Introduction to the tsfeatures package

Yangzhuoran Yang and Rob J Hyndman

2019-04-16

tsfeatures

The R package *tsfeatures* provides methods for extracting various features from time series data.

Installation

The **stable** version on R CRAN is coming soon.

You can install the **development** version from Github with:

```
# install.packages("devtools")
devtools::install_github("robjhyndman/tsfeatures")
```

Usage

The function `tsfeatures()` computes a tibble of time series features from a list of time series.

```
mylist <- list(sunspot.year, WWWusage, AirPassengers, USAccDeaths)
tsfeatures(mylist)
#> # A tibble: 4 x 20
#>   frequency nperiods seasonal_period trend   spike linearity curvature
#>       <dbl>    <dbl>           <dbl> <dbl>   <dbl>     <dbl>     <dbl>
#> 1         1        0               1 0.125 2.10e-5      3.58      1.11
#> 2         1        0               1 0.985 3.01e-8      4.45      1.10
#> 3        12        1              12 0.989 2.12e-8     11.0       1.10
#> 4        12        1              12 0.796 9.67e-7     -2.13      2.85
#> # … with 13 more variables: e_acf1 <dbl>, e_acf10 <dbl>, entropy <dbl>,
#> #   x_acf1 <dbl>, x_acf10 <dbl>, diff1_acf1 <dbl>, diff1_acf10 <dbl>,
#> #   diff2_acf1 <dbl>, diff2_acf10 <dbl>, seasonal_strength <dbl>,
#> #   peak <dbl>, trough <dbl>, seas_acf1 <dbl>
```

The default functions that `tsfeatures` uses to compute features are `frequency`, `stl_features`, `entropy` and `acf_features`. Each of them can produce one or more features. Detailed information of features included in the *tsfeatures* package are described below. Functions from other packages, or user-defined functions, may also be used.

```
# Function from outside of tsfeatures package being used
is.monthly <- function(x){
  frequency(x) == 12
}
tsfeatures(mylist, features = "is.monthly")
#> # A tibble: 4 x 1
#>   is.monthly
#>        <dbl>
#> 1          0
#> 2          0
#> 3          1
#> 4          1
```

List of features

FEATURES			
acf_features	max_kl_shift	compengine	outlierinclude_mdrmd
arch_stat	max_level_shift	ac_9	sampenc

FEATURES			
crossing_points	max_var_shift	binarize_mean	sampen_first
entropy	nonlinearity	embed2_incircle	spreadrandomlocal_meantaul
flat_spots	pacf_features	firstmin_ac	std1st_der
heterogeneity	stability	firstzero_ac	trev_num
holt_parameters	stl_features	fluctanal_prop_r1	walker_propcross
hurst	unitroot_kpss	histogram_mode	
hw_parameters	unitroot_pp	localsimple_taures	
lumpiness		motiftwo_entro3	

entropy

The spectral `entropy` is the Shannon entropy

$$-\int_{-\pi}^{\pi} \hat{f}(\lambda) \log \hat{f}(\lambda) d\lambda,$$

where $\hat{f}(\lambda)$ is an estimate of the spectral density of the data. This measures the "forecastability" of a time series, where low values indicate a high signal-to-noise ratio, and large values occur when a series is difficult to forecast.

```
entropy(AirPassengers)
#>   entropy
#> 0.5876078
```

lumpiness and stability

`Stability` and `lumpiness` are two time series features based on tiled (non-overlapping) windows. Means or variances are produced for all tiled windows. Then `stability` is the variance of the means, while `lumpiness` is the variance of the variances.

```
stability(AirPassengers)
#> stability
#> 0.9330704
lumpiness(AirPassengers)
#>  lumpiness
#> 0.01924803
```

