3.
Preparing the Data

Training Data

- **Goal**: forecast tomorrow's ridership based on the ridership of the past 8 weeks of data (56 days).
- The **inputs**: sequences containing 56 values from time steps $t-55$ to t.
  - For each input sequence, the model will output a single value: the forecast for time step $(t+1)$.
- We use every 56-day window from the past as training data, and the target for each window will be the value immediately following it.
- `tf.keras.utils.timeseries_dataset_from_array`() is a utility function in Keras which takes a time series as input, and builds a `tf.data.Dataset` containing all the windows of the desired length, and their corresponding targets.

Preparing Training Data

```
import tensorflow as tf

my_series = [0, 1, 2, 3, 4, 5]
my_dataset = tf.keras.utils.timeseries_dataset_from_array(
    my_series,
    targets=my_series[3:],  # the targets are 3 steps into the future
    sequence_length=3,
    batch_size=2
)
list(my_dataset)
```

```
[(<tf.Tensor: shape=(2, 3), dtype=int32, numpy=
  array([[0, 1, 2],
         [1, 2, 3]], dtype=int32)>,
  <tf.Tensor: shape=(2,), dtype=int32, numpy=array([3, 4], dtype=int32)>),
 (<tf.Tensor: shape=(1, 3), dtype=int32, numpy=array([[2, 3, 4]], dtype=int32)>,
  <tf.Tensor: shape=(1,), dtype=int32, numpy=array([5], dtype=int32)>)]
```

Training/Validation/Test Split

- Scale down the data by a factor of one million, to ensure the values are near the 0–1 range, and split it into training period, a validation period, and a test period:

```
rail_train = df["rail"]["2016-01":"2018-12"] / 1e6
rail_valid = df["rail"]["2019-01":"2019-05"] / 1e6
rail_test = df["rail"]["2019-06":] / 1e6
```

- When dealing with time series, you generally want to split across time.
  - In some cases you may be able to split along other dimensions, which will give you a longer time period to train on.
  - E.g., if you have data about the financial health of 10000 companies from 2001 to 2019, you might split this data across the different companies.

Preparing Data

```
seq_length = 56
train_ds = tf.keras.utils.timeseries_dataset_from_array(
    rail_train.to_numpy(),
    targets=rail_train[seq_length:],
    sequence_length=seq_length,
    batch_size=32,
    shuffle=True,
    seed=42
)
valid_ds = tf.keras.utils.timeseries_dataset_from_array(
    rail_valid.to_numpy(),
    targets=rail_valid[seq_length:],
    sequence_length=seq_length,
    batch_size=32
)
```

4.
Forecasting a Time Series

Forecasting Using a Linear Model

- We try a basic linear model with the Huber loss, which usually works better than minimizing the MAE directly. We also use early stopping.

```
model = tf.keras.Sequential([
    tf.keras.layers.Dense(1, input_shape=[seq_length])
])
early_stopping_cb = tf.keras.callbacks.EarlyStopping(
    monitor="val_mae", patience=50, restore_best_weights=True)
opt = tf.keras.optimizers.SGD(learning_rate=0.02, momentum=0.9)
model.compile(loss=tf.keras.losses.Huber(), optimizer=opt, metrics=["mae"])
history = model.fit(train_ds, validation_data=valid_ds, epochs=500,
                    callbacks=[early_stopping_cb])
```

- This model reaches a validation MAE, better than naive forecasting, but worse than the SARIMA model.

Forecasting Using a Simple RNN

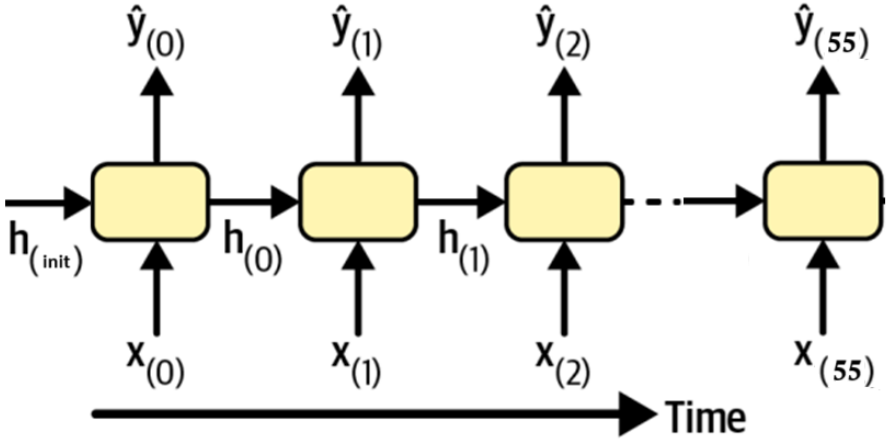
- The most basic RNN, contains a single recurrent layer with just one recurrent neuron:

```
model = tf.keras.Sequential([
    tf.keras.layers.SimpleRNN(1, input_shape=[None, 1])
])
```

