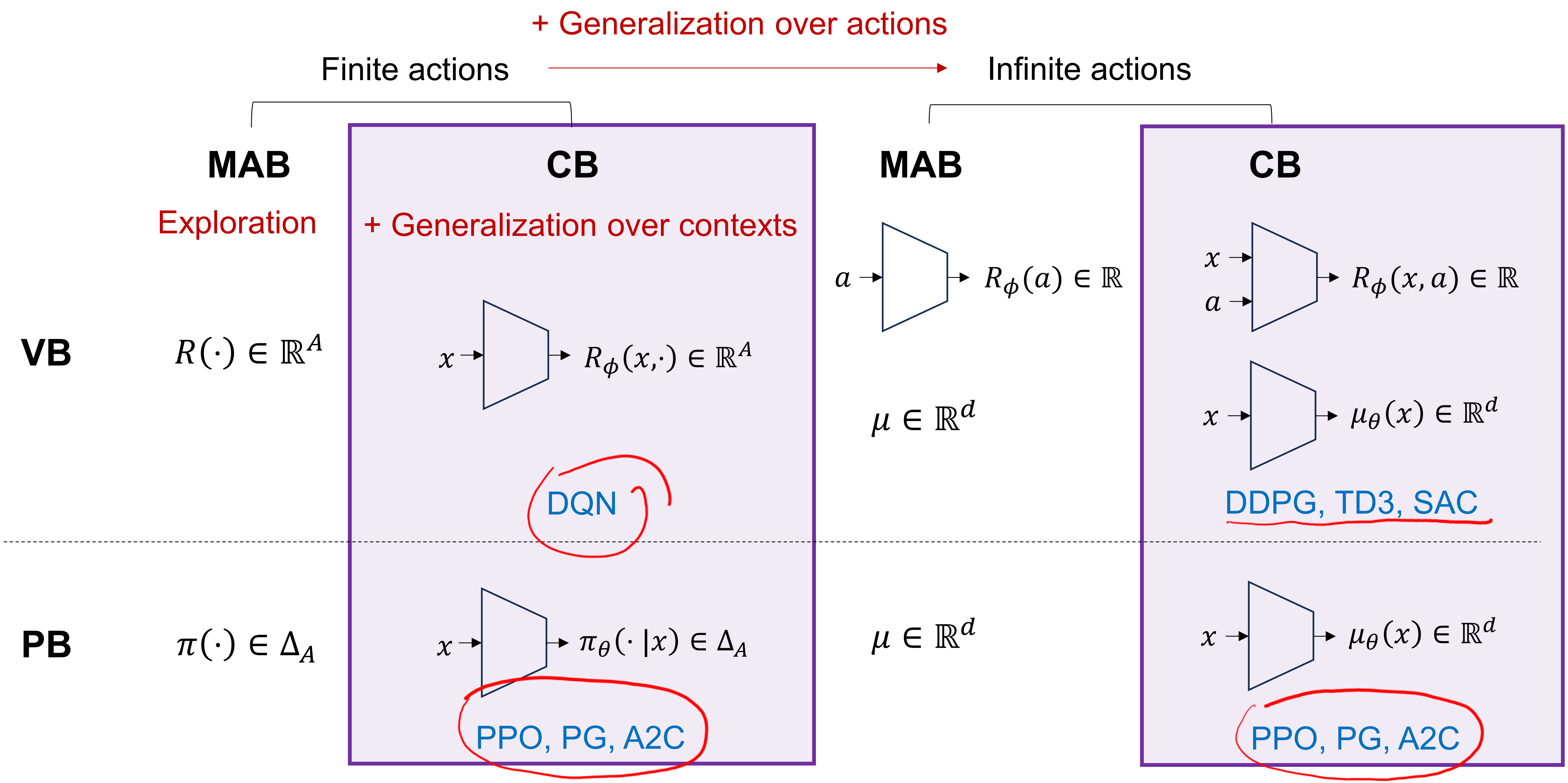
RL with Continuous Action Sets

Chen-Yu Wei

3 main challenges in online RL: Exploration, Generalization, (Temporal) Credit Assignment