max_level_shift, max_var_shift and max_kl_shift

These three features compute features of a time series based on sliding (overlapping) windows. `max_level_shift` finds the largest mean shift between two consecutive windows. `max_var-shift` finds the largest variance shift between two consecutive windows. `max_kl_shift` finds the largest shift in Kulback-Leibler divergence between two consecutive windows. Each feature returns a vector of 2 values: the size of the shift, and the time index of the shift.

```
max_level_shift(AirPassengers)
#>  max_level_shift time_level_shift
#>             54.5            117.0
max_var_shift(AirPassengers)
#>  max_var_shift time_var_shift
#>       2342.152        107.000
max_kl_shift(AirPassengers)
#>  max_kl_shift time_kl_shift
#>     0.1210444   122.0000000
```

crossing_points

`crossing points` are defined as the number of times a time series crosses the mean line.

```
crossing_points(AirPassengers)
#> crossing_points
#>               7
```

flat_spots

`flat_spots` are computed by dividing the sample space of a time series into ten equal-sized intervals, and computing the maximum run length within any single interval.

```
flat_spots(AirPassengers)
#> flat_spots
#>         18
```

hurst

We use a measure of the long-term memory of a time series (`hurst`), computed as 0.5 plus the maximum likelihood estimate of the fractional differencing order d given by Haslett & Raftery (1989). We add 0.5 to make it consistent with the Hurst coefficient. Note that the fractal dimension can be estimated as $D = 2 - \text{hurst}$.

```
hurst(AirPassengers)
#>     hurst
#> 0.9992466
```

unitroot_kpss and unitroot_pp

`unitroot_kpss` is a vector comprising the statistic for the KPSS unit root test with linear trend and lag one, and `unitroot_pp` is the statistic for the “Z-alpha” version of PP unit root test with constant trend and lag one.

```
unitroot_kpss(AirPassengers)
#> [1] 2.739474
```

```
unitroot_pp(AirPassengers)
#> [1] -6.565597
```

stl_features

`stl_features` Computes various measures of trend and seasonality of a time series based on an STL decomposition. The `mstl` function is used to do the decomposition.

`nperiods` is the number of seasonal periods in the data (determined by the frequency of observation, not the observations themselves) and set to 1 for non-seasonal data. `seasonal_period` is a vector of seasonal periods and set to 1 for non-seasonal data.

The size and location of the peaks and troughs in the seasonal component are used to compute strength of peaks (`peak`) and strength of trough (`trough`).

The rest of the features are modifications of features used in Kang, Hyndman & Smith-Miles (2017). We extend the STL decomposition approach (Cleveland et al.1990) to handle multiple seasonalities. Thus, the decomposition contains a trend, up to M seasonal components and a remainder component:

$$x_t = f_t + s_{1,t} + \cdots + s_{M.t} + e_t,$$

where f_t is the smoothed trend component, $s_{i,t}$ is the ith seasonal component and e_t is a remainder component. The components are estimated iteratively. Let $s_{i,t}^{(k)}$ be the estimate of s_i, t at the kth iteration, with initial values given as $s_{i,t}^{(0)} = 0$. The we apply an STL decomposition to $x_t - \sum_{j \neq 1}^{j=1}{}^{M} s_{j,t}^{k-1}$ to obtained updated estimates $s_{i,t}^{(k)}$ for $k = 1, 2, \ldots$. In practice, this converges quickly and only two iterations are required. To allow the procedure to be applied automatically, we set the seasonal window span for STL to be 21 in all cases. For a non-seasonal time series, we simply estimate $x_t = f_t + e_t$ where f_t is computed using Friedman's "super smoother" (Friedman 1984).

