

# **Actor-Critic Methods**

Chen-Yu Wei

# Review: Full-Information Policy Learning in MDPs

$$\theta_{k+1} = \operatorname{argmax}_{\theta} \underbrace{\left( V^{\pi_{\theta}}(\rho) - V^{\pi_{\theta_k}}(\rho) - \frac{1}{\eta} D(\theta, \theta_k) \right)}$$

$$\approx \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) (\pi_{\theta}(a|s) - \pi_{\theta_k}(a|s)) Q^{\pi_{\theta_k}}(s, a) = \mathbb{E}_{(s_i, a_i) \sim \pi_{\theta_k}} \left[ \frac{\pi_{\theta}(a_i|s_i) - \pi_{\theta_k}(a_i|s_i)}{\pi_{\theta_k}(a_i|s_i)} \boxed{Q^{\pi_{\theta_k}}(s_i, a_i)} \right]$$

$$\approx (\theta - \theta_k)^{\top} \underbrace{\sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left( \nabla_{\theta} \pi_{\theta}(a|s) \Big|_{\theta=\theta_k} \right) Q^{\pi_{\theta_k}}(s, a)}_{\text{PG/NPG}} \\ = \mathbb{E}_{(s_i, a_i)} \left[ \frac{\nabla_{\theta} \pi_{\theta}(a_i|s_i) \Big|_{\theta=\theta_k}}{\pi_{\theta_k}(a_i|s_i)} \boxed{Q^{\pi_{\theta_k}}(s_i, a_i)} \right]$$

PG/NPG: Estimate them using the empirical sum of reward in the trajectory (i.e., Monte Carlo estimator)

We can also use other estimators to balance bias and variance

# Actor-Critic Methods

Use value function approximation to estimate  $Q^{\pi_{\theta_k}}(s_i, a_i)$  or  $A^{\pi_{\theta_k}}(s_i, a_i)$

Use  $V_\phi(s)$ :  $\approx V^{\pi_{\theta_k}}(s)$   $\min_{\phi} \mathbb{E}_{(s,r,s') \sim \pi_{\theta_k}} \left[ \left( V_\phi(s) - r - \gamma V_\phi(s') \right)^2 \right]$

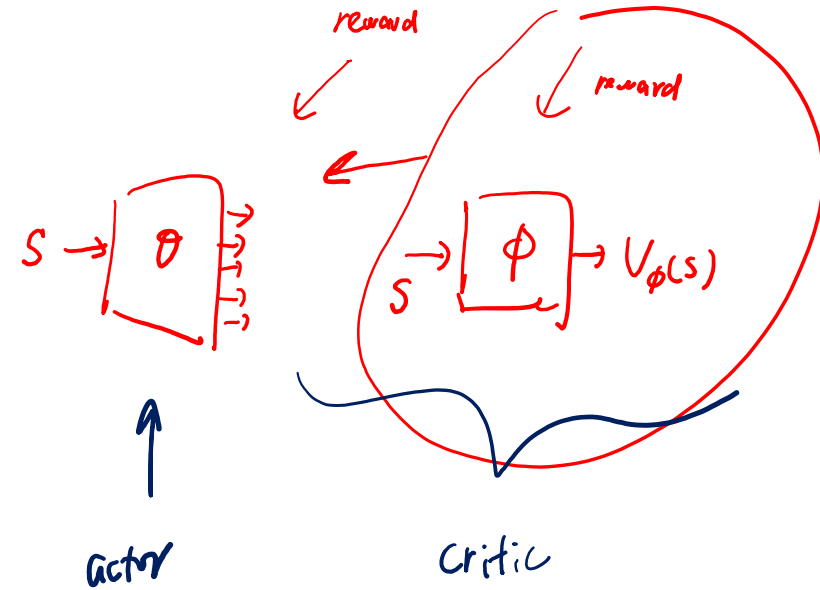
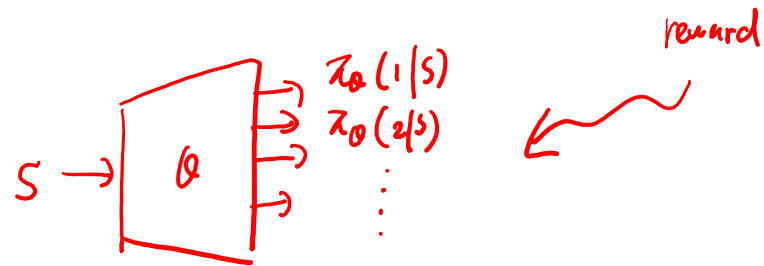
Use  $Q_\phi(s, a)$ :  $\approx Q^{\pi_{\theta_k}}(s, a)$   $\min_{\phi} \mathbb{E}_{(s,a,r,s',a') \sim \pi_{\theta_k}} \left[ \left( Q_\phi(s, a) - r - \gamma Q_\phi(s', a') \right)^2 \right]$

