Hierarchical time series forecasting in Emergency Medical Services

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Abstract

Accurate forecasts of ambulance demand are crucial inputs when planning and deploying staff and fleet. Such demand forecasts are required at national, regional and sub-regional levels, and must take account of the nature of incidents and their priorities. These forecasts are often generated independently by different teams within the organization. As a result, forecasts at different levels may be inconsistent, resulting in conflicting decisions and a lack of coherent coordination in the service. To address this issue, we exploit the hierarchical and grouped structure of the demand time series, and apply forecast reconciliation methods to generate both point and probabilistic forecasts that are coherent and use all the available data at all levels of disaggregation. The methods are applied to daily incident data from the Welsh Ambulance Service NHS Trust, from October 2015 to July 2019, disaggregated by nature of incident, priority, managing health board, and control area. We use an ensemble of forecasting models, and show that the resulting forecasts are better than any individual forecasting model. We validate the forecasting approach using time-series cross-validation.

Keywords: forecasting, healthcare, emergency services, forecast reconciliation, hierarchical time series, ambulance demand, attended incidents

1. Introduction

Inability to match the resources with the demand in Emergency Medical Services (EMS) results in an overcrowding care system. This is a serious problem causing challenging situations on patient flow with serious consequences on patients, staff and the entire care system (Ekström et al., 2015; Rostami-Tabar and Ziel, 2022). Demand forecasting in EMS is a vital element that helps to depict various courses of action to avoid the mismatch, which can result in massive savings in terms of patient safety and lives. An accurate daily demand forecasting enables planners and decision makers to manage resources to meet anticipated patients, reconfigure units, redeploy staff and vehicles, where necessary.

Demand forecasts at EMS are typically required at multiple levels of cross-sectional granularities to inform various planning and decision-making processes (Hulshof et al., 2012). There are some planning process at the national level (more strategic or long-term) such as workforce resource planning and budgeting; sub-national, regional or healthcare level (tactical or medium-term) such as temporary capacity expansions, resource sharing and staff-shift scheduling; and hospital or station level (operational or short-term) such as planning rosters for staff and ambulance deployment. Demand forecasts might also be required at different level for a specific group of interest such as nature of demand or priority. Moreover, the time series data in

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EMS has an inherent hierarchical and grouped structure to support such forecasting requirements. Demand for emergency medical services at the country level can be disaggregated in a geographical hierarchy into sub-national, regions, health boards, stations/hospitals or divided into groups such as nature of incidents or demand priority. Therefore, using forecasting methodologies that account for hierarchical and/or grouped structures of time series in EMS seems to be a natural fit.

However, despite a large number of studies dedicated to forecasting for EMS (Shi et al., 2022; Gul and Celik, 2020; Ibrahim et al., 2016; Wargon et al., 2009), this area is neglected and the main focus has been on producing forecasts at a single level, independently. Generating independent forecasts not only ignore the inherent hierarchical and/or grouped structure relationships of the time series demand but also results in a lack of consistency and coordination. Consistent forecasts of the EMS demand across all hierarchical and grouped levels are paramount for an effective planning and decision making. The hierarchical forecasting approaches can not only create consistent forecasts but also achieve more accurate forecasts than the independent (base) forecasts (Hyndman et al., 2011). Obtaining consistent forecasts at different levels is important as it helps to avoid making conflicting decisions. With hierarchical forecasting, plans at any level are based on coherent forecasts and therefore can be aligned. Implementing and sustaining improvements in EMS require alignments and coordination between different stakeholders, without which teams operate in isolation leading to conflicts, duplication work, rework, or work that runs counter to the overall goal to improve the quality of delivery service. Hierarchical forecasting framework can be used as a tool to improve coordination between teams across the care services at the national, sub-national, regional and local levels. To our knowledge, there is not only no research in the EMS forecasting to account for the hierarchical and grouped structure of the system but also in the entire field of forecasting for healthcare management.

In this paper, we address this gap by investigating the application of hierarchical forecasting approaches in the EMS using daily time series of verified incidents from 2015 to 2020 in a major ambulance service in Great Britain. The data has hierarchical and grouped structures with hierarchies at the national, control (i.e. sub-national), health board (i.e. regional) and groups by priority and nature of incidents. We produce not only the point forecast but also the forecast distribution across all levels, which is critical for an effective planning and associated risk management. We compare the point and probabilistic forecast accuracy of the independent forecasts, bottom-up and optimal reconciliation approaches. We first generate independent/based forecasts using Exponential Smoothing State Space (ETS), Generalized Linear Model (GLM), Poisson regression, a simple empirical distribution and an ensemble method followed by applying bottom-up and optimal reconciliation approaches. Forecast performance is assessed by the Root Mean Squared Scaled Error (RMSSE) for point forecasts and the Continuous Ranked Probability Score (CRPS) for the probabilistic forecasts. This paper complies with the principles of the reproducibility (Stodden and Miguez, 2013; Boylan et al., 2015). Therefore, the study could equally be applied to any healthcare service (e.g. emergency department, primary or social care) subject to the time series having a hierarchical and/or grouped structure, which is generally the case in the healthcare sector.

The remainder of this article is structured as follows: In section 2, we provide a brief review of the literature and discuss its limitation to position our work; in section 3, we present the experiment design describing the data set, forecasting methods and forecast evaluation metrics. In Section 4, we discuss the hierarchical time series forecasting approaches to generate both point and probabilistic forecasts. In section 5, we present and discuss our results; in section 7, we summarize our findings and present ideas for future research.

2. Research background

Emergency medical services (EMS) are a critical component in the delivery of urgent medical care to communities. An effective service delivery requires accurate resource planning that are generally replying on demand forecasts at operational, tactical and strategic levels.

