

# Kamerabasierte Fahrbahnerkennung zur automatisierten Fahrbahnhföhrung eines Modellauto

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Master Thesis – 02. November 2017

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TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

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Darmstadt, den 02. November 2017

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Bahri Enis Demirtel

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## Kurzfassung

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Das L<sup>A</sup>T<sub>E</sub>X-Dokument `sada_tudreport` ist eine Vorlage für schriftliche Arbeiten (Proseminar-, Projektseminar-, Studien-, Bachelor-, Master- und Diplomarbeiten, etc.) am Institut für Automatisierungstechnik der TU Darmstadt. Das Layout ist an die *Richtlinien zur Anfertigung von Studien- und Diplomarbeiten* [?] angepasst und durch Modifikation der Klasse `tudreport` realisiert, so dass in der Arbeit die gewohnten L<sup>A</sup>T<sub>E</sub>X-Befehle benutzt werden können. Die vorliegende Anleitung beschreibt die Klasse und gibt grundlegende Hinweise zum Verfassen wissenschaftlicher Arbeiten. Sie ist außerdem ein Beispiel für den Aufbau einer Studien-, Bachelor-, Master- bzw. Diplomarbeit.

**Schlüsselwörter:** Studienarbeit, Bachelorarbeit, Masterarbeit, Diplomarbeit, Vorlage, L<sup>A</sup>T<sub>E</sub>X-Klasse

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## Abstract

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The L<sup>A</sup>T<sub>E</sub>X document `sada_tudreport` provides a template for student's research reports and diploma theses (" Proseminar-, Projektseminar-, Studien-, Bachelor-, Master- und Diplomarbeiten") at the Institute of Automatic Control, Technische Universität Darmstadt. The layout is adapted to the "*Richtlinien zur Anfertigung von Studien- und Diplomarbeiten*" [?] and is implemented by modification of the standard `tudreport` class, so that common L<sup>A</sup>T<sub>E</sub>X commands can be used in the text. This manual describes the class and dwells on general considerations on how to write scientific reports. Additionally, it is an example for the structure of a thesis.

**Keywords:** Research reports, diploma theses, template, L<sup>A</sup>T<sub>E</sub>X class

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# Symbole und Abkürzungen

## Lateinische Symbole und Formelzeichen

Symbol	Beschreibung	Einheit
$I$	Strom	A
$R$	Widerstand	$\Omega$
$U$	Spannung	V

## Griechische Symbole und Formelzeichen

Symbol	Beschreibung	Einheit
$\Psi$	Datenmatrix	
$\sigma$	Standardabweichung	
$\omega$	Kreisfrequenz	$s^{-1}$

## Abkürzungen

Kürzel	vollständige Bezeichnung
Dgl.	Differentialgleichung
LS	Kleinste Quadrate ( <i>Least Squares</i> )
PRBS	Pseudo-Rausch-Binär-Signal ( <i>Pseudo Random Binary Signal</i> )
ZVF	Zustandsvariablenfilter



# 1 Introduction

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## 1.1 Introduction

As in all industries, technology in the automotive industry is continuing to develop day by day. For example, the number of sensors, and their corresponding features, is increasing exponentially. One such sensor is the color camera. To begin with, in the automotive industry, cameras were used only to assist drivers in parking and reversing.

Nowadays, however, one of the main functions of color cameras is lane detection, in both autonomous cars and in cars equipped with a lane departure warning system. In this master's thesis, the lanes will be detected and then formulated mathematically.

The results of this master's thesis will be utilized and expanded upon by the students who will participate in the Echtzeitsysteme Projektseminar at the Technical University of Darmstadt. One of the aims of this seminar is to attend the Carolo-Cup organized annually by the Technical University of Braunschweig. Because of that, the width, the curvature, and the changes of the curvature of the track used in this master's thesis are the same as those belonging to the track used in the Carolo-Cup. In a real-life situation, there are of course oftentimes more factors that can hinder lane detection, including shadows cast by trees, buildings, and other structures; sunlight directly entering the lens of the camera and similarly less-than-ideal lighting conditions; dirt and debris on the road surface; and so on.

Therefore, the lanes of the track must be detected in a sufficiently short amount of time and there should be no dead time between lane detection and mathematical formulation. Lane detection must also be sufficiently robust, so that it should not be disrupted by less-than-ideal lighting conditions.

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## 1.2 Problem Statement and Objective Target

Autonomous driving is a topic currently being actively researched. Research on autonomous driving can be conducted in two fundamental areas: lane detection and lane guidance. With regard to lane detection, there are different scientific techniques that can be utilized, according to the literature, all with their own advantages and disadvantages under different conditions. For example, some techniques are suitable for straight lines, but not for curves. Others are suitable for curves as well but do not function well under certain light conditions. Others still are quite robust and suitable for curves, yet are computationally intensive (resulting in a video

feed with significant gaps). In this master's thesis, my aim is to research and implement the most appropriate and effective method for use in the Carola-Cup.

