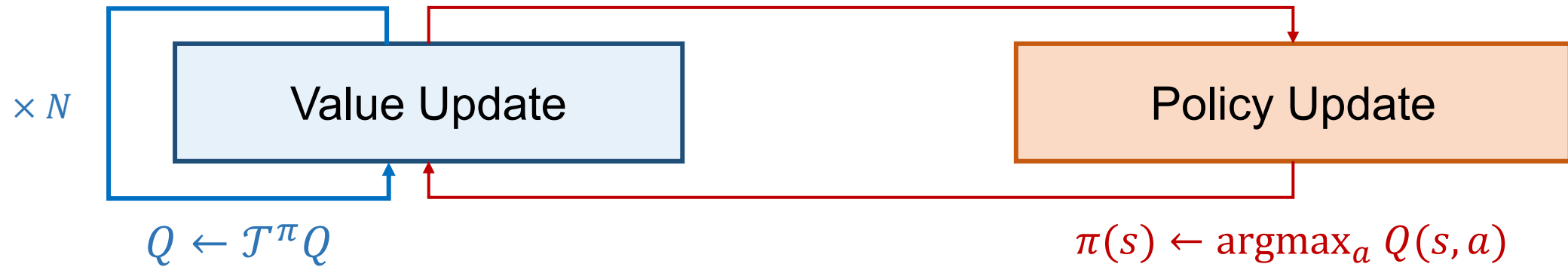


Model-Based RL

Chen-Yu Wei

Recap: Model-Free RL



$$Q \leftarrow \mathcal{T}^\pi Q \text{ means } Q(s, a) \leftarrow R(s, a) + \gamma \sum_{s', a'} P(s' | s, a) \pi(a' | s') Q(s', a') \quad \text{for all } s, a$$

Recap: Generalized Policy Iteration with Samples

For $k = 1, 2, \dots$

For $i = 1, 2, \dots, N$:

Choose action a_i with the current policy

Receive reward $r_i \sim R(s_i, a_i)$ and $s'_i \sim P(\cdot | s_i, a_i)$

$s_{i+1} = s'_i$ if episode continues, $s_{i+1} \sim \rho$ if episode ends

Push (s_i, a_i, r_i, s'_i) to \mathcal{B}

Draw (s, a, r, s') from \mathcal{B} , and use them to update policy/value

Empty \mathcal{B} if under on-policy training

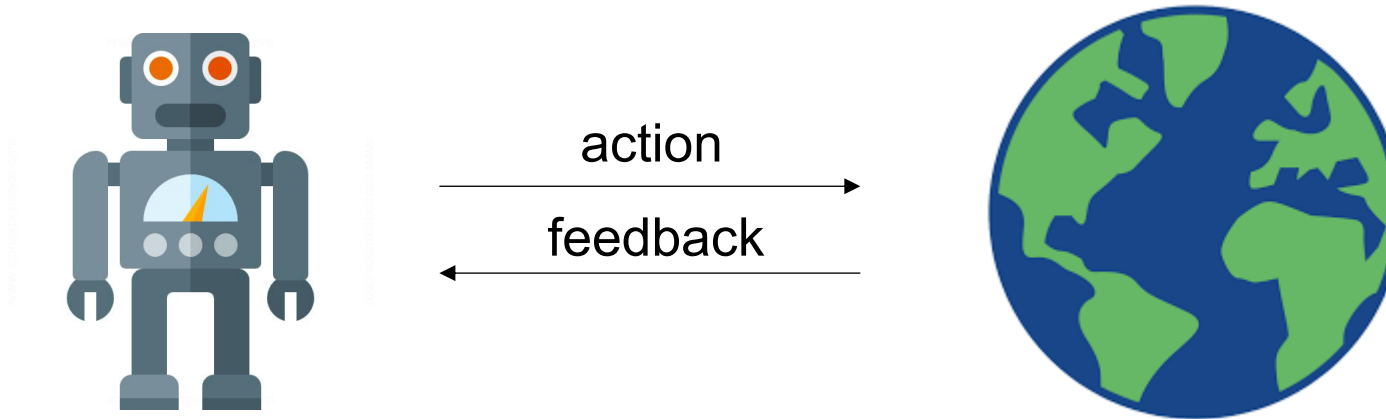
Data collection

Policy / value update

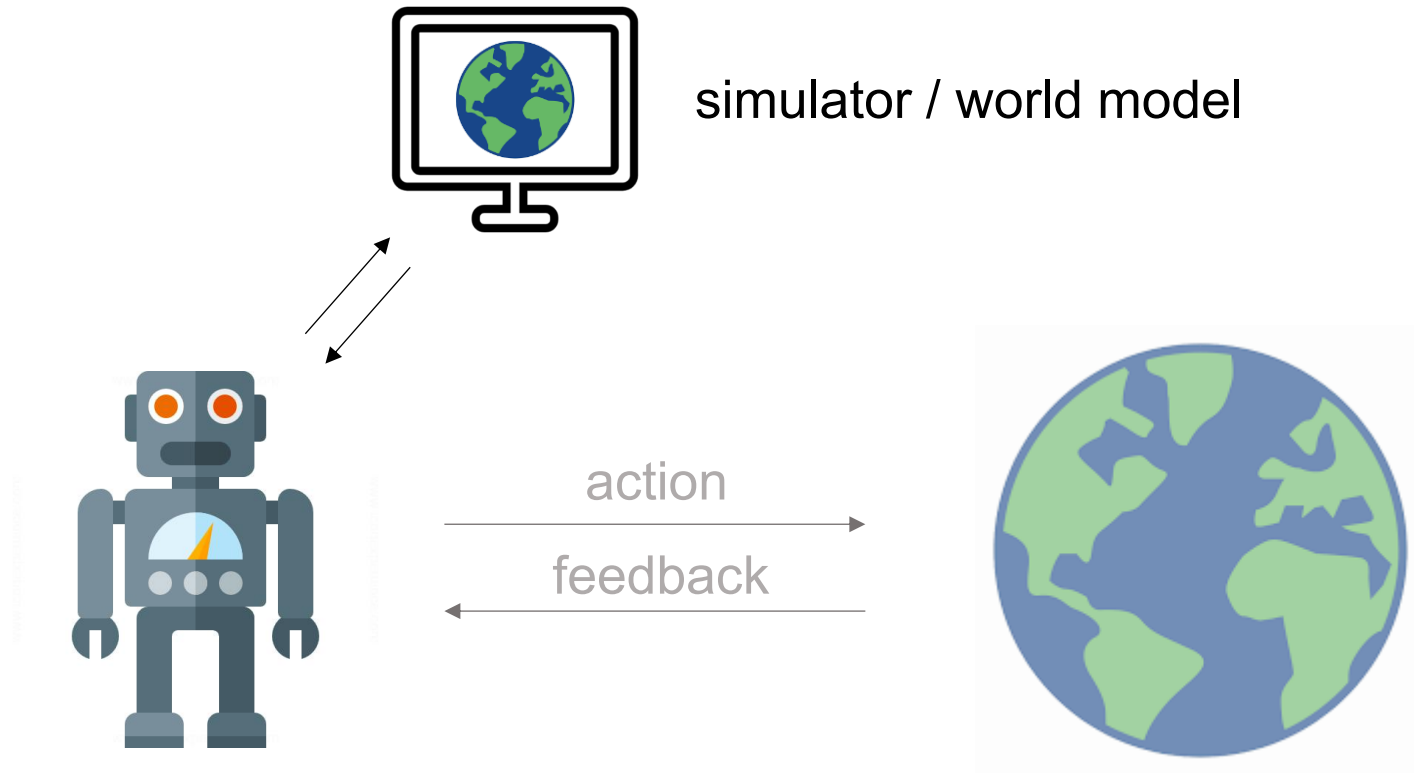
When Is Model-Based Method Helpful?

- Model (transition) is easy to learn
 - Deterministic transition could be easier to learn to stochastic one
 - System identification: known parameterized model with unknown parameters
- Model is known
 - The space/action space is too large for full policy/value iteration (Go, Chess)