Possible estimators for  $A^{\pi_{\theta_k}}(s, a)$ :

Let  $(s_1, a_1, r_1, s_2, a_2, r_2 \dots)$  be a trajectory starting from  $s_1 = s, a_1 = a$

	$Q_\phi(s_1, a_1) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot s)}[Q_\phi(s_1, a')]$
$\left\{ \begin{array}{l} r_1 + \gamma V_\phi(s_2) - V_\phi(s_1) \\ r_1 + \gamma r_2 + \gamma^2 V_\phi(s_3) - V_\phi(s_1) \\ \vdots \end{array} \right.$	$\left\{ \begin{array}{l} r_1 + \gamma Q_\phi(s_2, a_2) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot s)}[Q_\phi(s_1, a')] \\ r_1 + \gamma r_2 + \gamma^2 Q_\phi(s_3, a_3) - \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot s)}[Q_\phi(s_1, a')] \\ \vdots \end{array} \right.$

# Pure Policy-Based Methods vs. Actor-Critic Methods



# Actor-Critic with $Q_\phi$

(find  $\pi^*$ )  
(given  $\pi$ )

(off-policy)

$Q$ -learning:  $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha [r + \max_{a'} Q(s',a')]$   
 TD-learning:  $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha [r + \sum_{a'} \pi(a'|s) Q(s',a')]$

$\Rightarrow Q^*$

(on-policy)

$\downarrow$   
 $Q^\pi$

For  $k = 1, 2, \dots$

Use  $\pi_{\theta_k}$  to collect  $n$  trajectories

$$(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}), \dots, (s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)})$$

Define

$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \frac{\nabla_{\theta} \pi_{\theta} (a_h^{(i)} | s_h^{(i)})}{\pi_{\theta_k} (a_h^{(i)} | s_h^{(i)})} \overset{\text{red } Q}{\parallel} \underbrace{Q_{\phi_k} (s_h^{(i)}, a_h^{(i)})}_{\text{red } Q} \text{ or } \underbrace{\frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \sum_a \nabla_{\theta} \pi_{\theta} (a | s_h^{(i)})}_{\text{red } Q} \bigg|_{\theta=\theta_k} Q_{\phi_k} (s_h^{(i)}, a)$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \quad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left( Q_{\phi} (s_h^{(i)}, a_h^{(i)}) - r_h^{(i)} - \gamma Q_{\phi_k} (s_{h+1}^{(i)}, a_{h+1}^{(i)}) \right)^2 \bigg|_{\phi=\phi_k}$$

# Advantage Actor-Critic (A2C) = PG + $V_\phi$

For  $k = 1, 2, \dots$

Use  $\pi_{\theta_k}$  to collect  $n$  trajectories

$$(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}), \dots, (s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)})$$

