# Machine Learning

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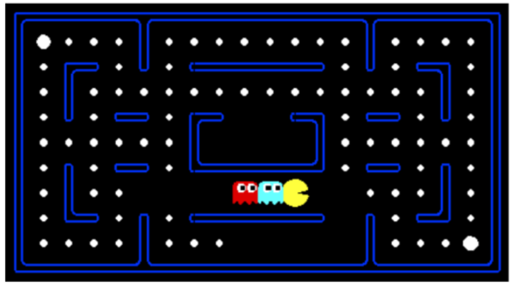
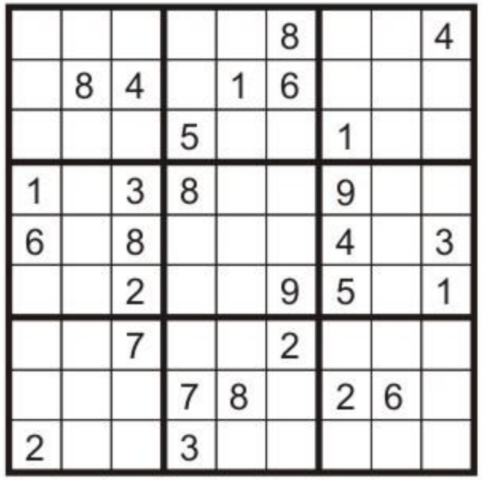
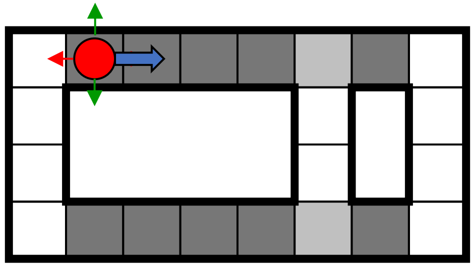
# What we have studied so far…

How are these “rules” or “model of the world” obtained?

- Given the **rule of a game** (and/or **behavior model of the opponents**), find the optimal solution (search, search in multi-player games)
- Given the **relation among variables**, find a satisfied solution (constraint satisfaction)
- Given the **relation among variables**, find the probability of certain events, or the most probable events (Bayes nets, HMM)
- Given a **knowledge base**, infer some facts (logic)

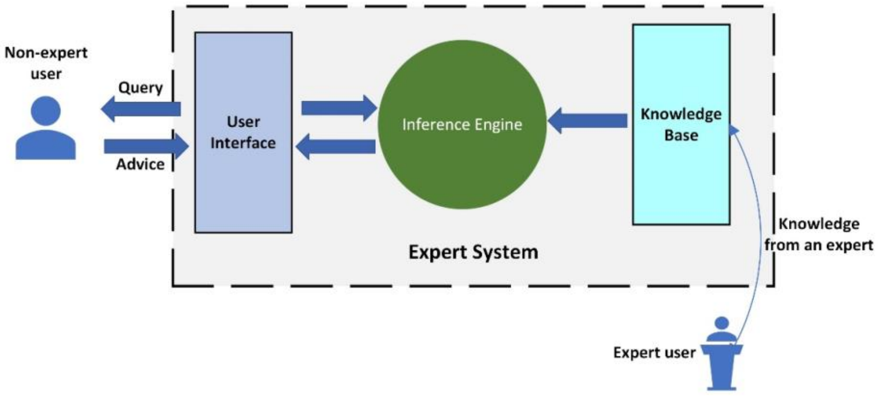






**Knowledge base**

If **has_hair**, then **mammal**.
If **mammal** and **has_hooves**, then **ungulate**.
If **has_feathers**, then **bird**.