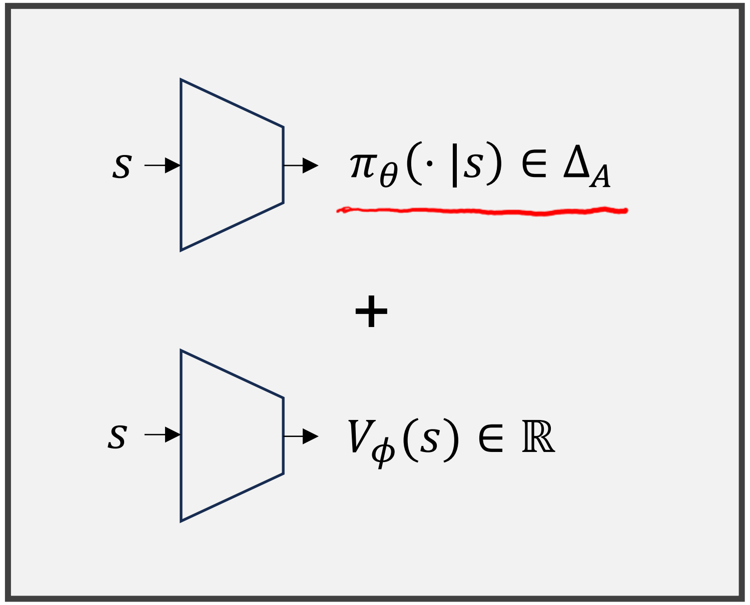


+ Generalization over actions

Finite actions → Infinite actions

	Finite actions: MAB	Finite actions: CB	Infinite actions: MAB	Infinite actions: CB
	Exploration	+ Generalization over contexts		
VB	$R(\cdot) \in \mathbb{R}^A$	$x \rightarrow R_\phi(x,\cdot) \in \mathbb{R}^A$ DQN	$a \rightarrow R_\phi(a) \in \mathbb{R}$ $\mu \in \mathbb{R}^d$	$x, a \rightarrow R_\phi(x,a) \in \mathbb{R}$ $x \rightarrow \mu_\theta(x) \in \mathbb{R}^d$ DDPG, TD3, SAC
PB	$\pi(\cdot) \in \Delta_A$	$x \rightarrow \pi_\theta(\cdot\,\vert x) \in \Delta_A$ PPO, PG, A2C	$\mu \in \mathbb{R}^d$	$x \rightarrow \mu_\theta(x) \in \mathbb{R}^d$ PPO, PG, A2C