Define

$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \nabla_{\theta} \log \pi_{\theta} (a_h^{(i)} | s_h^{(i)}) \Big|_{\theta=\theta_k} \underbrace{\left( r_h^{(i)} + \gamma V_{\phi_k}(s_{h+1}^{(i)}) - V_{\phi_k}(s_h^{(i)}) \right)}_{\substack{\text{Handwritten: } \frac{\nabla_{\theta} \log \pi_{\theta}(a_h^{(i)} | s_h^{(i)})}{\log \pi_{\theta}(a_h^{(i)} | s_h^{(i)})} \approx A^{\pi_k}(s_h^{(i)}, a_h^{(i)}) \\ \mathbb{E}(\cdot) = \sum_{s,a} d_{\rho}^{\pi_k}(s) \nabla_{\theta} \log \pi_{\theta}(a|s) A^{\pi_k}(s,a)}}$$

or any other advantage estimator in the previous slide

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \quad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left( V_{\phi} (s_h^{(i)}) - r_h^{(i)} - \gamma V_{\phi_k} (s_{h+1}^{(i)}) \right)^2 \Big|_{\phi=\phi_k}$$

$V_{\phi} \approx V^{\pi_k}$

# Proximal Policy Optimization (PPO) = NPG + $V_\phi$

For  $k = 1, 2, \dots$

Use  $\pi_{\theta_k}$  to collect  $n$  trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \dots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Perform updates

$$\theta_{k+1} \leftarrow \operatorname{argmax}_{\theta} \left\{ \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \frac{\pi_{\theta} \left( a_h^{(i)} \middle| s_h^{(i)} \right)}{\pi_{\theta_k} \left( a_h^{(i)} \middle| s_h^{(i)} \right)} \underbrace{\left( r_h^{(i)} + \gamma V_{\phi_k} \left( s_{h+1}^{(i)} \right) - V_{\phi_k} \left( s_h^{(i)} \right) \right)}_{\substack{\approx A^{\pi_{\theta_k}}(s_h) \\ \text{or any other advantage estimator in the previous slide}}} - \frac{1}{\eta} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \operatorname{KL} \left( \pi_{\theta} \left( \cdot \middle| s_h^{(i)} \right), \pi_{\theta_k} \left( \cdot \middle| s_h^{(i)} \right) \right) \right\}$$

$$\phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left( V_{\phi} \left( s_h^{(i)} \right) - r_h^{(i)} - \gamma V_{\phi_k} \left( s_{h+1}^{(i)} \right) \right)^2 \Big|_{\phi=\phi_k}$$

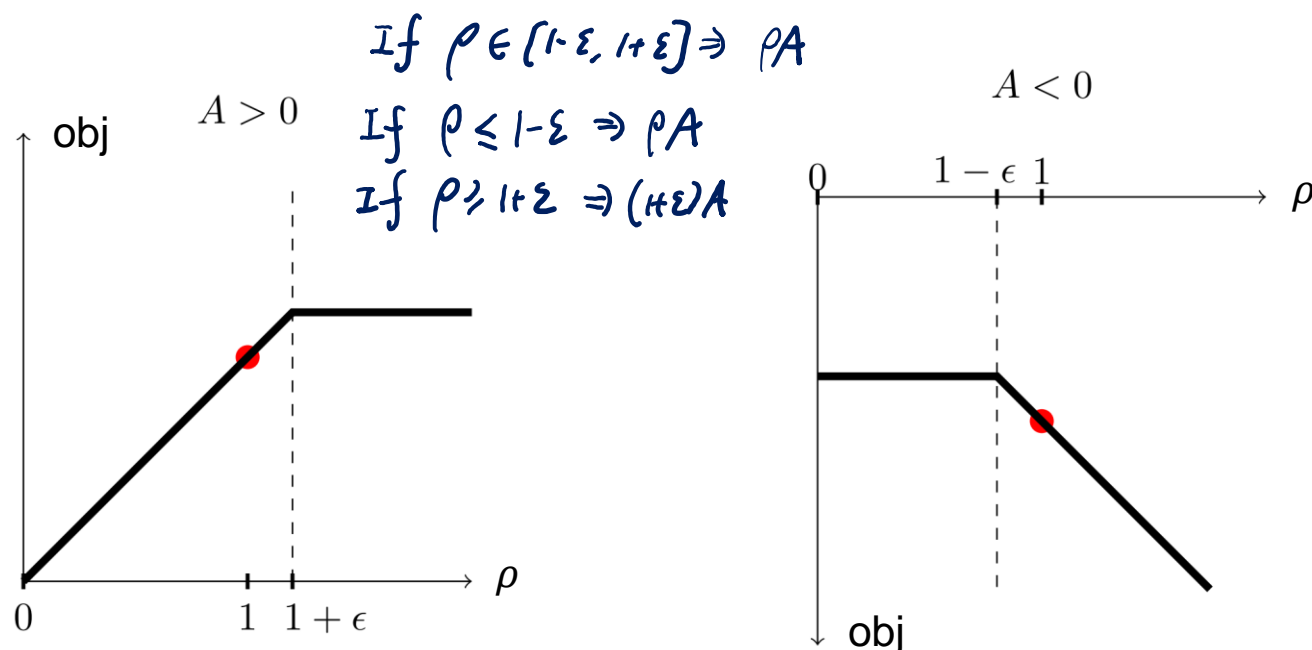
$$\frac{r(s) \mathbb{1}(a_+ = a)}{p_+(s)} \quad \frac{(r(s) - 1) \mathbb{1}(a_+ = a)}{p_+(s)}$$