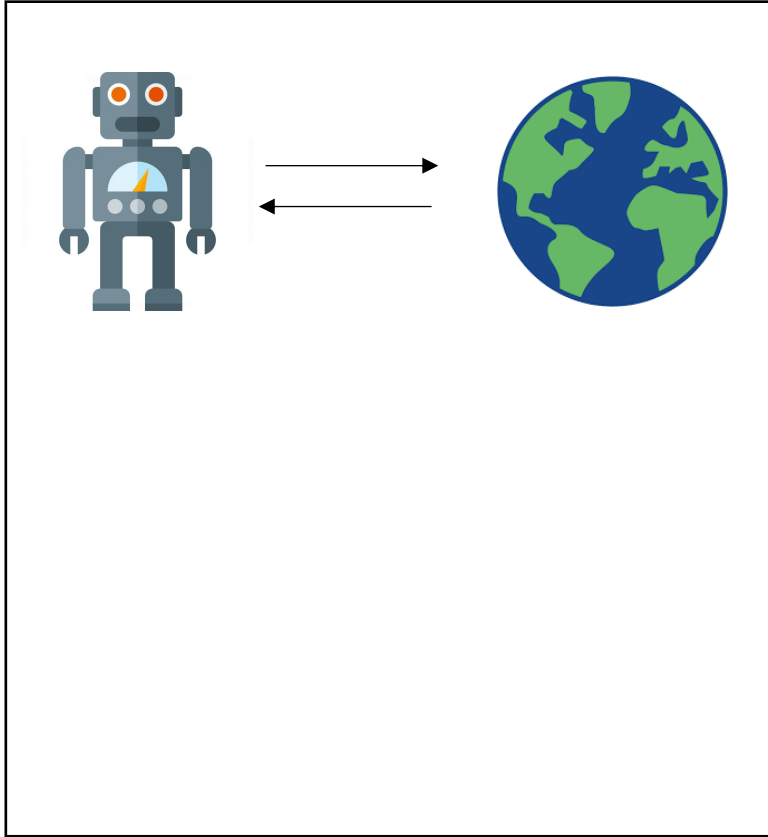
Model-Based RL



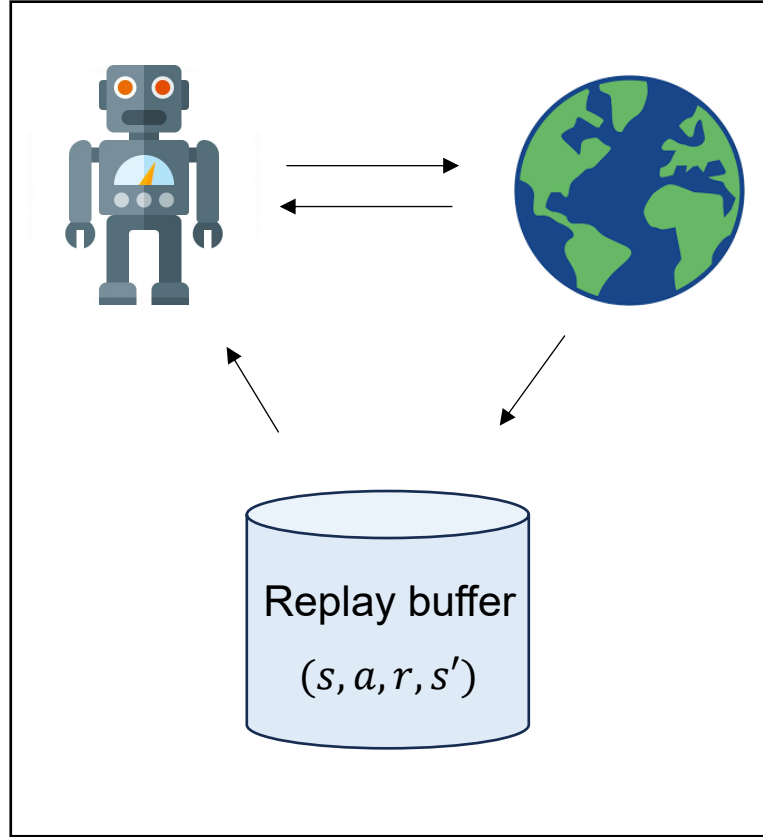
Model-Based RL



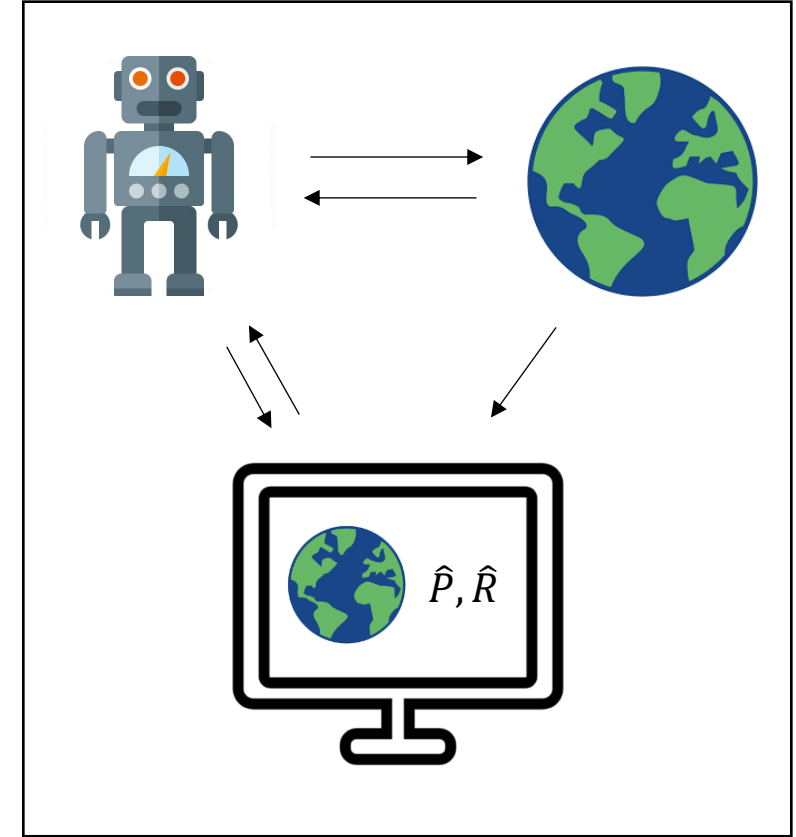
Comparison between training methods



Model-free On-policy



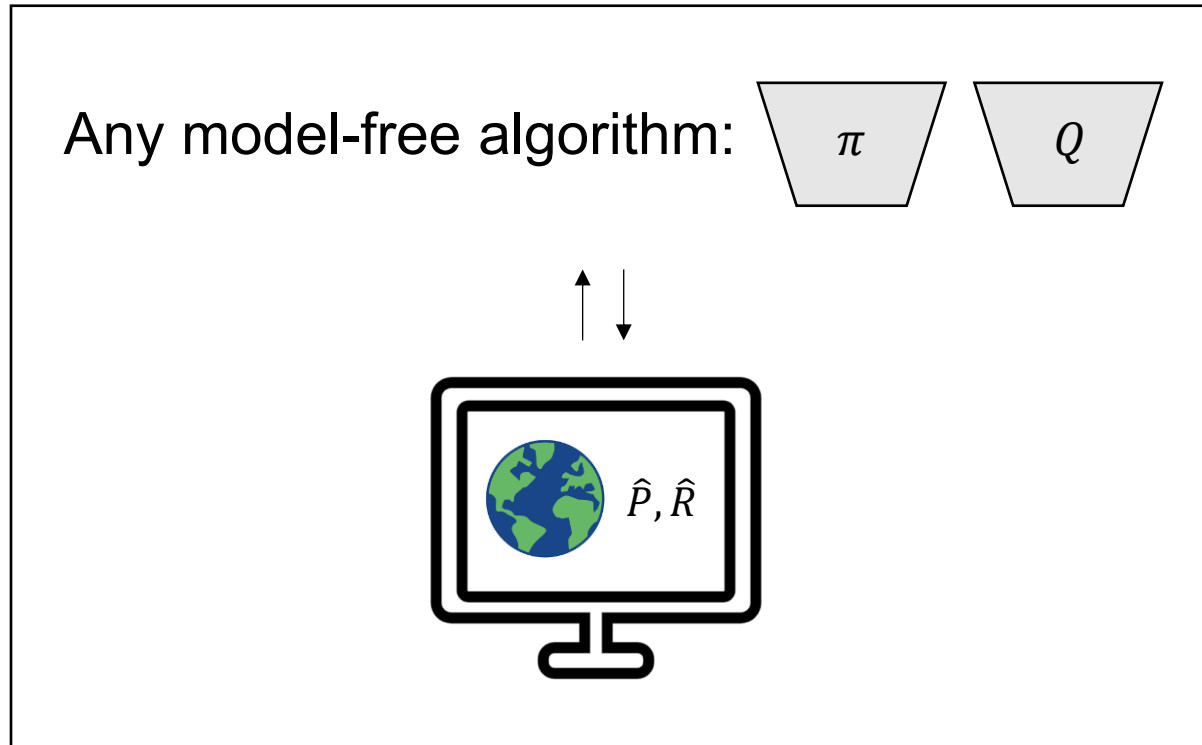
Model-free Off-policy



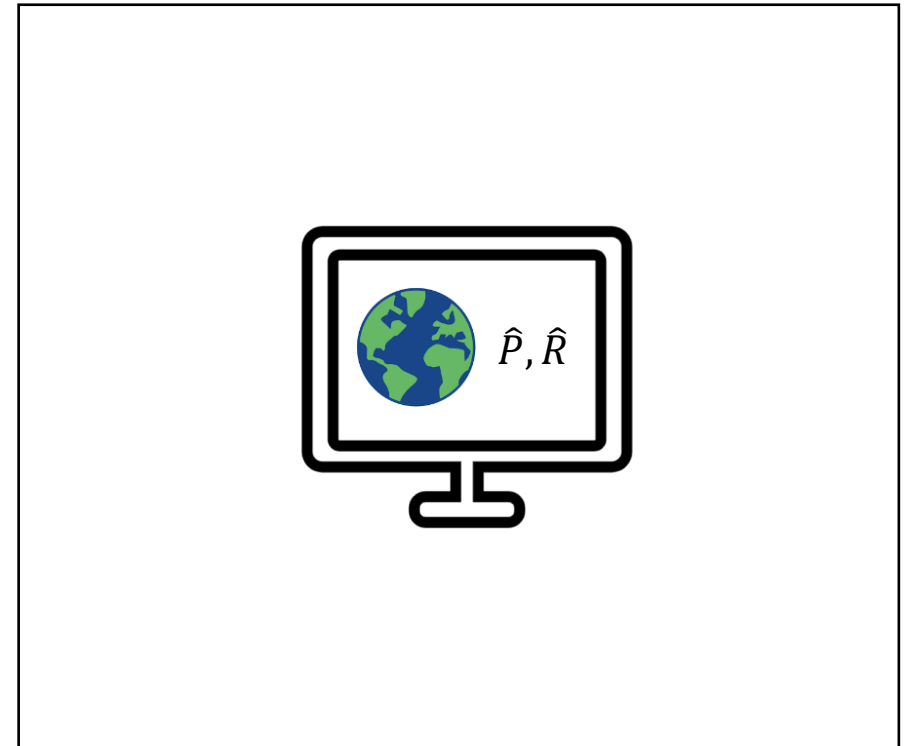
Model-based

Model-Based RL

Two ways to use the simulator / world model / model:



Model-assisted model-free learning



Planning with a model

1. Model-Assisted Model-Free Learning (Dyna-style)

For $k = 1, 2, \dots$

For $i = 1, 2, \dots, N$:

Choose action a_i with the current policy π_k

Receive reward $r_i \sim R(s_i, a_i)$ and $s'_i \sim P(\cdot | s_i, a_i)$

$s_{i+1} = s'_i$ if episode continues, $s_{i+1} \sim \rho$ if episode ends

Push (s_i, a_i, r_i, s'_i) to \mathcal{B}

Update model \hat{P}, \hat{R} with data in \mathcal{B}

Repeat several times:

Sample $(s, a) \sim \mathcal{B}$, or sample $s \sim \mathcal{B}$ and $a \sim \pi_k(\cdot | s)$ or uniform

Let $r \sim \hat{R}(s, a)$ and $s' \sim \hat{P}(\cdot | s, a)$

Update policy / value with sample (s, a, r, s')



Data collection

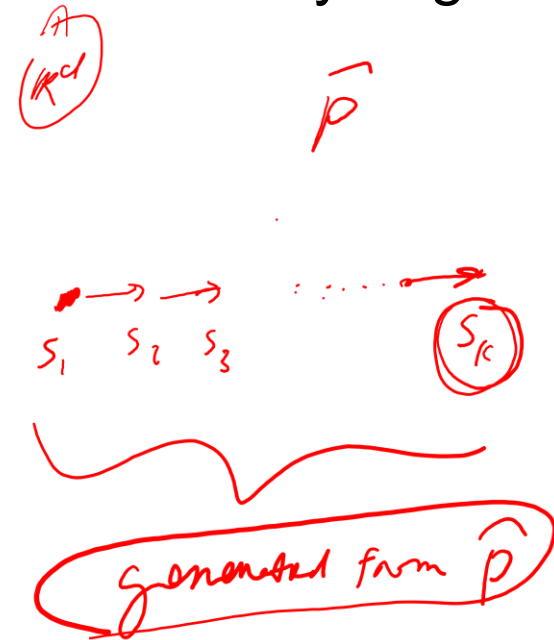
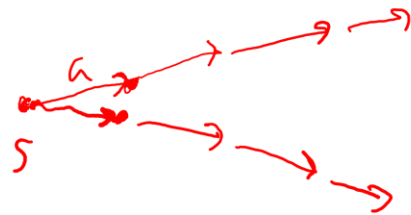


Model update

Policy / value update