There is a substantial number of studies on the application of time series forecasting in the Emergency Medical Services. Various areas have been the focus of the literature. Forecasting call volume arrivals is one of the major research topics. Ibrahim et al. (2016) provide an extensive review of the forecasting models in this context. Another important area is related to forecasting ambulance demand. Although the definition of demand might not be always clearly stated, however, this is typically referring to a situation where a physical resource has been deployed to respond to an incident. This might be also called *attended incidents*. Another demand related variable is verified incidents. These are all incidents that require an action: either send a physical vehicle, deal with via the Clinical Support Desk (e.g. calls), get an external (private) provider to respond to it, or send it through to other channels such as police, firefighters or general practitioners. Our study is aligned with this stream of the literature. Another similar area that is largely studied in the literature, is Emergency Department forecasting. We refer interested readers to Shi et al. (2022), Gul and Celik (2020) and Wargon et al. (2009) for extensive reviews of the literature on Emergency Department forecasting. Although crucial to EMS performance, Aringhieri et al. (2017) state that demand forecasting has received limited research attention in the EMS context. In this section, we provide a brief review of studies on forecasting ambulance demand in EMS.

There are generally two main streams of research related to forecasting ambulance demand in EMS: i) the first stream focuses on the application of time series methods and regression approaches on forecasting aggregate ambulance demand (Vile et al., 2012; Sasaki et al., 2010); ii) the second stream considers forecasting EMS demand in a more finer temporal and geographical granularities by employing temporal-spatial prediction methods (Zhou and Matteson, 2016; Zhou, 2016). The focus of our study is related to the first stream of research.

Sasaki et al. (2010) develop a multivariable regression model to estimate future EMS demands. In addition to the historical demand, the population census for different age groups and counts of the number of companies employing more than five people are included in the regression. The census variables describe groups who may be more likely to need an ambulance. A stepwise ordinary least squares regression analysis with SPSS is used for estimating the parameter and generating forecast. The only performance measure reported in this study is R^2. The research design of this study is not rigorous and the study is not reproducible. Vile et al. (2012) explore using a Singular Spectrum Analysis (SSA) method to generate forecasts of the EMS demand at the national level for 7-day, 14-day, 21-day and 28-day forecast horizons using data provided by the Welsh Ambulance Service Trust (WAST). The performance of this approach is compared with Auto-Regressive Moving Average (ARIMA) and Holt-winter time series methods using Root Mean Squared Error (RMSE). They concluded that point forecasts generated by SSA are more accurate for longer-term, but ARIMA and Holt-winter performance is superior for shorter-term horizons. Vile et al. (2016) further develop a decision support system to integrate forecasts generated by SSA. However, the study does not compare and contrast the performance of forecasting methods based on utility measures such as cost, resource utilization or response time. The tool contains options that allow generating forecasts at various levels of granularity, however, it ignore the hierarchical and grouped relationships structure, preventing aligned decision making and coordination.

Haugsbø Hermansen and Mengshoel (2021) investigate forecasting EMS demand in a high spatio-temporal resolution of 1×1km spatial regions and 1-hr time intervals using total incidents in Oslo, Norway from January 1st, 2015 up to and including February 11th, 2019. They used multi-layer perceptron (MLP) and long short-term memory (LSTM) models to forecast the EMS demand, and compare them to simple aggregation methods and baselines. The point forecast accuracy is evaluated using Mean Absolute Error (MAE) and Mean Squared Error (MSE) and the forecast distribution is measures by Categorical Cross-Entropy. They shows that while Neural Network models perform better in producing point forecast, a distribution baseline method based on spatial distribution of the incidents across all time steps provide more accurate forecast distribution. Zhou (2016) propose three methods based on Gaussian mixture models, kernel density estimation, and kernel warping to predict 4 weeks into future for a 1-km2 spatial region over an hour. Two years of incidents attended from Toronto, Canada (years 2007 and 2008 with 391,296 events) and Melbourne, Australia (years 2011 and 2012 with 696,975 events) are used to build the model and examine the performance on test data using mean negative log likelihood. They show that forecasts generated by the proposed methods are significantly more accurate than the current industry practice, a simple averaging formula. Grekousis and Liu (2019) investigate the combination of spatial analysis methods with data mining techniques based on an improved Hungarian algorithm and MLP neural network to identify the most likely locations of future emergency events. The proposed approach is tested using data of 2851 events attended by the EMS in Athens, Greece over 24 weeks. They show that 23.24% of real emergency events lie within 50 meter of the predicted ones and nearly 70% of the real emergency events lie no further away than 150 meter, which is rather accurate given the granularity of the problem at the city level.

We observed a number of limitations in the literature of EMS forecasting, that encourage us to undertake this research. These limitations are summarized as following:

1. Current studies ignore the inherent hierarchical and/or grouped structure of the time series data and the relationship between series at different levels of hierarchy. This may result in incoherent forecasts leading to misaligned planning and decision making. While the hierarchical forecasting methodology has been developed and applied in various domains over the past 10 years (Panagiotelis et al., 2022), it has never been explored in this area.
2. Current research is mainly concerned with generating point forecast at a single level of hierarchy. There is a lack of studies presenting the entire forecast distribution of daily ambulance demand for the entire hierarchy to inform the whole decision-making process and to better represent the uncertainty of future demand, providing a risk management tool for planners.
3. Reproducibility is still a major challenge in EMS forecasting, as it is unlikely to reproduce the results without the help of the authors of those papers.
4. Another limitation is related to the generated forecasts that are not integer counts. Since actual ambulance counts cannot be negative or fractions, ambulance demand forecasts should be the same. While this might not be an issue when producing forecasts at a single level, producing non-negative count forecasts in a hierarchical/grouped structure is challenging and requires further investigation in the future.