PPO / PG / A2C in Discrete / Continuous Action Sets

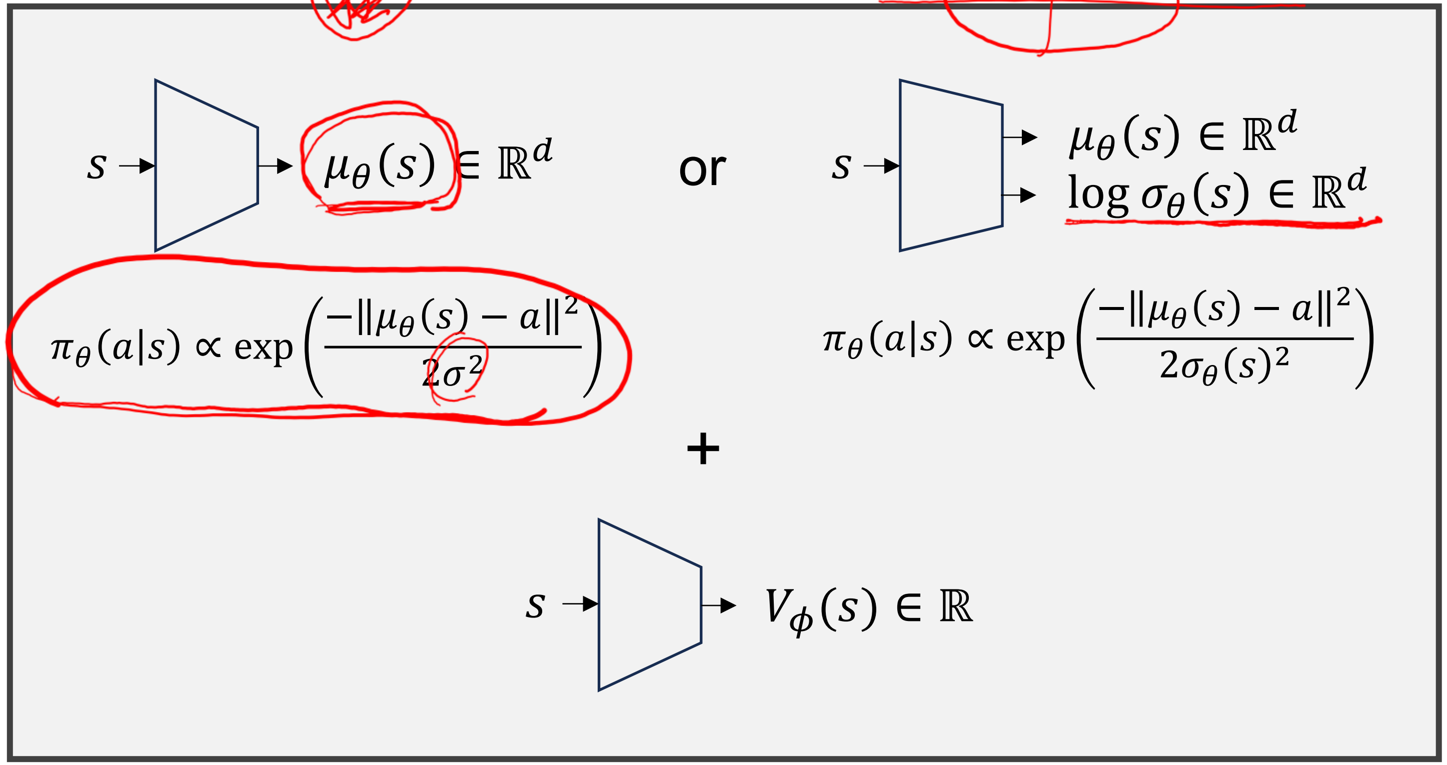


$s \to \pi_\theta(\cdot|s) \in \Delta_A$

+

$s \to V_\phi(s) \in \mathbb{R}$

Discrete actions



$s \to \mu_\theta(s) \in \mathbb{R}^d$

$$\pi_\theta(a|s) \propto \exp\left(\frac{-\|\mu_\theta(s) - a\|^2}{2\sigma^2}\right)$$

or

$s \to \mu_\theta(s) \in \mathbb{R}^d$, $\log \sigma_\theta(s) \in \mathbb{R}^d$

$$\pi_\theta(a|s) \propto \exp\left(\frac{-\|\mu_\theta(s) - a\|^2}{2\sigma_\theta(s)^2}\right)$$

+

$s \to V_\phi(s) \in \mathbb{R}$

Continuous actions

Algorithms involving a policy and value network where the value is used in the policy update are called **actor-critic** algorithms.

PPO / PG / A2C in Continuous Action Sets

$\pi_\theta(a|s_i)$ defined as $\mathcal{N}(\mu_\theta(s_i), \sigma^2 I)$ or $\mathcal{N}(\mu_\theta(s_i), \sigma_\theta(s_i)^2 I)$

$\exp(- \cdots)$

$$\theta_{k+1} \leftarrow \underset{\theta}{\operatorname{argmax}} \left\{ \sum_{i=1}^{N} \left(\frac{\pi_\theta(a_i|s_i)}{\pi_{\theta_k}(a_i|s_i)} A_i - \frac{1}{\eta} \mathrm{KL}\left(\pi_\theta(\cdot\,|s_i), \pi_{\theta_k}(\cdot\,|s_i) \right) \right) \right\}$$

$$\theta_{k+1} \leftarrow \theta_k + \eta \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta \log \pi_\theta(a_i|s_i) \Big|_{\theta=\theta_k} A_i$$

AC/PG

where A_i is a weighted average of the following (GAE):

$r_i + \gamma V_\phi(s_{i+1}) - V_\phi(s_i)$

$r_i + \gamma r_{i+1} + \gamma^2 V_\phi(s_{i+2}) - V_\phi(s_i)$

$r_i + \gamma r_{i+1} + \gamma^2 r_{i+2} + \gamma^3 V_\phi(s_{i+3}) - V_\phi(s_i)$

...