1. Model-Assisted Model-Free Learning (Dyna-style)

Why still sample s from the buffer? Why not generate s randomly or generate it from the trained model?



1. Model-Assisted Model-Free Learning (Dyna-style)

Some Dyna-style algorithms:

Gu et al., [Continuous deep Q-Learning with model-based acceleration](#), 2016. (MBA)

Feinberg et al., [Model-based value expansion](#), 2018. (MVE)

Janner et al., [When to trust your model: model-based policy optimization](#). 2019. (MBPO)

The performance of MB-RL is heavily influenced by how to represent and train \hat{P} efficiently, while making it predictive and scalable:

Hafner et al., [Mastering Diverse Domains through World Models](#). 2023. (DreamerV3)

2. Planning with A Model

If we have a model / simulator, how to decide the next action without having a trained policy / value network?

Exact / closed-form solution: finite-state-finite-action, linear system

$$s_{t+1} = \underbrace{A}_{\substack{\uparrow \\ \mathbb{R}^d}} s_t + \underbrace{B}_{\mathbb{R}^k} u_t + \underbrace{w_t}_{\text{noise}}$$

$$loss = s_t^T Q s_t + a_t^T R a_t$$



2. Planning with A Model

If we have a model / simulator, how to decide the next action without having a trained policy / value network?

