# Actor-Critic Methods

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# Review: Full-Information Policy Learning in MDPs

$$\theta_{k+1} = \operatorname*{argmax}_{\theta} \left( \underbrace{V^{\pi_\theta}(\rho) - V^{\pi_{\theta_k}}(\rho)} - \frac{1}{\eta} D(\theta, \theta_k) \right)$$

$$\approx \sum_{s,a} d_\rho^{\pi_{\theta_k}}(s) \big(\pi_\theta(a|s) - \pi_{\theta_k}(a|s)\big) Q^{\pi_{\theta_k}}(s,a) = \mathbb{E}_{\underset{\sim \pi_{\theta_k}}{(s_i,a_i)}} \left[ \frac{\pi_\theta(a_i|s_i) - \pi_{\theta_k}(a_i|s_i)}{\pi_{\theta_k}(a_i|s_i)} \boxed{Q^{\pi_{\theta_k}}(s_i, a_i)} \right]$$

$$\approx (\theta - \theta_k)^\top \underbrace{\sum_{s,a} d_\rho^{\pi_{\theta_k}}(s) \left( \nabla_\theta \pi_\theta(a|s)\Big|_{\theta=\theta_k} \right) Q^{\pi_{\theta_k}}(s,a)}$$

$$= \mathbb{E}_{(s_i,a_i)} \left[ \frac{\nabla_\theta \pi_\theta(a_i|s_i)|_{\theta=\theta_k}}{\pi_{\theta_k}(a_i|s_i)} \boxed{Q^{\pi_{\theta_k}}(s_i, a_i)} \right]$$

PG/NPG: Estimate them using the empirical sum of reward in the trajectory (i.e., Monte Carlo estimator)

We can also use other estimators to balance bias and variance

# Actor-Critic Methods

Use value function approximation to estimate $Q^{\pi_{\theta_k}}(s_i, a_i)$ or $A^{\pi_{\theta_k}}(s_i, a_i)$

Use $V_\phi(s)$ to approximate $V^{\pi_{\theta_k}}(s)$

Use $Q_\phi(s, a)$ to approximate $Q^{\pi_{\theta_k}}(s, a)$

Possible estimators for $A^{\pi_{\theta_k}}(s, a)$:

Let $(s_1, a_1, r_1, s_2, a_2, r_2 \ldots)$ be a trajectory starting from $s_1 = s, a_1 = a$

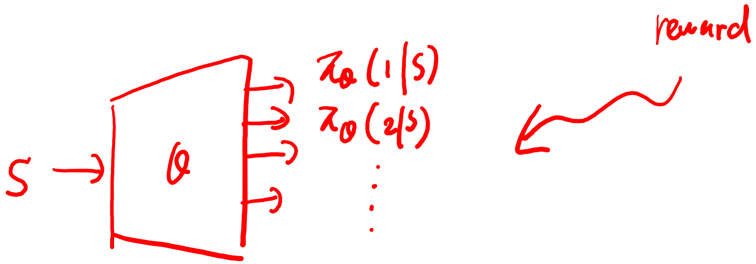
$$Q_\phi(s_1, a_1) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot|s)}[Q_\phi(s_1, a')]$$

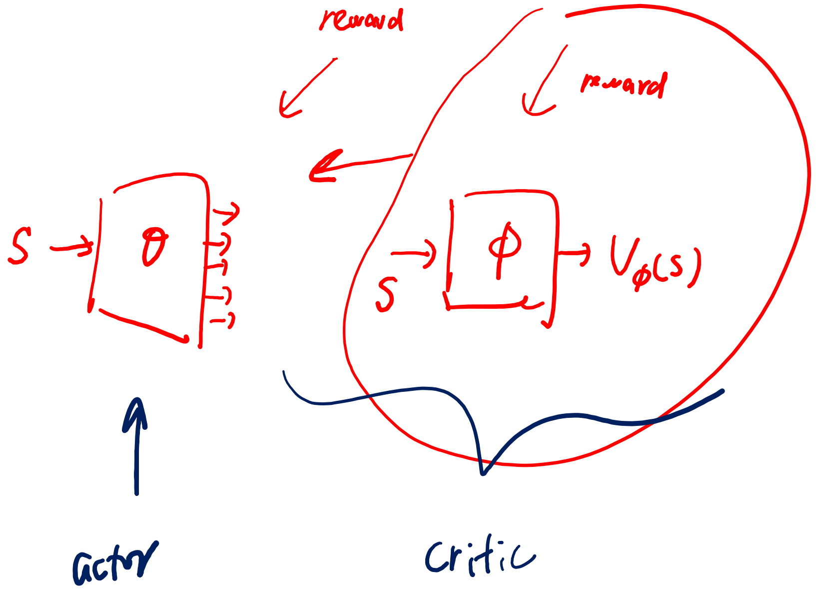
$$r_1 + \gamma V_\phi(s_2) - V_\phi(s_1) \qquad r_1 + \gamma Q_\phi(s_2, a_2) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot|s)}[Q_\phi(s_1, a')]$$

$$r_1 + \gamma r_2 + \gamma^2 V_\phi(s_3) - V_\phi(s_1) \qquad r_1 + \gamma r_2 + \gamma^2 Q_\phi(s_3, a_3) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot|s)}[Q_\phi(s_1, a')]$$

$$\vdots \qquad \vdots$$

# Pure Policy-Based Methods vs. Actor-Critic Methods



# Actor-Critic with $Q_\phi$

(find $\pi^*$) (off-policy) Q-learning: $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left[r + \max_{a'} Q(s',a')\right]$ $\Rightarrow Q^*$

(given $\pi$) TD-learning: $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left[r + \sum_{a'} \pi(a'|s')Q(s',a')\right]$ $\Rightarrow Q^\pi$ (on-policy)