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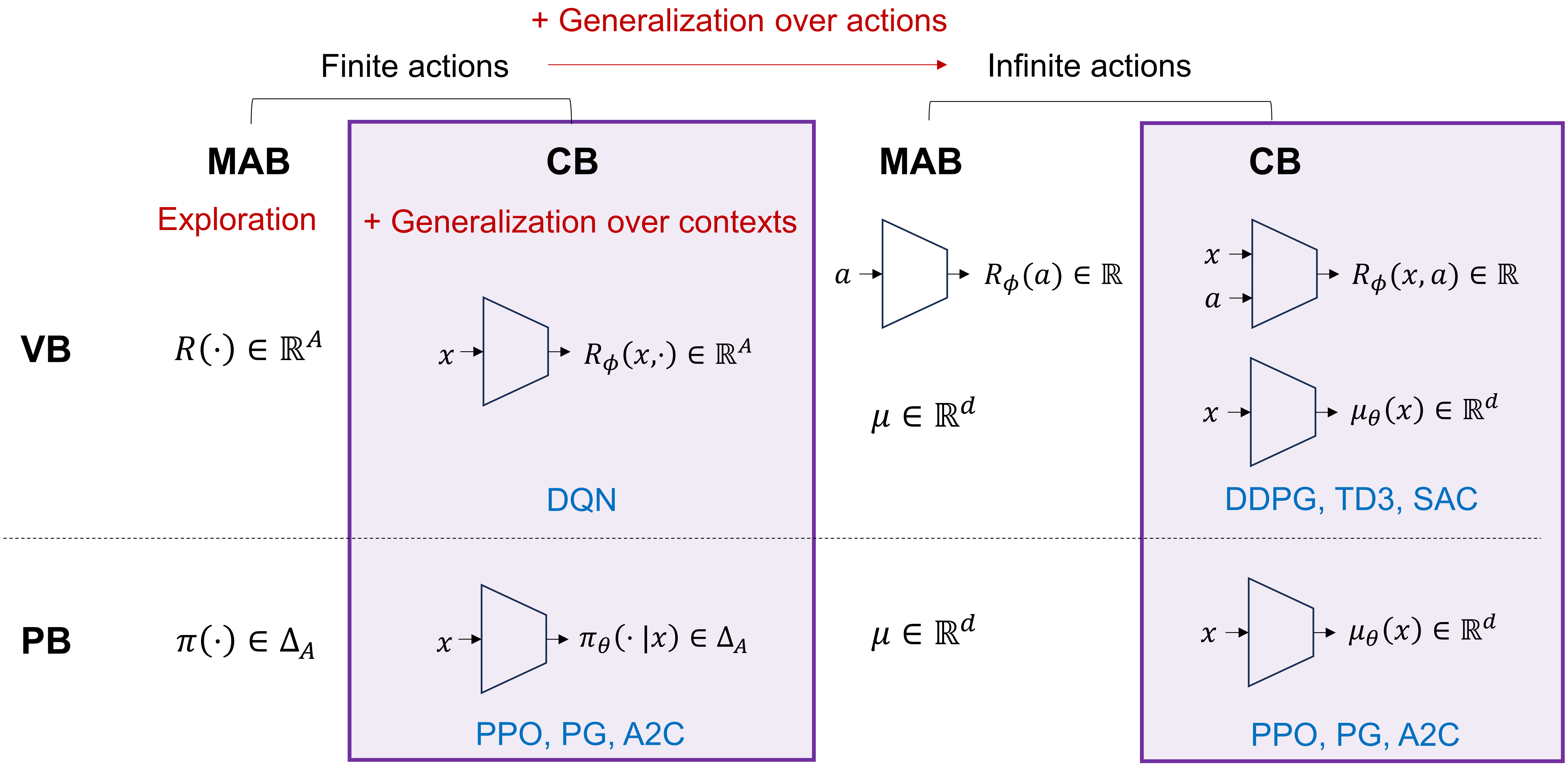