# Additional Technique 1: Clipped Objective (for PPO)

$$\rho := \frac{\pi_{\theta} (a_h^{(i)} | s_h^{(i)})}{\pi_{\theta_k} (a_h^{(i)} | s_h^{(i)})} \quad A := (r_h^{(i)} + \gamma V_{\phi_k} (s_{h+1}^{(i)}) - V_{\phi_k} (s_h^{(i)}))$$

$$\text{clip}_{[1-\epsilon, 1+\epsilon]}(\rho) = \min(\max(\rho, 1-\epsilon), 1+\epsilon)$$

Instead of using  $\rho A$  as the objective, use  $\min\{\rho A, \text{clip}_{[1-\epsilon, 1+\epsilon]}(\rho) A\}$



algorithm	avg. normalized score
No clipping or penalty	-0.39
Clipping, $\epsilon = 0.1$	0.76
<b>Clipping, <math>\epsilon = 0.2</math></b>	<b>0.82</b>
Clipping, $\epsilon = 0.3$	0.70
Adaptive KL $d_{\text{targ}} = 0.003$	0.68
Adaptive KL $d_{\text{targ}} = 0.01$	0.74
Adaptive KL $d_{\text{targ}} = 0.03$	0.71
Fixed KL, $\beta = 0.3$	0.62
Fixed KL, $\beta = 1.$	0.71
Fixed KL, $\beta = 3.$	0.72
Fixed KL, $\beta = 10.$	0.69



# Additional Technique 2: Entropy Bonus

In the objective of policy update, add a bonus term

$$H(\pi_{\theta}(\cdot | s)) = \sum_a \pi_{\theta}(a|s) \ln \frac{1}{\pi_{\theta}(a|s)}$$

For PPO:

$$\operatorname{argmax}_{\theta} \left\{ \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \frac{\pi_{\theta}(a_h^{(i)} | s_h^{(i)})}{\pi_{\theta_k}(a_h^{(i)} | s_h^{(i)})} A_h^{(i)} - \frac{1}{\eta} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \operatorname{KL}(\pi_{\theta}(\cdot | s_h^{(i)}), \pi_{\theta_k}(\cdot | s_h^{(i)})) + c \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \underbrace{H(\pi_{\theta}(\cdot | s_h^{(i)}))}_{\text{Entropy Bonus}} \right\}$$

—  $\operatorname{KL}(\pi_{\theta}(\cdot | s_h^{(i)}), \pi_{\text{unif}}(\cdot | s_h^{(i)}))$