This paper concerns the problem of hierarchical forecasting in EMS and generates and evaluates both point and probabilistic forecast across different levels of the hierarchy, hence addressing some important gaps identified in the literature.

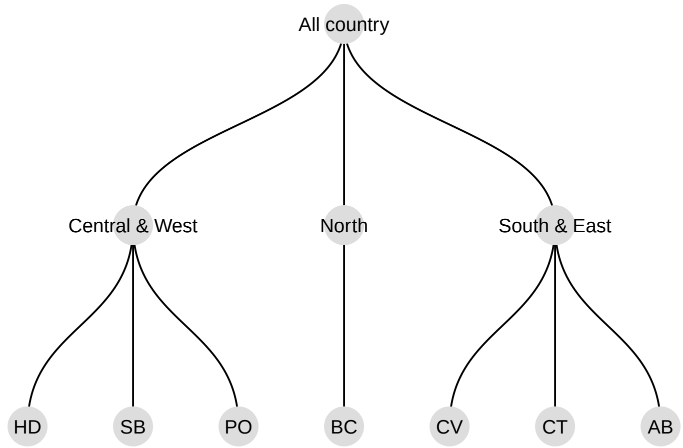
3. Experiment setup

We are interested in generating forecasts to inform the planning horizon of $ph = 42$ days, required by planners in the ambulance services trust. The forecast horizon in this study is $fh = 2 \times ph$ days ahead (2*42 days planning horizon). This is because the planning is generally frozen for ph days and considering a forecast horizon of ph days might not be helpful for planning. While forecasts are generated for $2 \times ph$ days ahead, performance evaluation is only assessed based on the last ph days and not the $2 \times ph$ days. The forecasts are produced for the holdout of 365 days using time series cross-validation (Hyndman and Athanasopoulos, 2021).

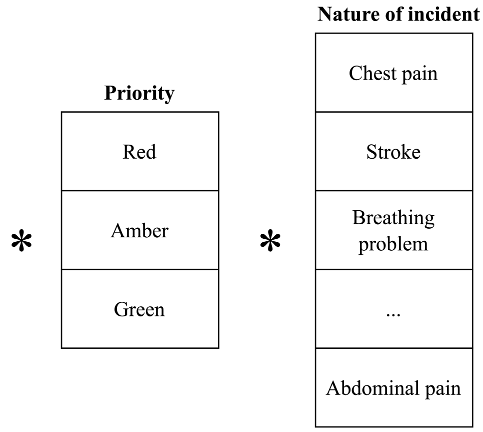
In the following section, we discuss the dataset, describe the forecasting methods used to generate base forecasts and present the point and probabilistic accuracy measures.

3.1. Data

The dataset used in this study is from a major ambulance service trust in Great Britain. It contains information relating to the daily number of attended incidents from 1 October 2015 to 31 July 2019, disaggregated by nature of incidents, priority, the health board managing the service and the control area (or region). Figure 1 depicts both the hierarchical and grouped structure of the data. Figure 1a illustrates the nested hierarchical structure based on control area and health board and Figure 1b shows the grouped structure by priority and the nature of incident.



(a) Hierarchical structure: Attended incidents in all country is disaggregated into 3 control areas /regions and then into 7 different healthboard at the bottom



(b) Grouped structure: Incidents could be grouped into priority (i.e. Red, Amber & Green) and the nature of attended incident (i.e. there are 35 different nature of incidents including chest pain , breathing problems, heart attack, stroke, and so on). The symbol * refers to the crossed attributes between hierarchical and grouped levels.

Figure 1: The hierarchical and grouped structure of attended incidents (ambulance demand).

Table 1 also displays the structure of data with the total number of series at each level. At the top level, we have the total attended incidents for the country. We can split these total attended incidents by control area, by health board, by priority or by nature of incident. There are 3 control areas breakdown by 7 local health boards. Attended incident data are categorized into 3 priority classes of red, amber and green. There are also 35 different nature of incidents such as chest pain, stroke, breathing problem, etc. In total, across all levels of disaggregation, there are 1530 time series.

Given the total number of time series, direct visual analysis is infeasible. Therefore, we first compute features of all 1530 time series (Kang et al., 2017) and display the strength of trend and weekly seasonality strength in Figure 2. Each point represents one time series with the strength of trend in x-axis and the strength of seasonality in y-axis. It is clear that there are some series showing strong trends and/or seasonality, corresponding to series at the higher levels of the hierarchy. The majority of series show low trend and seasonality. These are time series belonging to the bottom series, series related to the nature of incidents for a given control, health board and priority level. Bottom series are dominated by noise with little or no systematic patterns.

In addition to displaying the trend and seasonality features, we also visualize few time series at various levels of the aggregation. Figure 3 reveals different information such as trend, seasonality and noise. For example, some series depict seasonality and trend, whereas some other series report low volume of attended incidents and entropy, making them more volatile and difficult to forecast. At the level on nature of incidents combined with categories of other levels, there are many series that contain zeros with low counts. As such, the data set represents a diverse set of daily time series patterns.

We consider several forecasting models that account for the diverse patterns of the time series across the entire hierarchy. In developing the forecasting models, the time series of holidays are also used in addition to the attended incidents. We use public holidays, school holidays and Christmas Day and New Year's Day as predictors of incident attended. These type of holidays will affect peoples' activities and may increase or decrease the number of attended incidents.