+ Generalization over actions

Finite actions → Infinite actions

	Finite actions		Infinite actions	
	MAB	CB	MAB	CB
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VB	$R(\cdot) \in \mathbb{R}^A$	x → $R_\phi(x,\cdot) \in \mathbb{R}^A$ DQN	a → $R_\phi(a) \in \mathbb{R}$ $\mu \in \mathbb{R}^d$	x, a → $R_\phi(x,a) \in \mathbb{R}$ x → $\mu_\theta(x) \in \mathbb{R}^d$ DDPG, TD3, SAC
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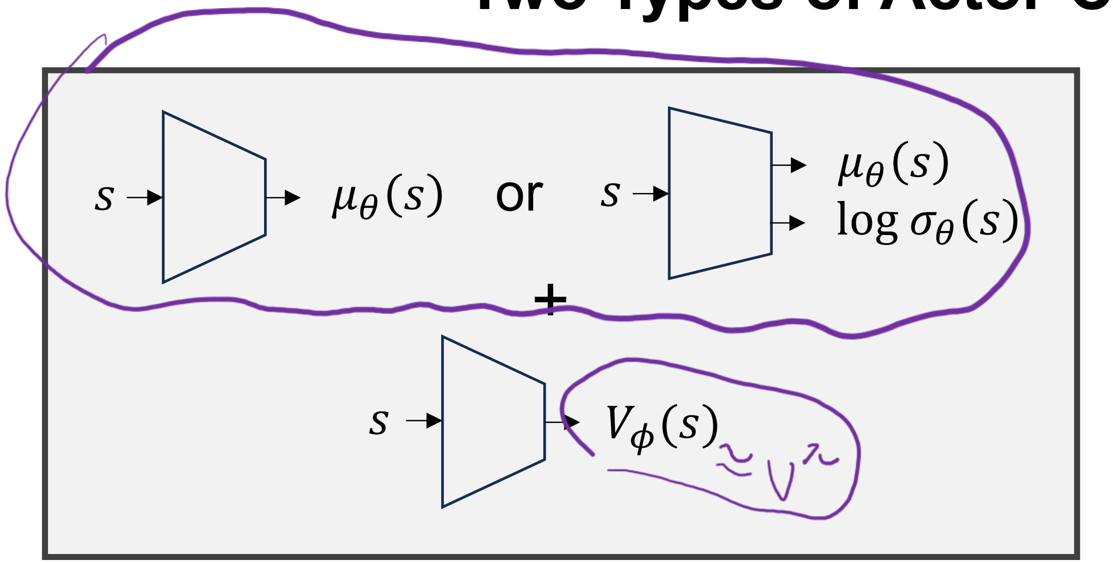
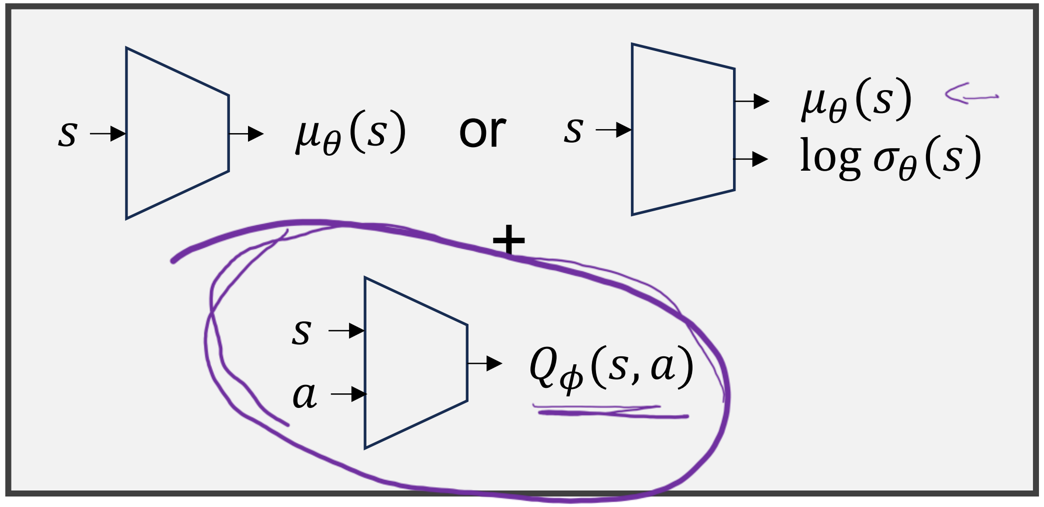
Two Types of Actor-Critic Algorithms



PPO / PG / A2C



DDPG / TD3 / SAC

Update θ with	$\frac{\pi_\theta(a_i \mid s_i)}{\pi_{\theta_k}(a_i \mid s_i)}\left(r_i + \gamma V_\phi(s_{i+1}) - V_\phi(s_i)\right)$	$\sum_a \pi_\theta(a \mid s_i) Q_\phi(s_i, a)$
Idea more aligned with	Policy-based bandits (forming unbiased reward estimator)	Value-based bandits (forming reward estimator from regression)
	Policy Iteration	Policy Iteration or Value Iteration – e.g. DQN
Training type	On-policy	On-policy or off-policy (using data collected from previous policies)

