

Actor-Critic Methods

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Review: Full-Information Policy Learning in MDPs

$$\theta_{k+1} = \operatorname{argmax}_{\theta} \underbrace{\left(V^{\pi_{\theta}}(\rho) - V^{\pi_{\theta_k}}(\rho) - \frac{1}{\eta} D(\theta, \theta_k) \right)}$$

$$\approx \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) (\pi_{\theta}(a|s) - \pi_{\theta_k}(a|s)) Q^{\pi_{\theta_k}}(s, a) = \mathbb{E}_{(s_i, a_i) \sim \pi_{\theta_k}} \left[\frac{\pi_{\theta}(a_i|s_i) - \pi_{\theta_k}(a_i|s_i)}{\pi_{\theta_k}(a_i|s_i)} \boxed{Q^{\pi_{\theta_k}}(s_i, a_i)} \right]$$

$$\approx (\theta - \theta_k)^{\top} \underbrace{\sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left(\nabla_{\theta} \pi_{\theta}(a|s) \Big|_{\theta=\theta_k} \right) Q^{\pi_{\theta_k}}(s, a)}_{\text{PG/NPG}} \\ = \mathbb{E}_{(s_i, a_i)} \left[\frac{\nabla_{\theta} \pi_{\theta}(a_i|s_i) \Big|_{\theta=\theta_k}}{\pi_{\theta_k}(a_i|s_i)} \boxed{Q^{\pi_{\theta_k}}(s_i, a_i)} \right]$$

PG/NPG: Estimate them using the empirical sum of reward in the trajectory (i.e., Monte Carlo estimator)

We can also use other estimators to balance bias and variance

Actor-Critic Methods

Use value function approximation to estimate $Q^{\pi_{\theta_k}}(s_i, a_i)$ or $A^{\pi_{\theta_k}}(s_i, a_i)$

Use $V_\phi(s)$: $\approx V^{\pi_{\theta_k}}(s)$ $\min_{\phi} \mathbb{E}_{(s,r,s') \sim \pi_{\theta_k}} \left[\left(V_\phi(s) - r - \gamma V_\phi(s') \right)^2 \right]$

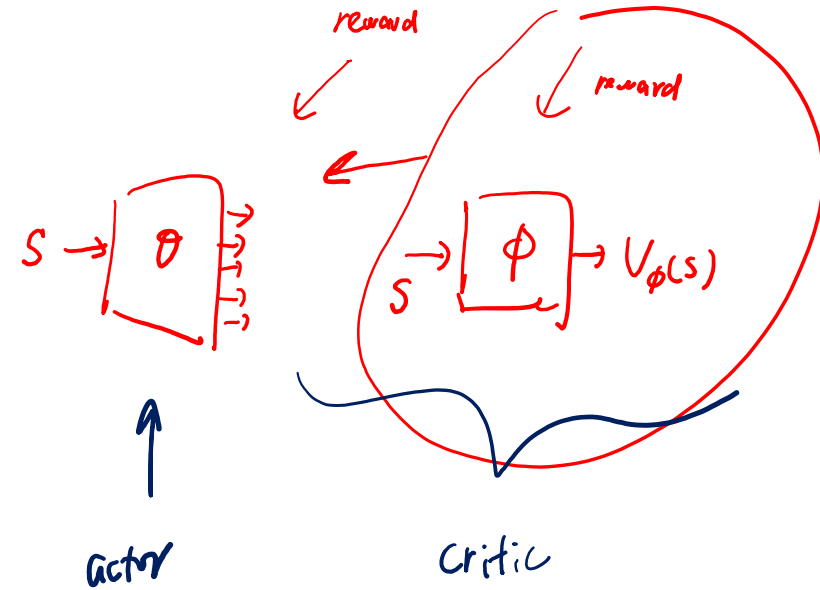
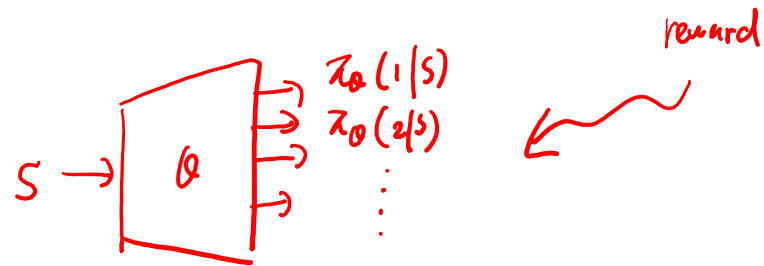
Use $Q_\phi(s, a)$: $\approx Q^{\pi_{\theta_k}}(s, a)$ $\min_{\phi} \mathbb{E}_{(s,a,r,s',a') \sim \pi_{\theta_k}} \left[\left(Q_\phi(s, a) - r - \gamma Q_\phi(s', a') \right)^2 \right]$

