?

Q-1) Probability of rolling a six sided dice is 1/6.

Probablity of rolling a six in all the 5 throws

$= (1/6)^5 = (1/7776) = 0.000129$

What would be the probablity of rolling a six in all 5 throws in exactly 1 of 500 trials?

→ As mentioned in ques, rolling 5 times is 1 trial.

Binomial probablity function : $P(x) = {}^nC_x \cdot p^x \cdot q^{n-x}$

$n = 500$ trials

$x =$ number of success $= 1$

$p =$ Probablity of success on single trial $= (1/6)^5$

$q =$ Probablity of failure on single trial $= 1 - p = 1 - (1/6)^5$

P exactly 1 success in 500 trials =

$$\frac{500}{1} \times \left(\frac{1}{6}\right)^5 \times \left(1 - \left(\frac{1}{6}\right)^5\right)^{500-1}$$

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$$= 500 \times \frac{1}{7776} \times \left(1 - \frac{1}{7776}\right)^{499}$$

$$= 0.06$$

Probability of getting a six in all 5 rolls in exactly 1 of 500 trials is 6 %.

→ Probability of not getting six in all 5 throws in atleast one of the 500 trials –

$P(\text{min at least 1 success}) = 1 - P(\text{no success})$

$= (1 - (1/6)^5)^{500}$

$= (1 - 1/7776)^{500}$

$= (7775/7776)^{500}$

$= 0.93771$

Probability of not getting a six in all 5 rolls in at least 1 of 500 trials is 93%.

∴ The probability of rolling a 6 in all 5 roles atleast one in 500 trials $\Rightarrow 1 - (7775/7776)^{500}$

$\Rightarrow 1 - 0.93771$

$= 0.06288$

∴ Probability is 0.06288 or 6.288%.

2. [10pt] A study found that the average amount of coffee consumed by college students is 3 cups per day. Assuming this consumption follows a normal distribution with a standard deviation of 0.8 cups, what is the probability that a randomly selected college student drinks between 2.2 and 3.8 cups of coffee per day?

Q-2) mean ($\bar{x}$) = 3 , std. deviation ($\sigma$) = 0.8

$x$ = coffee consumption amount per day

$x$ = 2.2 and $x$ = 3.8

→ Calculating z-score for $x = 2.2$ & $x = 3.8$

$$Z_x = \frac{x - \bar{x}}{\sigma}$$

$$Z_{2.2} = \frac{2.2 - 0.8}{0.8} = -1$$

$$Z_{3.8} = \frac{3.8 - 0.8}{0.8} = 1$$

→ looking for z-scores in a standard ~~deviation~~ normal distribution table :

$Z_{2.2} = -1$ , z score is 0.15866

$Z_{3.8} = 1$ , z score is 0.84134

$$P(Z \leq Z_{2.2}) = 0.15866$$

$$P(Z \leq Z_{3.8}) = 0.84134$$



→ Probability of randomly selected student who drinks between 2.2 and 3.8 cups

$$P(3.8 \geq x \geq 2.2) = P(Z \leq Z_{3.8}) - P(Z \leq Z_{2.2})$$

$$= 0.84134 - 0.15866$$

$$= 0.683$$

The probability of a student drinking between 2.2 and 3.8 is 68 %.

3. [10pt] 6 Digital Camera Prices The prices (in dollars) for a particular model of digital camera with 18.0 megapixels and a f/3.5–5.6 zoom lens are shown here for 10 randomly selected online retailers. Estimate the true mean price for this particular model with 95% confidence.
Prices: [999, 1499, 1997, 398, 591, 498, 798, 849, 449, 348]

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Q-3) $n = 10$, confidence level = 95%.