DDPG

Deep Deterministic Policy Gradient (DDPG)

```
For $k = 1,\ 2, \ldots$
    Use $\mu_\theta(s) + \mathcal{N}(0, \sigma^2)$ to collect samples and place them in **replay buffer**
    Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^{n}$ from the replay buffer
    $\phi \leftarrow \phi - \lambda \nabla_\phi \sum_{i=1}^{n} \left( Q_\phi(s_i, a_i) - r_i - \gamma Q_{\bar{\phi}}\left(s_i', \mu_{\bar{\theta}}(s_i{}')\right) \right)^2$
    $\theta \leftarrow \theta + \eta \sum_{i=1}^{n} \nabla_\theta Q_\phi\left(s_i, \mu_\theta(s_i)\right)$
    $\bar{\phi} \leftarrow \tau\phi + (1 - \tau)\bar{\phi},$        $\bar{\theta} \leftarrow \tau\theta + (1 - \tau)\bar{\theta}$
```

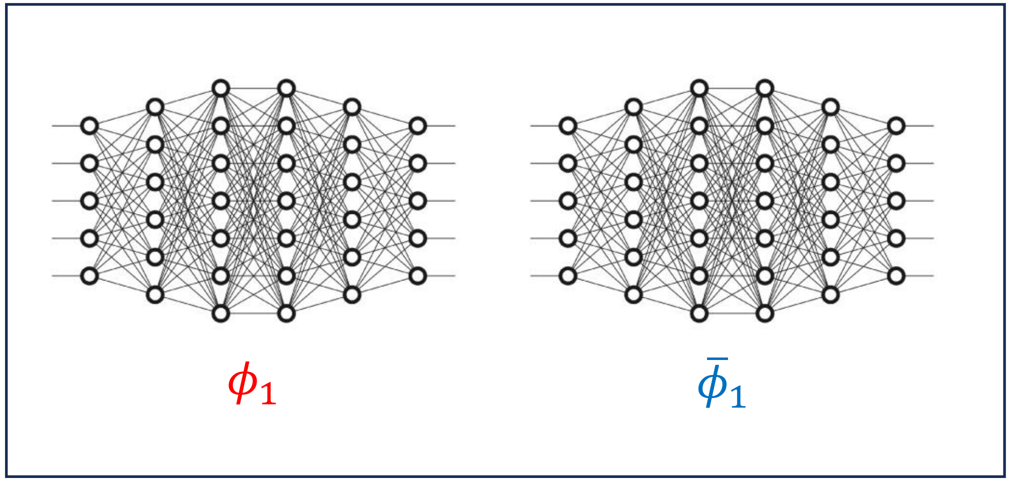
The bandit version of this algorithm: Page 11 here

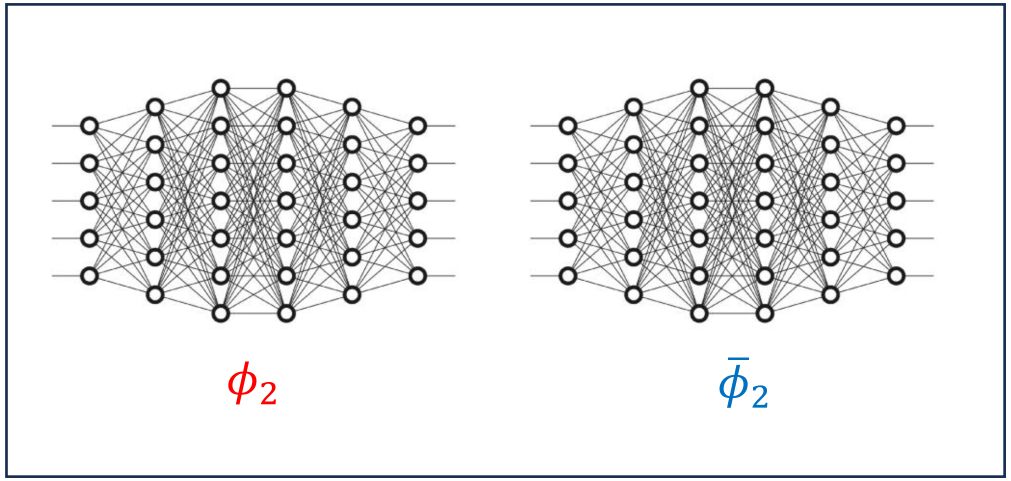
Lillicrap et al., Continuous control with deep reinforcement learning. 2015.