Search (for large state space without structure):

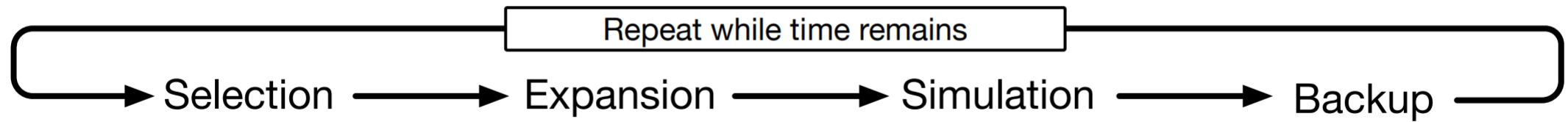
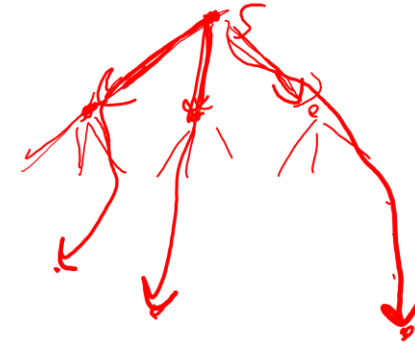
“Create the policy on the fly”: decide $\pi(\cdot | s)$ only when reaching s

This is often used when we want to enhance a **default** policy on the fly.



Monte-Carlo Tree Search (MCTS)

game of Go



Monte-Carlo Tree Search (MCTS)

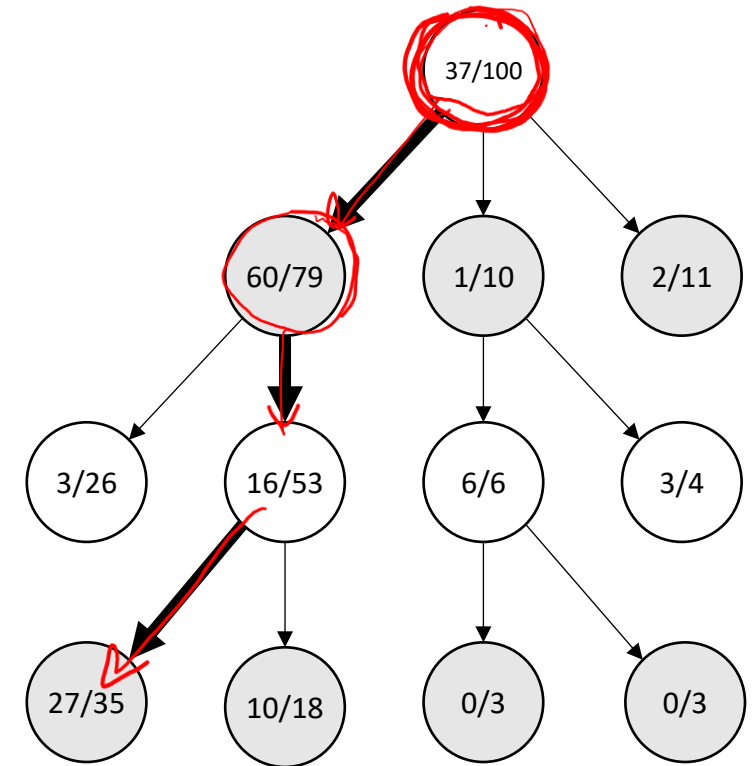
Selection

- Starting from the root node, execute **tree policy** until reaching a leaf node
- One effective tree policy is given by UCB1, which chooses an action based on

$$\frac{W(n)}{N(n)} + C \times \sqrt{\frac{\log N(\text{parent}(n))}{N(n)}}$$

$W(n)$: total #wins of all playouts that went through node n

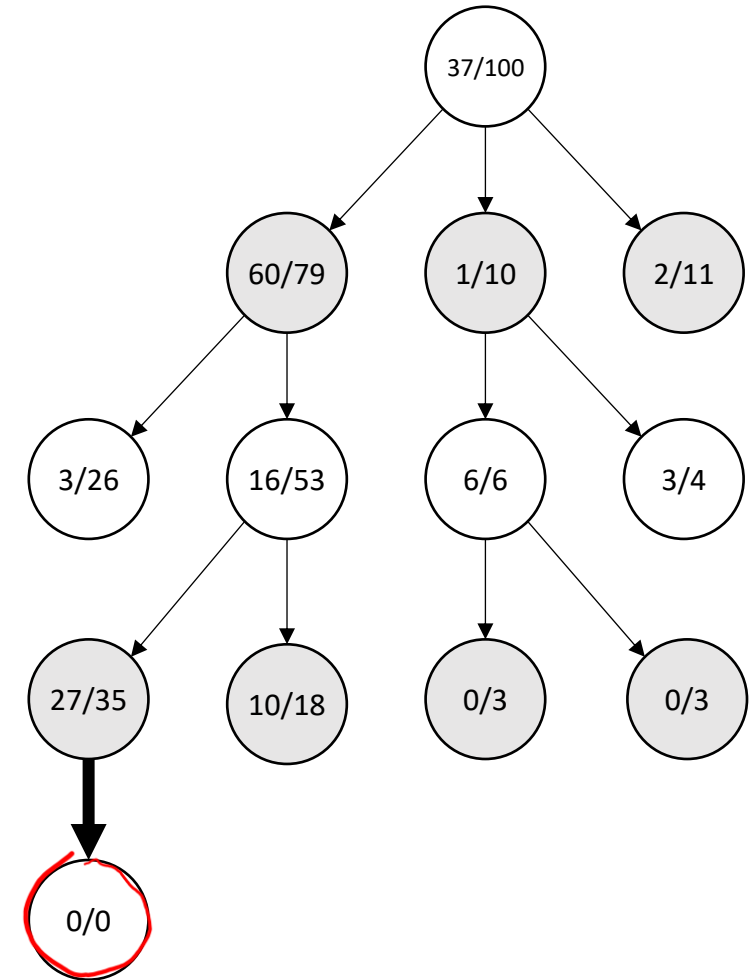
$N(n)$: total #playouts that went through node n



Monte-Carlo Tree Search (MCTS)