For $k = 1,\ 2, \ldots$

Use $\pi_{\theta_k}$ to collect $n$ trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \cdots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \ldots\ldots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \cdots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Define

$$g = \frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n} \frac{\nabla_\theta\, \pi_\theta\left(a_h^{(i)} \middle| s_h^{(i)}\right)\Big|_{\theta=\theta_k}}{\pi_{\theta_k}\left(a_h^{(i)} \middle| s_h^{(i)}\right)} \underbrace{\overset{Q^{\pi_{\theta_k}}\ \approx}{Q_{\phi_k}\left(s_h^{(i)}, a_h^{(i)}\right)}} \quad \text{or} \quad \underline{\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n}\sum_{a} \nabla_\theta\, \pi_\theta\left(a \middle| s_h^{(i)}\right)\Big|_{\theta=\theta_k} Q_{\phi_k}\left(s_h^{(i)}, a\right)}$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \qquad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_\phi \sum_{i=1}^{n}\sum_{h=1}^{\tau_n}\left(Q_\phi\left(s_h^{(i)}, a_h^{(i)}\right) - r_h^{(i)} - \gamma Q_{\phi_k}\left(s_{h+1}^{(i)}, a_{h+1}^{(i)}\right)\right)^2\Bigg|_{\phi=\phi_k}$$

# Advantage Actor-Critic (A2C) = PG + $V_\phi$

For $k = 1,\ 2, \ldots$

Use $\pi_{\theta_k}$ to collect $n$ trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \cdots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \ldots\ldots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \cdots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Define

$$= \frac{\nabla_\theta \pi_\theta\left(a_h^{(i)} \middle| s_h^{(i)}\right)\Big|_{\theta=\theta_k}}{\pi_{\theta_k}\left(a_h^{(i)} \middle| s_h^{(i)}\right)} \qquad \approx A^{\pi_k}\left(s_h^{(i)}, a_h^{(i)}\right)$$

$$\mathbb{E}_{s,a}(\cdot) = \sum d_\rho^{\pi_{\theta_k}}(s)\, \nabla_\theta \pi_\theta(a|s)\, A^{\pi_k}(s,a)$$

$$g = \frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n} \nabla_\theta \log \pi_\theta\left(a_h^{(i)} \middle| s_h^{(i)}\right)\Big|_{\theta=\theta_k} \underbrace{\left(r_h^{(i)} + \gamma V_{\phi_k}\left(s_{h+1}^{(i)}\right) - V_{\phi_k}\left(s_h^{(i)}\right)\right)}$$

or any other advantage estimator in the previous slide

Perform updates

$$V_\phi \approx V^{\pi_{\theta_k}}.$$

$$\theta_{k+1} \leftarrow \theta_k + \eta g \qquad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_\phi \frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n}\left(V_\phi\left(s_h^{(i)}\right) - r_h^{(i)} - \gamma V_{\phi_k}\left(s_{h+1}^{(i)}\right)\right)^2\Bigg|_{\phi=\phi_k}$$

Mnih et al., Asynchronous Methods for Deep Reinforcement Learning. 2016.

# Proximal Policy Optimization (PPO) = NPG + $V_\phi$

For $k = 1,\ 2, \ldots$

Use $\pi_{\theta_k}$ to collect $n$ trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \cdots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \ldots \ldots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \cdots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Perform updates

or any other advantage estimator in the previous slide

$$\theta_{k+1} \leftarrow \underset{\theta}{\operatorname{argmax}} \left\{ \frac{1}{n} \sum_{i=1}^{n} \sum_{h=1}^{\tau_n} \frac{\pi_\theta\left(a_h^{(i)} \middle| s_h^{(i)}\right)}{\pi_{\theta_k}\left(a_h^{(i)} \middle| s_h^{(i)}\right)} \left( r_h^{(i)} + \gamma V_{\phi_k}\left(s_{h+1}^{(i)}\right) - V_{\phi_k}\left(s_h^{(i)}\right) \right) - \frac{1}{\eta} \frac{1}{n} \sum_{i=1}^{n} \sum_{h=1}^{\tau_n} \mathrm{KL}\left( \pi_\theta\left(\cdot \middle| s_h^{(i)}\right), \pi_{\theta_k}\left(\cdot \middle| s_h^{(i)}\right) \right) \right\}$$

$$\phi_{k+1} \leftarrow \phi_k - \lambda \nabla_\phi \frac{1}{n} \sum_{i=1}^{n} \sum_{h=1}^{\tau_n} \left( V_\phi\left(s_h^{(i)}\right) - r_h^{(i)} - \gamma V_{\phi_k}\left(s_{h+1}^{(i)}\right) \right)^2 \Bigg|_{\phi=\phi_k}$$

Schulman et al., Proximal Policy Optimization Algorithms. 2017.

$\frac{r(a)\mathbb{1}(a_t=a)}{P_t(a)}$ $\quad$ $\frac{(r(a)-1)}{P_t(a)}\mathbb{1}(a_t=a)$

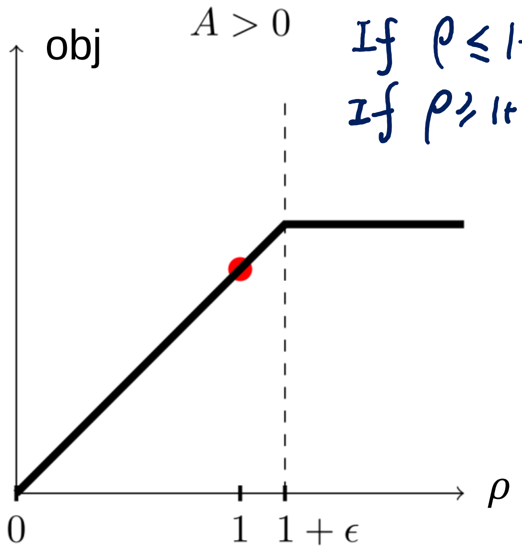
# Additional Technique 1: Clipped Objective (for PPO)

