Online EV Scheduling Algorithms for Adaptive Charging Networks with Global Peak Constraints

Bahram Alinia, Mohammad H. Hajiesmaili, Zachary J. Lee, Noel Crespi, and Enrique Mallada

Abstract—This paper tackles online scheduling of electric vehicles (EVs) in an adaptive charging network (ACN) with local and global peak constraints. Given the aggregate charging demand of the EVs and the peak constraints of the ACN, it might be infeasible to fully charge all the EVs according to their charging demand. Two alternatives in such resource-limited scenarios are to maximize the social welfare by partially charging the EVs (fractional model) or selecting a subset of EVs and fully charge them (integral model). The critical challenge is the need for online solution design since in practical scenarios the scheduler has no information of future arrivals of EVs in a time-coupled underlying problem. For the fractional model, we devise both offline and online algorithms. We prove that the offline algorithm is optimal. Using competitive ratio as the performance measure, we prove the online algorithm achieves a competitive ratio of 2. The integral model, however, is more challenging since the underlying problem is NP-hard due to 0/1 selection criteria of EVs. Hence, efficient solution design is challenging even in offline setting. We devise a low-complexity primal-dual scheduling algorithm that achieves a bounded approximation ratio. Built upon the offline approximate algorithm, we propose an online algorithm and analyze its competitive ratio in special cases. Extensive trace-driven experimental results show that the performance of the proposed online algorithms is close to the offline optimum, and outperform the existing solutions.

Index Terms—Electric Vehicle, Online Scheduling, Approximation Algorithm, Competitive Analysis

I. INTRODUCTION

TO promote quick adoption of renewable energy sources, electrification of vehicles is a trend that has been globally advocated in recent years. According to a Bloomberg report, EVs will account for more than half of the new car sales by 2040 [1]. Consequently, it is expected that demand from EV charging will constitute a considerable portion of total demand. Currently, transportation consumes 29% of total energy in the US, while electricity production consumes 40% [2].

EV charging demand is a typical example of a deferrable load, and there is often considerable flexibility in the charging schedule. This property makes the problem of EV charging scheduling important and there is an extensive research along this direction in the literature (see the recent survey [3], and references therein). In most of the existing works, the EV charging demand is treated as a constraint to the problem

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in a low-load regime [4]. Motivated by rapid proliferation of EVs, this work tackles the EV scheduling in high-load regime, where given the aggregate charging demand of the EVs and the peak constraints of the charging network, it is not feasible to fully charge all the EVs according to their charging demand.

More specifically, this paper studies resource-constrained EV charging scheduling in an adaptive charging network (ACN) governed by a single operator in a campus-scale location such as a university, a headquarters, etc. [5]. A notable example is the Caltech ACN [6] where individual charging ports are organized into several *charging stations* (CSs) which are dispersed in a *charging network* with the capability of adaptive charging of the EVs. The problem is different from EV charging scheduling with capacity constraint in single station scenarios [2], [4], [7]–[12] (we refer to Section II for detailed discussions on related work), because of the essential need to respect the aggregate peak demand of the ACN. Note that, the ACN operator might limit the total power drawn from EVs to control costs [13], [14], reserve the capacity for other loads, and/or participate in demand-response programs.

We consider a scenario with multiple EVs where each EV has different charging profile in terms of availability, charging demand, charging rate limit, and valuation of getting charged (for details see Section III-A2). We formulate an online EV charging scheduling problem with the goal of *selecting* and *scheduling* a subset of EVs such that: (1) the charging demand of the selected EVs are (fully or partially) satisfied; (2) the charging rate limit of EV batteries are respected, (3) the global peak constraint of the ACN is satisfied [6]; (4) the local peak constraint of each CS is respected; and finally, (5) the total revenue obtained by the valuation of the EVs is maximized.

There are two main challenges in the design and implementation of scheduling algorithms for EVs satisfying the goals mentioned above. Firstly, the problem calls for online scheduling design. In practice, EVs arrive to the CS in online fashion and the scheduler has no information about the arrival and demand of the future EVs. Secondly, the underlying optimization problem in integral model is NP-hard even in offline case (see Section V). This is because the problem is a mixed integer linear problem and a "time-expanded" extension of knapsack problem which is known as a classic NP-hard problem. In this paper, we tackle the challenge of online design by following competitive algorithm design [15] and the challenge of NP-hardness by pursuing approximation algorithm design [16] and make the following contributions:

▷ We first consider a *fractional model* (where EVs can be charged partially and the revenue is proportional accordingly) and design an optimal offline scheduling algorithm.

We then develop an efficient online algorithm in which no exact or stochastic information about the future EV arrivals is given. Despite its simplicity, the algorithm is proved to be 2-competitive with optimal offline solution, i.e., the revenue of the proposed online algorithm is at least $1/2$ of the offline optimum, regardless of input sequence. Even though there are competitive algorithms in the literature for similar problems, to the best of our knowledge, our algorithm is the first 2-competitive algorithm which considers the charging rate limits.

▷ We next study the more challenging scenario of the *integral model*, where EVs must receive all their demand to make revenue. We first propose a polynomial-time primal-dual offline approximate algorithm. We analyze the approximation ratio of the algorithm and by strengthening the linear relaxed version of the mixed integer problem [17], we obtain an approximation ratio of $\alpha = 1 + \sum_{j=1}^{m} \frac{p_j}{p_j - q_j} \cdot \frac{s}{s-1}$, where p_j is local peak constraint in station j, q_j is the maximum charging rate of the EVs in station j and s is a slackness parameter. We highlight that when $p_j \gg q_j$ and s is large enough, then $\alpha \approx m + 1$, where m is the number of stations in the ACN. Built on the basis of the offline algorithm, we devise an online algorithm, and discuss its competitive ratio in special cases.

▷ We conduct a set of simulations to evaluate the performance of our proposed algorithms. The results of online algorithms for both integral and fractional settings are close to the optimum (within 90% and 94% for integral and fractional models in a representative scenario). In addition, our algorithm outperforms the existing scheduling algorithm in Caltech ACN [6] by 35% for integral revenue model.

II. RELATED WORK

There is an extensive work in the general topic of EV demand management and scheduling [3] by considering different scenarios such as optimal operations with EV coordinations [18], EV scheduling with incorporation of renewable energy, and energy storage systems [19], [20]. In this section, we focus on the literature related to peak-constrained EV charging scheduling in single and multiple stations.

1) Peak-Constrained EV Charging Scheduling: There is an extensive literature on EV charging scheduling problem focusing on single CS [4], [7], while the local and global peak constraints are omitted or only the local peak is considered. As we discuss in Section III-B, the global optimal solution cannot be obtained by separately solving the single station problems. Hence, those solutions cannot be directly applied to the multiple CS scenario with global peak constraints.

Studies in [21]–[26] tackled charging scheduling problem in multiple CSs. The authors in [21] studied a global cost minimization EV charging scheduling problem. However, there is no limit on the maximum peak demand that the system can tolerate. Consequently, the peak can be arbitrary high and beyond the physical limit of the transformers in the charging station [6] leading to huge electricity bills [14]. Besides, the charger devices installed in the CSs have limitation on the maximum power that they can transfer in each time unit [27]. [24] considered a multi-microgrid system with global peak constraint where each microgrid has a CS and the goal is to manage electricity exchange between microgrids and main grid to minimize the operating costs. The authors assume that required information about individual EVs is available by forecast which may not represent a real scenario. A similar assumption is made in [25], [26] where the objective is to maximize total utility of EVs and aggregators in a distribution network. The authors in [22] use a similar model as in this paper, where both local and global peak constraints are considered. However, the authors solve the single-slot problem, which fails to provide a general solution taking into account EVs' arrival and departure times as considered in our study. Finally, as an alternative approach to control the peak, some studies directly target minimizing the peak [2], [28]. In [2], an online algorithm is developed for EV charging to minimize the peak and [28] proposed a valley filling method by leveraging V2G in peak hours. Although the peak is minimized in above works, it cannot guarantee that the minimized peak is tolerable by the underlying charging network.

2) Scheduling Under Demand Uncertainty: A main challenge in EV scheduling problem is to cope with demand uncertainty. Many studies including [2], [4], [8]–[12] addressed online scheduling problem with different objectives. [8], [11] studied the problem of maximizing social welfare considering the benefit for both users and service provider. [4] and [9] developed algorithms to minimize the charging price for the CS, where the proposed algorithm in [4] is 2.39-competitive. In [12], an online 2-competitive algorithm is proposed for a single CS, assuming that EVs are capable of being discharged in a negligible amount of time. The assumption, however, is not realistic for EVs.