Table 1: Number of time series in each level for the hierarchical & grouped structure of attended incidents

Level	Number of series
All country	1
Control	3
Health board	7
Priority	3
Priority * Control	9
Priority * Health board	21
Nature of incident	35
Nature of incident * Control	105
Nature of incident * Health board	245
Priority * Nature of incident	104
Control * Priority * Nature of incident	306
Control * Health board * Priority * Nature of incident (Bottom level)	691
Total	1530

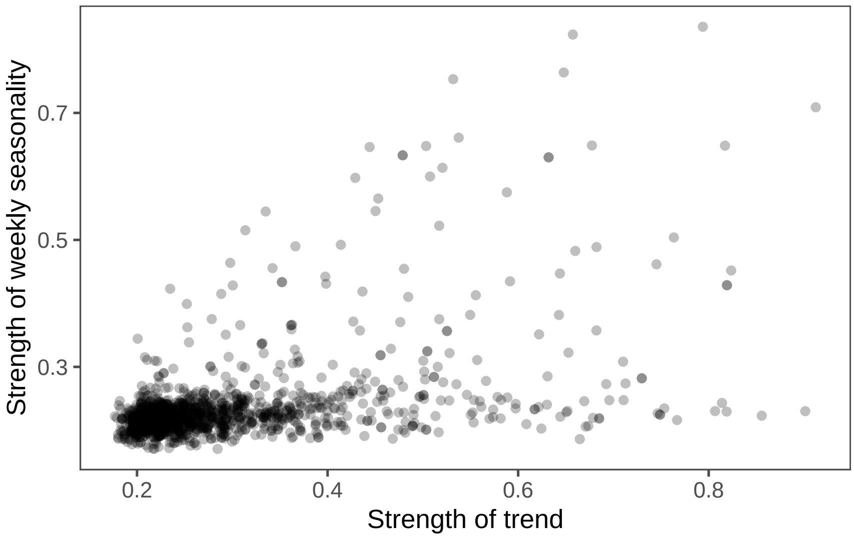


Figure 2: Time series features of attended incidents across all levels (1530 series)

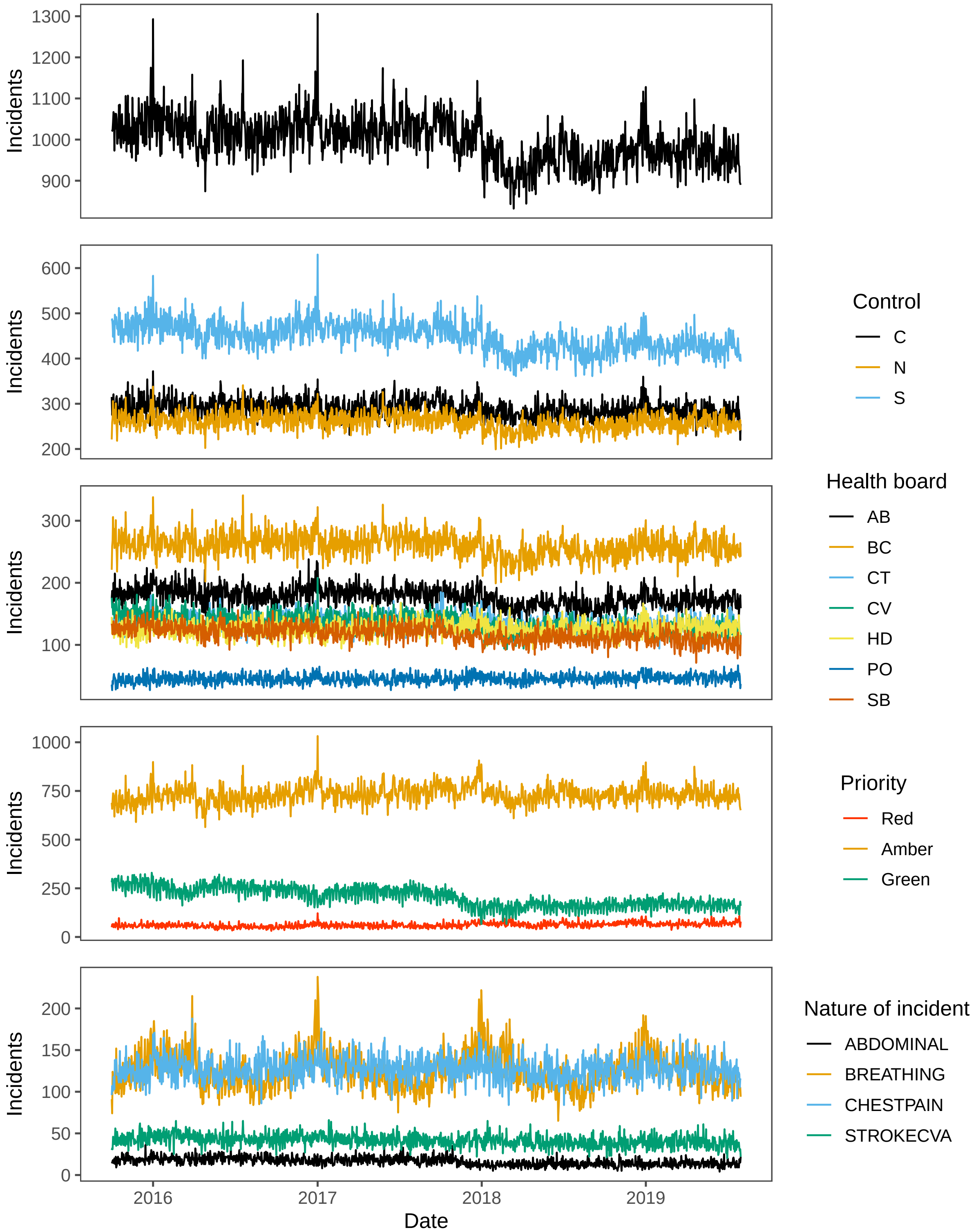


Figure 3: Time series of attended incidents at various levels.

