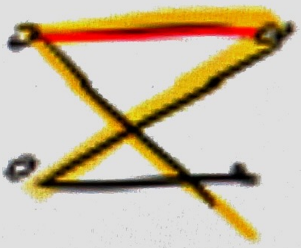


$K_{2,2}$

B

1.3. Matroid

Def.: <u>Matroid</u>

Let $(E, \mathcal{F})$ be an ISS with

M3) IF: $\forall X, Y \in \mathcal{F}$ & $|X| > |Y|$

THEN: $\exists x \in X \setminus Y$ such that

$$Y \cup \{x\} \in \mathcal{F}.$$

$(E, \mathcal{F})$ fulfills M1 – M3 it is called a <u>Matroid</u>.

<u>Examples:</u>

- E set of vectors in K^n

$\mathcal{F} = \{$lin. indep subsets of $E\}$ ✓

- Set of forests in a Graph $G(V,E)$ as $\mathcal{F}$. ✓

Theorem:

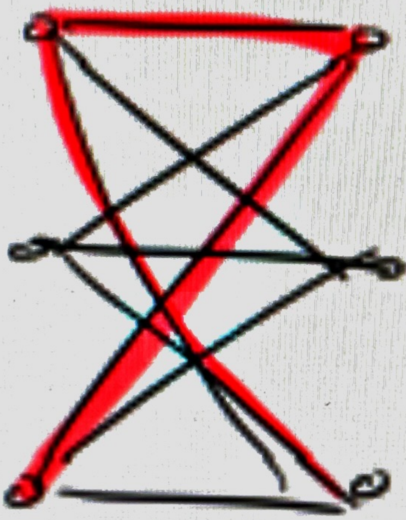
Let $(E,\mathcal{F})$ be an ISS.
Then the following three conditions are equivalent:

M3) $\forall X,Y \in \mathcal{F}$ & $|X| > |Y| \Rightarrow \exists x \in X \setminus Y : Y \cup \{x\} \in \mathcal{F}$.

M3') $\forall X,Y \in \mathcal{F}$ & $|X| = |Y|+1 \Rightarrow \exists x \in X \setminus Y : Y \cup \{x\} \in \mathcal{F}$

M3") $\forall X \subseteq E$: all Bases B of X have the same length $|B|$.

Example:



Proof: $\mathcal{M} \Leftrightarrow \mathcal{M}' \Leftrightarrow \mathcal{M}''$

Ring proof: $\mathcal{M} \underset{1.}{\Rightarrow} \mathcal{M}' \underset{2.}{\Rightarrow} \mathcal{M}'' \underset{3.}{\Rightarrow} \mathcal{M}$

1.: trivial

2.: Let $X \subset E$ & $A, B \subseteq X$

& Bases of X.

$\Rightarrow A, B \in \mathcal{F}$ if $|A| > |B|$

$\Rightarrow \exists \hat{A} \subseteq A : |\hat{A}| = |B| + 1$