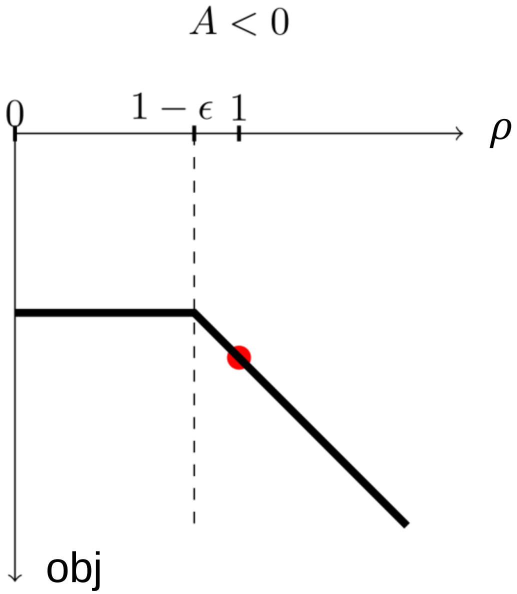
$$\rho := \frac{\pi_\theta\left(a_h^{(i)} \middle| s_h^{(i)}\right)}{\pi_{\theta_k}\left(a_h^{(i)} \middle| s_h^{(i)}\right)} \qquad A := \left(r_h^{(i)} + \gamma V_{\phi_k}\left(s_{h+1}^{(i)}\right) - V_{\phi_k}\left(s_h^{(i)}\right)\right)$$

$\mathrm{clip}_{[1-\epsilon,1+\epsilon]}(\rho) = \min(\max(\rho, 1-\epsilon), 1+\epsilon)$

Instead of using $\rho A$ as the objective, use $\min\{\rho A,\ \mathrm{clip}_{[1-\epsilon,1+\epsilon]}(\rho)A\}$

If $\rho \in [1-\epsilon, 1+\epsilon] \Rightarrow \rho A$
If $\rho \le 1-\epsilon \Rightarrow \rho A$
If $\rho \ge 1+\epsilon \Rightarrow (1+\epsilon)A$




If $\rho \in (1-\epsilon, 1+\epsilon] \Rightarrow \rho A$
If $\rho \le 1-\epsilon \Rightarrow (1-\epsilon)A$ (strange case)
If $\rho \ge 1+\epsilon \Rightarrow \rho A$

| algorithm | avg. normalized score |
|---|---|
| No clipping or penalty | -0.39 |
| Clipping, $\epsilon = 0.1$ | 0.76 |
| **Clipping,** $\epsilon = 0.2$ | **0.82** |
| Clipping, $\epsilon = 0.3$ | 0.70 |
| Adaptive KL $d_{\mathrm{targ}} = 0.003$ | 0.68 |
| Adaptive KL $d_{\mathrm{targ}} = 0.01$ | 0.74 |
| Adaptive KL $d_{\mathrm{targ}} = 0.03$ | 0.71 |
| Fixed KL, $\beta = 0.3$ | 0.62 |
| Fixed KL, $\beta = 1.$ | 0.71 |
| Fixed KL, $\beta = 3.$ | 0.72 |
| Fixed KL, $\beta = 10.$ | 0.69 |

Schulman et al., Proximal Policy Optimization Algorithms. 2017.

# Additional Technique 2: Entropy Bonus

In the objective of policy update, add a bonus term

$$H(\pi_\theta(\cdot\,|s)) = \sum_a \pi_\theta(a|s)\ln\frac{1}{\pi_\theta(a|s)}$$

For PPO:

$$\operatorname*{argmax}_\theta \left\{\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n}\frac{\pi_\theta\left(a_h^{(i)}\middle|s_h^{(i)}\right)}{\pi_{\theta_k}\left(a_h^{(i)}\middle|s_h^{(i)}\right)}A_h^{(i)} - \frac{1}{\eta}\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n}\mathrm{KL}\left(\pi_\theta\left(\cdot\middle|s_h^{(i)}\right),\pi_{\theta_k}\left(\cdot\middle|s_h^{(i)}\right)\right) + c\,\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n}\underbrace{H\left(\pi_\theta\left(\cdot\middle|s_h^{(i)}\right)\right)}_{-\mathrm{KL}\left(\pi_\theta\left(\cdot\middle|s_h^{(i)}\right),\pi_{\mathrm{unif}}\left(\cdot\middle|s_h^{(i)}\right)\right)}\right\}$$

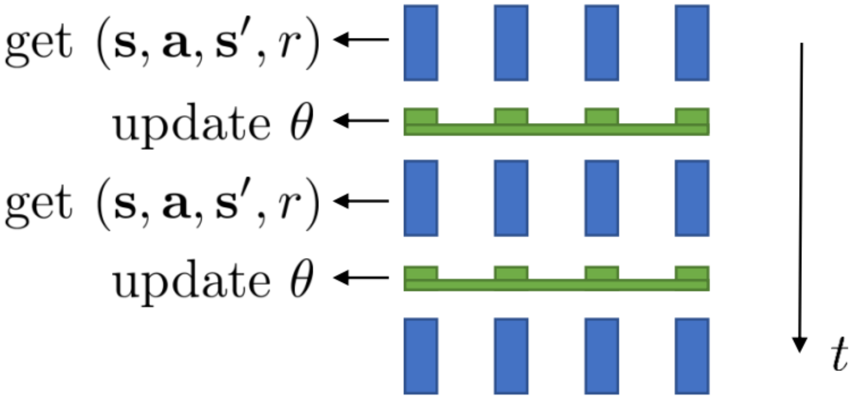
For A2C:

$$g = \frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n}\nabla_\theta \log\pi_\theta\left(a_h^{(i)}\middle|s_h^{(i)}\right)\Big|_{\theta=\theta_k}A_h^{(i)} + c\nabla_\theta\frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{\tau_n}H\left(\pi_\theta\left(\cdot\middle|s_h^{(i)}\right)\right)$$