Possible estimators for $A^{\pi_{\theta_k}}(s, a)$:

Let $(s_1, a_1, r_1, s_2, a_2, r_2 \dots)$ be a trajectory starting from $s_1 = s, a_1 = a$

	$Q_\phi(s_1, a_1) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot s)}[Q_\phi(s_1, a')]$
$\left\{ \begin{array}{l} r_1 + \gamma V_\phi(s_2) - V_\phi(s_1) \\ r_1 + \gamma r_2 + \gamma^2 V_\phi(s_3) - V_\phi(s_1) \\ \vdots \end{array} \right.$	$r_1 + \gamma Q_\phi(s_2, a_2) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot s)}[Q_\phi(s_1, a')]$
	$r_1 + \gamma r_2 + \gamma^2 Q_\phi(s_3, a_3) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot s)}[Q_\phi(s_1, a')]$
	\vdots

Pure Policy-Based Methods vs. Actor-Critic Methods



Actor-Critic with Q_ϕ

(find π^*)
(given π)

(off-policy)

$\Rightarrow Q^*$
Q-learning: $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha [r + \max_{a'} Q(s',a')]$
TD-learning: $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha [r + \sum_{a'} \pi(a'|s) Q(s',a')]$

(on-policy)

\downarrow
 Q^π

For $k = 1, 2, \dots$

Use π_{θ_k} to collect n trajectories

$$(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}), \dots, (s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)})$$

Define

$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \frac{\nabla_{\theta} \pi_{\theta} (a_h^{(i)} | s_h^{(i)})}{\pi_{\theta_k} (a_h^{(i)} | s_h^{(i)})} \overset{\text{red } Q}{\parallel} Q_{\phi_k} (s_h^{(i)}, a_h^{(i)}) \text{ or } \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \sum_a \nabla_{\theta} \pi_{\theta} (a | s_h^{(i)}) \big|_{\theta=\theta_k} Q_{\phi_k} (s_h^{(i)}, a)$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \quad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left(Q_{\phi} (s_h^{(i)}, a_h^{(i)}) - r_h^{(i)} - \gamma Q_{\phi_k} (s_{h+1}^{(i)}, a_{h+1}^{(i)}) \right)^2 \bigg|_{\phi=\phi_k}$$

Advantage Actor-Critic (A2C) = PG + V_ϕ

For $k = 1, 2, \dots$

Use π_{θ_k} to collect n trajectories

$$(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}), \dots, (s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)})$$

Define

$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \nabla_{\theta} \log \pi_{\theta} (a_h^{(i)} | s_h^{(i)}) \Big|_{\theta=\theta_k} \underbrace{\left(r_h^{(i)} + \gamma V_{\phi_k}(s_{h+1}^{(i)}) - V_{\phi_k}(s_h^{(i)}) \right)}_{\text{or any other advantage estimator in the previous slide}}$$

Handwritten notes: $\frac{\nabla_{\theta} \log \pi_{\theta}(a_h^{(i)} | s_h^{(i)})}{\log \pi_{\theta}(a_h^{(i)} | s_h^{(i)})} \approx A^{\pi_k}(s_h^{(i)}, a_h^{(i)})$ and $\mathbb{E}(\cdot) = \sum_{s,a} d_{\rho}^{\pi_k}(s) \nabla_{\theta} \log \pi_{\theta}(a|s) A^{\pi_k}(s,a)$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \quad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left(V_{\phi}(s_h^{(i)}) - r_h^{(i)} - \gamma V_{\phi_k}(s_{h+1}^{(i)}) \right)^2 \Big|_{\phi=\phi_k}$$