Our problem in this paper is unique from above works in many respects. First, we study the problem in an ACN where several CSs exist, while none of the above studies solve the problem under this setting. Second, the previous algorithms do not work for both integral and fractional charging models. In addition, [9], [10] put no limit on the charging rate of EVs which makes their solution impractical in real scenarios. Also, [2], [4], [8]–[10] do not consider the peak limit of the CS. Note that some studies [29] tackle the scenarios of satisfying the large demands (possibly EV demands) by participating in electricity market. This study assumes that the electricity price is given and fixed, and leaves the case of real-time pricing as the future direction. In addition, there are studies that incorporate prediction in online EV charging scheduling [30], [31]. The prediction-based approaches achieve satisfactory performance for the scenarios that follow prediction. Deviation from prediction models, however, degrades their performance. Our approach, on the other hand, has no assumptions on modeling/prediction, and in this way, is robust against any uncertainty in the instances to the problem. Finally, in [32], we considered a simplified EV charging scheduling in fractional model without global peak constraints and devised heuristic algorithms (without competitive and approximation analysis) with on-arrival commitment for EVs to notify the amount that they can receive by their departure.
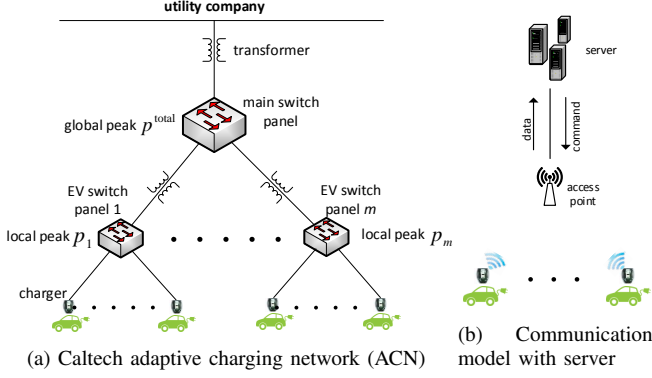
TABLE I: Summary of key notations

Notation	Description
T	Number of time slots, indexed by t
m	Number of CSs, indexed by j
n	Number of EVs, indexed by i
a_i	Arrival time of EV i
d_i	Departure time of EV i
D_i	Demand of EV i
v_i	Valuation of EV i for receiving its demand D_i
k_i	Maximum charging rate of EV i
$h(i)$	CS of EV i
q_j	Maximum k_i among all EVs in CS j
p_j	Maximum aggregate charging rate in station j
p^{total}	Maximum aggregate charging rate of all stations
y_i^t	**opt. variable**, The amount that EV i is charged at t

(a) Caltech adaptive charging network (ACN)

(b) Communication model with server

Fig. 1: System model [6].

III. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a time-slotted system model in which the time horizon is divided to T equal length slots $t = \{1, 2, \ldots, T\}$ (e.g., $T = 24$ with time slots of 1 hour length).

1) Charging Network: Our charging network model is inspired by the Caltech ACN [6] as illustrated in Fig. 1a. In the Caltech EV charging network (located in a parking garage), electricity is distributed through a two-level transformer architecture from a main switch panel to multiple EV switch panels (2 panels in the current Caltech ACN). Each EV switch panel then is connected to several chargers ($\approx$25 chargers per panel in the Caltech ACN). The total power drawn from the main switch panel by the charging network has a power limit of p^{total}, that is determined by the facility operator to control the costs, reserve the capacity for other loads, and/or participate in demand-response programs. In other words, p^{total}, which we refer to it as the *global peak*, hereafter, limits the maximum aggregate EV charging load at each time slot.

We assume that there are m EV switch panels that represent m CSs. In addition to the global peak constraint, each CS j has a capacity constraint on its total power drawn, indicated by p_j, and referred to it as the *local peak* constraint. The value of p_j is determined by the output power limit of the transformers installed between the main switch panel and EV switch panels and could be different for each EV switch panel. It is often observed that the charging demand of different CSs (EV switch panels in Fig. 1a) are well below the local peak constraints. To increase the flexibility due to heterogeneous charging demand of CSs, in the Caltech ACN, the global peak constraint of the main switch can be *over-provisioned*, i.e., p^{total} is less than the aggregate local peaks, ($\sum_j p_j \geq p^{\text{total}}$). While this increases flexibility, it also couples the problem of EV charging scheduling across different CSs. Our solutions in this paper will be centralized ones which can be obtained through communication between the CSs and a central server as illustrated in Fig. 1b.

2) EVs: There are n EVs in the system, indexed by i. EV i is represented by a charging profile $\langle a_i, d_i, v_i, D_i, k_i \rangle$ indicating its arrival time, departure time, willingness to pay, charging demand, and maximum charging rate, respectively. More specifically, the charging of EV i can be scheduled within its *availability window*, $[a_i, d_i]$. The charging rate at each slot is bounded by k_i, a parameter that depends on the physical constraints of the battery and on-board charger. It is assumed that the charging profile of each EV is feasible with respect to its maximum charging rate and a slackness parameter $s \geq 1$, which is the minimum ratio between the park time of the EV and its minimum charging time, i.e., $D_i \leq k_i(d_i - a_i + 1)/s$. The slackness parameter is imposed to tune the flexibility of the charging scheduling. In extreme case $s = 1$, the flexibility is minimum and the flexibility improves as s increases. We assume that EV owners select their CS, perhaps the nearest to them, and so the assignments are given to the problem. Define $h(i)$ as the CS of EV i. Moreover, q_j denotes the maximum k_i among all EVs in CS j, i.e., $q_j = \max_{h(i)=j} k_i$. Finally, v_i is the willingness to pay of EV i to receive its entire demand D_i before the departure time d_i. Note that in resource-constrained EV scheduling in ACN, it is not feasible to fulfill the entire demand of all EVs, hence, the problem turns into a resource allocation one with the goal of maximizing the aggregate value (a.k.a. utility) obtained from the EVs. In this way, each user can announce its willingness to pay to get charged which is denoted by v_i in the system model.

3) Revenue Models: We consider two revenue models: *(i) Fractional revenue model*: In this model, the fractional charging is allowed, i.e., the revenue from each EV is proportional to the fraction of the demand that is fulfilled [11] (see Eq. (2)). We tackle this model in Section IV. *(ii) Integral revenue model*: In this model, EV i pays v_i if it is fully charged and zero otherwise, i.e., there is no partial revenue for partial charging. This is the model that is considered in [8], [9]. We tackle this model in Section V.

B. Problem Formulation

We formulate an optimization problem to schedule the charging of the EVs with the objective of maximizing total revenue obtained from charged EVs while respecting local and global peak constraints. Note that each revenue model makes the underlying optimization problem fundamentally different. More specifically, the fractional charging model is a linear problem. Integral revenue model, however, turns the underlying charging scheduling problem to a Mixed Integer Linear Program (MILP). The integer nature originates from the 0/1-selection decision on EVs. We formulate Scheduling

TABLE II: Proposed algorithms and their properties.

Algorithm	Revenue model	Type	Optimality	Complexity
FCS	fractional	offline	Optimal	$O(n^2T + nT^2)$
ICS	integral	offline	$\left(1 + \sum_{j=1}^{m} \frac{p_j}{p_j - q_j} \cdot \frac{s}{s-1}\right)$-approximate	$O(nT \log T + n^2T)$
FOCS	fractional	online	2-competitive	$O(n^2T)$
IOCS	integral	online	$b(1 + \frac{p}{p-q} \cdot \frac{s}{s-1})$-competitive, $(m=1)$	$O(n^2T)$

Problem for Adaptive charging Network (SPAN) under fractional revenue model as follows:

$$\text{SPAN} : \max \quad \sum_{i=1}^{n} \frac{v_i}{D_i} \sum_{t=a_i}^{d_i} y_i^t$$

$$\text{s.t.} \quad \sum_{t=a_i}^{d_i} y_i^t \leq D_i, \qquad \forall i, \qquad (1a)$$

$$\sum_{i=1}^{n} y_i^t \leq p^{\text{total}}, \qquad \forall t, \qquad (1b)$$

$$\sum_{i:h(i)=j} y_i^t \leq p_j, \qquad \forall t,j, \quad (1c)$$

$$y_i^t \leq \frac{k_i}{D_i} \sum_{t'=a_i}^{d_i} y_i^{t'}, \quad \forall i,t, \quad (1d)$$

$$\text{vars.} \quad y_i^t \geq 0, \qquad \forall i,t,$$

where y_i^t is the amount that EV i is charged at slot t. Constraint (1a) ensures that the aggregate amount received by EV i is at most the demand D_i. The global and local peak constraints are represented by constraints (1b) and (1c), respectively.

The constraint (1d) enforces the maximum charging rate of EVs. The straightforward way to express this constraint is to simply state that at each time slot t, the charging rate of EV i should be less than or equal to its maximum charging rate, i.e., $y_i^t \leq k_i, \forall i, t$. However, for the sake of effective algorithm design for integral model and reducing the integrality gap of the relaxed linear problem, this constraint is strengthened in the form of Eq. (1d). Note that in case that the aggregated charging of EV i during its availability window is equal to its demand, i.e., $\sum_{t'=a_i}^{d_i} y_i^{t'} = D_i$, Eq. (1d) reduces to the simple form of $y_i^t \leq k_i$. This is a natural way in approximation algorithm design to improve the performance of the algorithms under linear-relaxation based design [17].