3.2. Forecasting methods

Given the presence of various significant patterns in the past attended incidents, we consider three different forecasting models to generate the base forecasts. Once the base forecasts are produced, hierarchical and grouped time series methods are used to reconcile them across the all levels. We briefly discussed forecasting models in the following sections, and the hierarchical forecasting methods are discussed in Section 4.

Naive: We start with a simple forecasting approach, assuming that the future days will be similar to past days. We use the empirical distribution of the past daily attended incidents to create the forecast distribution of future attended incidents. We consider the empirical distribution of the most recent year of historic data on a rolling basis to capture potential changes in behavior over time.

Actually we use all the data.

Exponential Smoothing State Space model (ETS): ETS models (Hyndman and Athanasopoulos, 2021) can combine trend, seasonality and error components in a time series through various forms such as additive, multiplicative or mixed. The trend component can be none ("N"), Additive ("A"), damped ("Ad") or multiplicative ("M"). The seasonality can be none ("N"), Additive ("A"), or multiplicative ("M"). The error term can also be additive ("A") or multiplicative ("M"). To forecast the attended incidents at each level, we use the `ETS()` function in the `fable` package (O'Hara-Wild et al., 2022) in R. To identify the best model for a given time series, the ETS function uses the corrected Akaike's Information Criterion (AICc).

Despite the popularity and the relevance of automatic `ETS` in this study, however it may produce forecast distributions that are non-integer and include negative values, despite the number of attended incidents being integer and non-negative. When using `ETS`, a time series transformation approach could be used to generate strictly positive forecasts, although forecast distributions will still be non-integer. An alternative is to use forecasting models that produce integer, non-negative forecasts. in the following section we present Generalized Linear Models (GLMs) and Poisson Regression to produce count base forecasts.

Generalized Linear Model (GLM): GLMs are a family of models developed to extend the concept of linear regression models. They perform a regression by modeling the response variable as coming from a particular member of the exponential family, and then transforming the mean of the response so that the transformed mean is a linear function of the predictors. One of the models that is frequently used in practice to generate count forecasts is the Poisson regression. We will consider forecasting attended incidents using the covariates spline trend, day of the week dummy variables (from Monday to Sunday), Fourier terms to capture yearly seasonality, public holidays (1 when is public holiday, 0 otherwise), school holidays (1 when is school holiday, 0 otherwise) and Christmas Day (1 when is Christmas Day, 0 otherwise) and New Year's Day (1 when is New Year's Day, 0 otherwise). We fit a Poisson regression model using the function `glm()` from the package in R, with the argument `family = poisson` to specify that we wish to fit a Poisson regression model.

Poisson Regression using tscount: We consider another Poisson regression model that takes into account serial dependence in addition to covariates used in the GLM model. To that end, we use `tsglm()` function in *tscount* package in R (Liboschik et al., 2017) to model the attended incidents. The logarithmic link function is used to ensure that the parameter of Poisson distribution is always positive. This model can be estimated via maximization of the likelihood function based on Poisson mass function. The regression model captures the short range serial dependence by including the three order autoregressive terms.

Ensemble method: we also use an ensemble method that combines forecasts generated from Naive, ETS, GLM and Poisson regression to form a mixture distribution (Wang et al., 2022).

3.3. Performance evaluation

To evaluate the performance of the various forecasting approaches, we split the data into a series of ten training and test sets. We use a time series cross-validation approach (Hyndman and Athanasopoulos, 2021), with a forecast horizon of 84 days, and each training set expanding in 42-day steps. The first training set uses all data up to 2018-04-25, and the first test set uses the 84 days beginning 2018-04-26. The second training set uses all data up to 2018-06-06, with the second test set using the following 84 days. The largest training set ends on 2019-05-09, with the test set ending on 2019-07-31. Model development and hyper-parameter tuning is performed using the training data only. Forecasting performance is evaluated using both point and probabilistic error measures. While we compute forecast errors for the entire 12 weeks, we are most interested in the last 42 days of each test set, because that corresponds to how forecasts are generated for planning in practice. Forecasting performance is evaluated using both point and probabilistic error measures.

Point forecast accuracy is measured via the Mean Squared Scaled Error (MSSE) and the Mean Absolute Scaled Error (MASE). The Mean Absolute Scaled Error (MASE) (Hyndman and Koehler, 2006) is calculated as:

$$\text{MASE} = \text{mean}(|q_j|),$$

where

$$q_j = \frac{e_j}{\dfrac{1}{T-m}\displaystyle\sum_{t=m+1}^{T} |y_t - y_{t-m}|},$$

and e_j is the point forecast error for forecast horizon j, $m = 7$ (as we have daily seasonal series), y_t is the observation for period t, and T is the sample size (the number of observations used for training the forecasting model). The denominator is the mean absolute error of the seasonal naive method in the fitting sample of T observations and is used to scale the error. Smaller MASE values suggest more accurate forecasts. Note that the measure is scale-independent, thus allowing us to average the results across series.

A related measure is MSSE (Hyndman and Athanasopoulos, 2021), which uses squared errors rather than absolute errors:

$$\text{MSSE} = \text{mean}(q_j^2),$$

where,

$$q_j^2 = \frac{e_j^2}{\dfrac{1}{T-m}\displaystyle\sum_{t=m+1}^{T} (y_t - y_{t-m})^2},$$

Again, this is scale-independent, and smaller MSSE values suggest more accurate forecasts.

