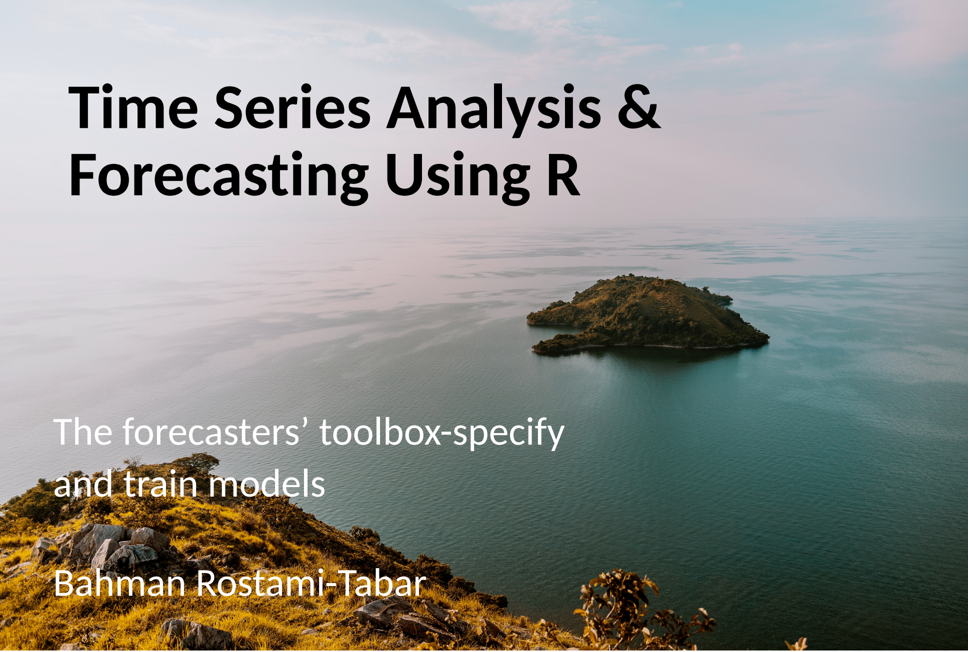

Time Series Analysis & Forecasting Using R

The forecasters’ toolbox-specify and train models

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Outline

Outline

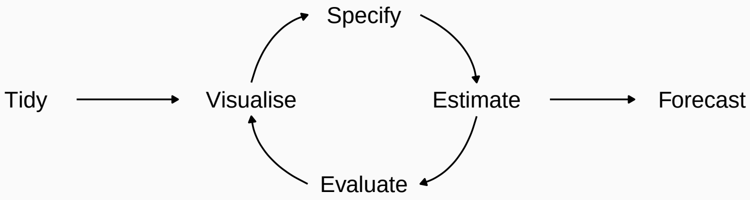
Learning outcome

You should be able to:

1. Explain simple forecasting methods (benchmarks)
2. Specify and estimate models using R functions in `fable`
3. Recognise and extract fitted values and residuals
4. Produce point and prediction interval forecasts

Outline

A tidy forecasting workflow



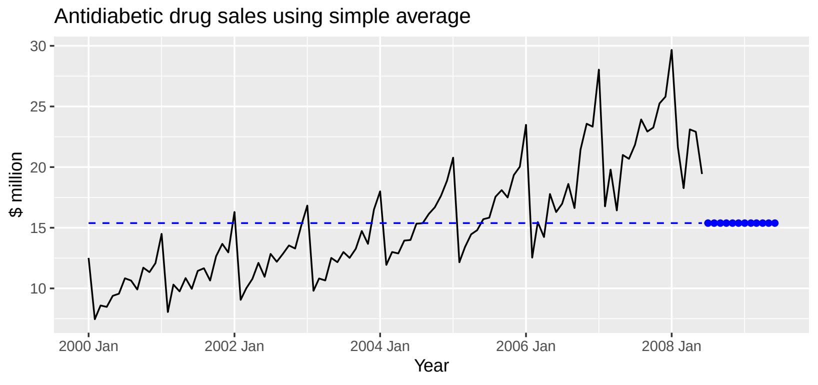
Outline

1. Learning outcome
2. A tidy forecasting workflow
3. Define the model (specify)
4. Train the model (estimate)
5. Fitted values and Residuals
6. Prediction intervals
7. lab session 4