The SPAN is an extension of formulated problem in [33] where a job scheduling problem in cloud applications is studied. It turns out that the resource allocation problem in cloud systems and the EV charging scheduling problem in a *single station* share similar structure. Indeed, each charging profile in our scheduling problem can be seen as a job in cloud system with a deadline, value, and CPU demand. The SPAN, however, comes with an additional constraint (1b), which makes it different from the problem in [33], such that the existing solution will not work in the new setting. More importantly, this paper studies both offline and online solutions for the problem in both fractional and integral settings, while [33] tackles only the offline integral model.

IV. FRACTIONAL REVENUE MODEL

In the fractional model, the revenue of the CS from EV i is directly proportional to the amount of resource that the EV is received, i.e.,

$$v_i^{\text{f}} = \frac{\sum_t y_i^t}{D_i} v_i, \qquad (2)$$

where v_i is the gain, if the entire demand D_i is fulfilled and v_i^{f} is the fractional gain.

We propose a simple algorithm, called FCS, with low computational complexity of $O(n^2T + nT^2)$ for the fractional model that finds the optimal solution in offline setting. Note that even though linear programs can be solved in polynomial time in general, the complexity of our proposed algorithm is much lower than the general linear program algorithms. Moreover, our proposed offline algorithm applies a valley-filling strategy to reduce the peak. A summary of proposed algorithms with their complexity and assumptions is given in Table II.

A. Optimal Offline Design

We refer the proposed algorithm as the FCS and summarize it as Algorithm 1. The FCS works in two phases. In the first phase (Section IV-A1), the algorithm decides on the amount of resource to be allocated to each EV within its availability window and reserves resources accordingly. In this phase, the details of allocation is not known. The actual resource allocation is done in the second phase (Section IV-A2) by setting variables y_i^t.

Before discussing the details of the algorithm, we give a definition and introduce some notations to facilitate our algorithm design.

Definition 1. *Time interval $[\delta, \delta']$ is a "super interval" for interval $[t, t']$ if $1 \leq \delta \leq t$ AND $t' \leq \delta' \leq T$. Moreover, $\mathcal{I}_{t,t'}$ is the set of all super intervals of interval $I_{t,t'}$ i.e., $\mathcal{I}_{t,t'} = \{[\delta, \delta'] : 1 \leq \delta \leq t \text{ AND } t' \leq \delta' \leq T\}$.*

The number of super intervals of an interval is at most T^2 and at least one (for interval $[1, T]$).

Let R_i be the amount of resource that is reserved for EV i by the FCS and $I_{t,t'}$ as time interval $[t, t']$. Then, assuming that charging demands are sorted in non-increasing order of their unit values, $A_j^i(t, t')$ is the aggregate residual resource in interval $I_{t,t'}$ at station j assuming that the reservation for EVs 1 to i is accomplished. We now explain in detail each phase of the algorithm.

1) Phase I-Reservation: In Line 1, the EVs are sorted in a non-increasing order of their unit values. In Line 5, the FCS processes demand of EV i, picked from top of the ordered list, and sets R_i as the amount to be reserved for EV i which will be allocated in Phase II. In Line 7, the residual resource of all intervals in set $\mathcal{I}_{a_i,d_i}$ decreases by R_i and EV i is added to the set of selected EVs.

Lemma 1. *Provided that for EV i we have*

$$R_i \leq \min \left\{ D_i, \min_{t,t'} A_j^{i-1}(t, t'), \forall t, t' : I_{t,t'} \in \mathcal{I}_{a_i,d_i} \right\} \qquad (3)$$

with a dummy $A_j^0(t, t')$ defined as $A_j^0(t, t') = (t' - t + 1) \times \min\{p^{\text{total}}, p_j\}$, then there is a feasible allocation to allocate R_i to EV i in its availability window $[a_i, d_i]$.

Algorithm 1: FCS

Input: n EVs with their profile, local and global peak constraints $p_j, j = 1, \ldots, m$ and p^{total}

Output: Optimal scheduling under fractional model

1 Sort charging requests in non-increasing order of their unit values, i.e., $\frac{v_1}{D_1} \geq \frac{v_2}{D_2} \geq \cdots \geq \frac{v_n}{D_n}$

2 $\mathcal{L} \leftarrow \emptyset$

3 //Phase I

4 **for** $i = 1, \ldots, n$ **do**

5    $R_i \leftarrow \min\{D_i, \min\limits_{t,t'} A_{h(i)}^{i-1}(t, t'), \forall t, t' : I_{t,t'} \in$
   $\mathcal{I}_{a_i, d_i}\}$

6   **if** $R_i > 0$ **then**

7      $A_{h(i)}^i(t, t') \leftarrow A_{h(i)}^{i-1}(t, t') - R_i, \forall t, t' : [t, t'] \in$
     $\mathcal{I}_{a_i, d_i}$

8      $\mathcal{L} \leftarrow \mathcal{L} \cup i$

9 //Phase II

10 Sort EVs in $\mathcal{L}$ in increasing order of their charging flexibility i.e., $\frac{(d_i - a_i + 1)k_i}{D_i}, i \in \mathcal{L}$.

11 **for** $i = 1, \ldots, |\mathcal{L}|$ **do**

12   Pick EV i from the sorted list $\mathcal{L}$.

13    $feasible \leftarrow \left(\sum_{t=a_i}^{d_i} \min\{k_i, A_{h(i)}^{i-1}(t, t)\}\right) - R_i$

14   **if** $feasible < 0$ **then**

15   Re-allocate previously allocated EVs such that $feasible \geq 0$

16   Arbitrarily allocate R_i to EV i in its availability window

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In Lemma 1, $A_j^0(t, t')$ indicates the available resource when no charging request is processed in $I_{t,t'}$. In Eq. (3), the second term, i.e., $\min_{t,t'} A_j^{i-1}(t, t')$, indicates the minimum remaining resource in all super intervals of interval I_{a_i, d_i}. For $i \geq 1$, $A_j^i(a_i, d_i)$ is defined as follows:

$$A_j^i(a_i, d_i) = \begin{cases} A_j^{i-1}(a_i, d_i) - R_i & j = h(i), \\ A_j^{i-1}(a_i, d_i) & j \neq h(i). \end{cases}$$

The optimal value of $R_i, i = 1, \ldots, n$ is set according to the following lemma:

Lemma 2. *Given n EVs sorted in a non-increasing order of the unit values, $v_1/D_1 \geq v_2/D_2 \geq \cdots \geq v_n/D_n$, and the value of R_i, where R_i is set after $R_{i-1}, i = 2, \ldots, n$ by Eq. (3), then,*

$$R_i = \min\left\{D_i, \min_{t,t'} A_{h(i)}^{i-1}(t, t'), \forall t, t' : I_{t,t'} \in \mathcal{I}_{a_i, d_i}\right\}, \forall i,$$

is the optimal value for R_i.

2) Phase II- Allocation: Lemma 1 shows that there is a feasible scheduling to allocate the reserved resources. However, despite its feasibility, it is not straightforward to find such a schedule. For example, assume that for EV i, $R_i = 10$ and $k_i = 4$. It is possible that all available resources are concentrated in a single time slot but EV i cannot use more than 4 kWh of it. In this situation, the previously allocated resources in interval I_{a_i, d_i} should be re-allocated such that the concentrated resources are *dispersed* and we

have $\sum_{t=a_i}^{d_i} \sigma_t \geq R_i$ where $\sigma_t = \min\{k_i, A_{h(i)}^{i-1}(t, t)\}$ is the maximum resource that can be allocated to EV i at time slot t. Since the total amount of allocated resource does not change in the interval, such dispersion is possible and can be done by a simple algorithm in which allocates $\min\{k_i, A_{h(i)}^{i-1}(t, t)\}$ starting from time slot $t = a_i$ until R_i units is allocated. To further reduce the peak of the system, we will develop SMARTALLOCATE algorithm (See Section V) which acts more intelligent so that Line 16 of the FCS can be replaced by "Run SMARTALLOCATE(i, R_i)".

Theorem 1. FCS *is an optimal solution under fractional revenue model.*

The following theorem characterizes the complexity of FCS.

Theorem 2. *The time complexity of FCS algorithm is $O(n^2 T + nT^2)$ where n is the number of EVs and T is number of time slots.*

B. Online Scenario

In this section, we devise an algorithm for the scenario that EVs arrive in online fashion. The scheduling decisions at each time slot are made given the information of available EVs and neither exact values nor stochastic modeling of future arrivals is available. Our goal is to obtain a *competitive ratio* for the online algorithm. A scheduling algorithm $\mathcal{A}$ is c-competitive for $c \geq 1$ if the revenue obtained by the optimal offline algorithm is at most c times the algorithm $\mathcal{A}$'s revenue for any input sequence [15].