Handwritten note: $V_{\phi} \approx V^{\pi_k}$

Proximal Policy Optimization (PPO) = NPG + V_ϕ

For $k = 1, 2, \dots$

Use π_{θ_k} to collect n trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \dots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Perform updates

$$\theta_{k+1} \leftarrow \operatorname{argmax}_{\theta} \left\{ \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \frac{\pi_{\theta} \left(a_h^{(i)} \middle| s_h^{(i)} \right)}{\pi_{\theta_k} \left(a_h^{(i)} \middle| s_h^{(i)} \right)} \underbrace{\left(r_h^{(i)} + \gamma V_{\phi_k} \left(s_{h+1}^{(i)} \right) - V_{\phi_k} \left(s_h^{(i)} \right) \right)}_{\substack{\approx A^{\pi_{\theta_k}}(s_h) \\ \text{or any other advantage estimator in the previous slide}}} - \frac{1}{\eta} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \operatorname{KL} \left(\pi_{\theta} \left(\cdot \middle| s_h^{(i)} \right), \pi_{\theta_k} \left(\cdot \middle| s_h^{(i)} \right) \right) \right\}$$

$$\phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left(V_{\phi} \left(s_h^{(i)} \right) - r_h^{(i)} - \gamma V_{\phi_k} \left(s_{h+1}^{(i)} \right) \right)^2 \bigg|_{\phi = \phi_k}$$

$$\frac{r(s) \mathbb{I}(a_+ = a)}{p_+(s)} \quad \frac{(r(s) - 1) \mathbb{I}(a_+ = a)}{p_+(s)}$$

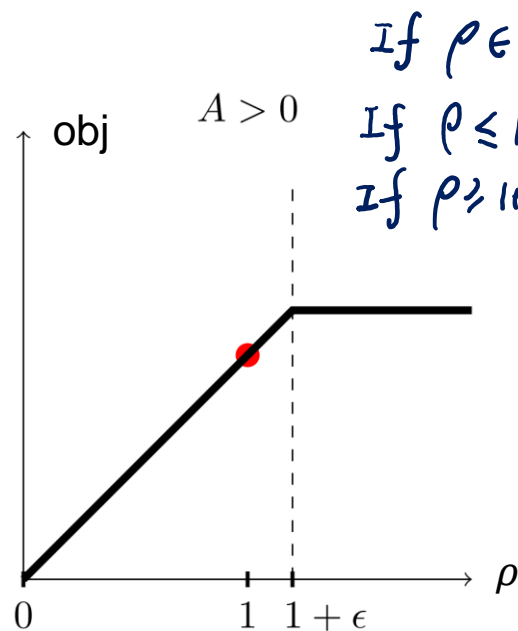
Additional Technique 1: Clipped Objective (for PPO)

$$\rho := \frac{\pi_{\theta} (a_h^{(i)} | s_h^{(i)})}{\pi_{\theta_k} (a_h^{(i)} | s_h^{(i)})}$$

$$A := \left(r_h^{(i)} + \gamma V_{\phi_k} (s_{h+1}^{(i)}) - V_{\phi_k} (s_h^{(i)}) \right)$$

$$\text{clip}_{[1-\epsilon, 1+\epsilon]}(\rho) = \min(\max(\rho, 1-\epsilon), 1+\epsilon)$$

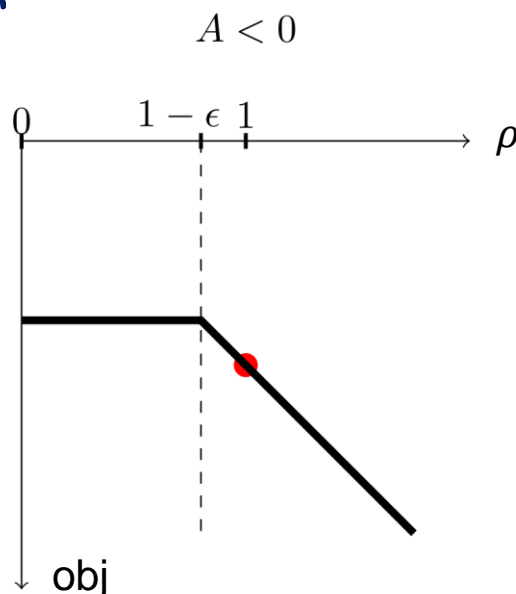
Instead of using ρA as the objective, use $\min\{\rho A, \text{clip}_{[1-\epsilon, 1+\epsilon]}(\rho) A\}$



