Appendix

A. Proof of Lemma 1

We prove the theorem by induction.

Base case: When $n = 1$, there is only one EV that can be easily scheduled in its interval since all charging profiles represent a feasible demand and all resources are free.

Induction step: Let $k \in \mathbb{Z}^+$ is given and the claim is true for $n = k$ i.e., EVs $1, \dots, k$ can be feasibly scheduled to receive their reserved resources. Now let $n = k + 1$. We claim that EV $k + 1$ can be feasibly scheduled in its interval. To prove, assume that this claim does not hold. Therefore, there should be at least one interval say $I_{t,t'}$ such that $A^{k+1}_{h(k+1)}(t, t') < 0$ and $A^{k}_{h(k+1)}(t, t') \geq 0$. Then, one of the following cases holds: *a*) $I_{t,t'} \notin \mathcal{I}_{a_{k+1},d_{k+1}}$ and *b*) $I_{t,t'} \in \mathcal{I}_{a_{k+1},d_{k+1}}$.

In case *a*, since $I_{t,t'} \notin \mathcal{I}_{a_{k+1},d_{k+1}}$, reserving resources in interval $I_{a_{k+1},d_{k+1}}$ does not affect remaining resource in $I_{t,t'}$. Therefore, we have $A^{k+1}_{h(k+1)}(t, t') = A^{k}_{h(k+1)}(t, t') \geq 0$.

In case *b*, according to Eq. (3), we have $R_{k+1} \leq A^{k}_{h(k+1)}(t, t')$. Also, $A^{k+1}_{h(k+1)}(t, t') = A^{k}_{h(k+1)}(t, t') - R_{k+1}$ which gives $A^{k}_{h(k+1)}(t, t') \geq 0$.

Consequently, in both cases $A^{k}_{h(k+1)}(t, t') \geq 0$ which is a contradiction. Therefore, the original claim holds.

B. Proof of Lemma 2

By induction.

Base case: When $n = 1$, the claim holds since $R_i > \min\{D_i, \min_{t,t'} A^{i-1}_{h(i)}(t, t'), \forall t, t' : [t, t'] \in \mathcal{I}_{a_i,d_i}\}$ is not feasible and the gain is maximized with maximum value of R_i i.e., $R_i = \min\{D_i, \min_{t,t'} A^{i-1}_{h(i)}(t, t'), \forall t, t' : [t, t'] \in \mathcal{I}_{a_i,d_i}\}$.

Induction step: Assume the claim holds for $n = k, k > 1$ i.e., $R^\star_i = \min\{D_i, \min_{t,t'} A^{i-1}_{h(i)}(t, t'), \forall t, t' : [t, t'] \in \mathcal{I}_{a_i,d_i}\}$, is the optimal value for $R_i, i = 1, \dots, k$. Now let $n = k + 1$. We claim that $R^\star_{k+1} = \Gamma$ where $\Gamma = \min\{D_{k+1}, \min_{t,t'} A^{k}_{h(i)}(t, t'), \forall t, t' : (t, t') \in \mathcal{I}_{a_{k+1},d_{k+1}}\}$. To prove, assume Γ is not the optimal value of R_{k+1}. Therefore, since according to the definition of R_{k+1} it always holds that $R_{k+1} \leq \Gamma$ thus, $R^\star_{k+1} < \Gamma$ and the amount of resource to be reserved for EV i is decreased by $\Gamma - R^\star_{k+1}$. This amount, can be only assigned to EVs $k + 2$ to n since R_i is set to its maximum value for $i = 1, \dots, k$. However, having $v_{k+1}/D_{k+1} \geq v_i/D_i, i = k + 2, \dots, n$ the optimal total revenue can be increased by setting $R_{k+1} = \Gamma$ which is a contradiction.