# Additional Technique 3: Parallel Sample Collection

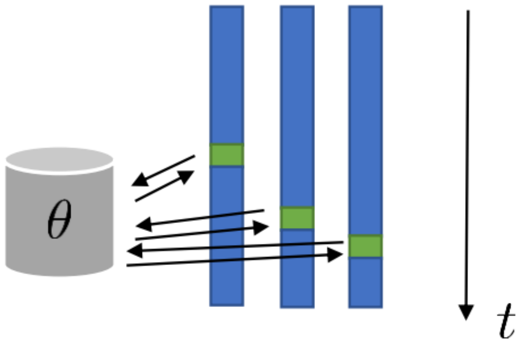
A2C

synchronized parallel actor-critic




A3C .

asynchronous parallel actor-critic




[Levine CS285 Lecture 6]

# Actor-Critic Summary

PG $\longrightarrow$ A2C

NPG $\longrightarrow$ PPO

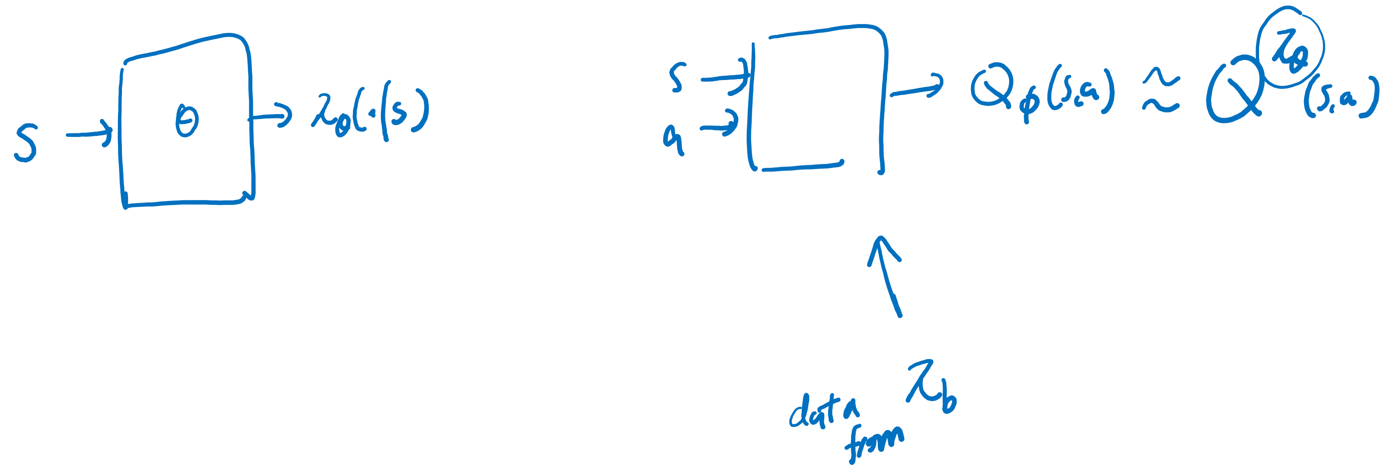
$s \rightarrow \boxed{\theta} \rightarrow \pi(\cdot|s)$

$s \rightarrow \boxed{\theta} \rightarrow \pi(\cdot|s)$ $\quad$ $s \rightarrow \boxed{\phi} \rightarrow V_\phi(s)$

$s, a \rightarrow \boxed{\phi} \rightarrow Q_\phi(s,a)$

# Off-Policy Actor-Critic

- Leveraging **off-policy evaluation** → allow reusing data



# Review: Full-Information Policy Learning in MDPs

$$\theta_{k+1} = \operatorname*{argmax}_{\theta} \left( \underbrace{V^{\pi_\theta}(\rho) - V^{\pi_{\theta_k}}(\rho)} - \frac{1}{\eta} D(\theta, \theta_k) \right)$$

$$\approx \sum_{s,a} d_\rho^{\pi_{\theta_k}}(s) \left( \pi_\theta(a|s) - \pi_{\theta_k}(a|s) \right) Q^{\pi_{\theta_k}}(s,a)$$

$$\approx (\theta - \theta_k)^\top \sum_{s,a} d_\rho^{\pi_{\theta_k}}(s) \left( \nabla_\theta \pi_\theta(a|s) \Big|_{\theta=\theta_k} \right) Q^{\pi_{\theta_k}}(s,a)$$

Use any off-policy policy evaluation methods to find $\phi_k$ such that $Q_{\phi_k}(s,a) \approx Q^{\pi_{\theta_k}}(s,a)$

Suppose that our $(s_i, a_i)$ samples are obtained from $\hat{\pi}$

# Off-Policy Actor-Critic

$$\theta_{k+1} = \underset{\theta}{\operatorname{argmax}} \left( \underbrace{V^{\pi_\theta}(\rho) - V^{\pi_{\theta_k}}(\rho)} - \frac{1}{\eta} D(\theta, \theta_k) \right)$$

$$\approx \sum_{s,a} d_\rho^{\pi_{\theta_k}}(s) \left( \pi_\theta(a|s) - \pi_{\theta_k}(a|s) \right) Q_{\phi_k}(s,a) \qquad = \mathbb{E}_{s \sim d_\rho^{\hat{\pi}}} \left[ \cancel{\frac{d_\rho^{\pi_{\theta_k}}(s)}{d_\rho^{\hat{\pi}}(s)}} \sum_a \left( \pi_\theta(a|s) - \pi_{\theta_k}(a|s) \right) Q_{\phi_k}(s,a) \right]$$

$$\approx (\theta - \theta_k)^\top \sum_{s,a} d_\rho^{\pi_{\theta_k}}(s) \left( \nabla_\theta \pi_\theta(a|s) \Big|_{\theta=\theta_k} \right) Q_{\phi_k}(s,a) \qquad = (\theta - \theta_k)^\top \mathbb{E}_{s \sim d_\rho^{\hat{\pi}}} \left[ \cancel{\frac{d_\rho^{\pi_{\theta_k}}(s)}{d_\rho^{\hat{\pi}}(s)}} \sum_a \nabla_\theta \pi_\theta(a|s) \Big|_{\theta=\theta_k} Q_{\phi_k}(s,a) \right]$$