TD3

Further Stabilizing DDPG (1/3): Twin Delayed DDPG

- Double Q-learning





Double Q-learning: When training ϕ_1, instead of using $Q_{\bar{\phi}_1}$ to evaluate the regression target, use $\cancel{Q_{\bar{\phi}_2}}$

TD3: $\min\left\{Q_{\bar{\phi}_1}, Q_{\bar{\phi}_2}\right\}$

Double Q-learning: Use independent samples to train ϕ_1 and ϕ_2

TD3: Use the same set of samples
(the independence between ϕ_1 and ϕ_2 only comes from random initialization)

Further Stabilizing DDPG (2/3): Twin Delayed DDPG

- Target policy smoothing

DDPG: use $Q_{\bar{\phi}}(s', \mu_{\bar{\theta}}(s'))$ as the regression target

TD3: sample $a' = \mu_{\bar{\theta}}(s') + \mathcal{N}(0, \sigma^2)$

use $Q_{\bar{\phi}}(s', a')$ as the regression target

Further Stabilizing DDPG (3/3): Twin Delayed DDPG

- Delayed policy updates: running multiple steps of value updates before running one step of policy update

Remark: all three changes make it harder for the policy $\mu_\theta(s)$ to exploit the error of the Q function $Q_\phi(s, a)$

Twin Delayed DDPG (TD3)

```
For $k = 1,\ 2, \dots$
    Use $\mu_\theta(s) + \mathcal{N}(0, \sigma^2)$ to collect samples and place them in replay buffer
    Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^n$ from the replay buffer
    For each sample $i$, draw $a_i' \sim \mu_{\bar{\theta}}(s_i{}') + \mathcal{N}(0, \sigma^2 I)$
    $\phi_j \leftarrow \phi_j - \lambda \nabla_{\phi_j} \sum_{i=1}^{n} \left( Q_{\phi_j}(s_i, a_i) - r_i - \gamma \min_{\ell=1,2} Q_{\bar{\phi}_\ell}(s_i', a_i') \right)^2 \qquad \forall j = 1,2$
    If $k \bmod M = 0$:
        $\theta \leftarrow \theta + \eta \sum_{i=1}^{n} \nabla_\theta Q_{\phi_1}(s_i, \mu_\theta(s_i))$
        $\bar{\theta} \leftarrow \tau\theta + (1-\tau)\bar{\theta}$
        $\bar{\phi}_j \leftarrow \tau\phi_j + (1-\tau)\bar{\phi}_j \qquad \forall j = 1,2$
```

Fujimoto et al., Addressing Function Approximation Error in Actor-Critic Methods. 2018.

SAC

Soft Actor-Critic (SAC)

- TD3 / DDPG: modeling $\mu_\theta(s)$ + additional noise for exploration
- SAC: modeling $\mu_\theta(s)$ and $\sigma_\theta(s)$ + adding entropy regularization

Entropy Bonus (≈ Boltzmann Exploration)

Bandit

$$\pi = \operatorname*{argmax}_{\pi} \sum_{a} \pi(a) R(a) + \alpha\, H(\pi) = \operatorname*{argmax}_{\pi} \mathbb{E}_{a\sim\pi}[R(a) - \alpha \log \pi(a)]$$

MDP

$$\begin{aligned}\pi &= \operatorname*{argmax}_{\pi}\ \mathbb{E}^{\pi}\left[\sum_{h=0}^{\infty} \gamma^{h} \left(\sum_{a} \pi(a|s_h) R(s_h, a) + \alpha\, H(\pi(\cdot\,|s_h))\right)\right] \\ &= \operatorname*{argmax}_{\pi}\ \mathbb{E}^{\pi}\left[\sum_{h=0}^{\infty} \gamma^{h}\ (R(s_h, a_h) - \alpha \log \pi(a_h|s_h))\right]\end{aligned}$$