Expand

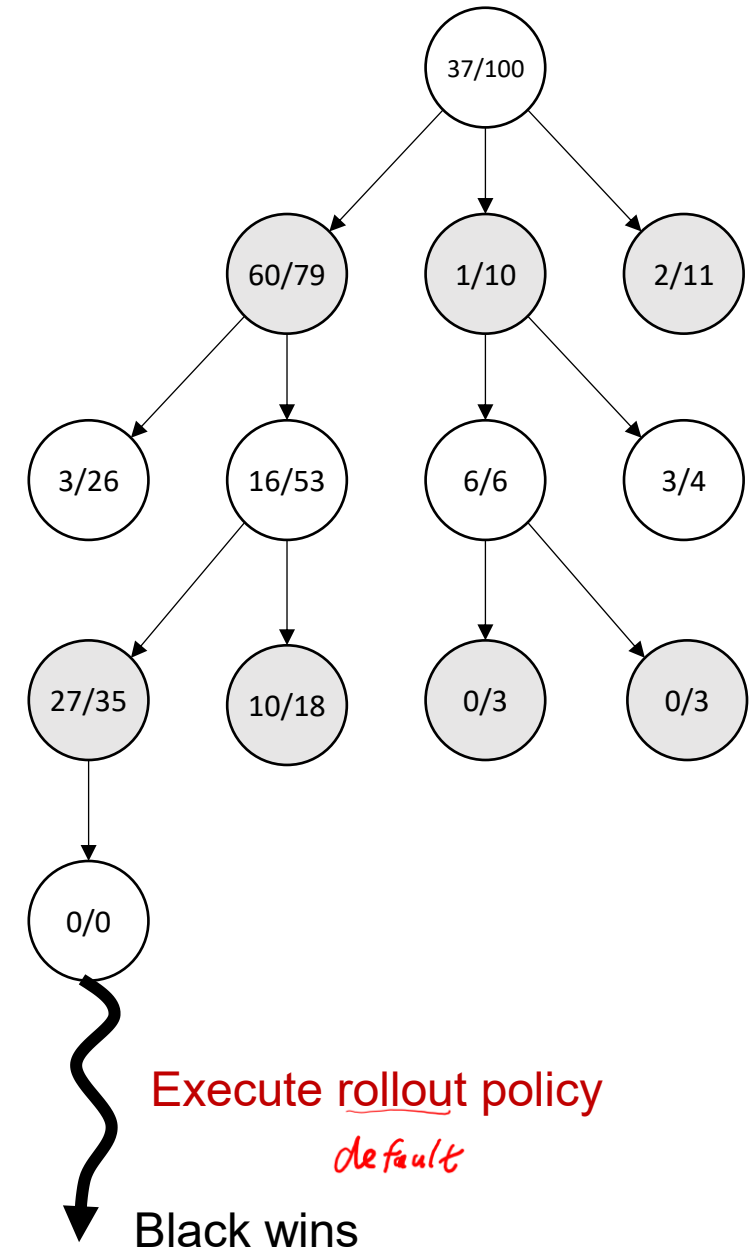
- On some iterations, grow the search tree from selected leaf nodes by adding one or more child nodes



Monte-Carlo Tree Search (MCTS)

Simulation

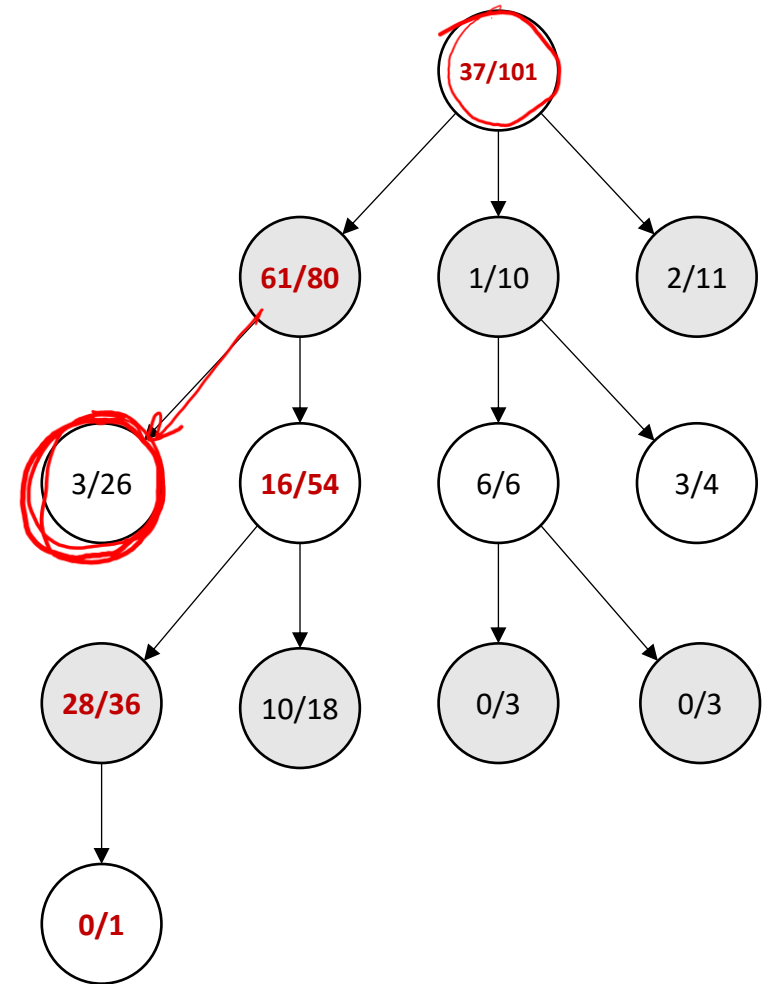
- From the selected or expanded node (if any), execute the **rollout policy** to the end of the game
- Rollout policy
 - Could be heuristics, such as “consider capture moves” in chess
 - Could be learned through neural networks by self-play



Monte-Carlo Tree Search (MCTS)

Backup

- Update the #wins and #playouts on nodes along the tree policy



Monte-Carlo Tree Search (MCTS)

Finally,

- Choose the action from the root node that has the largest visit count.
- After the opponent's move, start the same procedure from the new state (can keep the statistics from the previous state)

2. Planning with A Model

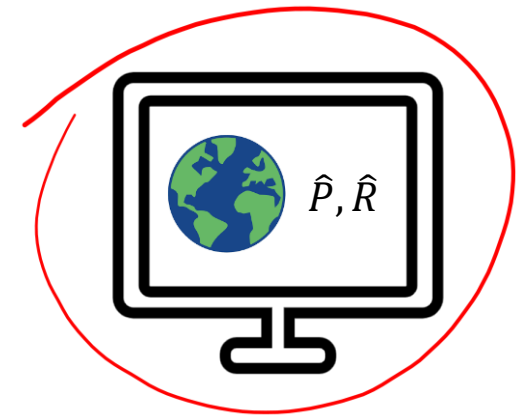
If we have a model / simulator, how to decide the next action without having a trained policy / value network?