Strength of trend (`trend`) and strength of seasonality (`seasonal.strength`) are defined as

$$\text{trend} = 1 - \frac{\text{Var}(e_t)}{\text{Var}(f_t + e_t)} \quad \text{and} \quad \text{seasonal.strength} = 1 - \frac{\text{Var}(e_t)}{\text{Var}(s_{i,t} + e_t)}.$$

If their values are less than 0, they are set to 0, while values greater than 1 are set to 1. For non-seasonal time series `seasonal.strength` is 0. For seasonal time series, `seasonal.strength` is an M-vector, where M is the number of periods. This is analogous to the way the strength of trend and seasonality were defined in Wang, Smith & Hyndman (2006), Hyndman, Wang & Laptev (2015) and Kang, Hyndman & Smith-Miles (2017).

`spike` measures the "spikiness" of a time series, and is computed as the variance of the leave-one-out variances of the remainder component e_t.

`linearity` and `curvature` measures the linearity and curvature of a time series calculated based on the coefficients of an orthogonal quadratic regression.

We compute the autocorrelation function of e_t, and `e_acf1` and `e_acf10` contain the first autocorrelation coefficient and the sum of the first ten squared autocorrelation coefficients.

```
stl_features(AirPassengers)
#>          nperiods   seasonal_period             trend             spike
#>         1.0000000        12.0000000         0.9886426         4.3879083
#>         linearity         curvature            e_acf1           e_acf10
#>      1325.3880968       131.3883022         0.5515403         1.0783947
#> seasonal_strength              peak            trough
#>         0.9252932         7.0000000        11.0000000
```

acf_features

We compute the autocorrelation function of the series, the differenced series, and the twice-differenced series. `acf_features` produces a vector comprising the first autocorrelation coefficient in each case, and the sum of squares of the first 10 autocorrelation coefficients in each case.

```
acf_features(AirPassengers)
#>      x_acf1     x_acf10  diff1_acf1 diff1_acf10  diff2_acf1 diff2_acf10
#>   0.9480473   5.6700871   0.3028553   0.4088376  -0.1910059   0.2507803
#>   seas_acf1
#>   0.7603950
```

pacf_features

We compute the partial autocorrelation function of the series, the differenced series, and the second-order differenced series. Then `pacf_features` produces a vector comprising the sum of squares of the first 5 partial autocorrelation coefficients in each case.

```
pacf_features(AirPassengers)
#>      x_pacf5 diff1x_pacf5 diff2x_pacf5    seas_pacf
#>    0.9670971    0.2122454    0.2476615   -0.1354311
```

holt_parameters and hw_parameters

`holt_parameters` Estimate the smoothing parameter for the level-alpha and the smoothing parameter for the trend-beta of Holt's linear trend method. `hw_parameters` considers additive seasonal trend: ETS(A,A,A) model, returning a vector of 3 values: alpha, beta and gamma.

```
holt_parameters(AirPassengers)
#>        alpha         beta
#> 0.9998999962 0.0001000071
hw_parameters(AirPassengers)
#>        alpha         beta        gamma
#> 0.9934803629 0.0001911792 0.0005800325
```

heterogeneity

The `heterogeneity` features measure the heterogeneity of the time series. First, we pre-whiten the time series to remove the mean, trend, and autoregressive (AR) information (Barbour & Parker 2014). Then we fit a $GARCH(1, 1)$ model to the pre-whitened time series, x_t, to measure for autoregressive conditional heteroskedasticity (ARCH) effects. The residuals from this model, z_t, are also measured for ARCH effects using a second $GARCH(1, 1)$ model.

- `arch_acf` is the sum of squares of the first 12 autocorrelations of $\{x_t^2\}$.
- `garch_acf` is the sum of squares of the first 12 autocorrelations of $\{z_t^2\}$.
- `arch_r2` is the R^2 value of an AR model applied to $\{x_t^2\}$.
- `garch_r2` is the R^2 value of an AR model applied to $\{z_t^2\}$.

