Pulsars and Double (Rainbow) Pendulums

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1 Predicting Pulsars using `Scikit-learn` Library

1.1 Neutron Stars

Neutron stars are the densest objects that astronomers can directly observe in the universe, which makes them quite an exotic, and exciting source to observe. Since the magnetic fields of these compact objects are trillions of times larger than that of the Earth's, they are able to create a very periodic pulse that can be detected across millions of light years of space [7]. When this high energy pulsing beam is aligned along the line of sight of an observer on Earth then this type of star is called a **pulsar** [Fig. 1]. Magnetars are simply neutron stars with magnetic fields about 1000x's larger than a regular neutron star [10].

Due to these extreme densities their radii typically range around 8-11 km [1], which is roughly the size of a metropolitan city [Fig. 2]. Despite their incredibly small size, being

[*]Thanks again for a truly challenging and rewarding course! ...and dance parties I guess

anywhere near a neutron star would cause *spaghettification* [4]. In other words, stretching the matter tremendously and transforming it into nuclear pasta 😕.

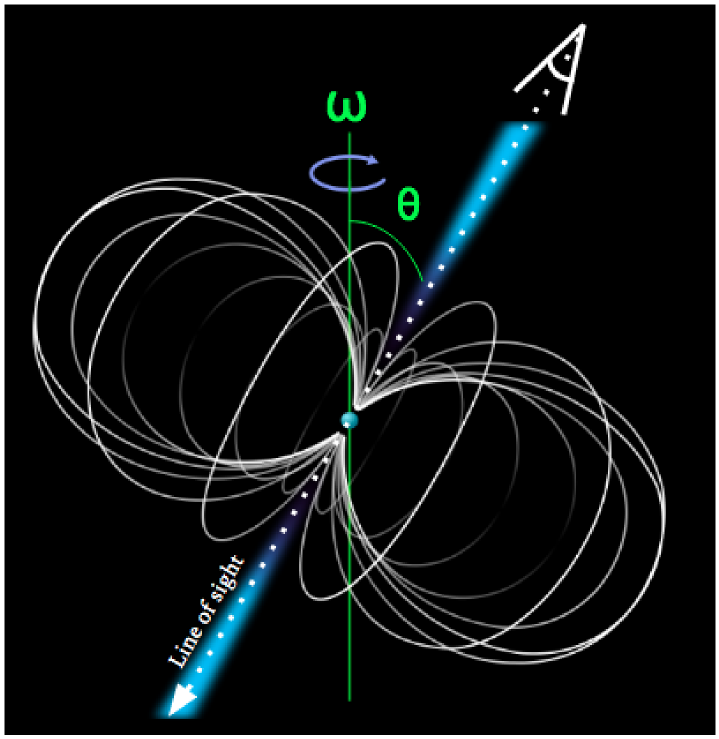

Figure 1: Schematic Diagram of a Pulsar. Credit: Wikipedia

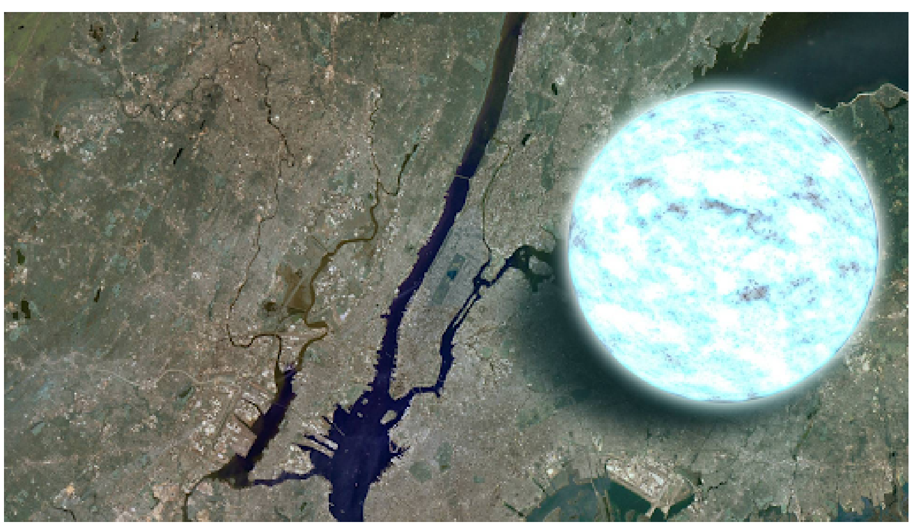

Figure 2: Neutron star size comparison with Brooklyn, New York. Credit: NASA

Fundamental physics of matter questions regarding densities larger than nuclear densities can only be tested via observation (due to our limitations in the laboratory). Understanding the composition behavior of dense material has great implications in relation to our understanding of nuclear physics, and astrophysics [8]. The modeling techniques used to determine the neutron star properties have improved tremendously over the past several years. The purpose of this project was to use a large dataset to classify whether luminous sources in the sky were pulsars, or something else. This task was accomplished by using the Scikit-learn library from scipy to simplify the data processing.