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### **1.3 Structure of Thesis**

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In Chapter 2, the fundamentals of lane detection are explained. All methods utilized in this thesis, along with their respective justifications, are also explained in this chapter. Some methods are also compared with regard to their advantages and disadvantages.

In Chapter 3, the steps of implementation are explained. The components can be divided broadly into the properties of the track, the hardware of the model car, and the software libraries and programs to be utilized. In this chapter, the program flow will also be explained in detail.

In Chapter 4, the results of the methods utilized will be compared. The computing time of all phases in this thesis will be presented and discussed. Also, all parameters utilized and their effects on this thesis will be also presented and discussed. In this chapter, the researcher will attempt to find an answer to the question, 'How can computing time be reduced?'.

In Chapter 5, the state of the art will be discussed. The other possible solutions for lane detection will also be explored here and their advantages and disadvantages will be compared.

In Chapter 6, all results of this master thesis will be presented and possible improvements and/or enhancements will be discussed.

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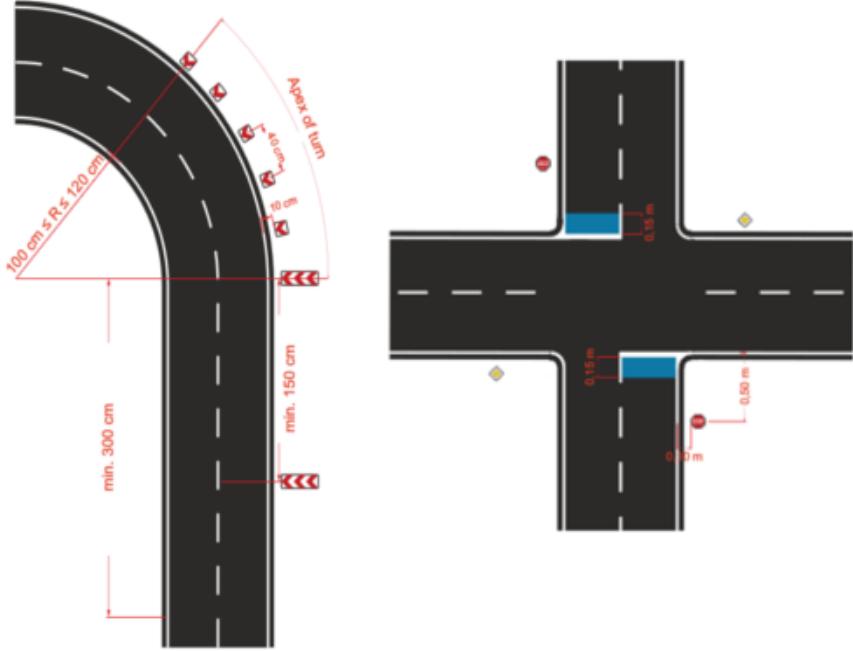
## 2 Fundamentals

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### 2.1 Properties of Truck at Carolo-Cup

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The Carolo-Cup is an annual competition at the Technical University of Braunschweig which are attended by students. Every year the truck and some properties of the competition are changing. For example, in the competitions until 2017 there was no traffic sign, but starting in 2017 there are also some traffic signs, speed limit zones, blocked areas and crosswalks for pedestrian. Because of this, in the competitions until 2017, there was only one way to understand who had the right of way. If there is a stop line on the road in front of an intersection, it means the car has to wait until the intersection is free. In the competitions starting from 2017, the intersections are in different parts: They are 'Intersections with stop lines', 'Intersections with give-way lines', 'Intersections with priority to right', 'Enforced crossing direction - give-way condition', 'Enforced crossing direction - right of way condition'. Except 'Intersections with priority to right', they all have traffic signs stating who has priority. If there is a no traffic sign, it means the right side always has priority.[1]



