

# Homework 4

6501 Reinforcement Learning (Spring 2024)

Submission deadline: 11:59pm, May 12

## Rules

- Collaboration is allowed but must be stated in precision per sub-problem. For example, comment in the sub-problem which idea is a result of a discussion with whom.
- Leveraging large language models is allowed but must be stated. Describe how you use them or provide precise prompts that you use.
- Please put all figures for experiments in a single pdf file. Submit the pdf to the Question 1 in Gradescope. Put all codes in a single .py file, and submit it to Question 2 in Gradescope.

## 1 Implementing DDPG / TD3

In this problem, we will implement actor-critic algorithms that work for continuous action sets. Specifically, we focus on deep deterministic policy gradient (DDPG) and twin-delayed DDPG (TD3). Download the starter code [here](#).

### 1.1 Environment

We focus on the Hopper environment, which is part of the Mujoco environment. The hopper is a two-dimensional one-legged figure that consist of four main body parts – the torso at the top, the thigh in the middle, the leg in the bottom, and a single foot on which the entire body rests ([Figure 1](#)). The goal is to make hops that move in the forward (right) direction by applying torques on the three hinges connecting the four body parts. More information about the environment can be found in <https://gymnasium.farama.org/environments/mujoco/hopper/>. You can search for some videos on the internet to see what the expected movement of Hopper looks like.

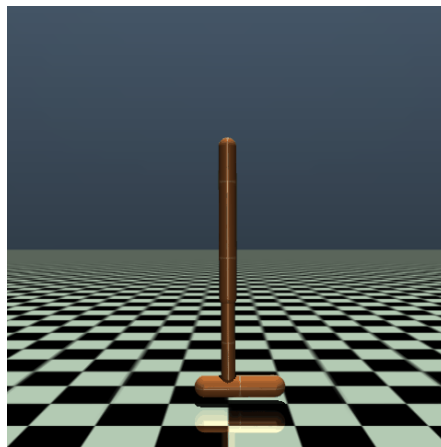


Figure 1: Hopper

**Observation / state space:** The observation / state space includes 11 features that describe the height of the hopper, angles of the joints, velocities, and angular velocities. The range of each feature is  $(-\infty, \infty)$ .

**Action space:** The action space is 3 dimensional. Each dimension represents the amount of torque applied to a joint. The range of each dimension is  $[-1, 1]$ .

**Rewards:** The reward is a sum of three parts. The first part is a constant for each step before termination. The second part measures how fast the hopper is moving in the positive  $x$  direction. The third part is a cost for taking large actions, i.e., the larger the magnitude of the action is, the larger the cost.

**Episode termination / truncation:** An episode terminates if the observation / state is outside a pre-defined range (usually means the hopper is fallen). When the number of steps reaches 1000, the episode will be truncated.

The Hopper environment requires you to install the Mujoco package, which might take some efforts. See the installation guide [here](#). If issues persist in the installation, you can try to work on alternative environments that also have continuous actions (for example, the continuous version of Lunar Lander).

## 1.2 DDPG

DDPG is an off-policy actor-critic algorithm for continuous action space. It can also be viewed as an adaptation of Q-learning for continuous actions. The pseudocode of DDPG is given in [Algorithm 1](#). Also see [actor-critic slides](#) Page 25 or [OpenAI spinning up](#).

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**Algorithm 1** DDPG

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**Hyper-Parameters:**  $\sigma, \eta, \lambda, \tau, N, M$ .

Initialize a replay buffer  $D$  with capacity  $N$ .

Initialize two actor networks (mappings from the state space to the action space) with weights  $\theta$  and  $\theta_{\text{tar}}$ .

Initialize two critic networks (mappings from the state-action space to a scalar value) with weights  $\phi$  and  $\phi_{\text{tar}}$ .

Initialize the environment with state  $s_1$ .

**for**  $t = 1, \dots$  **do**

    Select action

$$a_t = \Pi_{\mathcal{A}} (\mu(s_t; \theta_{\text{tar}}) + \mathcal{N}(0, \sigma^2 I))$$

    where  $\Pi_{\mathcal{A}}$  is the projection operator to the action space (in Hopper, simply clip each coordinate to  $[-1, 1]$ ). Execute action  $a_t$ , observe reward  $r_t$ , and observe the next state  $s_{t+1}$  if the episode hasn't terminate.

    Let  $s'_t$  be the terminal state if the episode terminates after  $t$ ; otherwise, let  $s'_t = s_{t+1}$ .

    Store transition  $(s_t, a_t, r_t, s'_t)$  in replay buffer  $D$ .

    Sample a random minibatch of transitions  $\{(s_j, a_j, r_j, s'_j)\}_{j=1}^M$  with size  $M$  from  $D$ .

    For each  $j$ , define

$$a'_j = \mu(s'_j; \theta_{\text{tar}}).$$

    Set

$$y_j = \begin{cases} r_j & \text{if } s'_j \text{ is the terminal state} \\ r_j + \gamma Q(s'_j, a'_j; \phi_{\text{tar}}) & \text{otherwise} \end{cases} \quad (1)$$

**Critic update:** Perform a gradient descent step on MSE loss:

$$\phi \leftarrow \phi - \eta \nabla_{\phi} \frac{1}{M} \sum_{j=1}^M (Q(s_j, a_j; \phi) - y_j)^2.$$

**Actor update:** Perform a gradient ascent step:

$$\theta \leftarrow \theta + \lambda \nabla_{\theta} \frac{1}{M} \sum_{j=1}^M Q(s_j, \mu(s_j; \theta); \phi).$$

**Target network update:**

$$\phi_{\text{tar}} \leftarrow \tau \phi + (1 - \tau) \phi_{\text{tar}},$$

$$\theta_{\text{tar}} \leftarrow \tau \theta + (1 - \tau) \theta_{\text{tar}}.$$

    If the episode is terminated, let  $s_{t+1}$  be the initial state of the next episode.