The proposed online algorithm for the fractional model, referred to as FOCS, is listed as Algorithm 2. The FOCS is a simple yet efficient algorithm that always selects EVs with highest unit value to allocate as follows. First, the algorithm sorts the available EVs at each time slot t based on their unit value (Line 2). In this step, the algorithm breaks ties based on EVs' deadline i.e., if two EVs have the same unit value, then the one with earliest deadline comes first in the sorted list. Next, the FOCS selects one EV at a time from the sorted list to allocate with maximum charging rate considering the EV's remaining demand, maximum charging rate, and peak constraints (Lines 3-4). The allocation is continued until all resources are allocated or there is no more EV that could be allocated. The time complexity of the FOCS is $O(n^2 T)$, determined by cost of its "**for**" loop multiplied by total number of times that the algorithm needs to be run.

Algorithm 2: FOCS: $\forall t \in \{1, 2, \ldots, T\}$

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1 $\mathcal{M}^t \leftarrow$ The set of available EVs that not received their entire demand by t

2 Sort EVs in set $\mathcal{M}^t$ indexed by $i = 1, \ldots, |\mathcal{M}^t|$:
   $v_1/D_1 \geq v_2/D_2 \geq \cdots \geq v_{|\mathcal{M}^t|}/D_{|\mathcal{M}^t|}$, where ties are broken based on users' deadline (giving priority to the users with earliest deadline)

3 **for** $i = 1, \ldots, |\mathcal{M}^t|$ **do**

4    $y_i^t \leftarrow \min\{k_i, D_i - \sum_{\tau=a_i}^t y_i^\tau, p_{h(i)} -$
   $\sum_{i':h(i')=h(i)} y_{i'}^t, p^{\text{total}} - \sum_{i'} y_{i'}^t\}$

Despite the simplicity of FOCS which makes it easy to implement, its performance is sound and within a constant

factor of the offline optimum. We now proceed to analyze the performance of the FOCS by first giving some preliminaries.

Fix an optimal scheduling and let $\mathcal{S}_{\text{FOCS},t}$ and $\mathcal{S}_{\text{OPT},t}$ be the sets of EVs selected by the FOCS and optimal solution at time slot t, respectively. Let y_i^t and z_i^t be the charging rate of EV i set by FOCS and OPT, respectively. We define Δ_i^t as follows:

$$\Delta_i^t = \begin{cases} \min\{z_i^t - y_i^t, r_{i,t}\} & i \in \mathcal{S}_{\text{OPT},t}, z_i^t > y_i^t, \\ 0 & \text{otherwise}, \end{cases} \quad (4)$$

where $r_{i,t}$ is the remaining demand of EV i by the end of time slot t. $\Delta_i^t > 0$ indicates that the optimal algorithm allocated Δ_i^t units more resources to EV i than the FOCS by time slot t that could be *feasibly* allocated by FOCS to the EV i. If for any EV $i \in \mathcal{S}_{\text{OPT},t}$ and time slot $t \in \mathcal{T}$ we have $y_i^t = z_i^t$, i.e., $\Delta_i^t = 0$, then the FOCS is obviously optimal because it gains whatever the optimal solution gains. We define *loss* of the FOCS imposed by EV i as follows:

$$l_{i,t} = \Delta_i^t \frac{v_i}{D_i}, \quad (5)$$

Note that the loss always takes a non-negative value as $\Delta_i^t \geq 0$. When FOCS sets charging rate of an EV less than its rate in the optimal solution, it gains $\Delta_i^t v_i / D_i$ less than the optimal solution from that EV. An upper bound for the distance between the optimal objective value (denote by OPT) and the revenue of the FOCS (denote by ALG) is summation of the losses over all time slots and all EVs, i.e.,

$$\text{OPT} - \text{ALG} \leq \sum_{t=1}^{T} \sum_{i \in \mathcal{S}_{\text{OPT},t}} l_{i,t}. \quad (6)$$

In addition to the amount of loss, the gain of the algorithm from charging alternative EVs should also be taken into account in comparison between OPT and ALG.

Let $i \in \mathcal{S}_{\text{OPT},t}$, $i \notin \mathcal{S}_{\text{ALG},t}$ and $g_{i,t}$ be the gain that the FOCS obtains from charging another EV instead of i at time slot t. We are going to show that in the FOCS, for each EV i with $l_{i,t} > 0$ there must be another EV (denote by i') where $y_{i'}^t \geq \Delta_i^t$ and $\frac{v_{i'}}{D_{i'}} \geq \frac{v_i}{D_i}$ which means in resource allocation phase for EV i, the FOCS allocated the difference Δ_i^t to another EV with the same or higher unit value. This can be proved by considering the fact that (i) the selected EVs have higher unit values than the unselected EVs, and (ii) the charging rate of the selected EVs are set to the maximum feasible value. Moreover, the FOCS is *"work-conserving"* i.e., it does not let any resource remain unused if there are some EVs that can use it.

Let $g_{i,t}$ denote the gain that FOCS obtains from allocating the same amount of resource that optimal algorithm allocated to EV i (with size z_i^t) to another EV(s). If $\Delta_i^t = 0$, the loss is zero. If $\Delta_i^t > 0$, then by the charging strategy that the FOCS uses we can conclude that (i) i is not finished by the FOCS, and (ii) no more resources from station $p_{h(i)}$ can be feasibly allocated, otherwise, the FOCS could allocate more resources to EV i. Therefore, the Δ_i^t units of the resource are allocated to one or multiple other EVs (denote them by set $\mathcal{J}_i^t$) by the FOCS. Moreover, it must hold that all the EVs in set $\mathcal{J}_i^t$ have a unit value equal to or higher than v_i/D_i which yields $g_{i,t} \geq l_{i,t}$, otherwise, the FOCS should not prefer the EVs in $\mathcal{J}_i^t$ to i. Since the result holds for any arbitrarily EV i, we can get the following:

$$\sum_{i=1}^{n} \sum_{t=1}^{T} l_{i,t} \leq \sum_{i=1}^{n} \sum_{t=1}^{T} g_{i,t}. \quad (7)$$

Moreover, the total gain of FOCS, i.e., ALG, is equal to sum of its gains from each single EV:

$$\text{ALG} = \sum_{i=1}^{n} \sum_{t=1}^{T} g_{i,t}. \quad (8)$$

With the above discussion and using Eqs. (6), (7) and (8) we are able to derive a competitive ratio of 2 for the FOCS.

Theorem 3. *The FOCS is 2-competitive.*

Remarks: When there is only one CS and EVs have no limit on their charging rate, the FOCS is identical to the FIRSTFIT algorithm [34] which is known to be 2-competitive for classic job scheduling problem. However, the charging rate limitation is crucial for EV charging problem. Moreover, [34] uses a "charging argument" to prove the competitive ratio of the proposed algorithm which cannot be directly applied to our problem. Thus, the FOCS extends the FIRSTFIT and makes it practical for the EV charging scenario. Moreover, the proof technique used for the competitive analysis of the FOCS is fundamentally different from the one used in [34].

V. INTEGRAL REVENUE MODEL

The MILP form of the SPAN in integral model is a generalized form of the $0/1$-knapsack problem which is a well-known NP-hard problem. To give an intuition, consider the scheduling problem in a single time slot (i.e, $T = 1$). Then, allocating power resources to the EVs is equivalent to allocating the capacity of knapsack to the items. In Section V-A, we propose a fast polynomial time offline approximation algorithm for the integral problem. In Section V-B, we extend the result and propose an online algorithm for the integral model.

A. Offline Scenario

We design our offline scheduling algorithm under integral model referred to as ICS, to solve the SPAN approximately. Since the performance analysis of the proposed algorithm relies on a dual fitting method and utilizes weak duality property, we first need to construct the dual problem of SPAN. Toward this, we introduce variables α, β, γ and π. Generally, in primal-dual approximation algorithm design, each constraint (resp. variable) in primal (resp. dual) problem is associated with a variable (resp. constraint) in dual (resp. primal) problem. In our case, constraints (1a) (1b), (1c) and (1d) in the SPAN are respectively associated with dual variables α, β, γ and π (for more details see [16]). The dual problem is formulated as follows:

$$\min \quad \sum_{i=1}^{n} D_i \alpha_i + \sum_{j=1}^{m} \sum_{t=1}^{T} p_j \beta(t) + \sum_{t=1}^{T} p^{\text{total}} \gamma(t)$$

$$\text{s.t.} \quad \alpha_i + \beta(t) + \gamma_i + \pi(t) - \frac{k_i}{D_i} \sum_{t'=a_i}^{d_i} \pi_i(t') \geq \frac{v_i}{D_i},$$

$$\forall i, t \in [a_i, d_i], \quad (9a)$$

$$\text{vars.} \quad \alpha_i, \beta_i, \gamma, \pi_i(t) \geq 0, \quad \forall i, t.$$

1) Explanation of the Main Algorithm: Our algorithm design is inspired by the basic algorithm proposed in [33]. The algorithm in [33], however, works for a single CS where arrival time of all EVs are the same and there is no global peak constraint. The ICS algorithm (listed as Algorithm 3) works in two phases. In the first phase it sorts the charging requests based on their unit values in a non-increasing order. Then, it selects most valuable demand. If the remaining resource is enough to cover the *entire* demand of the EV, it is admitted to receive the demand (Lines 6-7).