Some simple forecasting methods

`MEAN(y)`: Average method

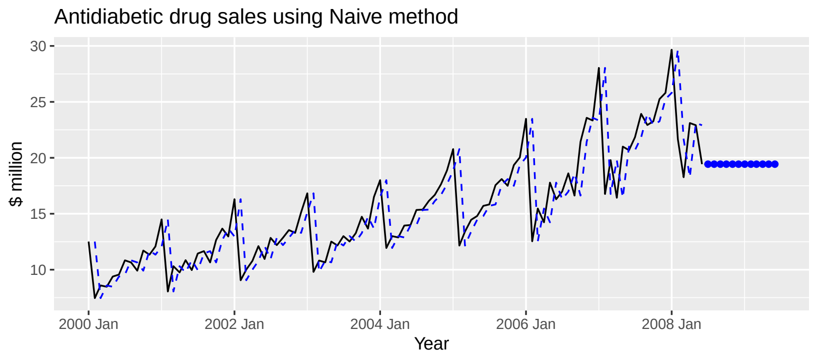
- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$



Some simple forecasting methods

`NAIVE(y)`: Naïve method

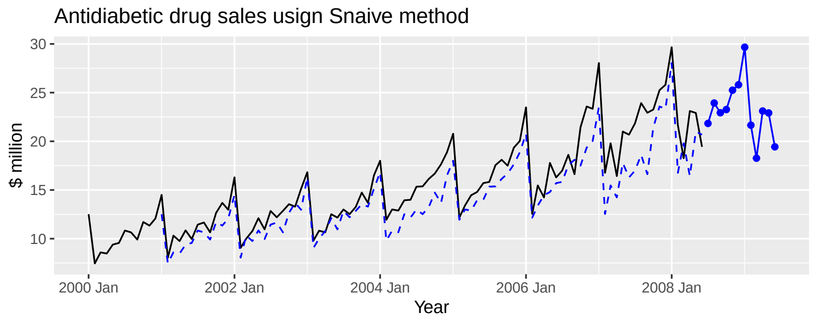
- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.



Some simple forecasting methods

`SNAIVE(y ~ lag(m))`: Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.



Model specification

- Model specification in fable supports a formula based interface
- A model formula in R is expressed using `response ~ terms`
  - the formula's left side describes the response
  - the right describes terms used to model the response.
- `Attention`: MODEL_NAME is in capital letters, e.g. `SNAIVE`

```
MODEL_NAME(response_variable ~ term1+term2+...)
SNAIVE(Beer ~ lag("year"))
```

Outline

Model estimation: template

The `model()` function trains models to data. - It returns a model table or a `mable` object.

```
# Fit the models
my_mable <- my_data %>%
  model(
    choose_name1 = MODEL_1(response_variable ~ term1+...),
    choose_name2 = MODEL_2(response_variable ~ term1+...),
    choose_name3 = MODEL_3(response_variable ~ term1+...),
    choose_name4 = MODEL_4(response_variable ~ term1+...)
  )
```

Model estimation

```r
# Fit the models
beer_fit <- aus_production %>%
  model(
    mean = MEAN(Beer),
    naive = NAIVE(Beer),
    snaive = SNAIVE(Beer, lag="year")
  )

#beer_fit <- beer_fit %>% stream(new_data),
#we can update the fitted models once we have new data
```

mable: a model object

```
beer_fit
```

```
## # A mable: 1 x 3
##        mean    naive    snaive
##     <model> <model>   <model>
## 1   <MEAN> <NAIVE> <SNAIVE>
```

- A `mable` is a model table, each cell corresponds to a fitted model.
- A mable contains
  - a row for each time series
  - a column for each model specification

Extract coefficients from `mable`

```
beer_fit %>% select(snaive) %>% report()
beer_fit %>% tidy()
beer_fit %>% glance()
```

- The `report()` function gives a formatted model-specific display.
- The `tidy()` function is used to extract the coefficients from the models.
- The `glance()` shows a summary from the models.
- We can extract information about some specific model using the `filter()` and `select()` functions.

Producing forecasts

- The `forecast()` function is used to produce forecasts from estimated models.
- **h** can be specified with a number (the number of future observations) or natural language (the length of time to predict).

```
beer_fc <- beer_fit %>%
  forecast(h = "3 years")
#h = "3 years" is equivalent to setting h = 12.
```