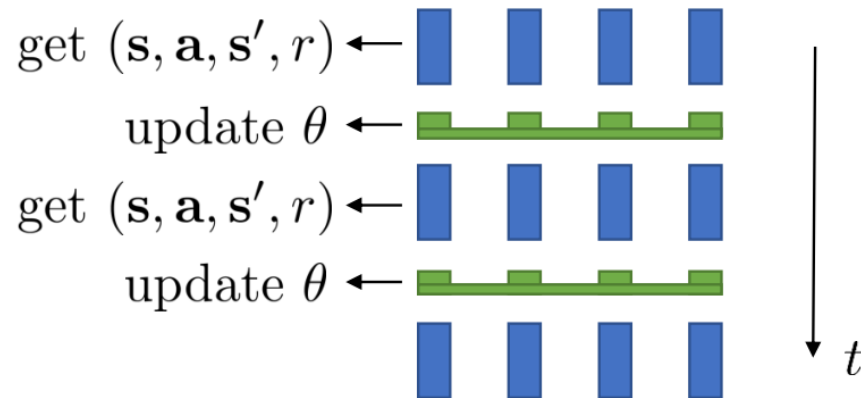
For A2C:

$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \nabla_{\theta} \log \pi_{\theta}(a_h^{(i)} | s_h^{(i)}) \Big|_{\theta=\theta_k} A_h^{(i)} + c \nabla_{\theta} \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} H(\pi_{\theta}(\cdot | s_h^{(i)}))$$

# Additional Technique 3: Parallel Sample Collection

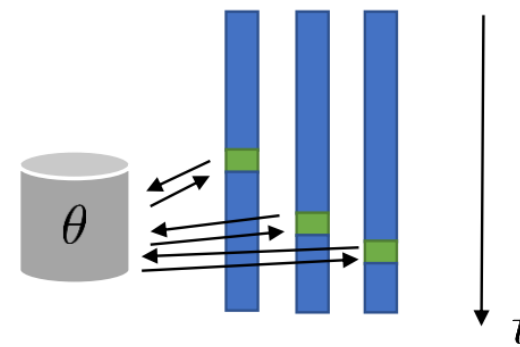
A2C

synchronized parallel actor-critic



A3C

asynchronous parallel actor-critic



# Actor-Critic Summary

PG  $\longrightarrow$  A2C

NPG  $\longrightarrow$  PPO

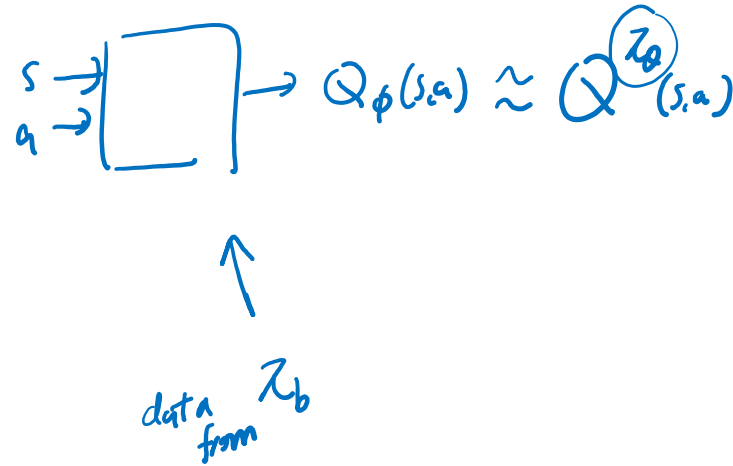
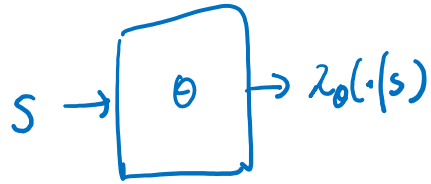
$$s \rightarrow \boxed{Q} \rightarrow \pi(s)$$

$$s \rightarrow \boxed{\theta} \rightarrow \pi(s) \quad s \rightarrow \boxed{\phi} \rightarrow V_{\phi}(s)$$