If $\rho \in [1-\epsilon, 1+\epsilon] \Rightarrow \rho A$

If $\rho \leq 1-\epsilon \Rightarrow \rho A$

If $\rho \geq 1+\epsilon \Rightarrow (1+\epsilon)A$



If $\rho \in (1-\epsilon, 1+\epsilon] \Rightarrow \rho A$

If $\rho \leq 1-\epsilon \Rightarrow (1-\epsilon)A$ (strange case)

If $\rho \geq 1+\epsilon \Rightarrow \rho A$

algorithm	avg. normalized score
No clipping or penalty	-0.39
Clipping, $\epsilon = 0.1$	0.76
Clipping, $\epsilon = 0.2$	0.82
Clipping, $\epsilon = 0.3$	0.70
Adaptive KL $d_{\text{targ}} = 0.003$	0.68
Adaptive KL $d_{\text{targ}} = 0.01$	0.74
Adaptive KL $d_{\text{targ}} = 0.03$	0.71
Fixed KL, $\beta = 0.3$	0.62
Fixed KL, $\beta = 1.$	0.71
Fixed KL, $\beta = 3.$	0.72
Fixed KL, $\beta = 10.$	0.69

Additional Technique 2: Entropy Bonus

In the objective of policy update, add a bonus term

$$H(\pi_{\theta}(\cdot | s)) = \sum_a \pi_{\theta}(a|s) \ln \frac{1}{\pi_{\theta}(a|s)}$$

For PPO:

$$\operatorname{argmax}_{\theta} \left\{ \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \frac{\pi_{\theta}(a_h^{(i)} | s_h^{(i)})}{\pi_{\theta_k}(a_h^{(i)} | s_h^{(i)})} A_h^{(i)} - \frac{1}{\eta} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \operatorname{KL}(\pi_{\theta}(\cdot | s_h^{(i)}), \pi_{\theta_k}(\cdot | s_h^{(i)})) + c \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \underbrace{H(\pi_{\theta}(\cdot | s_h^{(i)}))}_{\text{Entropy Bonus}} \right\}$$

— $\operatorname{KL}(\pi_{\theta}(\cdot | s_h^{(i)}), \pi_{\text{unif}}(\cdot | s_h^{(i)}))$

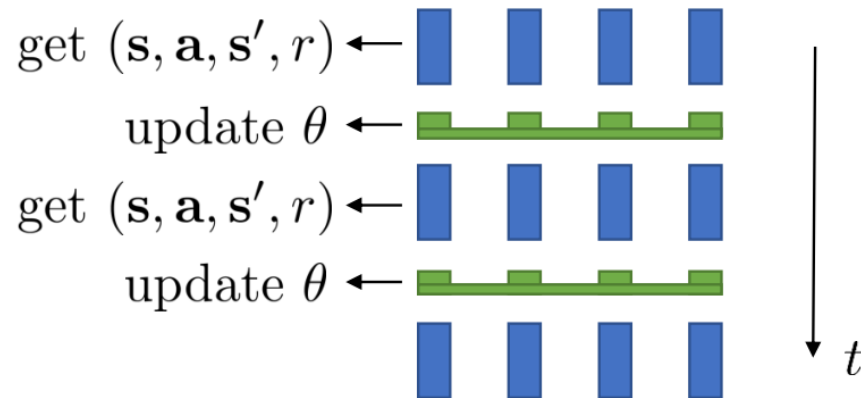
For A2C:

$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \nabla_{\theta} \log \pi_{\theta}(a_h^{(i)} | s_h^{(i)}) \Big|_{\theta=\theta_k} A_h^{(i)} + c \nabla_{\theta} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} H(\pi_{\theta}(\cdot | s_h^{(i)}))$$