- All recurrent layers in Keras expect 3D inputs of shape [*batch size*, *time steps*, *dimensionality*], where *dimensionality* is 1 for univariate time series and more for multivariate time series.
- The `input_shape` ignores the first dimension (i.e., the batch size).
- Recurrent layers can accept input sequences of any length, we can set the second dimension to `None`, which means “any size”.

Forecasting Using a Simple RNN

- The activation function is `tanh` by default.
- Recurrent layers in Keras only return the final output. To make them return one output per time step, set `return_sequences=True`.
- We compile, train, and evaluate the model, and find that it's no good at all: its validation MAE is greater than 100,000!



What went wrong?

- The model only has a single recurrent neuron, so the only data it can use to make a prediction at each time step is the input value at the current time step and the output value from the previous time step.
  - The RNN's memory is extremely limited: it's just a single number, its previous output.
  - The model only has three parameters (two weights plus a bias term).

- The time series contains values from 0 to about 1.4, but since the default activation function is `tanh`, the recurrent layer can only output values between –1 and +1.
  - There's no way it can predict values between 1.0 and 1.4.

A Larger RNN Layer

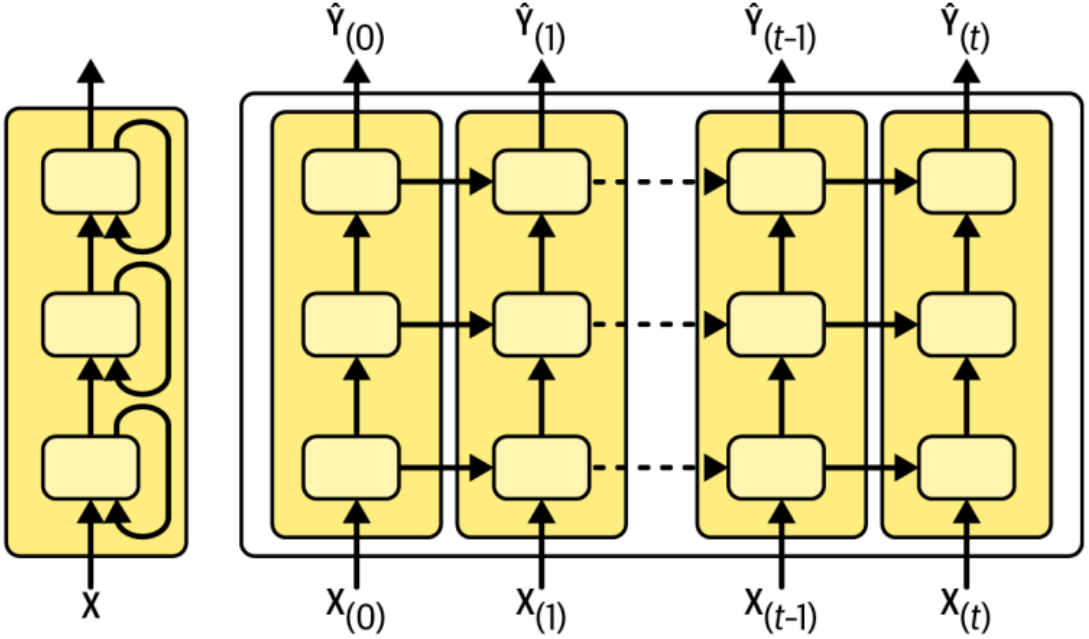
- Create a larger recurrent layer, with 32 recurrent neurons.
- A dense output layer to project the final output from 32 dimensions down to 1.

```
univar_model = tf.keras.Sequential([
    tf.keras.layers.SimpleRNN(32, input_shape=[None, 1]),
    tf.keras.layers.Dense(1)  # no activation function by default
])
```

- Its validation MAE reaches 27,703, better than the SARIMA model.
- We only normalized the time series, without removing trend and seasonality.
  - To get the best performance, you may want to try making the time series more stationary; e.g. using differencing.