Scheduling of the Selected EV: If the feasibility check passed (Line 6 in Algorithm 3), ICS calls sub-procedure SMARTALLOCATE to allocate required resources in interval $[a_i, d_i]$. Then, α_i is set to v_i/D_i in order to cover dual constraint in Eq. (9a) (Lines 7-8).

In SMARTALLOCATE, let us define $W(t, h(i)) = \sum_{i':h(i')=h(i)} y_{i'}^t$ as total workload at time slot t in CS $h(i)$ and $\bar{W}(t, h(i))$ as total available load to allocate at time slot t for CS $h(i)$. We always have $\bar{W}(t, h(i)) + W(t, h(i)) = p_{h(i)}, \forall t, i$. For scheduling, SMARTALLOCATE applies two main policies: 1) *valley-filling* and, 2) *right-to-left allocation*. With valley-filling, the slots with more available resources are preferred which helps to reduce the peak of the system. Right-to-left allocation is used when two or more time slots are equal in terms of their remaining resources. A ranking based approach is used to apply the aforementioned policies. To charge EV i, SMARTALLOCATE ranks time slots in interval $[a_i, d_i]$. Then, charging is done by allocating resources from the higher ranked time slot to lowest one. The rank of a time slot t is calculated based on remaining resources in the time slot (valley-filling) and value of t (right-to-left allocation).

Algorithm 3: ICS

1 **initialize**: $y \leftarrow 0, \alpha \leftarrow 0, \beta \leftarrow 0, \gamma \leftarrow 0, \pi \leftarrow 0$
2 Sort charging requests in non-decreasing order of their unit values: $v_1/D_1 \geq v_2/D_2 \geq \cdots \geq v_n/D_n$
3 **for** *(i=1...n)* **do**
4   **for** $t = a_i \ldots d_i$ **do**
5         $\sigma_t \leftarrow \min \Big\{ p_{h(i)} - \sum_{i':h(i')=h(i)} y_{i'}(t),$ $p^{\text{total}} - \sum_{i'=1}^n y_{i'}^t, k_i \Big\}$
6   **if** $D_i \leq \sum_{t=a_i}^{d_i} \sigma_t$ **then**
7   SMARTALLOCATE(i, D_i)
8         $\alpha_i \leftarrow v_i/D_i$
9   **else**
10   **if** $(\beta(d_i) = 0)$ **then**
11   BETACOVER(i)
12 **for** *(i=1...n)* **do**
13   **if** *EV i is not selected* **then**
14   RECONSIDER(i);

Dual Feasibility of the Non-selected EVs: If remaining power resource is not enough to fully charge EV i, i.e., $D_i > \sum_{t=a_i}^{d_i} \sigma_t$, the EV cannot be selected. However, we still need to satisfy constraint (9a) in dual problem which is done by calling BETACOVER(i). To cover the constraint (9a) for EV i, sum of dual variables for all $t \in [a_i, d_i]$ should be greater than or equal to v_i/D_i. BETACOVER(i) sets $\beta(t)$

to v_i/D_i for all time slots t in interval $[t_{\text{cov}}, R(d_i)]$ (Lines 3-4 of Algorithm 5). Observe that $\beta(t') \geq v_i/D_i, \forall t' < t_{\text{cov}}$ (with $t_{\text{cov}} > 1$) considering that the demands are sorted in a non-increasing order of the unit-values and $\beta(t')$ is already set to $v_{i'}/D_{i'}$ when processing the earlier charging demand of EV i' which is not selected. Hence, $v_{i'}/D_{i'} \geq v_i/D_i$, thereby $\beta(t) \geq v_i/D_i$, and the dual constraint in (9a) is satisfied. Lines 1-4 of BETACOVER is enough to cover the dual constraint. However, the algorithm continues in Lines 5-8 by setting a variable $\Phi_{i'}(t)$ for time slots $t = 1, \ldots, R(d_i)$ to a value dependent to amount of the resource that a selected EV i' received at slot t. $\Phi_{i'}(t)$ will be used in approximation analysis of the main algorithm and has no effect on the scheduling of EVs.

Algorithm 4: SMARTALLOCATE(i, D_i)

Input: EV i to receive D_i from CS $h(i)$
1 Rank time slots in interval $[a_i, d_i]$ such that for any t_1 and t_2: rank$(t_1) >$ rank(t_2) iff $\bar{W}(t_1, h(i)) > \bar{W}(t_2, h(i))$ OR $\bar{W}(t_1, h(i)) == \bar{W}(t_2, h(i)) \wedge t_1 > t_2$
2 **while** $\sum_{t'=a_i}^{d_i} y_i^{t'} \neq D_i$ **do**
3   Select time slot t with highest rank which is not selected before
4   Allocate $\min \Big\{ k_i, \bar{W}(t, h(i)), D_i - \sum_{\tau=a_i}^t y_i^\tau, p^{\text{total}} - \sum_{i'} y_{i'}^t \Big\}$ to EV i at t

Algorithm 5: BETACOVER(i)

1 $t_{\text{cov}} \leftarrow \min\{t : \beta(t) = 0\}$
2 $R(d_i) = \max\{t \geq d_i : \forall t' \in (d_i, t], \bar{W}(t') < q_{h(i)}\}$
3 **for** $(t = t_{\text{cov}} \ldots R(d_i))$ **do**
4     $\beta(t) \leftarrow v_i/D_i$
5 **for** $(t = 1 \ldots R(d_i))$ **do**
6   **for** $(i' = 1 \ldots n)$ **do**
7   **if** $y_{i'}^t > 0 \wedge \Phi_{i'}(t) = 0$ **then**
8             $\Phi_{i'}(t) \leftarrow \big[\frac{p_{h(i)}}{p_{h(i)}-k_i} \frac{s}{s-1} \big] \cdot \frac{v_i}{D_i} y_{i'}^t$

Algorithm 6: RECONSIDER(i)

1 $\mathcal{L} \leftarrow \emptyset$
2 $v_{\text{inc}} \leftarrow v_i$
3 $\sigma_t \leftarrow 0, t = a_i, \ldots, d_i$
4 **for** $(i' = i - 1 \ldots 1)$ **do**
5   **if** *EV i' is selected* $\wedge (h(i') = h(i)) \wedge (v_{\text{inc}} - v_{i'}) > 0$ **then**
6   Add EV i' to list $\mathcal{L}$
7         $v_{\text{inc}} \leftarrow v_{\text{inc}} - v_{i'}$
8   **for** $(t = a_i \ldots d_i)$ **do**
9             $\sigma_t \leftarrow \sigma_t + \min\{k_i, y_{i'}^t\}$
10 **if** $\sum_{t=a_i}^{d_i} \sigma_t \geq D_i$ **then**
11   Remove EVs in list $\mathcal{L}$ from charging schedule
12   SMARTALLOCATE(i, D_i)

Improving the Gain: In the second phase, the ICS tries to increase total value of selected EVs by calling RECONSIDER(i) on every unselected EV i (Lines 12-14). The RECONSIDER(i) (listed as Algorithm 6) checks that whether the total revenue can be increased by replacing some selected EVs with EV i or not (Lines 4-9). If such EVs are found, the algorithm stops their charging and allocates EV i using SMARTALLOCATE (Lines $10 - 12$).

2) Analysis: In primal-dual algorithm, the goal is to design an algorithm in a way that it produces a good solution for

primal problem (with primal value Γ) and a feasible solution for the dual problem (with dual value Λ). Then, assuming that the primal problem is a maximization problem, to prove that the algorithm is $c-$approximation (for $c \geq 1$), the important part is to show that $\Lambda \leq c\Gamma$. Then, based on weak duality theorem we have $\Lambda \geq$ OPT, and it is concluded that $\Gamma \geq \frac{1}{c} \times$ OPT where OPT is the optimal value. Based on the above understanding, the following theorem scrutinizes the approximation ratio of the ICS assuming that arrival times are the same for all EVs.

Theorem 4. ICS *algorithm is a* $\left(1 + \sum_{j=1}^{m} \frac{p_j}{p_j - q_j} \cdot \frac{s}{s-1}\right)$*-approximation when EVs have same arrival time.*

Note that in the case that the system is flexible enough, i.e., $s \gg 1$, and the maximum charging rates of stations are much bigger than those of EVs, i.e., $p_j \gg q_j, \forall j$, the approximation ratio approaches $m+1$. And in the case that there is one single station, the approximation ratio is 2.