To measure the forecast distribution accuracy, we calculate the Continuous Rank Probability Score (Gneiting and Katzfuss, 2014). It rewards sharpness and penalizes miscalibration, so it measures overall performance of the forecast distribution.

$$\text{CRPS} = \text{mean}(p_j),$$

where

$$p_j = \int_{-\infty}^{\infty} \left(G_j(x) - F_j(x)\right)^2 dx,$$

where $G_j(x)$ is the forecasted probability distribution function for forecast horizon j, and $F_j(x)$ is the true probability distribution function for the same period.

4. Hierarchical and grouped time series forecasting techniques

There are many applications in the healthcare and in particular in EMS where a collection of time series is available. These series are generally hierarchically organized based on multiple levels such as area/region, health board and/or are aggregated at different levels in groups based on nature of demand, priority of demand or some other attributes. While series could be strictly hierarchical or only grouped bases on some attributes, in many situation a more complex structures arise when attributes of interest are both nested and crossed, having hierarchical and grouped structure. This is also the case as discussed in Section 3.1.

4.1. Independent (base forecast)

A common practice in healthcare (and EMS) to predict hierarchical and grouped series relies on producing independent forecast, also refereed to as base forecast, typically by different teams as the need for such forecasts arise. This is also known as base forecast. We observe n time series at time t, across the entire hierarchical and grouped structure, written as y_t. The base forecasts of y_{T+h} given data $y_1, \ldots, y_T$ are denoted by $\hat{y}_h$ for h steps-ahead for all n series ($n = 1530$ in this study). Forecasts generated in this way are nor coherent.

4.2. Reconciliation methos

Traditionally, alternative approaches to produce coherent forecasts for hierarchical and grouped time series involves using bottom-up and top-down methods by generating forecasts at a single level and then aggregate or disaggregate. Top-down requires having a unique structure to disaggregated forecasts generated at top level by proportions. However, given that we have multiple grouped attributes combined with the hierarchical structure, there is no unique way to disaggregate top forecast. Hence the top-down can not be used in this case, so either we can do some kind of reconciliation or must define our own top-down method for each hierarchy. The recommended approach is to use reconciliation. In the following sections, we first discuss some notations and then present bottom-up and forecast reconciliation approach used in this study to generate coherent forecasts.

4.2.1. Notations

Let b_t be a vector of n_b "bottom-level" time series at time t, and let a_t be a corresponding vector of $n_a = n - n_b$ aggregated time series, where

$$a_t = Ab_t$$

and A is the $n_a \times n_b$ "aggregation" matrix specifying how the bottom-level series b_t are to be aggregated to form a_t. The full vector of time series is given by

$$y_t = \begin{bmatrix} a_t \\ b_t \end{bmatrix}.$$

This leads to the $n \times n_b$ "summing" or "structural" matrix given by

$$S = \begin{bmatrix} A \\ I_{n_b} \end{bmatrix}$$

such that $y_t = Sb_t$.

4.2.2. Bottom-up (BU) and linear reconciliation methods

Bottom-Up is a simple approach to generate coherent forecasts. It first involves creating the base forecasts for the bottom level series. These forecasts are then aggregated to the upper levels which results in generating coherent forecasts. BU approach can capture the dynamics of the series at the bottom level, but they may be noisy and difficult to forecast as well. BU approach is limited on using only forecasts at the bottom level and does not utilize all the information available across the hierarchical and grouped structure. Forecast reconciliation approaches fill this gap and combine and reconcile all the base forecasts in order to produce coherent forecasts.

Given the summing matrix and base forecasts, bottom-up and linear reconciliation methods can be written as $\tilde{y}_h = SG\hat{y}_h$ for different matrices G.

Optimal reconciled forecasts are obtained with $G = (S'W^{-1}S)^{-1}W^{-1}$, or $\tilde{y}_h = M\hat{y}_h$, where the $n \times n$ "mapping" matrix is given by $M = S(S'W^{-1}S)^{-1}W^{-1}$, where $\hat{y}_h$ are the h-step forecasts of y_{T+h} given data to time T, and W is an $n \times n$ positive definite matrix. Different choices for W lead to different solutions such as Ordinary Least Square (OLS) , Weighted Least Square (WLS) and Minimum Trace (MinT) (Wickramasuriya et al., 2019). We use the implementation of these methods in the fable package in R in the experiment.

5. Results and discussion

In this section, we compare the forecasting performance of Naive, ETS, GLM, Tscount and the ensemble model using base forecast and Minimum Trace (Mint) methods. We have also computed the forecast accuracy for Ordinary Least Square (OLS) and Weighted Least Square (WLS) approaches. However, they are not reported here because their accuracy is outperformed by Mint. We should also note that forecasts, and consequently their corresponding errors, are generated for the entire hierarchy and they could be reported at any level, if required.

The overall forecasting performance is reported in Table 2, in which the average forecast accuracy per model, method and the hierarchical level is presented. Reported forecast accuracy are averaged across all forecast horizons, rolling origins and series at each level. Table 2 presents both point and probabilistic forecast accuracy at total, control area, health board and bottom level series. Point forecast performance are reported using MASE and RMSSE in Table 2a and Table 2b, respectively. Probabilistic forecast accuracy is reported using CRPS in Table 2c. The bold entries in each table identify a combination of method and model that performs best for the corresponding level (i.e. each column), based on the smallest values of accuracy measures.