$$s \xrightarrow{a} \boxed{\phi} \rightarrow Q_{\phi}(s,a)$$

# Off-Policy Actor-Critic

- Leveraging **off-policy evaluation** → allow reusing data



# Review: Full-Information Policy Learning in MDPs

$$\begin{aligned}\theta_{k+1} &= \operatorname{argmax}_{\theta} \left( \underbrace{V^{\pi_{\theta}}(\rho) - V^{\pi_{\theta_k}}(\rho)}_{\text{}} - \frac{1}{\eta} D(\theta, \theta_k) \right) \\ &\approx \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left( \pi_{\theta}(a|s) - \pi_{\theta_k}(a|s) \right) Q^{\pi_{\theta_k}}(s, a) \\ &\approx (\theta - \theta_k)^{\top} \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left( \nabla_{\theta} \pi_{\theta}(a|s) \Big|_{\theta=\theta_k} \right) Q^{\pi_{\theta_k}}(s, a)\end{aligned}$$

Use any off-policy policy evaluation methods to find  $\phi_k$  such that  $Q_{\phi_k}(s, a) \approx Q^{\pi_{\theta_k}}(s, a)$

Suppose that our  $(s_i, a_i)$  samples are obtained from  $\hat{\pi}$

# Off-Policy Actor-Critic

$$\begin{aligned}
 \theta_{k+1} &= \operatorname{argmax}_{\theta} \left( \underbrace{V^{\pi_{\theta}}(\rho) - V^{\pi_{\theta_k}}(\rho)}_{\text{Handwritten: } \sum_s d_{\rho}^{\hat{\pi}}(s) \cdot \frac{d\rho^{\pi_{\theta_k}}(s)}{d\hat{\pi}(s)} \sum_a \dots} - \frac{1}{\eta} D(\theta, \theta_k) \right) \\
 &\approx \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left( \pi_{\theta}(a|s) - \pi_{\theta_k}(a|s) \right) Q_{\phi_k}(s,a) = \mathbb{E}_{s \sim \hat{\pi}} \left[ \frac{\cancel{d_{\rho}^{\pi_{\theta_k}}(s)}}{\cancel{d_{\rho}^{\hat{\pi}}(s)}} \sum_a \left( \pi_{\theta}(a|s) - \pi_{\theta_k}(a|s) \right) Q_{\phi_k}(s,a) \right] \\
 &\approx (\theta - \theta_k)^{\top} \sum_{s,a} d_{\rho}^{\pi_{\theta_k}}(s) \left( \nabla_{\theta} \pi_{\theta}(a|s) \Big|_{\theta=\theta_k} \right) Q_{\phi_k}(s,a) = (\theta - \theta_k)^{\top} \mathbb{E}_{s \sim \hat{\pi}} \left[ \frac{\cancel{d_{\rho}^{\pi_{\theta_k}}(s)}}{\cancel{d_{\rho}^{\hat{\pi}}(s)}} \sum_a \nabla_{\theta} \pi_{\theta}(a|s) \Big|_{\theta=\theta_k} Q_{\phi_k}(s,a) \right]
 \end{aligned}$$

Use any off-policy policy evaluation methods to find  $\phi_k$  such that  $Q_{\phi_k}(s,a) \approx Q^{\pi_{\theta_k}}(s,a)$

Suppose that our  $(s_i, a_i)$  samples are obtained from  $\hat{\pi}$

# Actor-Critic + Replay Buffer

For  $k = 1, 2, \dots$

Collect samples using  $\pi_{\theta_k}$ , and place them in the replay buffer

Sample a batch  $\{(s_i, a_i, r_i, s'_i)\}_{i=1}^n$  from replay buffer

Define

$$g = \frac{1}{n} \sum_{i=1}^n \sum_a \nabla_{\theta} \pi_{\theta}(a|s_i) \Big|_{\theta=\theta_k} Q_{\phi_k}(s_i, a) \quad \text{Note: not using } a_i \text{ here}$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g$$