Additional Technique 3: Parallel Sample Collection

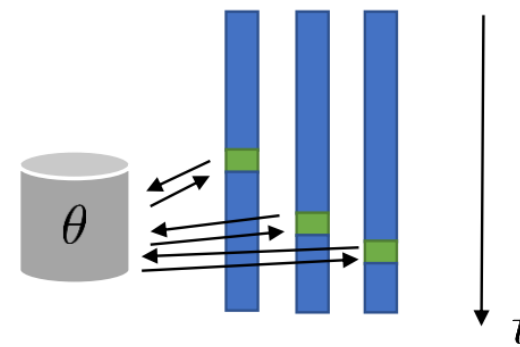
A2C

synchronized parallel actor-critic



A3C

asynchronous parallel actor-critic



Actor-Critic Summary

PG \longrightarrow A2C

NPG \longrightarrow PPO

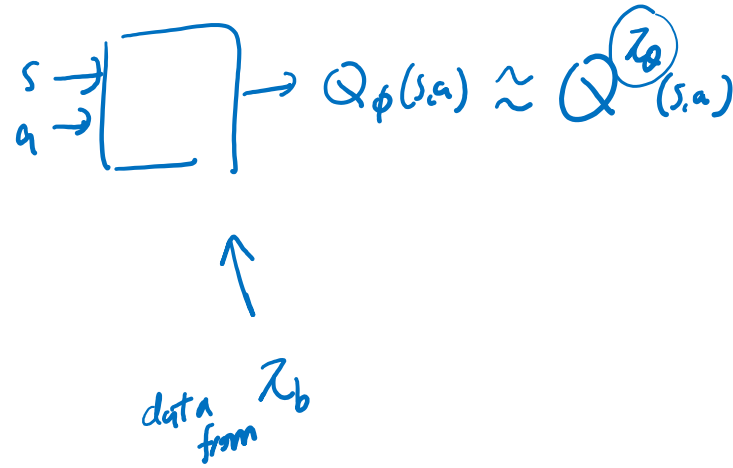
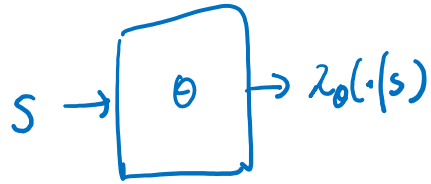
$$s \rightarrow \boxed{Q} \rightarrow \pi(s)$$

$$s \rightarrow \boxed{\theta} \rightarrow \pi(s) \quad s \rightarrow \boxed{\phi} \rightarrow V_{\phi}(s)$$

$$s \xrightarrow{a} \boxed{\phi} \rightarrow Q_{\phi}(s,a)$$

Off-Policy Actor-Critic

- Leveraging **off-policy evaluation** → allow reusing data



Review: Full-Information Policy Learning in MDPs

$$\begin{aligned}\theta_{k+1} &= \operatorname{argmax}_{\theta} \left(\underbrace{V^{\pi_{\theta}}(\rho) - V^{\pi_{\theta_k}}(\rho)}_{\text{}} - \frac{1}{\eta} D(\theta, \theta_k) \right) \\ &\approx \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left(\pi_{\theta}(a|s) - \pi_{\theta_k}(a|s) \right) Q^{\pi_{\theta_k}}(s, a) \\ &\approx (\theta - \theta_k)^{\top} \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left(\nabla_{\theta} \pi_{\theta}(a|s) \Big|_{\theta=\theta_k} \right) Q^{\pi_{\theta_k}}(s, a)\end{aligned}$$

Use any off-policy policy evaluation methods to find ϕ_k such that $Q_{\phi_k}(s, a) \approx Q^{\pi_{\theta_k}}(s, a)$

Suppose that our (s_i, a_i) samples are obtained from $\hat{\pi}$

Off-Policy Actor-Critic

$$\begin{aligned}
 \theta_{k+1} &= \operatorname{argmax}_{\theta} \left(\underbrace{V^{\pi_{\theta}}(\rho) - V^{\pi_{\theta_k}}(\rho)}_{\text{Handwritten: } \sum_s d_{\rho}^{\hat{\pi}}(s) \cdot \frac{d\rho^{\pi_{\theta_k}}(s)}{d\hat{\pi}(s)} \sum_a \dots} - \frac{1}{\eta} D(\theta, \theta_k) \right) \\
 &\approx \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left(\pi_{\theta}(a|s) - \pi_{\theta_k}(a|s) \right) Q_{\phi_k}(s,a) = \mathbb{E}_{s \sim \hat{\pi}} \left[\frac{\cancel{d_{\rho}^{\pi_{\theta_k}}(s)}}{\cancel{d_{\rho}^{\hat{\pi}}(s)}} \sum_a \left(\pi_{\theta}(a|s) - \pi_{\theta_k}(a|s) \right) Q_{\phi_k}(s,a) \right] \\
 &\approx (\theta - \theta_k)^{\top} \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left(\nabla_{\theta} \pi_{\theta}(a|s) \Big|_{\theta=\theta_k} \right) Q_{\phi_k}(s,a) = (\theta - \theta_k)^{\top} \mathbb{E}_{s \sim \hat{\pi}} \left[\frac{\cancel{d_{\rho}^{\pi_{\theta_k}}(s)}}{\cancel{d_{\rho}^{\hat{\pi}}(s)}} \sum_a \nabla_{\theta} \pi_{\theta}(a|s) \Big|_{\theta=\theta_k} Q_{\phi_k}(s,a) \right]
 \end{aligned}$$