The statistics obtained from $\{x_t^2\}$ are the ARCH effects, while those from $\{z_t^2\}$ are the GARCH effects. Note that the two R^2 values are used in the Lagrange-multiplier test of Engle (1982), and the sum of squared autocorrelations are used in the Ljung-Box test proposed by Ljung & Box (1978).

```
heterogeneity(AirPassengers)
#>   arch_acf garch_acf   arch_r2   garch_r2
#> 0.2295944 0.2277382 0.2106310 0.2101623
```

nonlinearity

The `nonlinearity` coefficient is computed using a modification of the statistic used in Teräsvirta's nonlinearity test. Teräsvirta's test uses a statistic $X^2 = T\log(\mathrm{SSE1}/\mathrm{SSE0})$ where SSE1 and SSE0 are the sum of squared residuals from a nonlinear and linear autoregression respectively. This is non-ergodic, so instead, we define it as $10X^2/T$ which will converge to a value indicating the extent of nonlinearity as $T \to \infty$. This takes large values when the series is nonlinear, and values around 0 when the series is linear.

```
nonlinearity(AirPassengers)
#> nonlinearity
#>    0.4238969
```

arch_stat

`arch_stat` Computes a statistic based on the Lagrange Multiplier (LM) test of Engle (1982) for autoregressive conditional heteroscedasticity (ARCH). The statistic returned is the R^2 value of an autoregressive model of order specified as lags applied to x^2.

```
arch_stat(AirPassengers)
#>   ARCH.LM
#> 0.9171945
```

compengine feature set

`compengine` calculate the features that have been used in the CompEngine database, using a method introduced in package `kctsa`.

The features involved can be grouped as autocorrelation, prediction, stationarity, distribution, and scaling, which can be computed using `autocorr_features`, `pred_features`, `station_features`, `dist_features`, and `scal_features`.

```
comp <- compengine(AirPassengers)
knitr::kable(comp)
```

	x
embed2_incircle_1	0.0000000
embed2_incircle_2	0.0000000
ac_9	0.6709483

	x
firstmin_ac	8.0000000
trev_num	-4902.1958042
motiftwo_entro3	1.1302445
walker_propcross	0.2027972
localsimple_mean1	2.0000000
localsimple_lfitac	3.0000000
sampen_first	Inf
std1st_der	33.7542815
spreadrandomlocal_meantaul_50	12.7800000
spreadrandomlocal_meantaul_ac2	38.9400000
histogram_mode_10	125.0000000
outlierinclude_mdrmd	0.4166667
fluctanal_prop_r1	0.7692308

embed2_incircle

`embed2_incircle` gives proportion of points inside a given circular boundary in a 2-d embedding space.

```
embed2_incircle(AirPassengers, boundary = 1e5)
#> [1] 0.2608696
```

ac_9

`ac_9` is just the autocorrelation at lag 9, included here for completion and consistency.

```
ac_9(AirPassengers)
#> [1] 0.6709483
```

firstmin_ac

`firstmin_ac` returns the time of first minimum in the autocorrelation function.

```
firstmin_ac(AirPassengers)
#> [1] 8
```

firstzero_ac

`firstzero_ac` returns the first zero crossing of the autocorrelation function.

```
firstzero_ac(AirPassengers)
#> [1] 52
```

trev_num

`trev_num` returns the numerator of the trev function of a time series, a normalized nonlinear autocorrelation. The time lag is set to 1.

```
trev_num(AirPassengers)
#> [1] -4902.196
```

motiftwo_entro3

Local motifs in a binary symbolization of the time series. Coarse-graining is performed. Time-series values above its mean are given 1, and those below the mean are 0. `motiftwo_entro3` returns the entropy of words in the binary alphabet of length 3.

```
motiftwo_entro3(AirPassengers)
#> [1] 1.130244
```

binarize_mean

`binarize_mean` converts an input vector into a binarized version. Time-series values above its mean are given 1, and those below the mean are 0.

```
str(binarize_mean(AirPassengers))
#>  num [1:144] 0 0 0 0 0 0 0 0 0 0 ...
```