C. Proof of Theorem 3

We first prove the theorem for single station scenario and then extend it to multiple stations. From (6), (7), and (8) we obtain the following result

$$\mathrm{OPT} - \mathrm{ALG} \leq \sum_{i=1}^{n} \sum_{t=1}^{T} g_{i,t} \leq \mathrm{ALG}, \tag{10}$$

hence, $\mathrm{OPT} \leq 2\mathrm{ALG}$.

In multi-station setting, the difference with the previous case is that when $\Delta^t_i > 0, i \in \{1, \dots, n\}$, then FOCS may allocate the difference Δ^t_i to one or multiple EVs in any CS that might not be $h(i)$. However, the inequality $l_{i,t} \leq g_{i,t}$ is still valid as FOCS is centralized and uses a single sorted list for all EVs. Using similar deductions as in the single station setting, it is easy to verify that the competitive ratio of 2 is preserved.

D. Proof of Theorem 4

Without loss of generality we assume that $a_i = 1, \forall i$, however, the proof holds for any other constant value for arrival time. We sum up all costs of covering dual constraints and then provide a bound for it.

Each EV is either selected or not selected. For the unselected EVs, $\sum_{j=1}^{m} \sum_{t=1}^{T} p_j \beta(t)$ determines the cost. When BETACOVER(i) is running as a result of charging request disapproval of EV i, for any previously accepted request i' the algorithm sets $\Phi_{i'}(t)$ to a value proportional to $y^t_{i'}$ for $t \leq R(d_i)$ (Line 8 of BETACOVER(i) algorithm). The followings are proved in [33] for a single station $h(i') = h(i) = j$:

$$\sum_{i'=1}^{n} \sum_{t=1}^{d_j} \Phi_{i'}(t) \leq \left[\frac{p_j}{p_j - q_j} . \frac{s}{s-1}\right] . \sum_{i'=1}^{n} v_{i'} \tag{11}$$

$$\sum_{t=1}^{T} p_j \beta(t) \leq \sum_{i'=1}^{n} \sum_{t=1}^{d_j} \Phi_{i'}(t). \tag{12}$$

For m CSs, we can obtain the following inequality based on (11) and (12),

$$\sum_{j=1}^{m} \sum_{t=1}^{T} p_j \beta(t) \leq \sum_{j=1}^{m} \Big(\frac{p_j}{p_j - q_j} . \frac{s}{s-1} \sum_{i:h(i)=j} v_i \Big). \tag{13}$$

Now for notational convenience let's define A_j, B_j and C as follows:

$$A_j = \frac{p_j}{p_j - q_j} . \frac{s}{s-1},$$
$$B_j = \sum_{i:h(i)=j} v_i,$$
$$C = \sum_{j=1}^{m} B_j = \sum_{i=1}^{n} v_i.$$

We can write the right hand side of Eq. (13) as follows:

$$\begin{aligned}
\sum_{j=1}^{m} A_j B_j &= \sum_{j=1}^{m} \Big[A_j \big(C - \sum_{i:h(i)\neq j} B_{h(i)}\big)\Big] \\
&= \sum_{j=1}^{m} A_j C - \sum_{j=1}^{m} \Big[A_j \sum_{i:h(i)\neq j} B_{h(i)} \Big] \\
&= \sum_{j=1}^{m} A_j C - \sum_{j=1}^{m} \Big[A_j \big(C - B_j\big)\Big] \\
&\leq \sum_{j=1}^{m} A_j C - \sum_{j=1}^{m} \Big[A_j \big(C - \max_j\{B_j\}\big)\Big] \\
&= \max_j\{B_j\} \sum_{j=1}^{m} A_j.
\end{aligned} \tag{14}$$

From (13) and (14) we get

$$\sum_{j=1}^{m}\sum_{t=1}^{T} p_j\beta(t) \le \max_j\{B_j\}\sum_{j=1}^{m}\frac{p_j}{p_j-q_j}.\frac{s}{s-1} \tag{15}$$