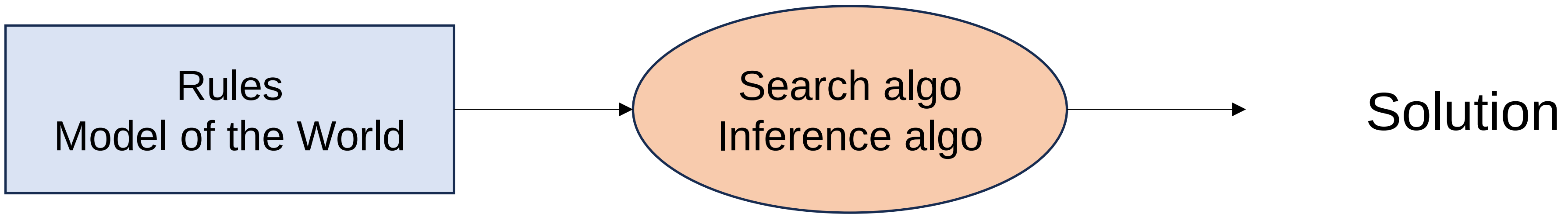
# Rules or Model of the World

- In games designed by human, we simply have the ground-truth rules
  - Pacman
  - Chess, Go
  - Sudoku

- Some are rules set by human based on their observations/knowledge of the world
  - Knowledge base in expert systems

- Some have appeared magically so far
  - The behavior model of the ghosts in Pacman
  - Probability tables in Bayes nets, HMM

# Machine Learning



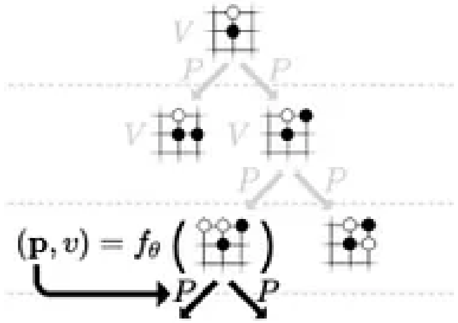

What we have taken for granted

We will discuss how to let machine **learn** these models through **data** collected from the world

**Machine Learning**

# Machine Learning

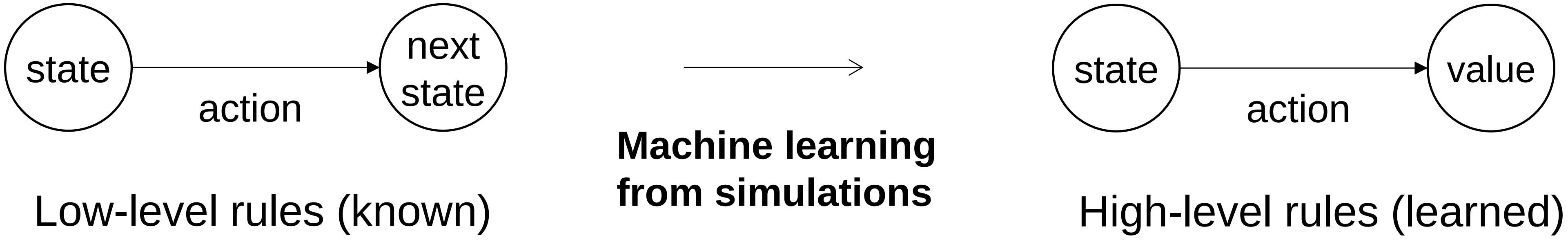
In some cases, even when the world model is designed by human and known, we still want to perform machine learning




**Evaluation function / Heuristic function**

Depth-limited search

Guide the search in games with large branching factors



# Naïve Bayes

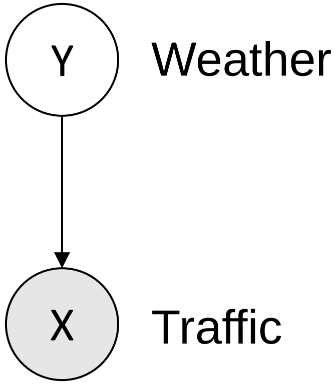
# Learning Simple Bayesian Networks

Suppose we have a set of data:

(Y, X) =

{ (sun, F) , (rain, F) , (rain, T) , (rain, T)
(sun, T) , (sun, F) , (sun, F) , (sun, T) }

**training**




P(Y)

P(X | Y)

How should we build the BN model?

| Y | P(Y) |
|---|---|
| Sun | 5/8 |
| Rain | 3/8 |

| Y | X | P(X \| Y) |
|---|---|---|
| Sun | T | 2/5 |
| Sun | F | 3/5 |
| Rain | T | 2/3 |
| Rain | F | 1/3 |

P(Y/X)

If now we observe X (traffic) = T, how to infer the Y (weather) distribution?

# How did we obtain the parameters?

Why do we model $\mathrm{P}(\mathrm{X} = \mathrm{T} \mid \mathrm{Y} = \mathrm{sun})$ as $\frac{\#\,(\mathrm{Y=sun, X=T})}{\#(\mathrm{Y=sun})}$ in the dataset?

**Maximum Likelihood Estimation** (MLE)

can be used in training any BNs with finite domains

set of all possible models

$\subset \mathbb{R}^6$

Likelihood

Pick $\underset{\boldsymbol{M}}{\operatorname{argmax}} \prod_{i=1}^{n} P_{\boldsymbol{M}}(x_i, y_i)$

Best explains the data (?)

--- has some drawbacks (discussed later)

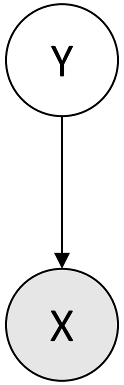
**Approximate inference**

We have the model, and thus the exact value of P(Y|X) is available. But because the exact computation is expensive, we approximate it with samples drawn from the model.

**Model learning**

We do not have the model, and try to build it from data drawn from the nature.

# Dealing with High-Dimensional Observation




Digit

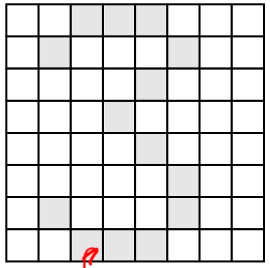
8x8 Bitmap

10 values

Y ∈ {0, 1, 2, …, 9}

$2^{64}$

X:



$\{0,1\}^{64}$

{0,1}

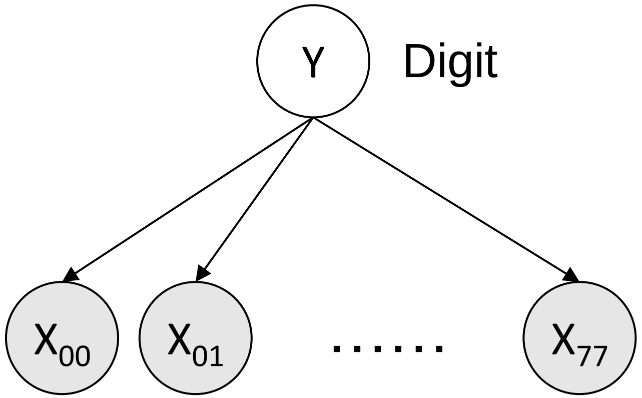
Number of parameters in this model?