Search (for large state space without structure):

“Create the policy on the fly”: decide $\pi(\cdot | s)$ only when reaching s

This is often used when we want to enhance a **default** policy on the fly.

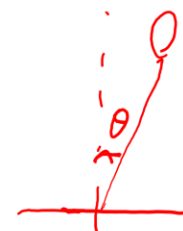
Plan for multiple steps, but execute only the first step.



linear system
$$S_{t+1} = AS_t + Ba_t$$

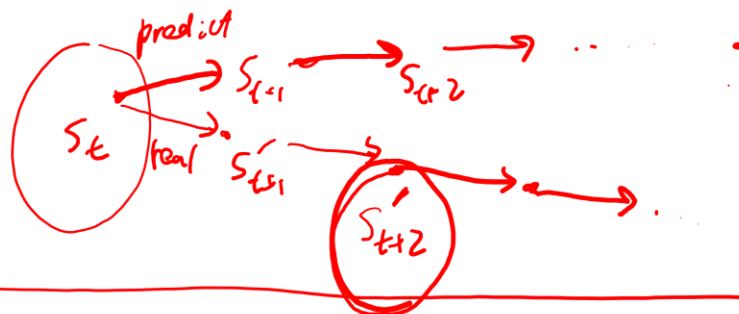
$$S_t \in \mathbb{R}^d, a_t \in \mathbb{R}^k$$

$$a = \begin{matrix} \text{dim} \\ \text{~~~~~} \\ \mathbb{R}^d \end{matrix} \left[K(A, B) \right] \begin{matrix} \text{dim} \\ \text{~~~~~} \\ \mathbb{R}^k \end{matrix} S$$

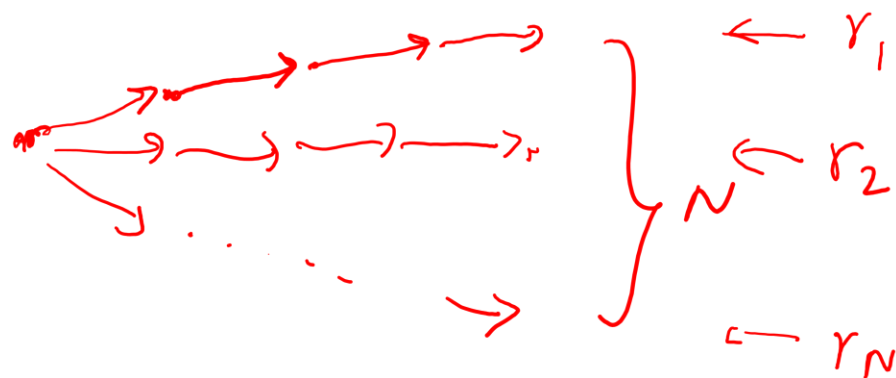


$$S = (\theta, \omega)$$

$a = \text{force}$



Best-of-N



$$i = \operatorname{argmax}_i r_i$$

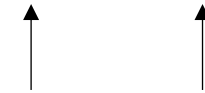
Uncertainty in Model-Based RL

Model Error / Failure of Generalization

As the model is learned with only finite samples, they could have large errors in uncovered areas.

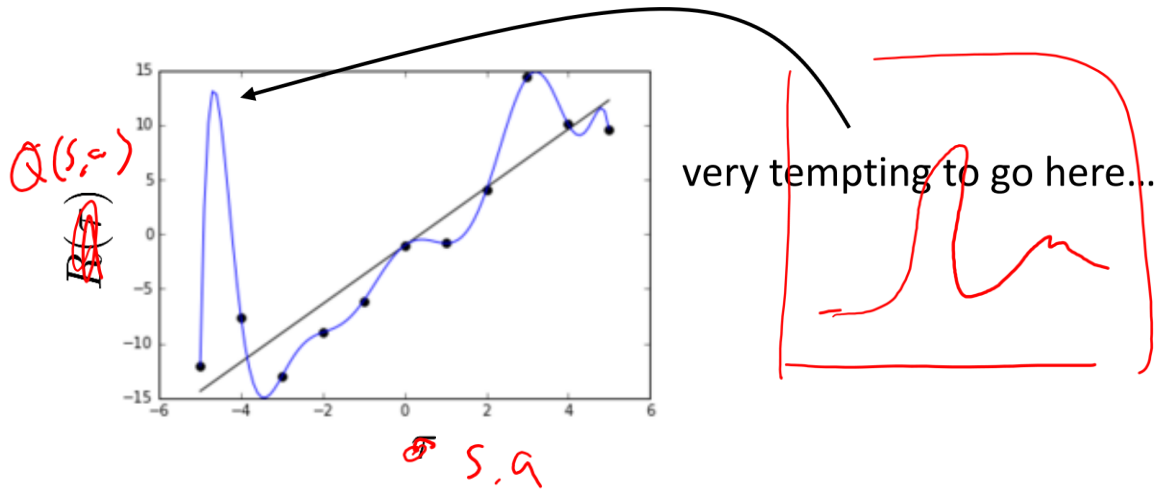
We can encounter the same issue in off-policy methods like DQN, DDPG.