Table 2a shows that forecast reconciliation (i.e. Mint) improves forecast accuracy at the higher levels of the hierarchy including total, control area and health board. However, it is does not result in accuracy improvement at the bottom level series, fro which base forecasts are more accurate. This might be due to the noisy structure of time series at the bottom level and the fact that base forecasts produced for the bottom of hierarchy are from well-specified forecasting models and therefore does not gain benefit from forecast reconciliation. Generally, if the bottom-level series are sufficiently smooth, base forecast is expected to outperform hierarchical reconciliation due to the better forecasts available at the level. It is also clear from Table 2a that the ensemble method improves forecast accuracy at total, control area and health board. However, this does not remain valid for bottom series where the Poisson regression by GLM outperforms other approaches.

Table 2b also reports point forecast accuracy but using RMSSE. The outperformance of forecast reconciliation using Mint at higher levels of hierarchy is also confirmed by RMSSE. However, for the bottom series, we observe that the base forecasts generated by ETS are the most accurate, followed by its reconciliations forecasts.

Table 2b presents the accuracy of the forecast distributions measures by CRPS, which considers both forecasting reliability and interval sharpness. The smaller the value of CRPS, the better the comprehensive performance. We observe that forecast reconciliation results in forecast improvement, regardless of the hierarchical level. Ensemble method is also more accurate fore higher levels, but ETS performs better at the bottom level. While results show that forecast reconciliation does not improve point forecast at the bottom level, however Table 2b indicates that it generates more accurate forecasts than base method. The outperformance of probabilistic forecast reconciliation at the bottom level may highlight an important fact. The reconciliation method may improve forecast accuracy at certain number of quantiles at the tails of forecast distribution, which are critical for managing risks.

Overall, our results indicate that forecast reconciliation using MinT method provides reliable forecasts, and all methods that reconcile using information at all levels of the forecast improve upon base (unreconciled)

Table 2: Average forecast perfomance calculated on out-of-sample with time series cross validation applied to attended incident data. The best approach is highlighted in bold.

(a) Point forecast accuracy using MASE

		MASE			
method	model	Total	Control areas	Health boards	Bottom
base	ensemble	0.7543	0.8432	0.8648	1.004
base	ets	0.9360	0.9133	0.8884	1.031
base	iglm	0.8609	0.9083	0.9039	**0.999**
base	naiveecdf	1.1396	1.0633	1.0389	1.017
base	tscount	0.8704	0.9109	0.9041	1.000
mint	ensemble	**0.7332**	**0.8346**	**0.8621**	2.251
mint	ets	0.8593	0.9001	0.8997	1.258
mint	iglm	0.8176	0.8805	0.8883	2.490
mint	naiveecdf	1.1395	1.0631	1.0389	2.648
mint	tscount	0.8218	0.8846	0.8913	2.516

(b) Point forecast accuracy using RMSSE

		RMSSE			
method	model	Total	Control areas	Health boards	Bottom
base	ensemble	0.7338	0.8335	0.8652	0.9885
base	ets	0.9465	0.9126	0.8915	**0.9795**
base	iglm	0.8383	0.9073	0.9125	0.9958
base	naiveecdf	1.0559	1.0213	1.0188	1.0081
base	tscount	0.8446	0.9090	0.9114	1.0069
mint	ensemble	**0.7153**	**0.8244**	**0.8621**	1.2405
mint	ets	0.8510	0.9002	0.9027	0.9879
mint	iglm	0.8015	0.8814	0.8972	1.3379
mint	naiveecdf	1.0558	1.0211	1.0188	1.4429
mint	tscount	0.8020	0.8849	0.9003	1.3515

(c) Probabilistic forecast accuracy using CRPS

		CRPS			
method	model	Total	Control areas	Health boards	Bottom
base	ensemble	12.44	5.681	3.385	0.2421
base	ets	13.96	5.974	3.430	0.2422
base	iglm	14.49	6.061	3.488	0.2420
base	naiveecdf	30.35	10.887	5.445	0.3000
base	tscount	14.98	6.124	3.500	0.2429
mint	ensemble	**12.09**	**5.664**	**3.383**	0.2455
mint	ets	13.28	5.884	3.472	**0.2412**
mint	iglm	13.35	5.812	3.394	0.2443
mint	naiveecdf	30.36	10.904	5.446	0.3103
mint	tscount	13.55	5.868	3.423	0.2483

forecasts, except the bottom level series. For bottom series with less obvious patterns, base forecasts near center of forecast distribution might be more accurate, while forecast reconciliation using Mint could improve accuracy in the tails of of the distribution.

In addition to the overall forecast accuracy presented in Table 2, we also report the point and probabilistic forecast accuracy measures for each forecast horizon in Figure 4. The figure focuses on the hierarchical levels important for decision making including total, control area and health board, however this could be calculated for any level. We only illustrate the results of the Mint method, given the its outperformance described in Table 2. For the illustration purpose, we report the average weekly forecast accuracy instead of the daily forecast horizon (i.e. 84 days including freezed planning (7 weeks = 42 days) plus future planing period (7 weeks = 42 days). Therefore, x-axis shows horizons from week 1 (h= 1,...,7) to week 12 (h= 78,...,84). The forecast horizon from week 7 to week 12 corresponds to the upcoming planning horizon, which is used by planners and decision making. For the point forecast, we can see that ensemble model performs much better than others regardless of forecast horizons. For the probabilistic forecasts, the distinction between forecasting models is less obvious. It is important to highlight that, all forecasting models outperform the Naive empirical distribution that is used as a Benchmark for both point and probabilsitic forecasts.