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**Tasks (50%)** Implement DDPG ([Algorithm 1](#)) to play Hopper. Plot the evolution of the episodic return over time (x-axis: number of episodes, y-axis: episodic return).

### 1.3 TD3

TD3 is a direct successor of DDPG, which mainly include three additional elements. See [actor-critic slides](#) Page 26-29 or [OpenAI spinning up](#). These three elements are:

- **Double-Q** Use two pairs of critic networks (so there are four critic networks in total:  $\phi_1, \phi_{\text{tar}1}, \phi_2, \phi_{\text{tar}2}$ ), and always use the minimum of the target networks  $\min_{\ell=1,2} Q(s, a; \phi_{\text{tar}\ell})$  as the regression target.

- **Delayed actor Update** Update actor less frequently than critic.
- **Target policy smoothing** Instead of using  $Q(s', \mu(s'; \theta_{\text{tar}}); \phi_{\text{tar}})$  as the regression target, use  $Q(s', a'; \phi_{\text{tar}})$  where  $a'$  is  $\mu(s'; \theta_{\text{tar}})$  plus noise.

The psuedocode of TD3 is given in [Algorithm 2](#), with the differences with DDPG highlighted in **this color**.

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**Algorithm 2** TD3

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**Hyper-Parameters:**  $\sigma, \eta, \lambda, \tau, N, M, c, \sigma', n$ .

Initialize a replay buffer  $D$  with capacity  $N$ .

Initialize two actor networks (mappings from the state space to the action space) with weights  $\theta$  and  $\theta_{\text{tar}}$ .

Initialize four critic networks (mappings from the state-action space to a scalar value) with weights  $\phi_1$  and  $\phi_{\text{tar}1}$ .

Initialize the environment with state  $s_1$ .

**for**  $t = 1, \dots$  **do**

    Select action

$$a_t = \Pi_{\mathcal{A}} (\mu_{\theta_{\text{tar}}}(s_t) + \mathcal{N}(0, \sigma^2 I))$$

where  $\Pi_{\mathcal{A}}$  is the projection operator to the action space (in Hopper, simply clip each coordinate to  $[-1, 1]$ ). Execute action  $a_t$ , observe reward  $r_t$ , and observe the next state  $s_{t+1}$  if the episode hasn't terminate.

Let  $s'_t$  be the terminal state if the episode terminates after  $t$ ; otherwise, let  $s'_t = s_{t+1}$ .

Store transition  $(s_t, a_t, r_t, s'_t)$  in replay buffer  $D$ .

Sample a random minibatch of transitions  $\{(s_j, a_j, r_j, s'_j)\}_{j=1}^M$  with size  $M$  from  $D$ .

**For each**  $j$ , **draw**

$$a'_j = \Pi_{\mathcal{A}} (\mu(s'_j; \theta_{\text{tar}}) + \epsilon_j)$$

    where  $\epsilon_j \sim \Pi_{[-c, c]}(\mathcal{N}(0, \sigma'^2 I))$  is a truncated Gaussian noise.

    Set

$$y_j = \begin{cases} r_j & \text{if } s'_j \text{ is the terminal state} \\ r_j + \gamma \min_{\ell \in \{1, 2\}} Q(s'_j, a'_j; \phi_{\text{tar}\ell}) & \text{otherwise} \end{cases} \quad (2)$$

**Critic update:** Perform a gradient descent step on MSE loss:

$$\forall \ell \in \{1, 2\} \quad \phi_{\ell} \leftarrow \phi_{\ell} - \eta \nabla_{\phi_{\ell}} \frac{1}{M} \sum_{j=1}^M (Q(s_j, a_j; \phi_{\ell}) - y_j)^2.$$

**if**  $t \bmod n = 0$  **then**

**Actor update:** Perform a gradient ascent step:

$$\theta \leftarrow \theta + \lambda \nabla_{\theta} \frac{1}{M} \sum_{j=1}^M Q(s_j, \mu(s_j; \theta); \phi_1). \quad (3)$$

**Target network update:**

$$\forall \ell \in \{1, 2\} \quad \begin{aligned} \phi_{\text{tar}\ell} &\leftarrow \tau \phi_{\ell} + (1 - \tau) \phi_{\text{tar}\ell}, \\ \theta_{\text{tar}} &\leftarrow \tau \theta + (1 - \tau) \theta_{\text{tar}}. \end{aligned}$$

    If the episode is terminated, let  $s_{t+1}$  be the initial state of the next episode.

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**Task (40%)** Implement TD3 ([Algorithm 2](#)) to play Hopper. Plot the episodic return over time (x-axis: number of episodes, y-axis: episodic return). Feel free to not entirely follow the original implementation of TD3, but just incorporate

similar ideas. For example, instead of only updating the actor every  $n$  steps, you might simply create a separation between the learning rates of actor and critic, which could produce a similar effect.

Also, it will be interesting to render the environment and see how your Hopper is performing (we do not have this part of starter code right now, but will try to include it afterwards).

You will get full score if the final episodic return of your Hopper can reach  $\gtrsim 1500$  or higher. Please refer to [1] to see the performance of the original implementation of TD3.

## 2 Survey (20%)

Thank you for staying in the class till the end! Please let us know any suggestions for the course so that we can improve in future semesters.

## References

- [1] Scott Fujimoto, Herke Hoof, and David Meger. Addressing function approximation error in actor-critic methods. In *International conference on machine learning*, pages 1587–1596. PMLR, 2018.