DQN TD3

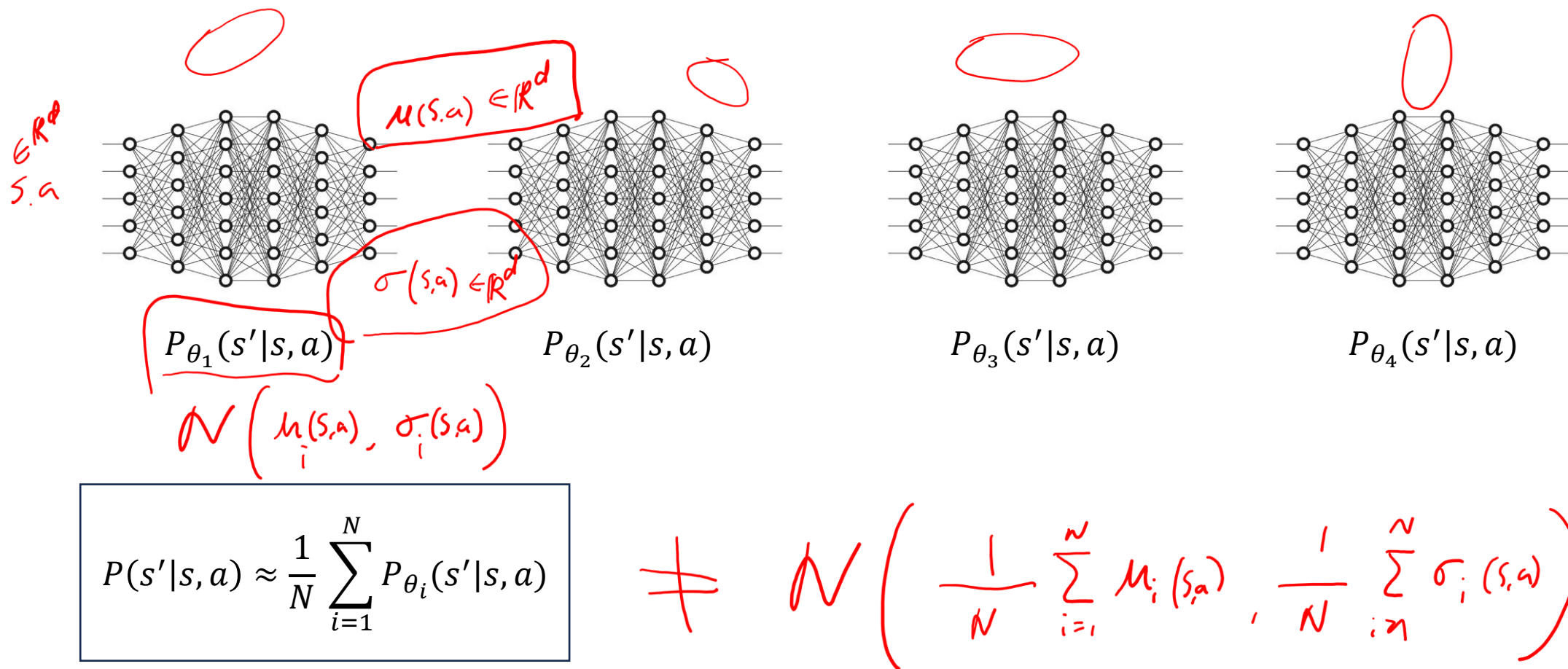


Solution: train 2 target networks and avoid the \max_a operator to exploit the error of a single network

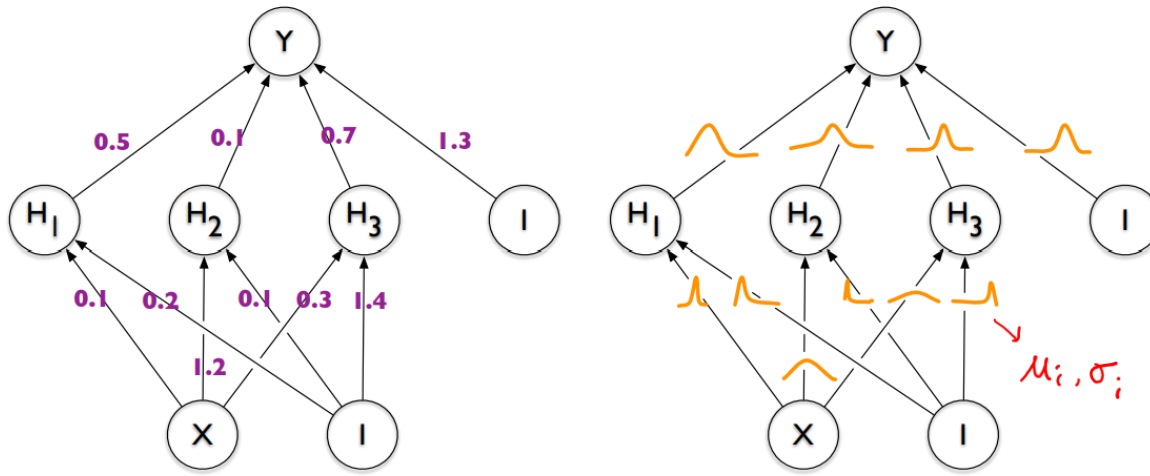
$$\max_a Q(s,a)$$



Ensemble Models



Bayesian Neural Network



Common approximation: $p(\theta) = \prod_i p(\theta_i)$

where $p(\theta_i) = \mathcal{N}(\mu_i, \sigma_i)$

expected weight uncertainty of the weight

Arrows point from the text above to the parameters μ_i and σ_i in the equation above.

Blundell et al., Weight Uncertainty in Neural Networks. 2015.
Gal et al., Concrete Dropout. 2017.

Aleatoric and Epistemic Uncertainty

- **Aleatoric uncertainty**

- Comes from inherent randomness or noise in the data (e.g., sensor noise, coin flips)
- **Irreducible** — cannot be removed even with more data

- **Epistemic uncertainty**

- Comes from lack of data or limited model capacity
- **Reducible** — can shrink with more data or better models

The “model uncertainty” here refers to **Epistemic uncertainty**.