B. Online Scenario

Due to the binary selection variable, the online solution design under integral model is more challenging than the one for fractional model. We propose the IOCS that is built upon the offline ICS. In particular, the IOCS calls the ICS at each time slot for the set of available EVs, however, any algorithm that is designed for offline integral model can be used alternatively. Hereinafter, $\mathcal{A}$ refers to the ICS or a similar algorithm.

Algorithm 7: IOCS: $\forall t \in \{1, 2, \ldots, T\}$

---

1 Let $\mathcal{A}$ be an algorithm that solves the problem with $a_1 = \cdots = a_n$
2 $\mathcal{R}^t \leftarrow$ set of EVs arrived at time slot t
3 $\mathcal{M}^t \leftarrow \mathcal{R}^t \cup \{i : t \in \mathcal{T}_i \text{ AND } \sum_{t'} y_i^{t'} < D_i\}$
4 Based on the residual in interval $[t, T]$, use algorithm $\mathcal{A}$ to allocate EVs in set $\mathcal{R}^t$ assuming no further arrivals
5 $\mathcal{S}_{\text{IOCS},t} \leftarrow \{i : i \in \mathcal{M}^t \text{ AND } i \text{ is admitted at } t\}$
6 $\widehat{\Gamma}_{\mathcal{A}, \mathcal{R}^t} \leftarrow \sum_{i \in \mathcal{S}_{\text{IOCS},t}} v_i$
7 Assume all reserved resources are freed at time slots $t, t+1, \ldots, T$
8 $r_{i,t} \leftarrow$ remaining demand of i at $t, \forall i$
9 $D'_i \leftarrow r_{i,t}, \quad v'_i \leftarrow \frac{r_{i,t}}{D_i} v_i, \forall i \in \mathcal{M}^t$
10 Run $\mathcal{A}$ on $\mathcal{M}^t$ using D'_i and $v'_i, \forall i$ and reconstruct $\mathcal{S}_{\text{IOCS},t}$
11 $\Gamma_{\mathcal{A}, \mathcal{M}^t} \leftarrow \sum_{i \in \mathcal{S}_{\text{IOCS},t}} v_i$ \\Use original values
12 **if** $\Gamma_{\mathcal{A}, \mathcal{M}^t} > \widehat{\Gamma}_{\mathcal{A}, \mathcal{R}^t}$ **then**
13 | Use the second schedule
14 **else**
15 | Use the first schedule

The IOCS is summarized as Algorithm 7. At slot t, the IOCS compares two scheduling results returned by $\mathcal{A}$ and chooses among them. In the first scheduling, the IOCS keeps all reserved resources in interval $[t, T]$ intact. Then, for utilizing the remaining resources, the algorithm runs $\mathcal{A}$ over arrived EVs at time slot t. In this case, the total revenue obtained by the *active* EVs (i.e., EVs that are available but not received their entire demand yet) is denoted by $\widehat{\Gamma}_{\mathcal{A}, \mathcal{R}^t}$ (Line 6 of the algorithm). In the second scheduling, the IOCS considers the case that it can sacrifice the previously admitted EVs by canceling their reservations and allocating the freed resources to more valuable demands. For this purpose, the algorithm modifies

the demand and valuation of the previously admitted EVs such that each demand is replaced by the EV's remaining demand, and the valuation of the EV is proportionally calculated based on the remaining demand (Line 9 of the algorithm) so that the unit values of EVs do not change. Then, the IOCS runs $\mathcal{A}$ over set of active EVs $\mathcal{M}^t$ where the corresponding gain is denoted by $\Gamma_{\mathcal{A}, \mathcal{M}^t}$. If $\Gamma_{\mathcal{A}, \mathcal{M}^t} > \widehat{\Gamma}_{\mathcal{A}, \mathcal{R}^t}$, the IOCS forgets the previously admitted EVs and follows the second scheduling.

The following theorem characterizes the competitive ratio of the IOCS in a special case.

Theorem 5. *Let $\mathcal{A}$ be* ICS *in* IOCS *algorithm and $m = 1$. Assuming that EVs are released in b distinct groups where arrival time of EVs in each group are the same, the* IOCS *is $b\left(1 + \frac{p}{p-q} \frac{s}{s-1}\right)$-competitive with optimal offline solution, where p is the station peak and $q = \max_i k_i, i = 1, \ldots, n$.*

VI. SIMULATION RESULTS

Simulation Setup and Overview: We consider charging scheduling of EVs during a period of 12 time slots of length 1 hour (e.g., from 08:00 to 20:00). We gathered information of 12 popular EV models in the market to use in the simulation. Each EV model is characterized by its battery capacity as shown in Table III and maximum charging rate. Tesla models can get charged with up to 100 kW using Tesla super chargers. Also, all other models have rapid DC charging capability (up to 50 kW) with CHAdeMO method. This setting for charging rates is in accordance with future DCFC systems that can have several fast DC chargers installed in the charging station. The peak intervals include 08:00-10:00, 12:00-14:00, and 18:00-20:00 which is in accordance to NHTS survey [4], [35]. The probability of arriving an EV in the peak hours is two times higher than the off-peak periods. Demands are uniform random values from $[\frac{1}{2}U_i, U_i]$ where U_i is the battery capacity for EV i in Table III. The deadline of each EV is set according to slackness parameter s (with default value of 1.2), its demand and maximum charging rate of the battery as $d_i = a_i + \lceil \frac{D_i s}{k_i} \rceil - 1$. EVs are assigned to different CSs randomly and given as input to the algorithms. The willingness to pay by each user for one kWh of power is a random uniform number from interval $[\frac{1}{2}z, \frac{3}{2}z]$ where $z = \$0.11$ is national average price of electricity in the US [36]. In the simulation, the results are plotted with 95% confidence level and each point represents average result of 50 random scenarios. Table IV explains the comparison algorithms including current implemented algorithm in Caltech ACN referred to as IOLP [6] and its adapted version for the fractional revenue model FOLP. The IOLP and FOLP algorithms runs as follows: at each time slot assume that there will be no further arrivals and solve the optimization problem (e.g., by commercial solvers) for the current set of EVs and their remaining demands. In Table IV, the letters "I", "F" and "O" in front of the algorithms' name refer to integral, fractional and online types, respectively.

The measured performance metrics are total revenue, percentage of EVs that received all their demand, and total peak. To calculate optimal solution in integral revenue model, we used Gurobi solver [37].

Evaluation Based on Total Revenue: Fig. 2 depicts the comparison results based on total revenue under fractional and
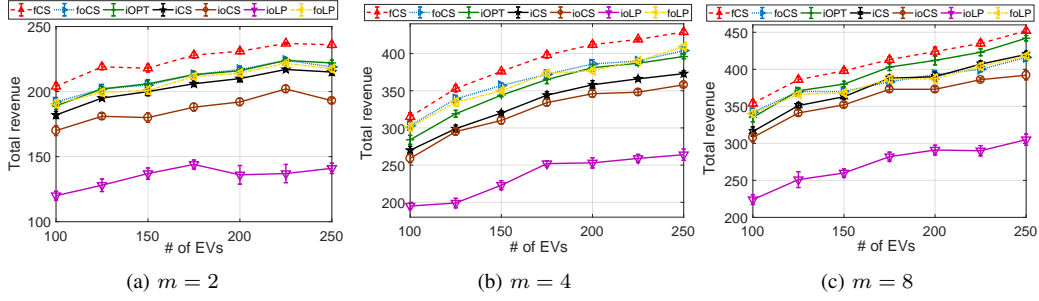
Fig. 2: Comparison results for 2, 4, and 8 CSs for fractional and integral revenue models.

(a) Total revenue
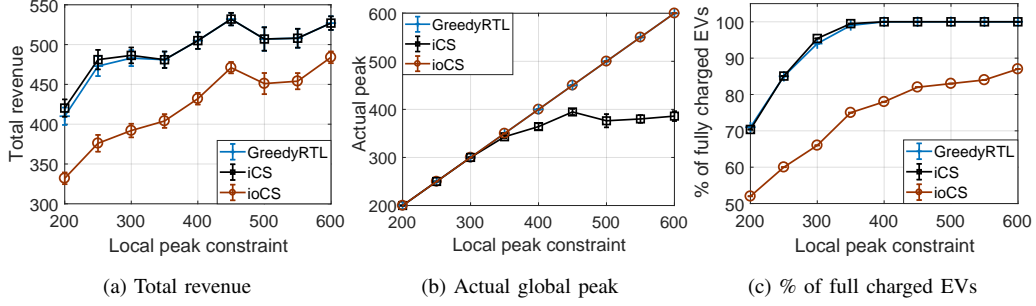
(b) Actual global peak

(c) % of full charged EVs

Fig. 3: Comparison in terms of total revenue, actual peak, and percentage of fully charged EVs by varying local peak value.

TABLE III: EV models and their battery capacity.