Use any off-policy policy evaluation methods to find $\phi_k$ such that $Q_{\phi_k}(s,a) \approx Q^{\pi_{\theta_k}}(s,a)$

Suppose that our $(s_i, a_i)$ samples are obtained from $\hat{\pi}$

# Actor-Critic + Replay Buffer

For $k = 1,\ 2, \ldots$

Collect samples using $\pi_{\theta_k}$, and place them in the replay buffer

Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^{n}$ from replay buffer

Define

$$g = \frac{1}{n}\sum_{i=1}^{n}\sum_{a} \nabla_\theta \pi_\theta(a|s_i)\Big|_{\theta=\theta_k} Q_{\phi_k}(s_i, a)$$

Note: not using $a_i$ here

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g$$

Off-policy TD → unstable (more on this later)

$$\phi_{k+1} \leftarrow \phi_k - \lambda \nabla_\phi \frac{1}{n}\sum_{i=1}^{n}\left(Q_\phi(s_i, a_i) - r_i - \gamma\, \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot|s_i')}\left[Q_{\phi_k}(s_i', a')\right]\right)^2\Bigg|_{\phi=\phi_k}$$

# Dealing with Continuous Action Sets

# Review: Linear Bandits and One-Point Gradient Estimator

Feasible set $\mathcal{A} \subseteq \mathbb{R}^d$

For $t=1,\cdots,T$:

Learner choose $a_t \in \mathcal{A}$

Environment reveals $f_t(a_t)$ , where $f_t : \mathcal{A} \to \mathbb{R}$

Ideal update

$$a_{t+1} \leftarrow a_t + \eta \nabla f_t(a_t)$$

$a_t - v$ $\quad a_t \quad$ $a_t + v$

(1d)

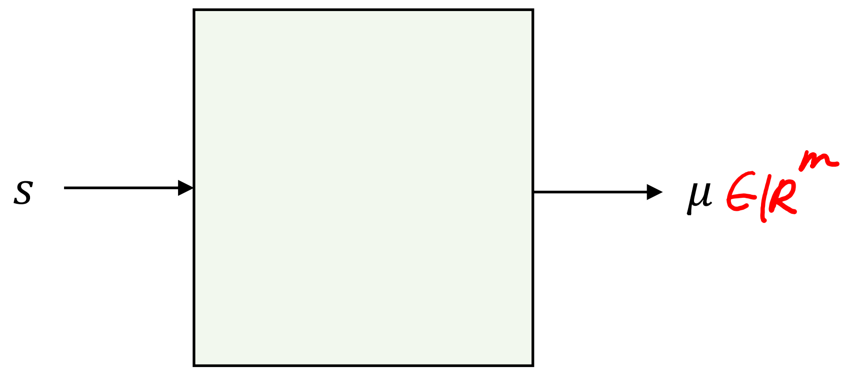
$$\nabla f_t(a_t) \approx \frac{f_t(a_t+v) - f_t(a_t-v)}{2v}$$

$$= \mathbb{E}\, \frac{f_t(\tilde{a}_t)\, s}{v} = \frac{f_t(\tilde{a}_t)(\tilde{a}_t - a_t)}{v^2}$$

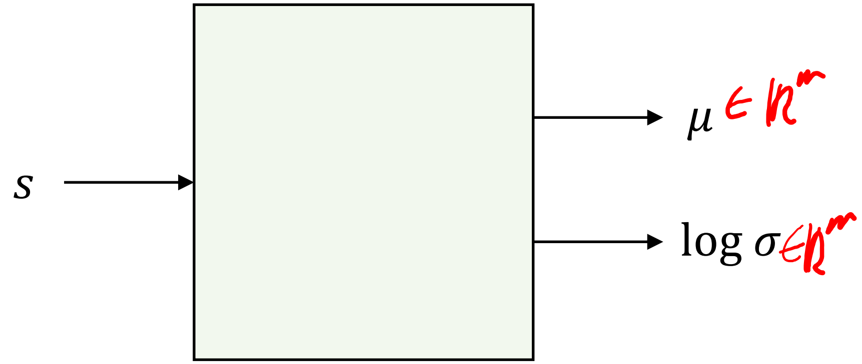
where $\tilde{a}_t = \begin{cases} a_t + v, & \text{w.p. } \frac{1}{2} \\ a_t - v, & \text{w.p. } \frac{1}{2} \end{cases}$

$$s = \begin{cases} 1, & \tilde{a}_t = a_t + v \\ -1, & \tilde{a}_t = a_t - v \end{cases}$$

# Policy Network for Continuous Action Sets



# Policy Network for Continuous Action Sets




$a_i = \mu_i + \varepsilon \sigma_i$ , where $\varepsilon \sim \mathcal{N}(0,1)$

# A2C / PPO with Continuous Action Sets

$\int_a \pi_\theta(a|s)\, da = 1$

$\mu_\theta$

$\pi_\theta(a) = \text{const} \cdot \exp\left(\frac{-(a-\mu_\theta)^2}{2\sigma^2}\right)$

$\mu_\theta \in \mathbb{R}^m$

$\theta \in \mathbb{R}^n$

$$g = \frac{1}{n}\sum_{i=1}^{n} \nabla_\theta \log \pi_\theta(a_i|s_i)\Big|_{\theta=\theta_k} A_i$$

$\log \pi_\theta(a) = \text{const} - \frac{(a-\mu_\theta)^2}{2\sigma^2}$

$\nabla_\theta \log \pi_\theta(a) = \underbrace{\nabla_\theta \mu_\theta}_{\mathbb{R}^{n\times m}} \underbrace{\frac{(a-\mu_\theta)}{\sigma^2}}_{m} \in \mathbb{R}^n$

$$\theta_{k+1} \leftarrow \underset{\theta}{\operatorname{argmax}} \left\{ \frac{1}{n}\sum_{i=1}^{n} \frac{\pi_\theta(a_i|s_i)}{\pi_{\theta_k}(a_i|s_i)} A_i - \frac{1}{\eta}\frac{1}{n}\sum_{i=1}^{n} \mathrm{KL}\left(\pi_\theta(\cdot\,|s_i), \pi_{\theta_k}(\cdot\,|s_i)\right) \right\}$$