| Y | P(Y) |
|---|---|
| | |

| X | Y | P(X\|Y) |
|---|---|---|
| | | |

$10 \times 2^{64}$

# Dealing with High-Dimensional Observation

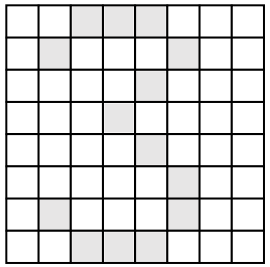



Digit

$\{0, 1\}$

$Y \in \{0, 1, 2, \ldots, 9\}$

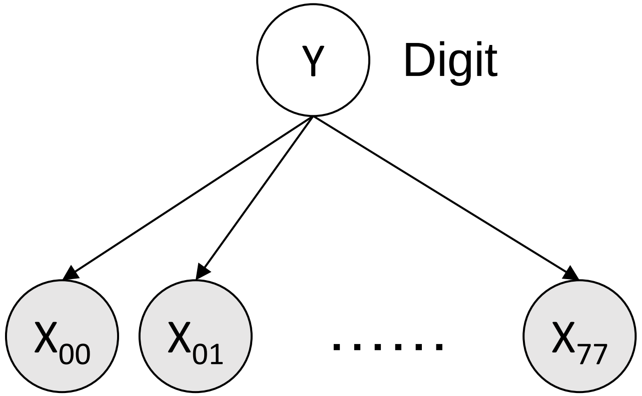
X:



Number of parameters in this model?

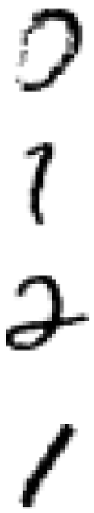
$10 \times 2 \times 64$

# Dealing with High-Dimensional Observation




**Training:**

1) Get dataset

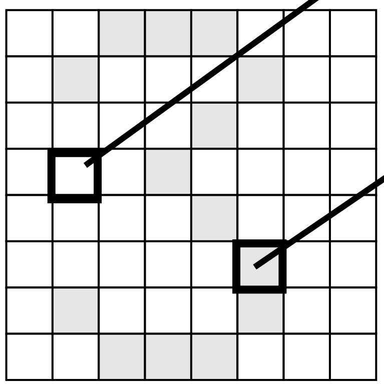


0
1
2
1

2) Match model with empirical frequency

P(Y)

| | |
|---|---|
| 1 | 0.1 |
| 2 | 0.1 |
| 3 | 0.1 |
| 4 | 0.1 |
| 5 | 0.1 |
| 6 | 0.1 |
| 7 | 0.1 |
| 8 | 0.1 |
| 9 | 0.1 |
| 0 | 0.1 |



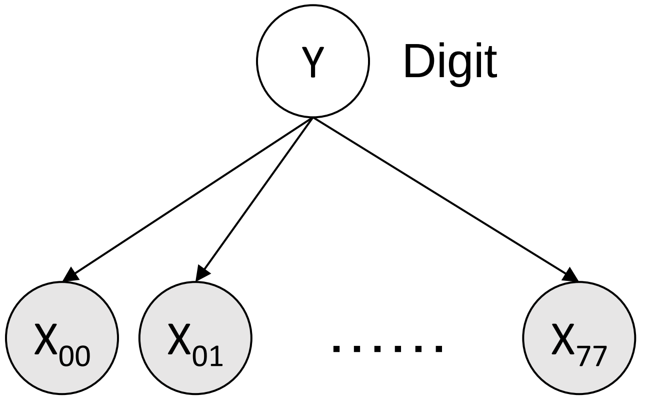
$P(X_{31}=on \mid Y)$

| | |
|---|---|
| 1 | 0.01 |
| 2 | 0.05 |
| 3 | 0.05 |
| 4 | 0.30 |
| 5 | 0.80 |
| 6 | 0.90 |
| 7 | 0.05 |
| 8 | 0.60 |
| 9 | 0.50 |
| 0 | 0.80 |

$P(X_{55}=on \mid Y)$

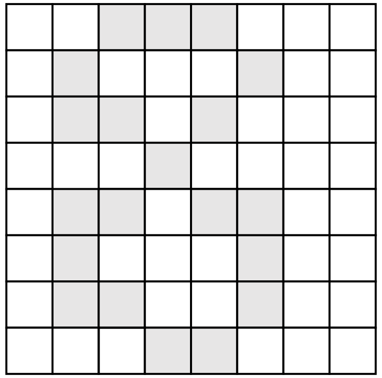
| | |
|---|---|
| 1 | 0.05 |
| 2 | 0.01 |
| 3 | 0.90 |
| 4 | 0.80 |
| 5 | 0.90 |
| 6 | 0.90 |
| 7 | 0.25 |
| 8 | 0.85 |
| 9 | 0.60 |
| 0 | 0.80 |