walker_propcross

Simulates a hypothetical walker moving through the time domain. The hypothetical particle (or ‘walker’) moves in response to values of the time series at each point. The walker narrows the gap between its value and that of the time series by 10. `walker_propcross` returns the fraction of time series length that walker crosses time series.

```
walker_propcross(AirPassengers)
#> [1] 0.2027972
```

localsimple_taures

Simple predictors using the past trainLength values of the time series to predict its next value. `localsimple_taures` returns the first zero crossing of the autocorrelation function of the residuals from this Simple local time-series forecasting.

```
localsimple_taures(AirPassengers)
#> [1] 2
```

sampen_first and sampenc

`sampen_first` returns the first Sample Entropy of a time series where the embedding dimension is set to 5 and the threshold is set to 0.3. `sampenc` is the underlying function to calculate the first sample entropy with optional dimension and threshold settings.

```
sampen_first(AirPassengers)
#> [1] Inf
sampenc(AirPassengers, M = 5, r = 0.3)
#> [1] Inf
```

std1st_der

`std1st_der` returns the standard deviation of the first derivative of the time series.

```
std1st_der(AirPassengers)
#> [1] 33.75428
```

spreadrandomlocal_meantaul

100 time-series segments of length l are selected at random from the time series and the mean of the first zero-crossings of the autocorrelation function in each segment is calculated using `spreadrandomlocal_meantaul`.

```
spreadrandomlocal_meantaul(AirPassengers)
#> [1] 13.28
```

histogram_mode

`histogram_mode` measures the mode of the data vector using histograms with a given number of bins (default to 10) as suggestion.

```
histogram_mode(AirPassengers, numBins = 10)
#> [1] 125
```

outlierinclude_mdrmd

`outlierinclude_mdrmd` measures the median as more and more outliers are included in the calculation according to a specified rule, of outliers being furthest from the mean.

The threshold for including time-series data points in the analysis increases from zero to the maximum deviation, in increments of 0.01*sigma (by default), where sigma is the standard deviation of the time series.

At each threshold, proportion of time series points included and median are calculated, and outputs from the algorithm measure how these statistical quantities change as more extreme points are included in the calculation.

`outlierinclude_mdrmd` essentially returns the median of the median of range indices.

```
outlierinclude_mdrmd(AirPassengers)
#> [1] 0.4166667
```

fluctanal_prop_r1

`fluctanal_prop_r1` implements fluctuation analysis. It fits a polynomial of order 1 and then returns the range. The order of fluctuations is 2, corresponding to root mean square fluctuations.

```
fluctanal_prop_r1(AirPassengers)
#> [1] 0.7692308
```

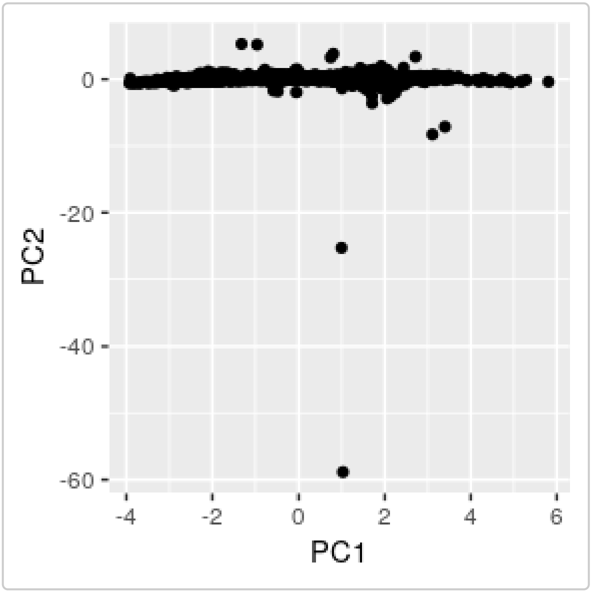
Reproducing papers

Hyndman, Wang and Laptev (ICDM 2015)