$a$

# Recall: Actor-Critic with $Q_\phi$ Critic

$s \to \boxed{\phantom{x}} \to \mu_\theta$ (deterministic)

$\to \boxed{\phantom{x}} \to \pi_\theta(1|s), \;\vdots\;, \pi_\theta(A|s)$

For $k = 1,\ 2, \dots$

Use $\pi_{\theta_k}$ to collect samples $\{(s_i, a_i, r_i, s_i{}')\}_{i=1}^{n}$

Define $$g = \frac{1}{n}\sum_{i=1}^{n}\sum_{a} \nabla_\theta\, \pi_\theta(a|s_i)\Big|_{\theta=\theta_k} Q_{\phi_k}(s_i, a)$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \qquad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_\phi \sum_{i=1}^{n}\Big(Q_\phi(s_i, a_i) - r_i - \gamma\, \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot|s_i')}\big[Q_{\phi_k}(s_i{}', a')\big]\Big)^2\Bigg|_{\phi=\phi_k}$$

# Deterministic Policy Gradient Theorem

Policy: $\mu_\theta(s) \in \mathbb{R}^m$

$$V^{\pi_{\theta+\Delta\theta}}(\rho) - V^{\pi_\theta}(\rho) = \sum_s d_\rho^{\pi_{\theta+\Delta\theta}}(s) \sum_a \left( \underline{\underline{\pi_{\theta+\Delta\theta}(a|s)}} - \underline{\underline{\pi_\theta(a|s)}} \right) Q^{\pi_\theta}(s,a)$$

$$= \sum_s d_\rho^{\pi_{\theta+\Delta\theta}}(s) \left( Q^{\pi_\theta}(s, \mu_{\theta+\Delta\theta}(s)) - Q^{\pi_\theta}(s, \mu_\theta(s)) \right)$$

$$\Rightarrow \quad \nabla_\theta V^{\pi_\theta}(\rho) = \sum_s d_\rho^{\pi_\theta}(s) \; \nabla_\theta \left[ Q^{\pi_\theta}(s, \mu_\theta(s)) \right]$$

$$\left\{ \sum_s d_\rho^{\pi_\theta}(s) \left[ \sum_a \nabla_\theta \pi_\theta(a|s) \, Q^{\pi_\theta}(s,a) \right] \right\}$$

# Deterministic Policy Gradient

A.C.

$Q_{\phi_k} \approx Q^{\mu_{\theta_k}}$

For $k = 1,\ 2, \ldots$

Q-learning

Use $\mu_{\theta_k}$ to collect samples $\{(s_i, a_i, r_i, s_i')\}_{i=1}^{n}$

$Q \leftarrow (1-\alpha) Q + \alpha \left( r + \gamma \max_{a'} Q(s', a') \right)$

Define $\quad g = \frac{1}{n} \sum_{i=1}^{n} \nabla_\theta Q_{\phi_k}\left(s_i, \mu_{\theta_k}(s_i)\right)\Big|_{\theta=\theta_k}$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \qquad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_\phi \sum_{i=1}^{n} \left(Q_\phi(s_i, a_i) - r_i - \gamma Q_{\phi_k}(s_i', \mu_{\theta_k}(s_i'))\right)^2 \Bigg|_{\phi=\phi_k}$$

$\mu_{\theta_k}(s) \approx \arg\max_a Q_{\phi_k}(s, a)$

# Two Viewpoints for the Deterministic PG Algorithm

# Deep Deterministic Policy Gradient (DDPG)

For $k = 1,\ 2, \ldots$

Use $\mu_{\theta_k}(s) + \mathcal{N}(0, \sigma^2)$ to collect samples and place them in replay buffer

Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^{n}$ from the replay buffer

$$\theta \leftarrow \theta + \eta \sum_{i=1}^{n} \nabla_\theta Q_\phi\big(s_i, \mu_\theta(s_i)\big)$$

$$\phi \leftarrow \phi - \lambda \nabla_\phi \sum_{i=1}^{n} \Big(Q_\phi(s_i, a_i) - r_i - \gamma Q_{\phi_{\text{tar}}}\big(s_i', \mu_{\theta_{\text{tar}}}(s_i')\big)\Big)^2$$

0.001

$$\theta_{\text{tar}} \leftarrow \tau\theta + (1-\tau)\theta_{\text{tar}}$$

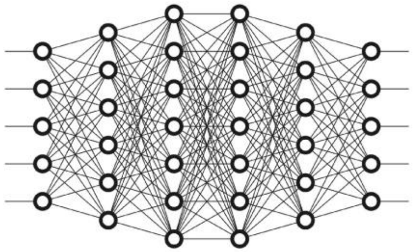
$$\phi_{\text{tar}} \leftarrow \tau\phi + (1-\tau)\phi_{\text{tar}}$$

**Elements:** replay buffer, target network, action noise

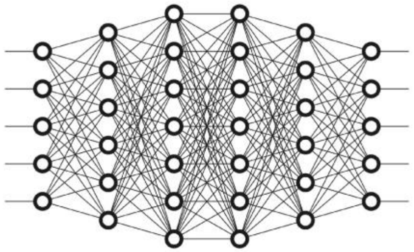
Lillicrap et al., Continuous control with deep reinforcement learning. 2015.