For the selected EVs, the covering cost is determined by the term $\sum_i D_i\alpha_i$ in the dual objective which equals to $\sum_{i\in\mathcal{S}} v_i$ where $\mathcal{S}$ is the set of selected EVs. Therefore, the total cost of covering dual constraints equals to

$$\begin{aligned}\Lambda &= \sum_{i\in\mathcal{S}} v_i + \max_j\{B_j\}\sum_{j=1}^{m}\frac{p_j}{p_j-q_j}.\frac{s}{s-1}\\ &\le \sum_{i\in\mathcal{S}} v_i + \sum_{i\in\mathcal{S}} v_i\sum_{j=1}^{m}\frac{p_j}{p_j-q_j}.\frac{s}{s-1}\\ &= \Big[1+\sum_{j=1}^{m}\frac{p_j}{p_j-q_j}.\frac{s}{s-1}\Big]\sum_{i\in\mathcal{S}} v_i \end{aligned} \tag{16}$$

Given that the primal value obtained from ICS is $\Gamma = \sum_{i\in\mathcal{S}} v_i$, we get

$$\Lambda \le \Big[1+\sum_{j=1}^{m}\frac{p_j}{p_j-q_j}.\frac{s}{s-1}\Big]\Gamma. \tag{17}$$

Finally, considering the fact that $\Lambda \ge$ OPT, we conclude that ICS is $\Big[1+\sum_{j=1}^{m}\frac{p_j}{p_j-q_j}.\frac{s}{s-1}\Big]$-approximation.

E. Proof of Theorem 5

In the worst case $b = T$ i.e., there are T groups where at each time slot, a group of EVs arrive. Observe that

$$\text{OPT} \le \rho\Gamma_{\mathcal{A},\mathcal{R}^1} + \cdots + \rho\Gamma_{\mathcal{A},\mathcal{R}^T}.$$

Put it simply, the increase in optimal gain at each time slot is at most equal to maximum gain that can be obtained from arrived EVs at time slot t. Moreover, according to the IOCS, the gain of the algorithm obtained from set of active jobs at each time slot t, Γ^t_{IOCS}, is as follows:

$$\Gamma^t_{\text{IOCS}} = \max\{\widehat{\Gamma}_{\mathcal{A},\mathcal{R}^t}, \Gamma_{\mathcal{A},\mathcal{M}^t}\}.$$

Therefore,

$$\Gamma_{\mathcal{A},\mathcal{R}^t} \le \Gamma^t_{\text{IOCS}}, t = 1,\ldots,T$$

Thus, OPT $\le T\Big(1+\frac{p}{p-q}\frac{s}{s-1}\Big)$. If $b < T$, we can obtain OPT $\le b\Big(1+\frac{p}{p-q}\frac{s}{s-1}\Big)$ by the same analysis.

F. Proof of Theorem 1

By utilizing Lemma 2, FCS sets R_i to its optimal value for EV $i, i = 1,\ldots,n$. Based on Theorem 1, since it is feasible to allocate R_i to EV $i, i = 1,\ldots,n$, the total gain by FCS is optimal.

G. Proof of Theorem 2

The algorithm starts by sorting the charging profiles which costs $O(n\log n)$. Then, in the first "for" loop in Lines $4-8$, the algorithm calculates R_i for $i = 1,\ldots,n$. This requires us to check that for each EV i, there are enough available resources in all time intervals in the set $\mathcal{I}_{a_i,d_i}$. By definition, number of these times slots is $(T-d_i+1)a_i$ which is $O(T^2)$ and their length varies from 1 to T. Therefore, the complexity of the first "for" is $O(nT^2)$. Finally, in the second "for" loop where the algorithm makes re-allocations, it should check all previously allocated EVs in their availability interval which can be done in $O(n^2T)$ and dominates the cost of sub procedure SMARTALLOCATE. Therefore, the total cost is $O(n^2T + nT^2)$.