# Dealing with High-Dimensional Observation




**Inference:**

After training, now given a bitmap, decide its likelihood to be each digit

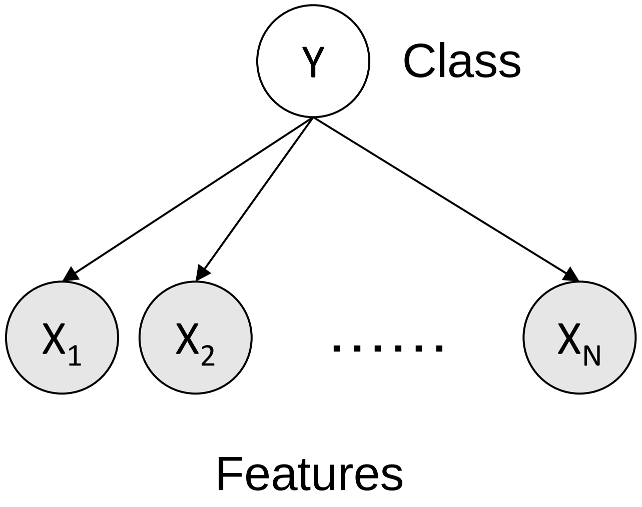


$P(Y \mid x_{00}, x_{01}, \ldots, x_{77})$

$\propto p(Y, x_{00}, x_{01}, \cdots, x_{77})$

$= p(Y)\, p(x_{00} \mid Y)\, p(x_{01} \mid Y) \cdots p(x_{77} \mid Y)$

# General Naïve Bayes Model




Finite domains for $Y$ and $X_i$

**Training:**

1) Get dataset consisting of $(X, Y) = (X_1, \ldots, X_N, Y)$ pairs

2) Train model $P(Y)$, $P(X_i \mid Y)$ with maximum likelihood estimation (= empirical frequency)

(more options discussed later)

**Inference:**

Given x,

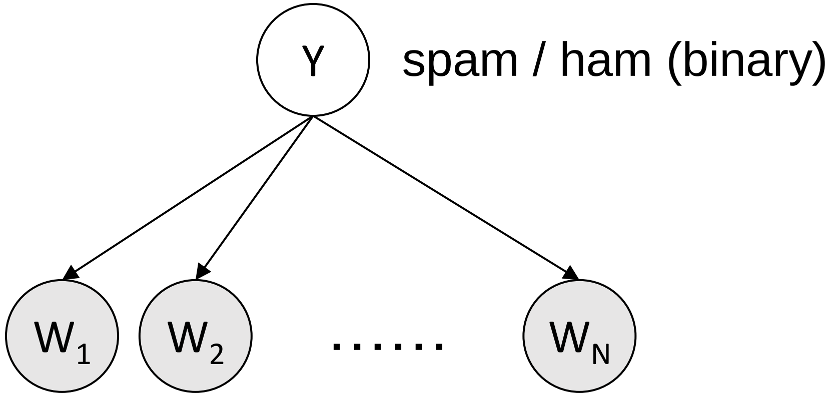
Infer $P(Y \mid x) \propto P(Y)\, P(x_1 \mid Y)\, P(x_2 \mid Y) \ldots P(x_N \mid Y)$

# Example: Spam Filtering

Training data:

Collection of emails, labeled spam or ham

Model (**bag-of-word**):




Special assumption (not in the digit example):
$P(W_i \mid Y)$ is identical for every i

→ This is why it is called bag-of-world (word ordering does not matter)

Dear Sir.

First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret. …

TO BE REMOVED FROM FUTURE MAILINGS, SIMPLY REPLY TO THIS MESSAGE AND PUT "REMOVE" IN THE SUBJECT.