Model	Battery	Model	Battery
Mitsubishi i-MiEV	16 kWh	Citroen C-Zero	14 kWh
Peugeot iOn	16 kWh	Honda Clarity	25.5 kWh
Hyundai Kona	64 kWh	Nissan LEAF	40 kWh
Hyundai Ioniq	28 kWh	BMW i3	22/33 kWh
Tesla Model S/X	60/100 kWh	Kia Soul EV	27 kWh

TABLE IV: Acronyms for the algorithms

Notation	Description
IOPT	Optimal value under integral revenue model
ICS	Proposed offline algorithm for SPAN under integral revenue model
FCS	Proposed *optimal* algorithm for SPAN under fractional revenue model
IOCS	Proposed online algorithm for SPAN under integral revenue model
FOCS	Proposed online algorithm for SPAN under fractional revenue model
IOLP	Algorithm in [6] (works under integral revenue model)
FOLP	Adaptation of algorithm in [6] for fractional revenue model
GreedyRTL	The algorithm in [33] for single station scenario without global peak constraint (works under integral revenue model)

integral models while the total number of EVs increases from 100 to 250 for 2, 4, and 8 CSs. The local and global peak constraints are set to 50 kWh and 200 kWh, respectively.

The proposed algorithms compared to the optimal offline solution of integral model (IOPT) and IOLP.Recall that FCS is optimal offline solution. The notable observations are as follows: (i) the general trend in scenario of Fig. 2 is that by increasing the number of EVs, total revenue increases. This is because with more number of EVs, the scheduler has more freedom to choose more valuable EVs. (ii) as explained in Section IV, under fractional charging model, better results are expected due to increased scheduling flexibility in CSs.

According to the simulation data that we extracted from Fig. 2, the gain obtained by FCS and FOCS in Fig. 2 are respectively 12% and 10% more than the gain of the ICS and IOCS. (iii) In Fig. 2a and Fig. 2b, sum of the local peaks is less than or equal to the global peak while in Fig. 2c this sum is two times greater than the global peak. Consequently, the total revenue significantly improved when the number of CSs is increased from 2 to 4 while there is a slight improvement from 4 to 8 CSs as the global peak constraint prevents the algorithms from charging more EVs. (iv) in integral revenue model, the proposed IOCS algorithm acts significantly better than IOLP. In particular, IOCS improves IOLP by 38%, 36%, and 32% for $m = 2$, $m = 4$ and $m = 8$, respectively. In fractional revenue model, however, there is a very slight difference between FOCS and FOLP. (v) ICS approximates IOPT by 97%, 94%, and 95% for $m = 2$, $m = 4$, and $m = 8$, respectively. On average, IOCS is 90% of IOPT and, FOCS is 94% of its optimal offline solution, FCS. (vi) finally, the results depict that ICS achieves much better results in practice as compared to the theoretical approximation ratio that characterizes the performance in worst-case scenario.

Comparison Based on Actual Peak: The constraint set in the SPAN assures that any feasible solution respects the local and global peak constraints. An efficient scheduling algorithm may take a further step by not only satisfying the peak constraints but to further reduce the peak as much as possible. The proposed offline algorithms (i.e., ICS and FCS) apply valley-filling policy to reduce the peak. The online algorithms (i.e., IOCS and FOCS) do not apply the same policy as they do not have future knowledge to be able to balance allocated resources. To investigate the effect of employed valley-filling strategy, we conducted a set of simulations by varying local peak constraints. The results of ICS is compared to IOCS and GreedyRTL [33] where the latter is an approximation scheduling for single station scenario with EVs having same

arrival time (see the explanations after formulating the SPAN in Section III-B). The results are shown in Fig. 3. Along with total revenue in Fig. 3a, we also report total *actual* peak in Fig. 3b and percentage of fully charged EVs in Fig. 3c. In Fig. 3a, the results for ICS and GreedyRTL are almost identical while IOCS is 90% of the other two algorithms, on average. When $p_j \geq 400$, the total revenue for ICS and GreedyRTL does not increase in Fig. 3a and percentage of fully charged EVs is 100 for both algorithms according to Fig. 3c. From this point and onward, the scheduling is not challenging for offline methods to obtain optimal answer because of resource sufficiency. However, it is still a challenge to control total actual peak for the system. As a result of valley-filling policy, the value of actual peak of ICS in Fig. 3b remains almost unchanged for $p_j \geq 400$. IOCS and GreedyRTL, however, continuously increase the peak demand, since they are not using any peak shaving approach.

VII. CONCLUSION

This paper proposed offline and online algorithms for the EV charging scheduling problem under fractional and integral revenue models in an adaptive charging network (ACN). The problem is different, and more challenging than the existing single station EV charging scheduling problems since it requires respecting the aggregate peak charging demand of the ACN. As the notable contributions, our proposed online algorithm for fractional revenue model achieves constant competitive ratio of 2. Moreover, the offline integral algorithm achieves a theoretical bound on the optimality gap and approximates the optimum by 92%, on average in experiments. As a future work, we plan to study the problem under posted pricing mechanism where the charging station publishes the unit price of the power (which can be varied over time) and the users can accept or reject the offer. Another interesting future direction is to tackle the EV charging scheduling in market-based scenarios where the prices change over time.

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APPENDIX

A. Details of the RECONSIDER(i)

Before giving the details of RECONSIDER(i), we first explain the intuition behind this algorithm. Note that if in the scheduling problem we set $T = 1, m = 1, k_i = p^{\text{total}}, a_i = d_i = 1 \ \forall i$, then the problem is equal to the well-known 0-1 knapsack problem [16]. In the knapsack problem, a widely used greedy approach sorts items based on their unit values and selects items accordingly. It turns out that in this approach the approximation factor can be arbitrarily bad. For example, consider a knapsack problem with two items with $v_1 = 2, v_2 = p^{\text{total}}, D_1 = 1$, and $D_2 = p^{\text{total}}$. Given these values we have $v_1/D_1 > v_2/D_2$. To maximize total value of selected items, the optimal solution chooses item 2 while greedy algorithm selects item 1 which results in a worst-case approximation factor of c/OPT in general where c is a constant (in this example $c = 2$). To resolve it, one approach is to *re-consider* unselected items after running greedy algorithm and replace some selected items in the knapsack with unselected ones and then check whether the result is improved or not. In a simple case, only the largest unselected item can be examined which makes a significant theoretical improvement by providing a worst case approximation factor of $\text{OPT}/2$.

ICS algorithm leverages the same idea but using a more intelligent replacing method called RECONSIDER(i). RECONSIDER(i) is called on every unselected EV i. It tries to find some selected EVs that if they are replaced by EV i, total revenue from the selected EVs increases.

B. Proof of Lemma 1

We prove the theorem by induction.

Base case: When $n = 1$, there is only one EV that can be easily scheduled in its interval since all charging profiles represent a feasible demand and all resources are free.

Induction step: Let $k \in \mathbb{Z}^+$ is given and the claim is true for $n = k$ i.e., EVs $1, \ldots, k$ can be feasibly scheduled to receive their reserved resources. Now let $n = k + 1$. We claim that EV $k + 1$ can be feasibly scheduled in its interval. To prove, assume that this claim does not hold. Therefore, there should be at least one interval say $I_{t,t'}$ such that $A_{h(k+1)}^{k+1}(t,t') < 0$ and $A_{h(k+1)}^{k}(t,t') \geq 0$. Then, one of the following cases holds: a) $I_{t,t'} \notin \mathcal{I}_{a_{k+1},d_{k+1}}$ and b) $I_{t,t'} \in \mathcal{I}_{a_{k+1},d_{k+1}}$.

In case a, since $I_{t,t'} \notin \mathcal{I}_{a_{k+1},d_{k+1}}$, reserving resources in interval $\mathcal{I}_{a_{k+1},d_{k+1}}$ does not affect remaining resource in $I_{t,t'}$. Therefore, we have $A_{h(k+1)}^{k+1}(t,t') = A_{h(k+1)}^{k}(t,t') \geq 0$.

In case b, according to Eq. (3), we have $R_{k+1} \leq A_{h(k+1)}^{k}(t,t')$. Also, $A_{h(k+1)}^{k+1}(t,t') = A_{h(k+1)}^{k}(t,t') - R_{k+1}$ which gives $A_{h(k+1)}^{k+1}(t,t') \geq 0$.

Consequently, in both cases $A_{h(k+1)}^{k+1}(t,t') \geq 0$ which is a contradiction. Therefore, the original claim holds.

C. Proof of Lemma 2

By induction.

Base case: When $n = 1$, the claim holds since $R_i > \min\{D_i, \min_{t,t'} A_{h(i)}^{i-1}(t,t')\}, \forall t, t' : [t,t'] \in \mathcal{I}_{a_i,d_i}\}$ is not feasible and the gain is maximized with maximum value of R_i i.e., $R_i = \min\{D_i, \min_{t,t'} A_{h(i)}^{i-1}(t,t'), \forall t, t' : [t,t'] \in \mathcal{I}_{a_i,d_i}\}$.