Forecasting Using a Deep RNN

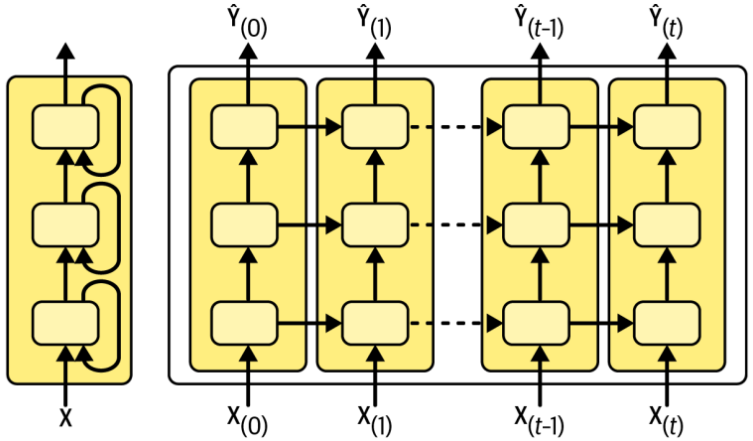
- To implement a deep RNN, stack `SimpleRNN` layers.



Forecasting Using a Deep RNN

```
deep_model = tf.keras.Sequential([
    tf.keras.layers.SimpleRNN(32, return_sequences=True, input_shape=[None, 1]),
    tf.keras.layers.SimpleRNN(32, return_sequences=True),
    tf.keras.layers.SimpleRNN(32),
    tf.keras.layers.Dense(1)
])
```

- If you train and evaluate this model, you will find that it reaches an MAE of about 31,211.
  - This RNN is too large for our task.



Forecasting Multivariate Time Series

```
df_mulvar = df[["bus", "rail"]] / 1e6  # use both bus & rail series as input
df_mulvar["next_day_type"] = df["day_type"].shift(-1)  # we know tomorrow's type
df_mulvar = pd.get_dummies(df_mulvar)  # one-hot encode the day type
df_mulvar.head()
```

date	bus	rail	next_day_type_A	next_day_type_U	next_day_type_W
2001-01-01	0.297192	0.126455	False	False	True
2001-01-02	0.780827	0.501952	False	False	True
2001-01-03	0.824923	0.536432	False	False	True
2001-01-04	0.870021	0.550011	False	False	True
2001-01-05	0.890426	0.557917	True	False	False

Train/Validation/Test Split

```
mulvar_train = df_mulvar["2016-01":"2018-12"]
mulvar_valid = df_mulvar["2019-01":"2019-05"]
mulvar_test = df_mulvar["2019-06":]
```

```
train_mulvar_ds = tf.keras.utils.timeseries_dataset_from_array(
    mulvar_train.to_numpy(),  # use all 5 columns as input
    targets=mulvar_train["rail"][seq_length:],  # forecast only the rail series
    sequence_length=seq_length,
    batch_size=32,
    shuffle=True,
    seed=42
)
valid_mulvar_ds = tf.keras.utils.timeseries_dataset_from_array(
    mulvar_valid.to_numpy(),
    targets=mulvar_valid["rail"][seq_length:],
    sequence_length=seq_length,
    batch_size=32
)
```

Create the Model

```
mulvar_model = tf.keras.Sequential([
    tf.keras.layers.SimpleRNN(32, input_shape=[None, 5]),
    tf.keras.layers.Dense(1)
])
```

- It reaches a validation MAE of 22,062.
- Using a single model for multiple related tasks often results in better performance than using a separate model for each task.
  - Features learned for one task may be useful for the other tasks.
  - Having to perform well across multiple tasks prevents the model from overfitting (it's a form of regularization).

5.
Handling Long Sequences

The Unstable Gradients Problem in RNNs

- Many of the previous tricks can also be used for RNNs: good parameter initialization, faster optimizers, dropout, …
- Non-saturating activation functions (e.g., ReLU) may not help. Why?
  - Suppose gradient descent updates the weights in a way that increases the outputs slightly at the first time step.
  - Because the same weights are used at every time step, the outputs at the second time step may also be slightly increased, and those at the third, and so on until the outputs explode—and a non-saturating activation function does not prevent that.
- You can reduce this risk by using a smaller learning rate, or you can use a saturating activation function like the hyperbolic tangent.

The Unstable Gradients Problem in RNNs

- Batch normalization cannot be used as efficiently with RNNs as with deep feedforward nets.
  - You cannot use it between time steps, only between recurrent layers.
  - You can apply BN between layers by adding a `BatchNormalization` layer before each recurrent layer, but it will slow down training, and it may not help much.

- Another form of normalization often works better with RNNs is *layer normalization*.
  - It is very similar to batch normalization, but instead of normalizing across the batch dimension, layer normalization normalizes across the features dimension.

