So, after removing the two null productions $A \to \epsilon$ and $B \to \epsilon$, the modifi ed grammar will become

$$S \to ABAC/ABC/AAC/BAC/BC/C$$
$$A \to aA/a$$
$$B \to bB/b$$
$$C \to c$$

There is no null production in the grammar.

Example 6.33

Remove the ϵ production from the following grammar.

$$S \to aS/A$$
$$A \to aA/\epsilon$$

Solution: In the previous grammar, there is a null production $A \to \epsilon$. But in the language set generated by the grammar, there is a null string which can be generated by the following way $S \to A \to \varepsilon$. As null string is in the language set, and so the null production cannot be removed from the grammar.

6.6 Linear Grammar

A grammar is called linear grammar if it is context free, and the RHS of all productions have at most one non-terminal.

As an example, $S \to abSc/\epsilon$, the grammar of $(ab)^n c^n, n \geq 0$.
There are two types of linear grammar:

1. Left linear grammar: A linear grammar is called left linear if the RHS non-terminal in each productions are at the left end. In a linear grammar if all productions are in the form $A \to B\alpha$ or $A \to \alpha$, then that grammar is called left linear grammar. Here, A and B are non-terminals and α is a string of terminals.
2. Right linear grammar: A linear grammar is called right linear if the RHS non-terminal in each productions are at the right end. In a grammar if all productions are in the form $A \to B\alpha$ or $A \to \alpha$, then that grammar is called right linear grammar. Here A and B are non-terminals and α is a string of terminals.

A linear grammar can be converted to regular grammar. Take the following example.

Example 6.34

Convert the following linear grammar into Regular Grammar.

$$S \to S \to baS/aA$$
$$S \to A \to bbA/bb$$

Solution: Consider two non-terminals B and C with production $B \to aS\, and C \to bA$. The grammar becomes

$$S \to S \to bB/aA$$
$$S \to A \to bC/bb$$
$$S \to B \to aS$$
$$S \to C \to bA$$