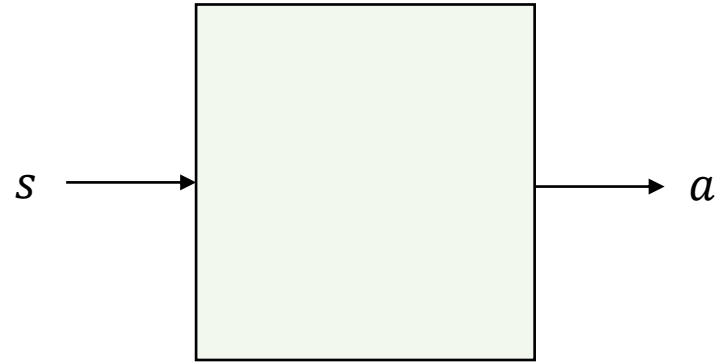
$$\phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \frac{1}{n} \sum_{i=1}^n \left( Q_{\phi}(s_i, a_i) - r_i - \gamma \mathbb{E}_{a' \sim \pi_{\theta_k}(\cdot|s'_i)} [Q_{\phi}(s'_i, a')] \right)^2 \Big|_{\phi=\phi_k}$$

# **Dealing with Continuous Action Sets**

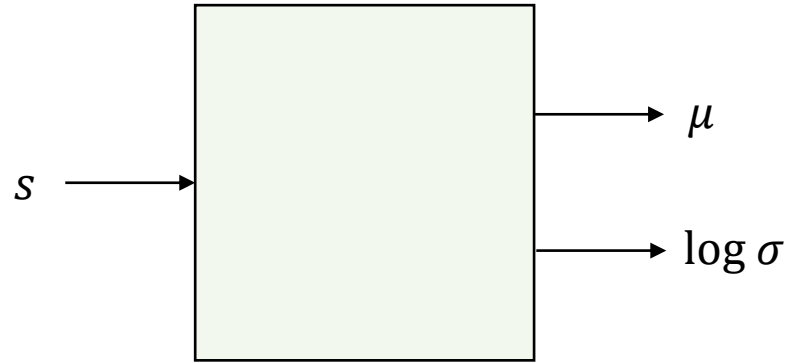


# **Review: Linear Bandits and One-Point Gradient Estimator**

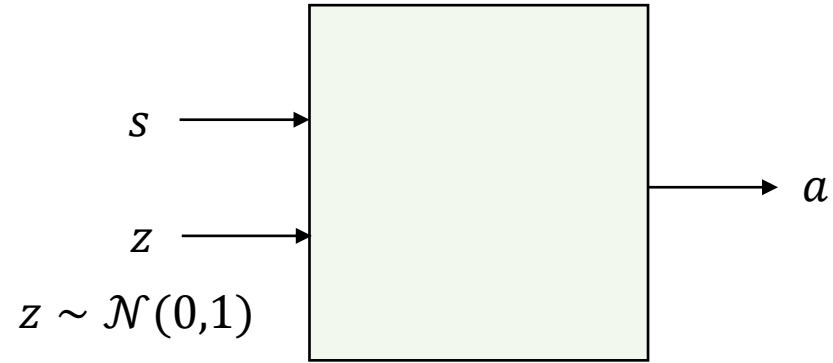
# Policy Network for Continuous Action Sets



# Policy Network for Continuous Action Sets



Explicitly models  $\pi_{\theta}(a|s)$



Implicitly modeling  $\pi_{\theta}(a|s)$

**Option 1:** making  $\sigma$  part of policy parameters

**Option 2:** making  $\sigma$  a hyper-parameters  
(decreases over time)

can sample from it, but do not  
know the function  $\pi_{\theta}(\cdot |s)$

# A2C / PPO with Continuous Action Sets

$$g = \frac{1}{n} \sum_{i=1}^n \nabla_{\theta} \log \pi_{\theta}(a_i | s_i) \Big|_{\theta=\theta_k} A_i$$

$$\theta_{k+1} \leftarrow \operatorname{argmax}_{\theta} \left\{ \frac{1}{n} \sum_{i=1}^n \frac{\pi_{\theta}(a_i | s_i)}{\pi_{\theta_k}(a_i | s_i)} A_i - \frac{1}{\eta} \frac{1}{n} \sum_{i=1}^n \operatorname{KL} \left( \pi_{\theta}(\cdot | s_i), \pi_{\theta_k}(\cdot | s_i) \right) \right\}$$