Short-Term Memory Problem

- Due to the transformations that the data goes through when traversing an RNN, some information is lost at each time step.
  - After a while, the RNN's state contains virtually no trace of the first inputs.
- To tackle this problem, the *long short-term memory* (LSTM) cell was proposed in 1997.
- The LSTM cell can be used very much like a basic cell, except it will perform much better; training will converge faster, and it will detect longer-term patterns in the data:

```
lstm_model = tf.keras.models.Sequential([
    tf.keras.layers.LSTM(32, return_sequences=True, input_shape=[None, 5]),
    tf.keras.layers.Dense(14)
])
```

LSTM Cell

$\mathbf{h}_{(t)}$: short-term state

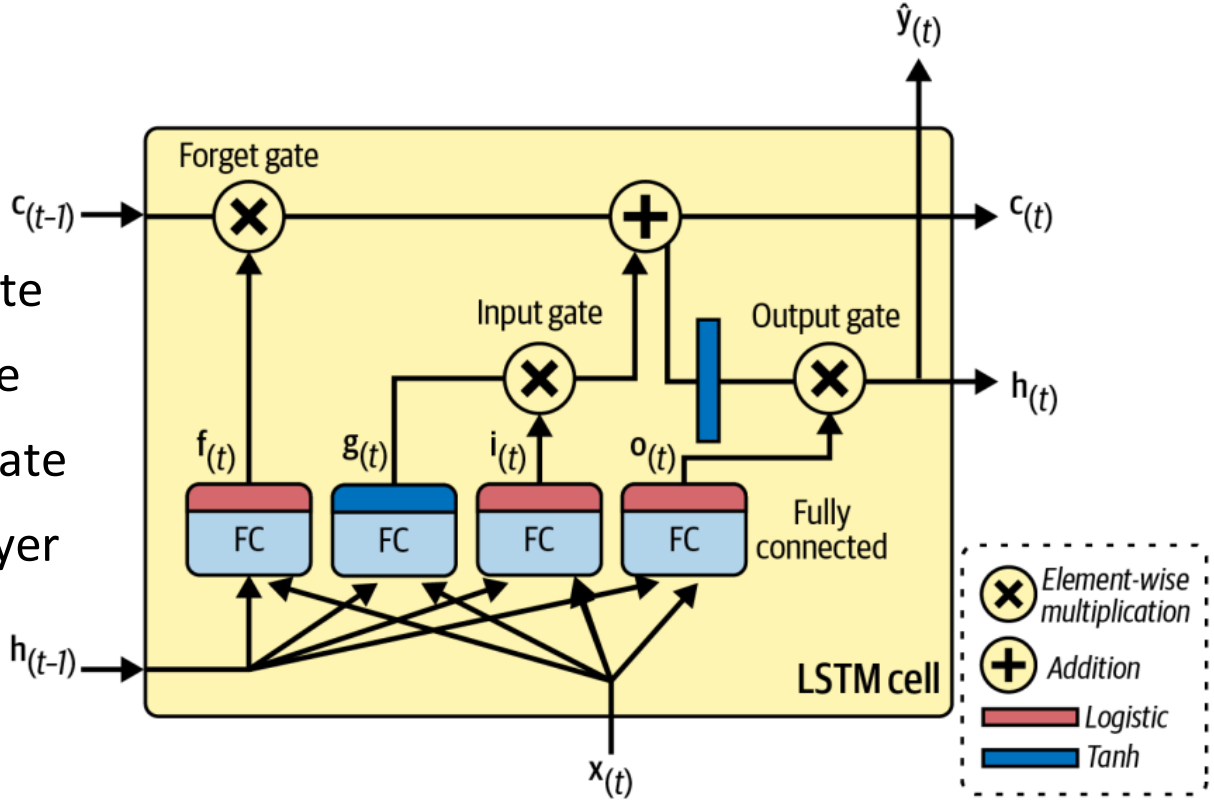
$\mathbf{c}_{(t)}$: long-term state

$\mathbf{f}_{(t)}$: controller of forget gate

$\mathbf{i}_{(t)}$: controller of input gate

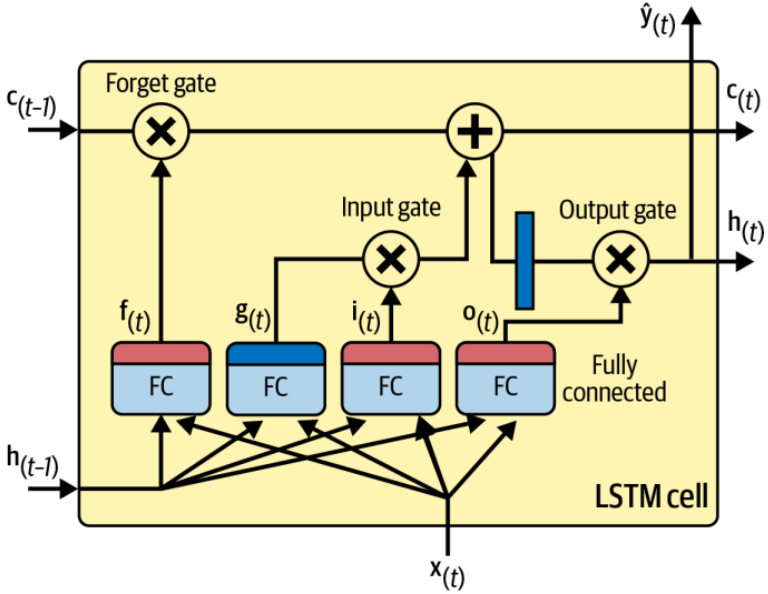
$\mathbf{o}_{(t)}$: controller of output gate

$\mathbf{g}_{(t)}$: output of the main layer