Induction step: Assume the claim holds for $n = k, k > 1$ i.e., $R_i^\star = \min\{D_i, \min_{t,t'} A_{h(i)}^{i-1}(t,t'), \forall t, t' : [t,t'] \in \mathcal{I}_{a_i,d_i}\}$, is the optimal value for $R_i, i = 1, \ldots, k$. Now let $n = k + 1$. We claim that $R_{k+1}^\star = \Gamma$ where $\Gamma = \min\{D_{k+1}, \min_{t,t'} A_{h(i)}^{k}(t,t'), \forall t, t' : (t,t') \in \mathcal{I}_{a_{k+1},d_{k+1}}\}$. To prove, assume Γ is not the optimal value of R_{k+1}. Therefore, since according to the definition of R_{k+1} it always holds that $R_{k+1} \leq \Gamma$ thus, $R_{k+1}^\star < \Gamma$ and the amount of resource to be reserved for EV i is decreased by $\Gamma - R_{k+1}^\star$. This amount, can be only assigned to EVs $k + 2$ to n since R_i is set to its maximum value for $i = 1, \ldots, k$. However, having $v_{k+1}/D_{k+1} \geq v_i/D_i, i = k + 2, \ldots, n$ the optimal total revenue can be increased by setting $R_{k+1} = \Gamma$ which is a contradiction.

D. Proof of Theorem 3

We first prove the theorem for single station scenario and then extend it to multiple stations. From (6), (7), and (8) we obtain the following result

$$\text{OPT} - \text{ALG} \leq \sum_{i=1}^{n} \sum_{t=1}^{T} g_{i,t} \leq \text{ALG}, \tag{10}$$

hence, $\text{OPT} \leq 2\text{ALG}$.

In multi-station setting, the difference with the previous case is that when $\Delta_i^t > 0, i \in \{1, \ldots, n\}$, then FOCS may allocate the difference Δ_i^t to one or multiple EVs in any CS that might not be $h(i)$. However, the inequality $l_{i,t} \leq g_{i,t}$ is still valid as FOCS is centralized and uses a single sorted list for all EVs. Using similar deductions as in the single station setting, it is easy to verify that the competitive ratio of 2 is preserved.

E. Proof of Theorem 4

Without loss of generality we assume that $a_i = 1, \forall i$, however, the proof holds for any other constant value for arrival time. We sum up all costs of covering dual constraints and then provide a bound for it.

Each EV is either selected or not selected. For the unselected EVs, $\sum_{j=1}^{m} \sum_{t=1}^{T} p_j \beta(t)$ determines the cost. When BETACOVER(i) is running as a result of charging request disapproval of EV i, for any previously accepted request i' the algorithm sets $\Phi_{i'}(t)$ to a value proportional to $y_{i'}^t$ for $t \leq R(d_i)$ (Line 8 of BETACOVER(i) algorithm). The followings are proved in [33] for a single station $h(i') = h(i) = j$:

$$\sum_{i'=1}^{n} \sum_{t=1}^{d_j} \Phi_{i'}(t) \leq \left[\frac{p_j}{p_j - q_j} \cdot \frac{s}{s-1} \right] \cdot \sum_{i'=1}^{n} v_{i'} \tag{11}$$

$$\sum_{t=1}^{T} p_j \beta(t) \leq \sum_{i'=1}^{n} \sum_{t=1}^{d_j} \Phi_{i'}(t). \tag{12}$$

For m CSs, we can obtain the following inequality based on (11) and (12),

$$\sum_{j=1}^{m}\sum_{t=1}^{T} p_j \beta(t) \le \sum_{j=1}^{m}\left(\frac{p_j}{p_j - q_j}\cdot\frac{s}{s-1}\sum_{i:h(i)=j} v_i\right). \quad (13)$$

Now for notational convenience let's define A_j, B_j and C as follows:

$$A_j = \frac{p_j}{p_j - q_j}\cdot\frac{s}{s-1},$$
$$B_j = \sum_{i:h(i)=j} v_i,$$
$$C = \sum_{j=1}^{m} B_j = \sum_{i=1}^{n} v_i.$$

We can write the right hand side of Eq. (13) as follows:

$$\sum_{j=1}^{m} A_j B_j = \sum_{j=1}^{m}\left[A_j\left(C - \sum_{i:h(i)\ne j} B_{h(i)}\right)\right]$$
$$= \sum_{j=1}^{m} A_j C - \sum_{j=1}^{m}\left[A_j \sum_{i:h(i)\ne j} B_{h(i)}\right]$$
$$= \sum_{j=1}^{m} A_j C - \sum_{j=1}^{m}\left[A_j\left(C - B_j\right)\right]$$
$$\le \sum_{j=1}^{m} A_j C - \sum_{j=1}^{m}\left[A_j\left(C - \max_{j}\{B_j\}\right)\right]$$
$$= \max_{j}\{B_j\}\sum_{j=1}^{m} A_j. \quad (14)$$

From (13) and (14) we get

$$\sum_{j=1}^{m}\sum_{t=1}^{T} p_j \beta(t) \le \max_{j}\{B_j\}\sum_{j=1}^{m}\frac{p_j}{p_j - q_j}\cdot\frac{s}{s-1} \quad (15)$$

For the selected EVs, the covering cost is determined by the term $\sum_i D_i \alpha_i$ in the dual objective which equals to $\sum_{i\in\mathcal{S}} v_i$ where $\mathcal{S}$ is the set of selected EVs. Therefore, the total cost of covering dual constraints equals to

$$\Lambda = \sum_{i\in\mathcal{S}} v_i + \max_{j}\{B_j\}\sum_{j=1}^{m}\frac{p_j}{p_j - q_j}\cdot\frac{s}{s-1}$$
$$\le \sum_{i\in\mathcal{S}} v_i + \sum_{i\in\mathcal{S}} v_i \sum_{j=1}^{m}\frac{p_j}{p_j - q_j}\cdot\frac{s}{s-1}$$
$$= \left[1 + \sum_{j=1}^{m}\frac{p_j}{p_j - q_j}\cdot\frac{s}{s-1}\right]\sum_{i\in\mathcal{S}} v_i \quad (16)$$

Given that the primal value obtained from ICS is $\Gamma = \sum_{i\in\mathcal{S}} v_i$, we get

$$\Lambda \le \left[1 + \sum_{j=1}^{m}\frac{p_j}{p_j - q_j}\cdot\frac{s}{s-1}\right]\Gamma. \quad (17)$$

Finally, considering the fact that $\Lambda \ge \text{OPT}$, we conclude that ICS is $\left[1 + \sum_{j=1}^{m}\frac{p_j}{p_j - q_j}\cdot\frac{s}{s-1}\right]$-approximation.

F. Proof of Theorem 5

In the worst case $b = T$ i.e., there are T groups where at each time slot, a group of EVs arrive. Observe that

$$\text{OPT} \le \rho\Gamma_{\mathcal{A},\mathcal{R}^1} + \cdots + \rho\Gamma_{\mathcal{A},\mathcal{R}^T}.$$

Put it simply, the increase in optimal gain at each time slot is at most equal to maximum gain that can be obtained from arrived EVs at time slot t. Moreover, according to the IOCS, the gain of the algorithm obtained from set of active jobs at each time slot t, Γ_{IOCS}^t, is as follows:

$$\Gamma_{\text{IOCS}}^t = \max\{\widehat{\Gamma}_{\mathcal{A},\mathcal{R}^t}, \Gamma_{\mathcal{A},\mathcal{M}^t}\}.$$

Therefore,

$$\Gamma_{\mathcal{A},\mathcal{R}^t} \le \Gamma_{\text{IOCS}}^t, t = 1, \ldots, T$$

Thus, $\text{OPT} \le T\left(1 + \frac{p}{p-q}\frac{s}{s-1}\right)$. If $b < T$, we can obtain $\text{OPT} \le b\left(1 + \frac{p}{p-q}\frac{s}{s-1}\right)$ by the same analysis.

G. Proof of Theorem 1

By utilizing Lemma 2, FCS sets R_i to its optimal value for EV $i, i = 1, \ldots, n$. Based on Theorem 1, since it is feasible to allocate R_i to EV $i, i = 1, \ldots, n$, the total gain by FCS is optimal.

H. Proof of Theorem 2

The algorithm starts by sorting the charging profiles which costs $O(n\log n)$. Then, in the first "for" loop in Lines $4 - 8$, the algorithm calculates R_i for $i = 1, \ldots, n$. This requires us to check that for each EV i, there are enough available resources in all time intervals in the set $\mathcal{I}_{a_i, d_i}$. By definition, number of these times slots is $(T - d_i + 1)a_i$ which is $O(T^2)$ and their length varies from 1 to T. Therefore, the complexity of the first "for" is $O(nT^2)$. Finally, in the second "for" loop where the algorithm makes re-allocations, it should check all previously allocated EVs in their availability interval which can be done in $O(n^2 T)$ and dominates the cost of sub procedure SMARTALLOCATE. Therefore, the total cost is $O(n^2 T + nT^2)$.