# Further Stabilizing DDPG (1/3)

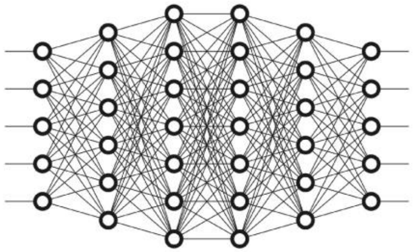
- Double Q-learning



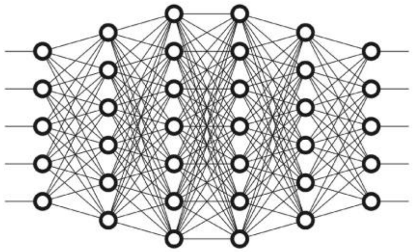
$\phi_1$



$\phi_{\text{tar1}}$



$\phi_2$



$\phi_{\text{tar2}}$

**Double Q-learning:** When training $\phi_1$, instead of using $Q_{\phi_{\text{tar1}}}$ to evaluate the regression target, use $\cancel{Q_{\phi_{\text{tar2}}}}$

**TD3:** $\min\left\{Q_{\phi_{\text{tar1}}}, Q_{\phi_{\text{tar2}}}\right\}$

**Double Q-learning:** Use independent samples to train $\phi_1$ and $\phi_2$

**TD3:** Use the same set of samples
(the independence between $\phi_1$ and $\phi_2$ only comes from random initialization)

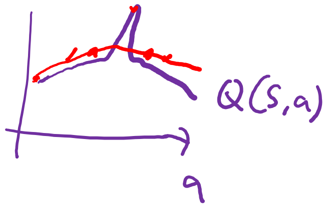
# Further Stabilizing DDPG (2/3)

- Target policy smoothing

**DDPG:** use $Q_{\phi_{\text{tar}}}(s', \mu_{\theta_{\text{tar}}}(s'))$ as the regression target

**TD3:** sample $a' = \mu_{\theta_{\text{tar}}}(s') + \mathcal{N}(0, \sigma^2)$

use $Q_{\phi_{\text{tar}}}(s', a')$ as the regression target



# Further Stabilizing DDPG (3/3)

- Delayed policy updates:  running multiple steps of value updates before running one step of policy update

# Twin Delayed DDPG (TD3)

For $k = 1,\ 2, \ldots$

Use $\mu_\theta(s) + \mathcal{N}(0, \sigma^2)$ to collect samples and place them in replay buffer

Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^n$ from the replay buffer

For each sample $i$, draw $a_i' \sim \mu_{\theta_{\text{tar}}}(s_i{}') + \mathcal{N}(0, \sigma^2 I)$

$$\phi_j \leftarrow \phi_j - \lambda \nabla_{\phi_j} \sum_{i=1}^n \left(Q_{\phi_j}(s_i, a_i) - r_i - \gamma \min_{\ell=1,2} Q_{\phi_{\text{tar}\ell}}(s_i', a_i')\right)^2 \qquad \forall j = 1,2$$

If $k \bmod M = 0$:

$$\theta \leftarrow \theta + \eta \sum_{i=1}^n \nabla_\theta Q_\phi\big(s_i, \mu_\theta(s_i)\big)$$

$$\theta_{\text{tar}} \leftarrow \tau\theta + (1-\tau)\theta_{\text{tar}}$$

$$\phi_{\text{tar}j} \leftarrow \tau\phi_j + (1-\tau)\phi_{\text{tar}j} \qquad \forall j = 1,2$$

Fujimoto et al., Addressing Function Approximation Error in Actor-Critic Methods. 2018.

# Soft Actor-Critic (SAC)


- TD3 / DDPG: modeling a deterministic policy + additional noise for exploration
- SAC:  modeling a randomized policy (by adding entropy as an exploration bonus)
- TD3 / DDPG vs. SAC is similar to $\epsilon$-greedy vs. Boltzmann exploration

# Entropy Bonus

## Bandit

$H(\pi) = \boxed{-\sum_a \pi(a) \log \pi(a)}$

$$\pi = \underset{\pi}{\operatorname{argmax}} \sum_a \pi(a)R(a) + \alpha\, H(\pi) = \underset{\pi}{\operatorname{argmax}}\, \mathbb{E}_{a\sim\pi}[R(a) - \alpha \log \pi(a)]$$

$\log \frac{1}{\pi(a)}$

## MDP

$\pi(a) = \exp\left(\frac{1}{\alpha} R(a)\right)$

$\sum_{h=0}^{\infty} \gamma^h \sum_a \pi(a|s_h) R(s_h, a) + \sum_{h=1}^{\infty} \gamma^h \alpha H(\pi(\cdot|s_h))$

$$\pi = \underset{\pi}{\operatorname{argmax}}\ \mathbb{E}^{\pi}\left[\sum_{h=0}^{\infty} \gamma^h \left(\sum_a \pi(a|s_h)R(s_h, a) + \alpha\, H(\pi(\cdot\,|s_h))\right)\right]$$

$$= \underset{\pi}{\operatorname{argmax}}\ \mathbb{E}^{\pi}\left[\sum_{h=0}^{\infty} \gamma^h\ (R(s_h, a_h) - \alpha \log \pi(a_h|s_h))\right]$$

# Bellman Equation with Entropy Bonus

$$Q^{\pi}(s,a) = \left[R(s,a) - \alpha \log \pi(a|s)\right] + \gamma \mathop{\mathbb{E}}_{s' \sim P(\cdot|s,a)} \left[ V^{\pi}(s') \right]$$

(in SAC)

$$Q^{\pi}(s,a) = R(s,a) + \gamma \mathop{\mathbb{E}}_{s' \sim P(\cdot|s,a)} \left[ V^{\pi}(s') + \alpha H(\pi(\cdot|s')) \right] \quad \checkmark$$

# TD3 vs. SAC

- Value update

**TD3:** Sample $a' \sim \mu_\theta(s') + \mathcal{N}(0, \sigma^2)$

Use $Q_{\phi_{\mathrm{tar}}}(s', a')$ as the regression target

**SAC:** Sample $a' \sim \pi_\theta(\cdot\,|s') = \mu_\theta(s') + \mathcal{N}(0, \sigma_\theta^2(s'))$

Use $Q_{\phi_{\mathrm{tar}}}(s', a') - \alpha \log \pi_\theta\,(a'|s')$ as the regression target