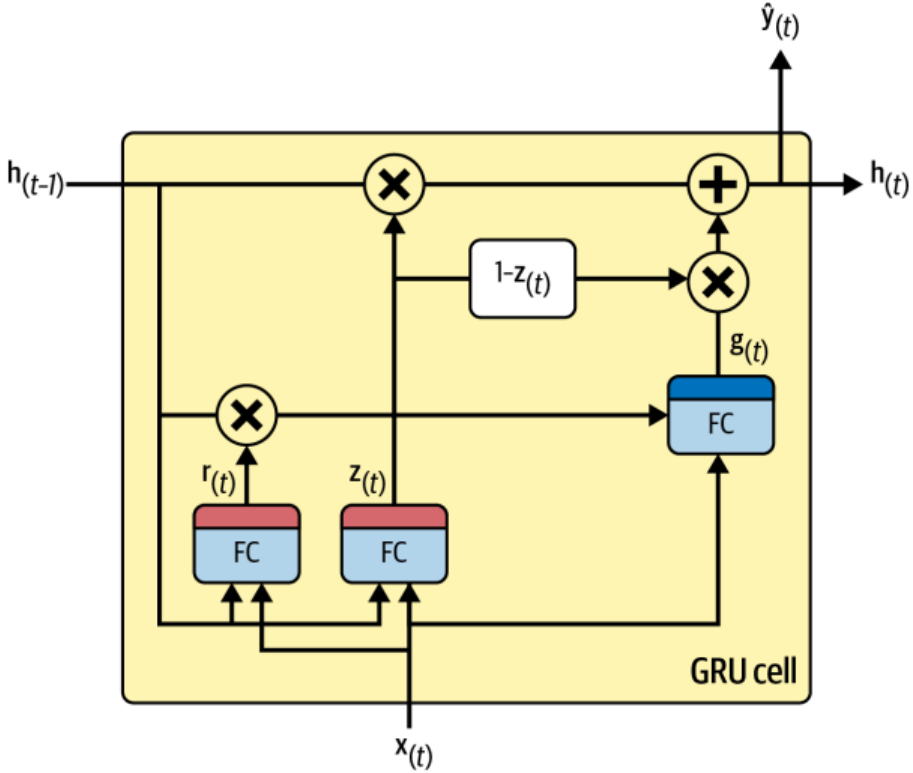
Where new memories come from?

- The main FC layer has the usual role of analyzing the current inputs $\mathbf{x}_{(t)}$ and the previous short-term state $\mathbf{h}_{(t-1)}$.
- The 3 other layers are *gate controllers*:
  - *forget gate* controls which parts of the long-term state should be erased.
  - *input gate* controls which parts of $\mathbf{g}_{(t)}$. should be added to the long-term state.
  - *output gate* controls which parts of the long-term state should be read and output at this time step.



GRU Cell

- The *gated recurrent unit* (GRU) cell is a simplified version of the LSTM cell:
  - Both state vectors are merged into a single vector $\mathbf{h}_{(t)}$.
  - A single gate controller $\mathbf{z}_{(t)}$ controls both the forget gate and the input gate.
  - There is no output gate.




```
gru_model = tf.keras.Sequential([
    tf.keras.layers.GRU(32, return_sequences=True, input_shape=[None, 5]),
    tf.keras.layers.Dense(14)
])
```