Use any off-policy policy evaluation methods to find ϕ_k such that $Q_{\phi_k}(s,a) \approx Q^{\pi_{\theta_k}}(s,a)$

Suppose that our (s_i, a_i) samples are obtained from $\hat{\pi}$

Actor-Critic + Replay Buffer

For $k = 1, 2, \dots$

Collect samples using π_{θ_k} , and place them in the replay buffer

Sample a batch $\{(s_i, a_i, r_i, s'_i)\}_{i=1}^n$ from replay buffer

Define

$$g = \frac{1}{n} \sum_{i=1}^n \sum_a \nabla_{\theta} \pi_{\theta}(a|s_i) \Big|_{\theta=\theta_k} Q_{\phi_k}(s_i, a) \quad \text{Note: not using } a_i \text{ here}$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g$$

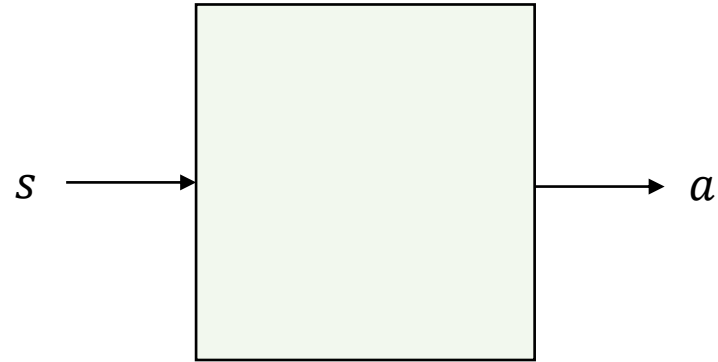
$$\phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \frac{1}{n} \sum_{i=1}^n \left(Q_{\phi}(s_i, a_i) - r_i - \gamma \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot|s'_i)} [Q_{\phi}(s'_i, a')] \right)^2 \Big|_{\phi=\phi_k}$$

Off-policy TD \rightarrow unstable (more on this later)

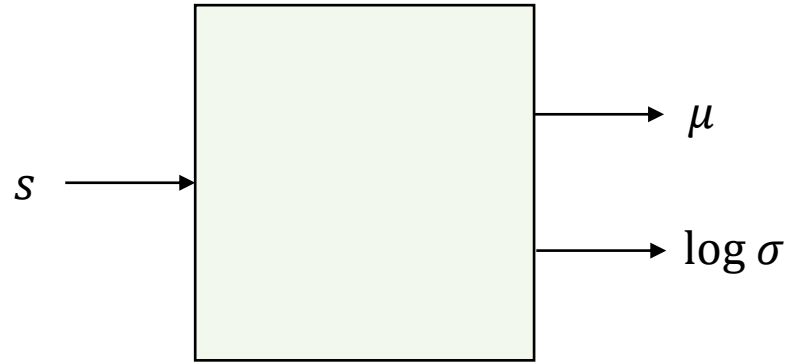
Dealing with Continuous Action Sets

Review: Linear Bandits and One-Point Gradient Estimator

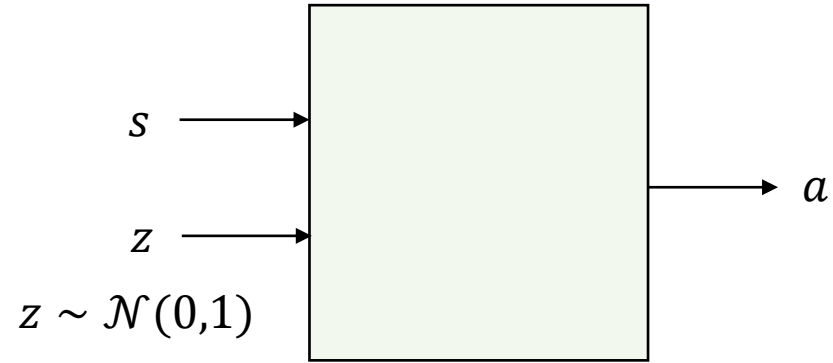
Policy Network for Continuous Action Sets