# Soft Actor-Critic (SAC)

For $k = 1,\ 2, \ldots$

Use $\mu_\theta(s) + \mathcal{N}(0, \sigma^2_{\theta(s)})$ to collect samples and place them in replay buffer

Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^n$ from the replay buffer

For each sample $i$, draw $a_i' \sim \mu_\theta(s_i') + \mathcal{N}(0, \sigma_\theta^2(s_i'))$

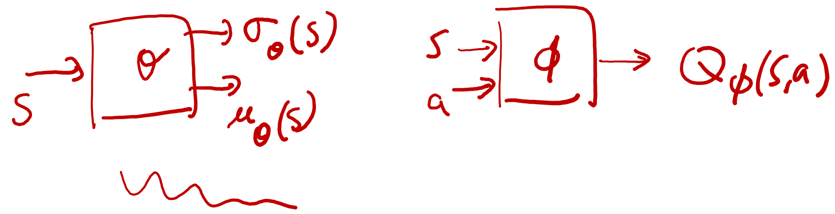
$$\phi_j \leftarrow \phi_j - \lambda \nabla_{\phi_j} \sum_{i=1}^{n} \left( Q_{\phi_j}(s_i, a_i) - r_i - \gamma \left( \min_{\ell=1,2} Q_{\phi_{\text{tar}\ell}}(s_i', a_i') + \alpha \log \pi_\theta(a_i' | s_i') \right) \right)^2 \quad \forall j = 1,2$$

Perform Policy ($\theta$) Update (to be specified later)

$$\phi_{\text{tar}j} \leftarrow \tau \phi_j + (1 - \tau) \phi_{\text{tar}j} \quad \forall j = 1,2$$

Haarnoja et al., Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor. 2018.

# TD3 vs. SAC




- Policy update

**TD3:** Do not view $-\alpha \log \pi_\theta (a|s)$ as part of the reward

Simply perform $\theta \leftarrow \theta + \eta \nabla_\theta Q_\phi(s, \mu_\theta(s))$

**SAC:** View $-\alpha \log \pi_\theta (a|s)$ as part of the reward

Perform the following:

Let $a_\theta(s) = \mu_\theta(s) + \epsilon \sigma_\theta(s)$ where $\epsilon \sim \mathcal{N}(0,1)$

Perform $\theta \leftarrow \theta + \eta \nabla_\theta \big( Q_\phi(s, a_\theta(s)) - \alpha \log \pi_\theta(a_\theta(s)|s) \big)$

$$\nabla_\theta \left( \int \pi_\theta(a|s)\, Q_\phi(s,a)\, da - \alpha \int \pi_\theta(a|s) \log \pi_\theta(a|s)\, da \right)$$

# Policy Gradient with Entropy Bonus

$$\nabla_\theta \int_a \underline{\pi_\theta(a|s)} \left( Q_\phi(s,a) - \alpha \log \pi_\theta(a|s) \right) \underline{da}$$

$$\boxed{a \sim \pi_\theta(\cdot|s)}$$

$$\Updownarrow$$

$$\boxed{\begin{array}{l} \varepsilon \sim N(0,1) \\ a = \mu_\theta(s) + \varepsilon \sigma_\theta(s) \end{array}}$$

$$= \nabla_\theta \mathop{E}_{a \sim \pi(\cdot|s)} \left[ Q_\phi(s,a) - \alpha \log \pi_\theta(a|s) \right]$$

$$= \nabla_\theta \mathop{E}_{\varepsilon \sim N(0,1)} \left[ \underline{Q_\phi(s, \mu_\theta(s) + \varepsilon \sigma_\theta(s)) - \alpha \log \pi_\theta(\mu_\theta(s) + \varepsilon \sigma_\theta(s) | s)} \right]$$

estimator can be constructed as:

1. draw $\varepsilon \sim N(0,1)$
2. use $\nabla_\theta [\quad\quad\quad\quad]$

# The Reparameterization Trick

① inverse propensity weighting

$$\nabla_\theta \mathop{\mathbb{E}}_{a \sim \pi_\theta(\cdot|s)} \left[ Q_\phi(s,a) - \alpha \log \pi_\theta(a|s) \right]$$

- First draw $\tilde{a} \sim \pi_\theta(\cdot|s)$
- construct $\dfrac{\nabla_\theta \pi_\theta(\tilde{a}|s)}{\pi_\theta(a|s)} \left[ Q_\phi(s,\tilde{a}) - \alpha \log \pi_\theta(\tilde{a}|s) \right]$

② Reparametrization

# Soft Actor-Critic (SAC)

For $k = 1,\ 2, \ldots$

Use $\mu_\theta(s) + \mathcal{N}(0, \sigma^2)$ to collect samples and place them in replay buffer

Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^n$ from the replay buffer

For each sample $i$, draw $a_i' \sim \mu_\theta(s_i') + \mathcal{N}(0, \sigma_\theta^2(s_i'))$

$$\phi_j \leftarrow \phi_j - \lambda \nabla_{\phi_j} \sum_{i=1}^n \left( Q_{\phi_j}(s_i, a_i) - r_i - \gamma \left( \min_{\ell=1,2} Q_{\phi_{\text{tar}\ell}}(s_i', a_i') + \alpha \log \pi_\theta(a_i' | s_i') \right) \right)^2 \quad \forall j = 1,2$$

Let $a_\theta(s_i) = \mu_\theta(s_i) + \epsilon \sigma_\theta(s_i)$ where $\epsilon \sim \mathcal{N}(0, I)$

$$\theta \leftarrow \theta + \eta \sum_{i=1}^n \nabla_\theta \left( Q_\phi(s, a_\theta(s_i)) - \alpha \log \pi_\theta(a_\theta(s_i) | s_i) \right)$$

$$\phi_{\text{tar}j} \leftarrow \tau \phi_j + (1 - \tau) \phi_{\text{tar}j} \quad \forall j = 1,2$$

Haarnoja et al., Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor. 2018.