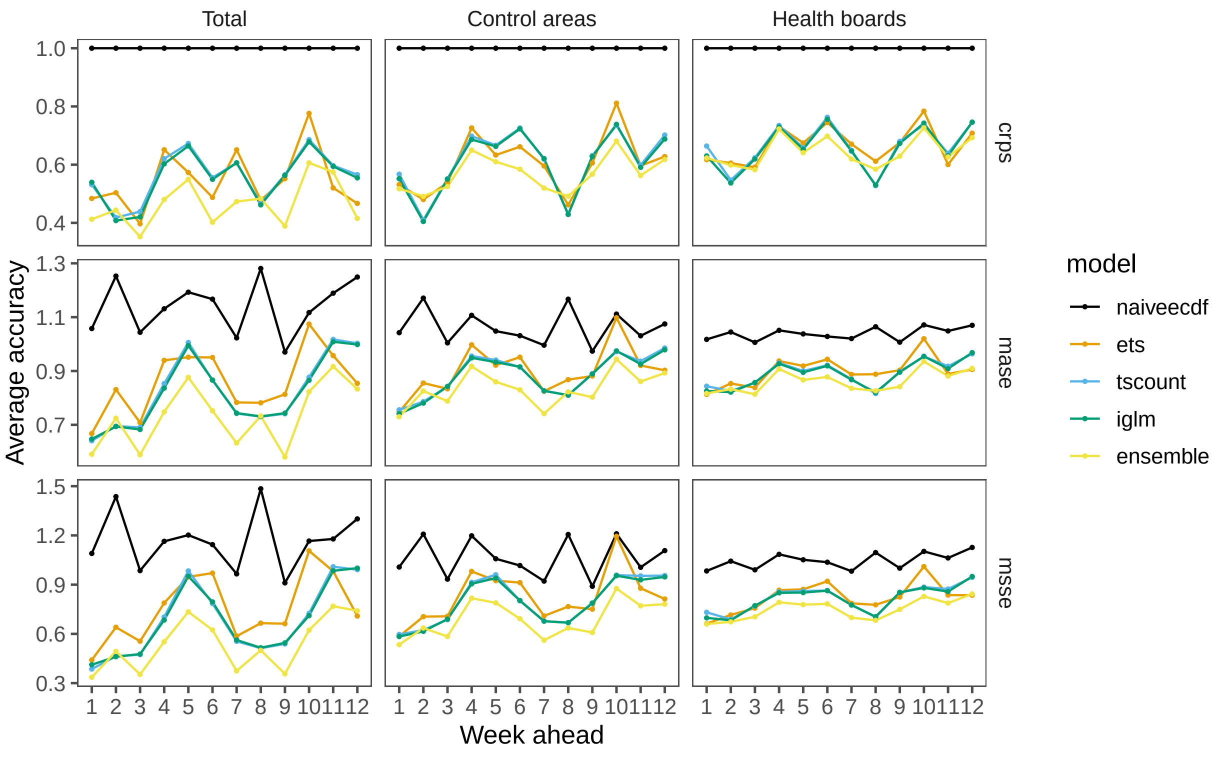


Figure 4: Average accuracy by week for 12 weeks. CRPS is relative to a naive ECDF. MASE and MSSE are relative to the corresponding values for the training set.

6. Conclusion

Forecasting problems at Emergency Medical Services have often inherent hierarchical and grouped structures. For example, looking at time series of arrival calls in a clinical desk service, verified incidents, or attended incidents in a country, they could be disaggregated by various attributes of interest. Total demand in the country could be disaggregated by region, then within each region by health board, within each health board, by station/hospital, and so on down to the post code area. Alternative structure may arise when attributes of interest are crossed rather than nested. For example, the total demand could be disaggrgated by priority (e.g Red, Amber, Green) or nature of incidents. It is also natural to have a mixed structures, for example the total demand could be disaggregated by priority and also by health board.

Despite the inherent hierarchical structure of the forecasting problem in EMS, the common practice is to produce point forecast for each time series independently. This practice may lead to the lack of coordination and possibly undesirable and conflicting outcomes. Furthermore, due to the asymmetric impact of resource allocation in this area, quantifying forecast uncertainty through probabilistic forecasts is also of value as it enables planners to manage associated risks. In this paper, we investigate the application of hierarchical forecasting methods for producing a probabilistic forecast of daily incident attended up to 84 days ahead, using different forecasting methods.

Our results indicates that forecast reconciliation can not only contribute to a more coordinated efforts through producing coherent forecasts when it comes to using forecasting to inform decisions in EMS, but also it can result in forecast accuracy improvement. Our proposed forecasting models combined with reconciliation approaches outperform the empirical distribution benchmark. We show that a substantial forecast improvement can be achieved at higher levels of hierarchical and grouped structure time series by applying forecast reconciliation methods. When a point forecast is of interest at the bottom level of series, we observe that reconciliation may not improve the forecast accuracy, if the bottom series are noisy and lack systematic patterns. However, forecast reconciliation may result in more forecast results for bottom series, if we are interested in the tails of forecast distribution rather than just center measures like mean (i.e. point forecast). Producing consistent forecast are also crucial for informing planning activities, we also demonstrate that proposed models produce consistent forecasts across all forecast horizons. Therefore, we recommend that forecast recociliation approaches to be adopted for routine use in EMS, whenever hierarchical and/or grouped time series data need to be forecasted. Moreover, we found that using an ensemble forecasting model, combining al models developed in this paper, instead of using each individualy, works remarkably well for our mixed hierarchical & grouped stucture.

Further research could investigate the practical benefits of probabilistic hierarchical forecasting in EMS. Linking forecasts with its utilities (response time, resource utilisation, etc) can offer an opportunity to maximise benefits through more holistic planning approach. While, we generated count attend incident for the base forecast usign Poisson regression models, however the reconciles forecast is snot yet count. This could be also an avenue for further research.

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