TD3 vs. SAC

- Value update

TD3: Sample $a' \sim \mu_\theta(s') + \mathcal{N}(0, \sigma^2)$

Use $Q_{\bar{\phi}}(s', a')$ as the regression target

SAC: Sample $a' \sim \pi_\theta(\cdot\,|s') = \mu_\theta(s') + \mathcal{N}(0, \sigma_\theta^2(s'))$

Use $Q_{\bar{\phi}}(s', a') - \alpha \log \pi_\theta\,(a'|s')$ as the regression target

Soft Actor-Critic (SAC)

```
For $k = 1,\ 2, \dots$
    Use $\mu_\theta(s) + \mathcal{N}(0, \sigma_\theta^2(s))$ to collect samples and place them in replay buffer
    Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^n$ from the replay buffer
    For each sample $i$, draw $a_i' \sim \mu_\theta(s_i{}') + \mathcal{N}(0, \sigma_\theta^2(s_i{}'))$
    $\phi_j \leftarrow \phi_j - \lambda \nabla_{\phi_j} \sum_{i=1}^{n} \left( Q_{\phi_j}(s_i, a_i) - r_i - \gamma \left( \min_{\ell=1,2} Q_{\bar{\phi}_\ell}(s_i', a_i') - \alpha \log \pi_\theta(a_i' | s_i{}') \right) \right)^2 \quad \forall j = 1,2$
    Perform Policy ($\theta$) Update (to be specified later)
    $\bar{\phi}_j \leftarrow \tau \phi_j + (1 - \tau) \bar{\phi}_j \quad \forall j = 1,2$
```

Haarnoja et al., Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor. 2018.

TD3 vs. SAC

- Policy update

TD3: Do not view $-\alpha \log \pi_\theta\ (a|s)$ as part of the reward

Only train $\mu_\theta(s)$

$$\theta \leftarrow \theta + \eta \nabla_\theta Q_\phi(s, \mu_\theta(s))$$

SAC: View $-\alpha \log \pi_\theta\ (a|s)$ as part of the reward

Train both $\mu_\theta(s)$ and $\log \sigma_\theta(s)$

Sample $a_\theta(s) = \mu_\theta(s) + \epsilon \sigma_\theta(s)$ where $\epsilon \sim \mathcal{N}(0,1)$

$$\theta \leftarrow \theta + \eta \nabla_\theta \big(Q_\phi(s, a_\theta(s)) - \alpha \log \pi_\theta(a_\theta(s)|s)\big)$$

Soft Actor-Critic (SAC)

Further using $\pi_\theta(a|s) = \frac{1}{(2\pi\sigma_\theta(s)^2)^{d/2}} \exp\left(-\frac{\|a-\mu_\theta(s)\|^2}{\sigma_\theta(s)^2}\right)$

```
For $k = 1,\ 2, \dots$
    Use $\mu_\theta(s) + \mathcal{N}(0, \sigma_\theta^2(s))$ to collect samples and place them in replay buffer
    Sample a batch $\{(s_i, a_i, r_i, s_i')\}_{i=1}^n$ from the replay buffer
    For each sample $i$, draw $a_i' \sim \mu_\theta(s_i{}') + \mathcal{N}(0, \sigma_\theta^2(s_i{}'))$
    $\phi_j \leftarrow \phi_j - \lambda \nabla_{\phi_j} \sum_{i=1}^n \left(Q_{\phi_j}(s_i, a_i) - r_i - \gamma\left(\min_{\ell=1,2} Q_{\bar{\phi}_\ell}(s_i', a_i') + \alpha \log \pi_\theta(a_i'|s_i{}')\right)\right)^2 \quad \forall j = 1,2$
        Let $a_\theta(s_i) = \mu_\theta(s_i) + \epsilon\sigma_\theta(s_i)$   where $\epsilon \sim \mathcal{N}(0, I)$
        $\theta \leftarrow \theta + \eta \sum_{i=1}^n \nabla_\theta\left(Q_{\phi_1}(s, a_\theta(s_i)) - \alpha \log \pi_\theta(a_\theta(s_i)|s_i)\right)$
    $\bar{\phi}_j \leftarrow \tau\phi_j + (1-\tau)\bar{\phi}_j \quad \forall j = 1,2$
```

Haarnoja et al., Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor. 2018.