Here we replicate the analysis in Hyndman, Wang & Laptev (ICDM 2015). However, note that crossing_points, peak and trough are defined differently in the *tsfeatures* package than in the Hyndman et al (2015) paper. Other features are the same.

```
library(tsfeatures)
library(dplyr)

yahoo <- yahoo_data()


hwl <- bind_cols(
         tsfeatures(yahoo,
           c("acf_features","entropy","lumpiness",
             "flat_spots","crossing_points")),
         tsfeatures(yahoo,"stl_features", s.window='periodic', robust=TRUE),
         tsfeatures(yahoo, "max_kl_shift", width=48),
         tsfeatures(yahoo,
           c("mean","var"), scale=FALSE, na.rm=TRUE),
         tsfeatures(yahoo,
           c("max_level_shift","max_var_shift"), trim=TRUE)) %>%
  select(mean, var, x_acf1, trend, linearity, curvature,
         seasonal_strength, peak, trough,
         entropy, lumpiness, spike, max_level_shift, max_var_shift, flat_spots,
         crossing_points, max_kl_shift, time_kl_shift)


# 2-d Feature space
library(ggplot2)
hwl_pca <- hwl %>%
  na.omit() %>%
  prcomp(scale=TRUE)
```

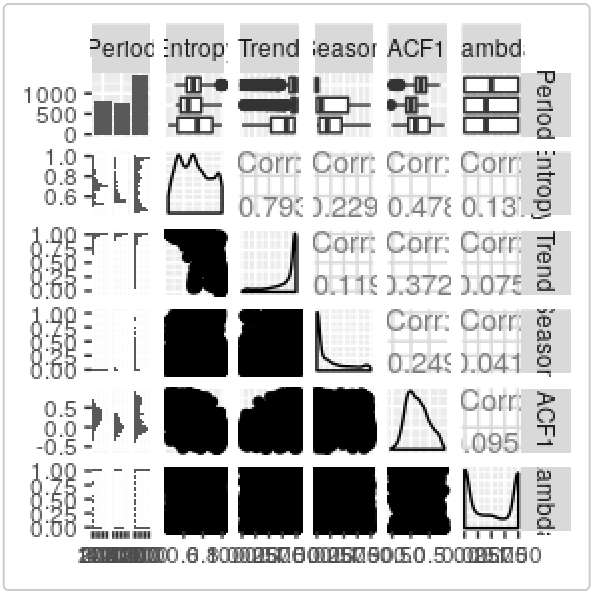
```
hwl_pca$x %>%
  as_tibble() %>%
  ggplot(aes(x=PC1, y=PC2)) +
    geom_point()
```



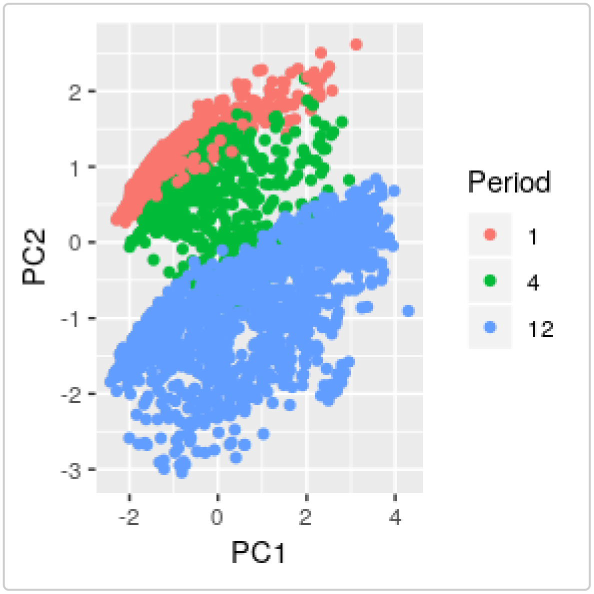
Kang, Hyndman & Smith-Miles (IJF 2017)