1.2 Machine Learning

In the Python notebook code I chose to use the scikit learn library to take a machine learning approach to classifying stellar sources as pulsars [3]. In general, a machine learning problem considers a dataset with n samples and then tries to predict properties of unknown data. If each sample is more than a single number, then the sample is a multi-dimensional entry (aka multivariate data), and is said to have several attributes or features.

The two types of learning problems are *unsupervised* learning and *supervised* learning. In unsupervised learning the machine attempts to train itself without any particular target value. In supervised learning the machine is given a sample dataset with attributes (i.e. target value), so it can learn and test the validity of its approach. In this process it creates a testing and training set. The two types of approaches supervised learning takes are the classification and regression methods.

1.3 Pulsar Coding

As a continuation of the neutron star project from the midterm, I looked through papers and once again found out how difficult it was to find a data set that I could easily code. So I decided to switch it up and see if I could find code from GitHub that would be interesting to reproduce and discuss [9]. In addition upon doing further research about the "target class" variable, I found a link from Kagle, which contained the dataset I used from the High Time Resolution Universe Survey. I had quite a bit of problems getting this csv file, so I resorted to going through the url method of importing the dataset. In order to classify the sources I used both the classification and regression methods to analyze the dataset, and compile a list of target classes. In conclusion, based out of the $\approx$ 18000 points only 9.16% were actually pulsars, with a total of 1639.

2 Tracing the path of a Double Pendulum with `odeint` function, and animation

2.1 Chaos Theory

By definition chaos refers to complete disorder, and confusion. However, in the context of physics, Chaos Theory is the science of the (seemingly) unpredictable. Any small changes in the initial conditions of a system causes the behavior to appear random [5]. While most fields of science investigate predictable phenomena like gravity, electricity, or chemical reactions, Chaos Theory deals with nonlinear things that are effectively impossible to predict or control, like turbulence, weather, the stock market, and so on.

In order to break down the unpredicatable, these phenomena can often be described by fractal mathematics, which captures the infinite complexity of nature. Not only does fractal mathematics often occur in nature, such as landscapes, clouds, plants, etc., it can also be modeled in the complex plane. Recognizing the chaotic, fractal nature of our world can give us new insights, which can help us avoid actions that are detrimental to our long-term well-being. On a much simpler scale, this theory can be applied to the system of a **double pendulum** to model the chaotic motion [Fig. 3].

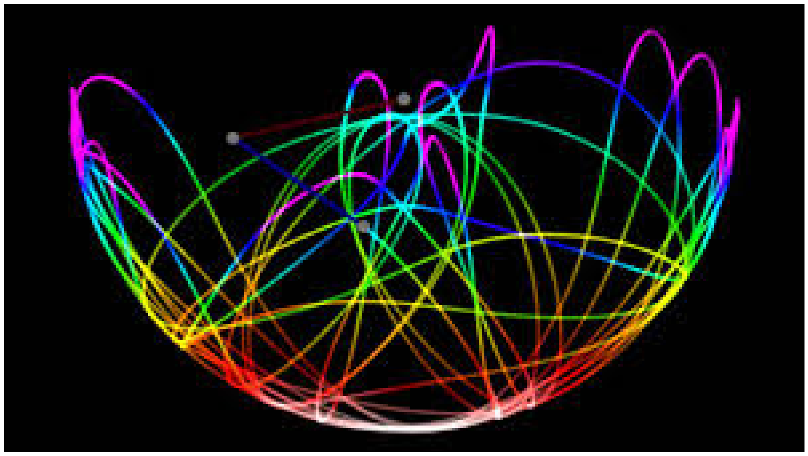

Figure 3: Path of a Double Pendulum for some initial condition. Credit: Youtube

2.2 Classical Mechanics

One of the standard problems in the classical mechanics course is the double pendulum. The apparatus consists of a pendulum attached to a fixed point while another pendulum is attached to the end of the first pendulum [Fig. 4]. When constrained to move in a single plane, and the rods have negligible mass, the problem can be simplified and solved numerically using generalized coordinates, and Lagrangian formulation [6].

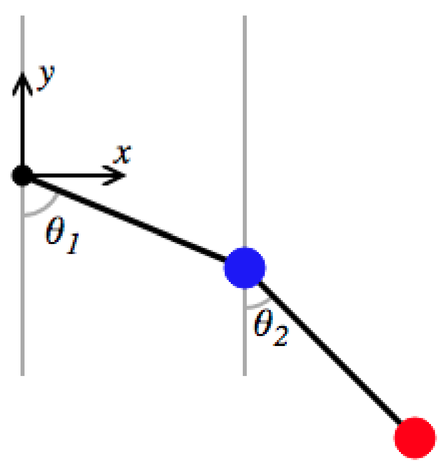

Figure 4: Schematic Diagram of a Double Pendulum [2].