# Recall: Actor-Critic without need for inverse weighting

Actor-critic with  $Q_\phi(s, a)$  function approximation

For  $k = 1, 2, \dots$

Use  $\pi_{\theta_k}$  to collect  $n$  trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \dots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Define 
$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \sum_a \nabla_{\theta} \pi_{\theta} \left(a | s_h^{(i)}\right) \Big|_{\theta=\theta_k} Q_{\phi_k} \left(s_h^{(i)}, a\right)$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \qquad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left( Q_{\phi} \left( s_h^{(i)}, a_h^{(i)} \right) - r_h^{(i)} - \gamma Q_{\phi_k} \left( s_{h+1}^{(i)}, a_{h+1}^{(i)} \right) \right)^2 \Big|_{\phi=\phi_k}$$

# Deterministic Policy Gradient Theorem

# Deterministic Policy Gradient Algorithm

For  $k = 1, 2, \dots$

Use  $\pi_{\theta_k}$  to collect  $n$  trajectories

$$\left(s_1^{(1)}, a_1^{(1)}, r_1^{(1)}, \dots, s_{\tau_1}^{(1)}, a_{\tau_1}^{(1)}, r_{\tau_1}^{(1)}\right), \dots, \left(s_1^{(n)}, a_1^{(n)}, r_1^{(n)}, \dots, s_{\tau_n}^{(n)}, a_{\tau_n}^{(n)}, r_{\tau_n}^{(n)}\right)$$

Define 
$$g = \frac{1}{n} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \nabla_{\theta} Q_{\phi_k} \left( s_h^{(i)}, \pi_{\theta} \left( s_h^{(i)} \right) \right) \Bigg|_{\theta=\theta_k}$$

Perform updates

$$\theta_{k+1} \leftarrow \theta_k + \eta g \qquad \phi_{k+1} \leftarrow \phi_k - \lambda \nabla_{\phi} \sum_{i=1}^n \sum_{h=1}^{\tau_n} \left( Q_{\phi} \left( s_h^{(i)}, a_h^{(i)} \right) - r_h^{(i)} - \gamma Q_{\phi_k} \left( s_{h+1}^{(i)}, a_{h+1}^{(i)} \right) \right)^2 \Bigg|_{\phi=\phi_k}$$

# **Two Viewpoints for the Deterministic PG Algorithm**



# Deep Deterministic Policy Gradient (DDPG)

For  $k = 1, 2, \dots$

Use  $\pi_\theta$  to collect samples and place them in replay buffer

Sample a batch  $\{(s_i, a_i, r_i, s'_i)\}_{i=1}^n$  from the replay buffer

$$\theta \leftarrow \theta + \eta \sum_{i=1}^n \nabla_{\theta} Q_{\phi}(s_i, \pi_{\theta}(s_i))$$

$$\phi \leftarrow \phi - \lambda \nabla_{\phi} \sum_{i=1}^n \left( Q_{\phi}(s_i, a_i) - r_i - \gamma Q_{\phi_{\text{tar}}}(s'_i, \pi_{\theta_{\text{tar}}}(s'_i)) \right)^2$$

$$\theta_{\text{tar}} \leftarrow \tau \theta + (1 - \tau) \theta_{\text{tar}}$$

$$\phi_{\text{tar}} \leftarrow \tau \phi + (1 - \tau) \phi_{\text{tar}}$$

# **Twin Delayed DDPG (TD3)**

# Soft Actor-Critic (SAC)