Compute the features used in Kang, Hyndman & Smith-Miles (IJF 2017). Note that the trend and ACF1 are computed differently for non-seasonal data in the *tsfeatures* package than in the Kang et al (2017). `tsfeatures` uses `mstl` which uses `supsmu` for the trend calculation with non-seasonal data, whereas Kang et al used a penalized regression spline computed using `mgcv` instead. Other features are the same.

```
library(tsfeatures)
library(dplyr)
library(tidyr)
library(forecast)

M3data <- purrr::map(Mcomp::M3,
  function(x) {
      tspx <- tsp(x$x)
      ts(c(x$x,x$xx), start=tspx[1], frequency=tspx[3])
  })
khs_stl <- function(x,...) {
  lambda <- BoxCox.lambda(x, lower=0, upper=1, method='loglik')
  y <- BoxCox(x, lambda)
  c(stl_features(y, s.window='periodic', robust=TRUE, ...), lambda=lambda)
}


khs <- bind_cols(
  tsfeatures(M3data, c("frequency", "entropy")),
  tsfeatures(M3data, "khs_stl", scale=FALSE)) %>%
  select(frequency, entropy, trend, seasonal_strength, e_acf1, lambda) %>%
  replace_na(list(seasonal_strength=0)) %>%
  rename(
    Frequency = frequency,
    Entropy = entropy,
    Trend = trend,
    Season = seasonal_strength,
    ACF1 = e_acf1,
    Lambda = lambda) %>%
  mutate(Period = as.factor(Frequency))


# Fig 1 of paper
khs %>%
  select(Period, Entropy, Trend, Season, ACF1, Lambda) %>%
  GGally::ggpairs()
```



```
  # 2-d Feature space (Top of Fig 2)
  khs_pca <- khs %>%
    select(-Period) %>%
    prcomp(scale=TRUE)
  khs_pca$x %>%
    as_tibble() %>%
    bind_cols(Period=khs$Period) %>%
    ggplot(aes(x=PC1, y=PC2)) +
      geom_point(aes(col=Period))
```



Resources

Hyndman, R. J., Wang, E., & Laptev, N. (2015, November). Large-scale unusual time series detection. In Data Mining Workshop (ICDMW), 2015 IEEE International Conference on (pp. 1616-1619). IEEE.

Kang, Y., Hyndman, R. J., & Li, F. (2018). Efficient generation of time series with diverse and controllable characteristics.

Haslett, J., & Raftery, A. E. (1989). Space-time modelling with long-memory dependence: Assessing Ireland's wind power resource. Applied Statistics, 1-50.

Barbour, A. J., & Parker, R. L. (2014). psd: Adaptive, sine multitaper power spectral density estimation for R. Computers & Geosciences, 63, 1-8.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica: Journal of the Econometric Society, 987-1007.

Ljung, G. M., & Box, G. E. (1978). On a measure of lack of fit in time series models. Biometrika, 65(2), 297-303.

Kang, Y., Hyndman, R. J., & Smith-Miles, K. (2017). Visualising forecasting algorithm performance using time series instance spaces. International Journal of Forecasting, 33(2), 345-358.

Cleveland, R. B., Cleveland, W. S., McRae, J. E., & Terpenning, I. (1990). STL: A Seasonal-Trend Decomposition. Journal of Official Statistics, 6(1), 3-73.

Friedman, JH (1984). *A variable span scatterplot smoother*. Technical Report 5. Laboratory for Computational Statistics, Stanford University.

Wang, X, KA Smith & RJ Hyndman (2006). Characteristic-based clustering for time series data. Data Mining and Knowledge Discovery 13(3), 335–364.

Kang, Y., Hyndman, R. J., & Smith-Miles, K. (2017). Visualising forecasting algorithm performance using time series instance spaces. International Journal of Forecasting, 33(2), 345-358.

License