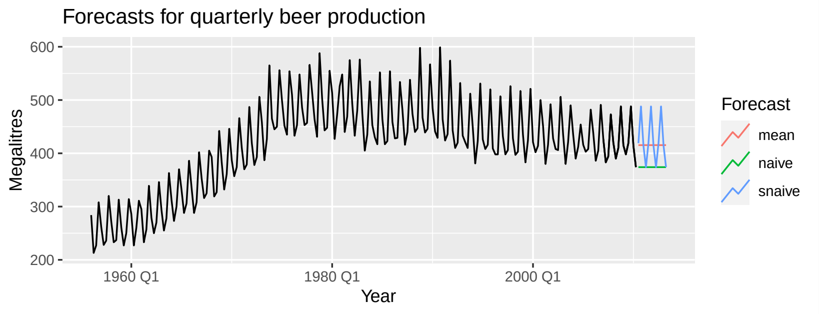
Producing forecasts

```
## # A fable: 36 x 4 [1Q]
## # Key:     .model [3]
##   .model Quarter          Beer .mean
##   <chr>    <qtr>        <dist> <dbl>
## 1 mean   2010 Q3 N(415, 7409)  415.
## 2 mean   2010 Q4 N(415, 7409)  415.
## 3 mean   2011 Q1 N(415, 7409)  415.
## 4 mean   2011 Q2 N(415, 7409)  415.
## # ... with 32 more rows
```

A `fable` is a forecast table with point forecasts and distributions.

Visualising forecasts

```
beer <- aus_production |> select(Beer)
# Plot forecasts against actual values
beer_fc %>%
  autoplot(beer, level = NULL) +
    autolayer(filter_index(aus_production, "2007 Q1" ~ .), color = "black") +
    ggtitle("Forecasts for quarterly beer production") +
    xlab("Year") + ylab("Megalitres") +
    guides(colour=guide_legend(title="Forecast"))
```



Outline

Fitted values

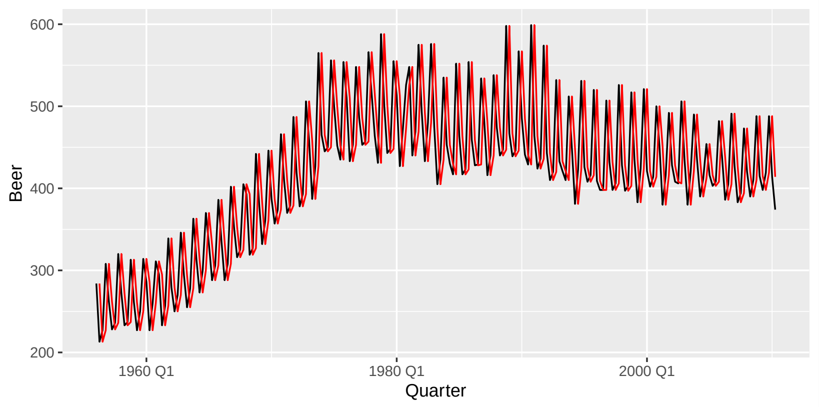
- $\hat{y}_{T|T-1}$ is the forecast of y_T based on observations $y_1, \dots, y_T - 1$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_T \equiv \hat{y}_{T|T-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_T = \bar{y}$ for average method.
- $\hat{y}_T = y_{T-1} + (y_T - y_1)/(T - 1)$ for drift method.
- $\hat{y}_T = y_{T-1}$ for naive method.

Fitted values

```
beer_fit %>% select(naive) %>% augment() %>%
  ggplot(aes(x=Quarter, y=Beer))+
  geom_line()+
  geom_line(aes(y=.fitted), colour="red")
```



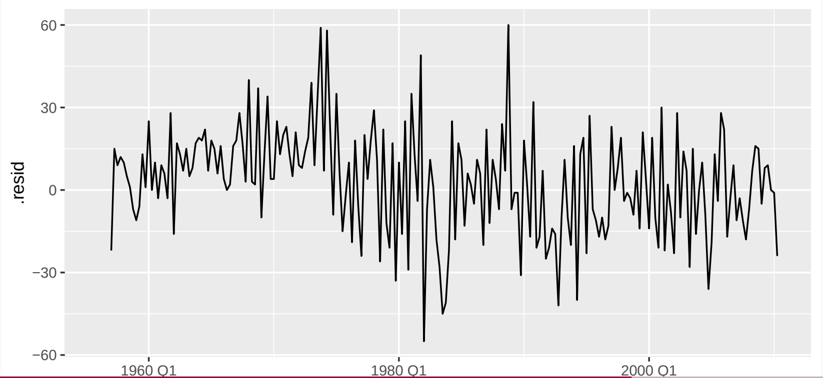
Residuals

- The “residuals” in a time series model are what is left over after fitting a model.
- Residuals are useful in checking whether a model has adequately captured the information in the data.