99 MILLION EMAIL ADDRESSES
FOR ONLY $99

Ok, Iknow this is blatantly OT but I'm beginning to go insane. Had an old Dell Dimension XPS sitting in the corner and decided to put it to use, I know it was working pre being stuck in the corner, but when I plugged it in, hit the power nothing happened.

# Example: Spam Filtering

- Model: $$P(Y, W_1 \dots W_n) = P(Y) \prod_i P(W_i|Y)$$
- What are the parameters?

$P(Y)$

| | |
|---|---|
| ham | 0.66 |
| spam | 0.33 |

$P(W|\text{spam})$

| | |
|---|---|
| the | 0.0156 |
| to | 0.0153 |
| and | 0.0115 |
| of | 0.0095 |
| you | 0.0093 |
| a | 0.0086 |
| with | 0.0080 |
| from | 0.0075 |
| ... | |

$P(W|\text{ham})$

| | |
|---|---|
| the | 0.0210 |
| to | 0.0133 |
| of | 0.0119 |
| 2002 | 0.0110 |
| with | 0.0108 |
| from | 0.0107 |
| and | 0.0105 |
| a | 0.0100 |
| ... | |

# Spam Example

$$\log\left( P(Y) \prod_i P(w_i|Y) \right) = \boxed{\log P(Y) + \sum_i \log(w_i|Y)}$$

| Word | P(w\|spam) | P(w\|ham) | Tot Spam | Tot Ham |
|---|---|---|---|---|
| (prior) | 0.33333 | 0.66666 | -1.1 | -0.4 |

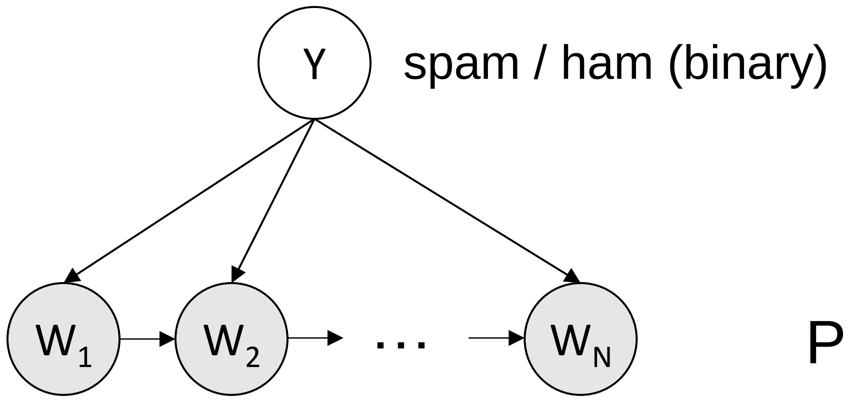
P(spam | w) = 98.9

# Another Possible Model

May slightly improve accuracy

e.g., Earn money vs. Earn degree

with the price of larger model (usually requires more data to train)




$P(W_i \mid Y, W_{i-1})$

# Overfitting and Generalization

# If using Maximum Likelihood Estimation…

For a new bitmap:

$P(\text{features}, C = 2)$

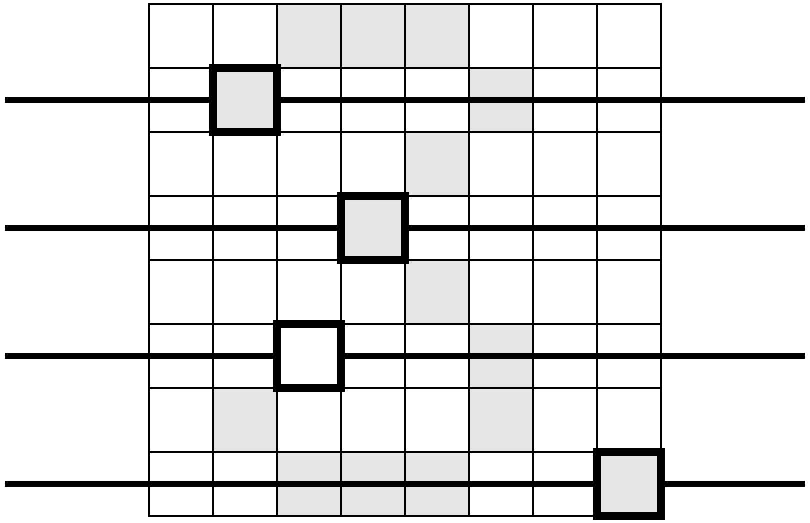
$P(C = 2) = 0.1$

$P(\text{on}|C = 2) = 0.8$

$P(\text{on}|C = 2) = 0.1$

$P(\text{off}|C = 2) = 0.1$

$P(\text{on}|C = 2) = 0.01$



$P(\text{features}, C = 3)$

$P(C = 3) = 0.1$

$P(\text{on}|C = 3) = 0.8$

$P(\text{on}|C = 3) = 0.9$

$P(\text{off}|C = 3) = 0.7$

$P(\text{on}|C = 3) = 0.0$

*2 wins!!*

# If using Maximum Likelihood Estimation…

Prediction determined by *relative* probabilities:

$$\frac{P(W|\text{ham})}{P(W|\text{spam})}$$

```
south-west : inf
nation     : inf
morally    : inf
nicely     : inf
extent     : inf
seriously  : inf
...
```

$$\frac{P(W|\text{spam})}{P(W|\text{ham})}$$

```
screens    : inf
minute     : inf
guaranteed : inf
$205.00    : inf
delivery   : inf
signature  : inf
...
```

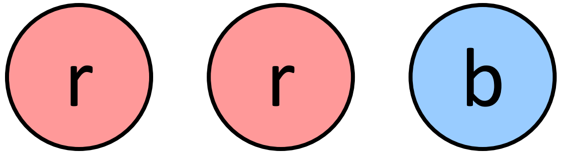
# Overfitting

- Relative frequency parameters will overfit the training data!
  - Just because we never saw a 3 with pixel (15,15) on during training doesn't mean we won't see it at test time
  - Unlikely that every occurrence of "minute" is 100% spam
  - Unlikely that every occurrence of "seriously" is 100% ham
  - What about all the words that don't occur in the training set at all?
  - In general, we can't give unseen events zero probability

- For Naïve Bayes we use smoothing to address this issue
  - A special case of the general concept of "regularization"

# Laplace Smoothing

- Laplace's estimate:
  - Pretend you saw every outcome k extra times




$$P_{LAP,k}(x) = \frac{c(x) + k}{N + k|X|}$$

  - What's Laplace with k = 0?
  - k is the strength of the prior

$$P_{LAP,0}(X) =$$

$$P_{LAP,1}(X) =$$

$$P_{LAP,100}(X) =$$

- Laplace for conditionals:
  - Smooth each condition independently:

$$P_{LAP,k}(x|y) = \frac{c(x,y) + k}{c(y) + k|X|}$$

# Real NB: Smoothing

- For real classification problems, smoothing is critical
- New odds ratios:

$$\frac{P(W|\text{ham})}{P(W|\text{spam})} \qquad \frac{P(W|\text{spam})}{P(W|\text{ham})}$$

```
helvetica : 11.4
seems     : 10.8
group     : 10.2
ago       :  8.4
areas     :  8.3
...
```

```
verdana : 28.8
Credit  : 28.4
ORDER   : 27.2
<FONT>  : 26.9
money   : 26.5
...
```

# Tuning on Held-Out Data

- Now we've got two kinds of unknowns
  - Parameters: the probabilities P(X|Y), P(Y)
  - Hyperparameters: e.g. the amount / type of smoothing to do, k, $\alpha$

- What should we learn where?
  - Learn parameters from training data
  - Tune hyperparameters on different data
  - For each value of the hyperparameters, train and test on the held-out data
  - Choose the best value and do a final test on the test data