As shown above the system consists of two masses m_1, and m_2, two rods with lengths l_1, and l_2, while moving in the x-y plane with a gravitational field g [2]. In addition, the generalized coordinates are θ_1, and θ_2, which are the angles in relation to the rod and the vertical. The components of the mass positions, x and y, and their velocities, $\dot{x}$ and $\dot{y}$, are...

$$x_1 = l_1 sin\theta_1 \longrightarrow \dot{x_1} = l_1\dot{\theta}_1 cos\theta_1$$

$$y_1 = -l_1 cos\theta_1 \longrightarrow \dot{y_1} = l_1\dot{\theta}_1 sin\theta_1$$

$$x_2 = l_1 sin\theta_1 + l_2 sin\theta_2 \longrightarrow \dot{x_2} = l_1\dot{\theta}_1 cos\theta_1 + l_2\dot{\theta}_2 cos\theta_2$$

$$y_2 = -l_1 cos\theta_1 - l_2 cos\theta_2 \longrightarrow \dot{y_2} = l_1\dot{\theta}_1 sin\theta_1 + l_2\dot{\theta}_2 sin\theta_2$$

While the potential V, kinetic energies T, and Lagrangian $\mathcal{L} = T - V$ are...

$$V = m_1 gy + m_2 gy = -(m_1 + m_2)l_1 gcos\theta_1 - m_2 l_2 gcos\theta_2$$

$$T = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m_1(\dot{x_1^2} + \dot{y_1}^2) + \frac{1}{2}m_2(\dot{x_2^2} + \dot{y_2}^2)$$
$$= \frac{1}{2}m_1{l_1}^2\dot{\theta_1}^2 + \frac{1}{2}m_2[{l_1}^2\dot{\theta_1}^2 + {l_2}^2\dot{\theta_2}^2 + 2l_1 l_2\dot{\theta}_1\dot{\theta}_2 cos(\theta_1 - \theta_2)]$$

$$\mathcal{L} = \frac{1}{2}(m_1 + m_2){l_1}^2\dot{\theta_1}^2 + \frac{1}{2}m_2{l_2}^2\dot{\theta_2}^2 + m_1 l_1 l_2\dot{\theta}_1\dot{\theta}_2 cos(\theta_1 - \theta_2) + (m_1 + m_2)l_1 gcos\theta_1 + m_2 gl_2 cos\theta_2$$

The associated Euler-Lagrange equations are...

$$\frac{d}{dt}(\frac{\partial\mathcal{L}}{\partial\dot{\theta_1}}) - (\frac{\partial\mathcal{L}}{\partial\theta_1}) = (m1+m2)l_1\ddot{\theta_1} + m_2 l_2\ddot{\theta_2}cos(\theta_1 - \theta_2) + m_2 l_2\dot{\theta_2}^2 sin(\theta_1 - \theta_2) + (m_1 + m_2)gsin\theta_1 = 0$$

$$\frac{d}{dt}(\frac{\partial\mathcal{L}}{\partial\dot{\theta_2}}) - (\frac{\partial\mathcal{L}}{\partial\theta_2}) = m_2 l_2\ddot{\theta_2} + m_2 l_1\ddot{\theta_1}cos(\theta_1 - \theta_2) + m_2 l_1\dot{\theta_1}^2 sin(\theta_1 - \theta_2) + m_2 gsin\theta_2 = 0$$

Since the scipy.integrate odeint function is best suited for first-order differential equations, the double derivatives of θ must be redefined. Using the conversions $z_1 \equiv \dot{\theta_1} \Rightarrow \ddot{\theta_1} = \dot{z_1}$, and $z_2 \equiv \dot{\theta_2} \Rightarrow \ddot{\theta_2} = \dot{z_2}$, the equations of motion become...

$$\dot{z_1} = \frac{m_2 g sin\theta_2 cos(\theta_1 - \theta_2) - m_2 sin(\theta_1 - \theta_2[l_1 {z_1}^2 cos(\theta_1 - \theta_2) + l_2 {z_2}^2] - (m_1 + m_2) g sin\theta_1}{l_1[m_1 + m_2 sin^2(\theta_1 - \theta_2)]}$$

$$\dot{z_2} = \frac{(m_1 + m_2)[l_1 {z_1}^2 sin(\theta_1 - \theta_2) - g sin\theta_2 + g sin\theta_1 cos(\theta_1 - \theta_2)] + m_2 l_2 {z_2}^2 sin(\theta_1 - \theta_2) cos(\theta_1 - \theta_2)}{l_2[m_1 + m_2 sin^2(\theta_1 - \theta_2)]}$$

2.3 Double Pendulum Coding

Using the above mentioned equations, and definitions, I was able to create my own set of 301 images to capture this motion. The reason I chose to do this physics problem is that I worked on modeling this system in undergrad using Mathematica, and I wanted to see how much my phyics and coding skills have developed since then. Given enough time I would have liked to make an interactive simulation such as `https://www.myphysicslab.com/pendulum/double-pendulum-en.html`, where users would be able to control different parameters using the ipywidgets module. I ended up using this link `https://scipython.com/blog/the-double-pendulum/#rating-74`, to work through the problem. The one problem I ran into was that I wasn't sure how to animate these images in a simple way within a python notebook. As a result, I chose a online gif generator to compile the images, and create a gif file. Another prossibility I considered was including the animation on a plot to see the motion but the trails do a good job of tracing the paths [Fig. 5].

Figure 5: Random image out of the 301 images used to create the gif file

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