Policy Network for Continuous Action Sets



Explicitly models $\pi_{\theta}(a|s)$



Implicitly modeling $\pi_{\theta}(a|s)$

Option 1: making σ part of policy parameters

Option 2: making σ a hyper-parameters
(decreases over time)

can sample from it, but do not
know the function $\pi_{\theta}(\cdot |s)$

A2C / PPO with Continuous Action Sets

$$g = \frac{1}{n} \sum_{i=1}^n \nabla_{\theta} \log \pi_{\theta}(a_i | s_i) \Big|_{\theta=\theta_k} A_i$$

$$\theta_{k+1} \leftarrow \operatorname{argmax}_{\theta} \left\{ \frac{1}{n} \sum_{i=1}^n \frac{\pi_{\theta}(a_i | s_i)}{\pi_{\theta_k}(a_i | s_i)} A_i - \frac{1}{\eta} \frac{1}{n} \sum_{i=1}^n \operatorname{KL} \left(\pi_{\theta}(\cdot | s_i), \pi_{\theta_k}(\cdot | s_i) \right) \right\}$$

Recall: Actor-Critic without need for inverse weighting

Actor-critic with $Q_\phi(s, a)$ function approximation

For $k = 1, 2, \dots$

Use π_{θ_k} to collect n trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \dots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Define
$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \sum_a \nabla_{\theta} \pi_{\theta} \left(a | s_h^{(i)}\right) \Big|_{\theta=\theta_k} Q_{\phi_k} \left(s_h^{(i)}, a\right)$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \qquad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left(Q_{\phi} \left(s_h^{(i)}, a_h^{(i)} \right) - r_h^{(i)} - \gamma Q_{\phi_k} \left(s_{h+1}^{(i)}, a_{h+1}^{(i)} \right) \right)^2 \Big|_{\phi=\phi_k}$$

Deterministic Policy Gradient Theorem

Deterministic Policy Gradient Algorithm

For $k = 1, 2, \dots$

Use π_{θ_k} to collect n trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \dots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Define
$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \nabla_{\theta} Q_{\phi_k} \left(s_h^{(i)}, \pi_{\theta} \left(s_h^{(i)} \right) \right) \Bigg|_{\theta=\theta_k}$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \qquad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left(Q_{\phi} \left(s_h^{(i)}, a_h^{(i)} \right) - r_h^{(i)} - \gamma Q_{\phi_k} \left(s_{h+1}^{(i)}, a_{h+1}^{(i)} \right) \right)^2 \Bigg|_{\phi=\phi_k}$$

Two Viewpoints for the Deterministic PG Algorithm

Deep Deterministic Policy Gradient (DDPG)

For $k = 1, 2, \dots$

Use π_θ to collect samples and place them in replay buffer

Sample a batch $\{(s_i, a_i, r_i, s'_i)\}_{i=1}^n$ from the replay buffer

$$\theta \leftarrow \theta + \eta \sum_{i=1}^n \nabla_\theta Q_\phi(s_i, \pi_\theta(s_i))$$

$$\phi \leftarrow \phi - \lambda \nabla_\phi \sum_{i=1}^n \left(Q_\phi(s_i, a_i) - r_i - \gamma Q_{\phi_{\text{tar}}}(s'_i, \pi_{\theta_{\text{tar}}}(s'_i)) \right)^2$$

$$\theta_{\text{tar}} \leftarrow \tau \theta + (1 - \tau) \theta_{\text{tar}}$$

$$\phi_{\text{tar}} \leftarrow \tau \phi + (1 - \tau) \phi_{\text{tar}}$$

Twin Delayed DDPG (TD3)

Soft Actor-Critic (SAC)