Prices = [999, 1499, 1997, 398, 591, 498, 798, 849, 449, 348]

$$\bar{x} = \frac{999 + 1499 + 1997 + 398 + 591 + 498 + 798 + 849 + 449 + 348}{10}$$

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{2569262.4}{9}}$$

$$= 534.29$$

| $(x_i - \bar{x})^2$ |
|---|
| 24460.96 |
| 430860.96 |
| 1332639.36 |
| 197669.16 |
| 63302.56 |
| 118749.16 |
| 40.96 |
| 154920.96 |
| 244629.16 |
| 2569262.4 |

t-value for 95% confidence level with degree of freedom $(n-1)$, 9 is 2.262

→ Finding ~~true~~ true mean :

$$\bar{x} = \mu = \bar{x} \pm t\left(s/\sqrt{n}\right)$$

$$\bar{x} + t\left(s/\sqrt{n}\right) \leq \mu \leq \bar{x} + t\left(s/\sqrt{n}\right)$$

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$\mu = 842.6 \pm 2.262 \left( 534.29 / \sqrt{10} \right)$

$= 842.6 \pm 168.95 \times 2.262$

$= 842.6 \pm 382.16$

$460.44 \le \mu \le 1224.76$

$\therefore$ True mean price of camera with 95% confidence lies between 460.44 and 1224.76

4. [10pt] The average number of books read by a person in a year is reported to be 12. A 'reader' is defined as a person who reads at least one book in a year. A random sample of 50 readers from a local community library showed that the average number of books read per person was 13.4. The population standard deviation is 4.5 books. At the 0.01 level of significance, can it be concluded that this sample represents a significant difference from the national average?

Q-4) $\mu = 12$ (Avg no. of books read by person in a year)

$\bar{x} = 13.4$ (sample mean)

$n = 50$ (sample size)

$\sigma = 4.5$ (Population standard deviation)

$\alpha = 0.01$ (significance level)

$H_0$ (null hypothesis) = Avg no. of books read by a person in a year is 12.

$H_1$ (Alt hypothesis) = Avg no. of books read by a person in a year is 13.4

$n > 30$ and $\sigma$ is known, using z-test

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{13.4 - 12}{4.5/\sqrt{50}} = 2.19 \approx 2.20$$

When $\alpha = 0.01$, critical value is 2.58

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The calculated z-value is 2.20 which is less than critical value 2.58. Fail to reject the null hypothesis.

with 0.01 level of signifance, means with 99% confidence conclude this sample doesnt represent a significant difference from the national averge.

5. [10pt] A statistics professor is used to having a variance in his class grades of no more than 100. He feels that his current group of students is different, and so he examines a random sample of midterm grades as shown. At $\alpha = 0.05$, can it be concluded that the variance in grades exceeds 100?

The grades: [92.3, 89.4, 76.9, 65.2, 49.1, 96.7, 69.5, 72.8, 67.5, 52.8, 88.5, 79.2, 72.9, 68.7, 75.8]

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Q-5) $n = 15$ (sample size)

$\alpha = 0.05$ (significance level)

$\sigma^2 = 100$ (Population variance

grades = 92.3, 89.4, 76.9, 65.2, 49.7, 96.7, 69.5, 72.8, 67.5, 52.8, 88.5, 79.2, 72.9, 68.7, 75.8

Null hypothesis ($H_0$) = variance in class grades is no more than 100

Alt hypothesis ($H_1$) = variance in class grades is more than 100.

$$\bar{x} = \frac{\text{sum of all grades}}{n}$$

$$\bar{x} = \frac{92.3 + 89.4 + 76.9 + 65.2 + 49.1 + 96.7 + 69.5 + 72.8 + 67.5 + 52.8 + 88.5 + 79.2 + 72.9 + 68.7 + 75.8}{15}$$

$\bar{x} = 74.48$

$$s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1}$$

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$(x_i - \bar{x})^2$

317.31
222.40
5.82
86.24
644.48
493.43
24.86
2.84
48.81
470.31
176.37
22.21
2.51
33.48
1.72

2572.85

$= \frac{2572.85}{14}$

$= 183.77$

Critical chi-squared value for degree of freedom = 15 and significance level of 0.05 is 23.68, critical value.

Calculating the test stats

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2} = \frac{14 \times 183.77}{100} = 25.72$$

$\therefore 25.72 > 23.68$, rejecting the null hypothesis.

With 0.05 significance level we can conclude the variance in grades ~~increases~~ exceeds 100.