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Residuals

```
#beer_fit %>% fitted
# `augment()` fucntion gets residuals and fitted values
beer_fit %>% select(snaive) %>% augment() %>%
  ggplot(aes(x=Quarter, y=.resid))+
  geom_line()
```



Extract fitted values and residuals

```
beer_fit %>% augment()
```

```
## # A tsibble: 654 x 6 [1Q]
## # Key:       .model [3]
##    .model Quarter  Beer .fitted .resid .innov
##    <chr>    <qtr> <dbl>   <dbl>  <dbl>  <dbl>
##  1 mean   1956 Q1   284    415. -131.  -131. 
##  2 mean   1956 Q2   213    415. -202.  -202. 
##  3 mean   1956 Q3   227    415. -188.  -188. 
##  4 mean   1956 Q4   308    415. -107.  -107. 
##  5 mean   1957 Q1   262    415. -153.  -153. 
##  6 mean   1957 Q2   228    415. -187.  -187. 
##  7 mean   1957 Q3   236    415. -179.  -179. 
##  8 mean   1957 Q4   320    415.  -95.4  -95.4
##  9 mean   1958 Q1   272    415. -143.  -143. 
## 10 mean   1958 Q2   233    415. -182.  -182. 
```

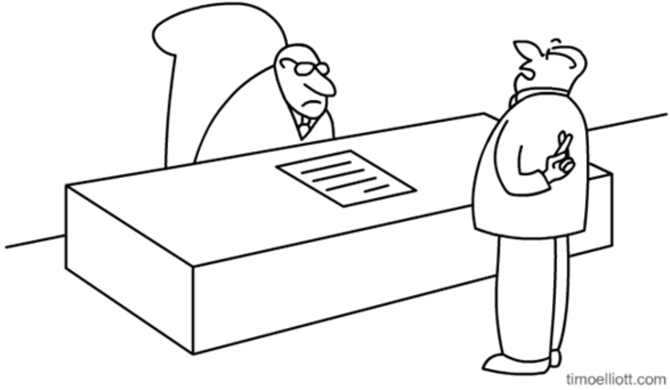
Outline

What is wrong with point forecasts?

- Resource allocation in A&E is assymmetric
  - The cost of over-allocating resources(over estimation) can vastly differ from the cost of under-allocating(under estimation)
- The disadvantage of point forecast:
  - it ignores additional information in future demand;
  - it does not explain uncertainties around future demand
  - it can not deal with assymmetric.

Importance of providing interval forecast

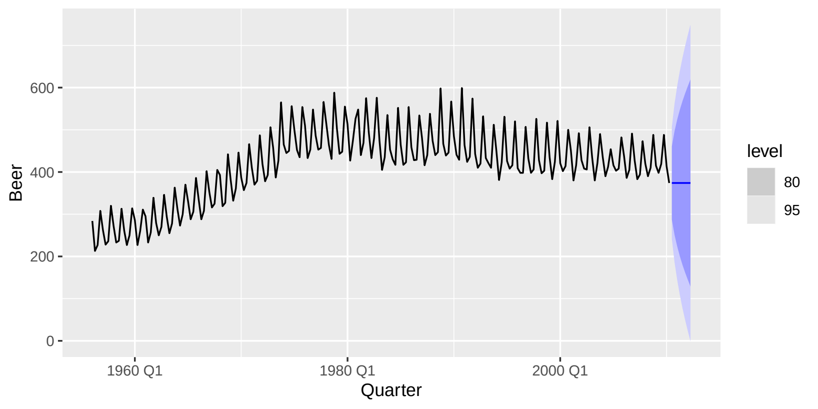
Point forecasts are often useless without a measure of uncertainty



"Yes sir, you can absolutely trust those numbers"

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability
- It consists of an upper and a lower limit between which the future value is expected to lie



Prediction intervals

- Assuming forecast errors are normally distributed, then a c% PI is:

$$\hat{y}_{T+h|T} \pm c\hat{\sigma}_h$$

where the multiplier c depends on the coverage probability and $\hat{\sigma}_h$ is the st dev of the h-step distribution.

Prediction intervals

- Forecast intervals can be extracted using the `hilo()` function
- Use `level` argument to control coverage.

```
fit <- aus_production %>% model(NAIVE(Beer))
forecast(fit) %>% hilo(level = c(80, 95))
```

```
## # A tsibble: 8 x 6 [1Q]
## # Key:       .model [1]
##   .model   Quarter          Beer .mean                 `80%`
##   <chr>      <qtr>        <dist> <dbl>                <hilo>
## 1 NAIVE(~ 2010 Q3   N(374, 4580)   374 [287.2735, 460.7265]80
## 2 NAIVE(~ 2010 Q4   N(374, 9159)   374 [251.3502, 496.6498]80
## 3 NAIVE(~ 2011 Q1 N(374, 13739)   374 [223.7853, 524.2147]80
## 4 NAIVE(~ 2011 Q2 N(374, 18319)   374 [200.5470, 547.4530]80
```

Outline

lab session 4