**Abbildung 2.1:** Left: Markings for sharp turns at Carolo-Cup. Right: Intersections with stop lines at Carolo-Cup[1]

## 2.2 Inverse Perspective Mapping

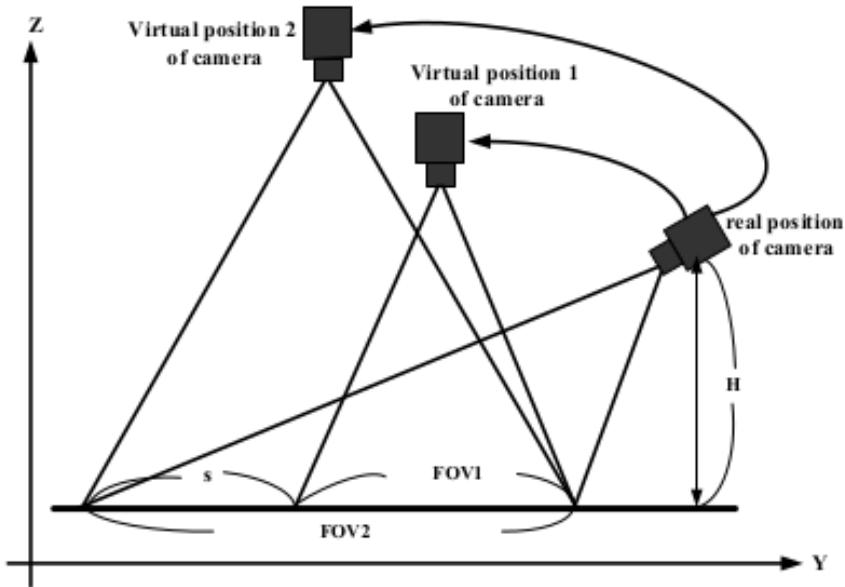
Inverse Perspective Mapping(IPM) is an algorithm which is able to obtain accurate bird's-eye view images from the sequential of forward looking cameras. With the IPM algorithm, each image pixel is remapped, and a new array of pixels is created where the lines in perspective are transformed into straight lines and objects are distorted. IPM is one of the most used methods in lane detection. In lane detection, IPM ensures that the lanes are shown vertical and parallel to each other. On the other hand, because of the re-mapping of pixels, IPM is a computationally expensive method. Because of this reason, in some cases in this master's thesis, rather than remapping all pixels of the images, only the pixels relevant to the lane and accordingly, the fitted curve, were remapped. Thanks to this, in some cases, a lot of computing time was saved.

In order to use the IPM method, the intrinsic and extrinsic parameters of camera are necessary to process images for coordinate transformation and calibration.

- **Intrinsic Parameters :** Intrinsic parameters are camera-specific. It includes information of the focal length ( $f_x, f_y$ ) and optical centers ( $c_x, c_y$ ). It is also called a camera matrix. Although the intrinsic parameters are camera-specific, once the camera is calibrated, the modified intrinsic parameters can be stored for future purposes. It is expressed as a  $3 \times 3$  matrix:

$$\text{camera matrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

- **Extrinsic Parameters :** Extrinsic parameters are dependent on the camera position. The parameters are H and  $\theta$ . H is the distance between the camera and ground.  $\theta$  is the camera tilt angle.



**Abbildung 2.2:** Related Positions of the Camera [2]

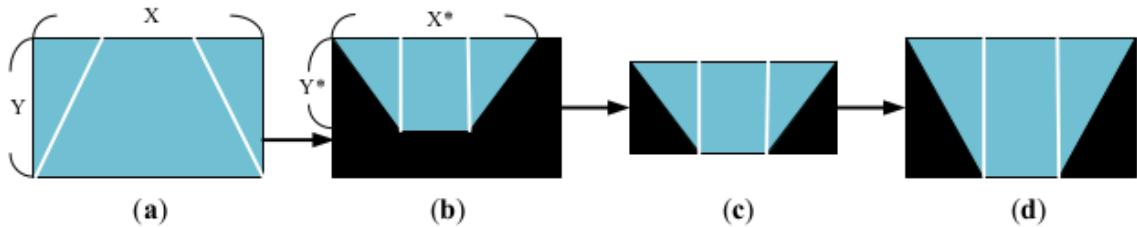
As seen at ??, the camera on the car has field of view 2(FOV2) at the real position of the camera but in this case, the view is not a bird's-eye view, so if the same FOV is to be observed from a bird's-eye view, IPM will virtually change the position to Virtual Position 2 of the camera. In this case, although the camera is at its real position, it will appear as though it is at Virtual Position 2. For that, the image coordinates must also be changed. Below, the steps of IPM calculations from the paper of [2] will be detailed.

In the formula, the original image coordinates will be defined as  $(x, y)$ , the destination image coordinates will be defined as  $(x^*, y^*)$ , the distance between the ground and the camera will be defined as  $H$ , the focal length of camera will be defined as  $f$ , and the tilt angle of camera will be defined as  $\theta$ .

$$x^* = H \frac{x \sin \theta + f \cos \theta}{-y \cos \theta + f \sin \theta} ; y^* = H \frac{y \sin \theta + f \cos \theta}{-y \cos \theta + f \sin \theta}$$

In this equation, the transformed component values of  $x^*$  and  $y^*$  may be less than or equal to zero. Because of this reason, a constant  $d$  is defined as  $|H(\sin \theta + \cos \theta)/(f \sin \theta - \cos \theta)| + 1$ . This means that the coordinate point in the original source image has been mapped into the point of the destination image coordinate system. Below there is the proposed equation :

$$x^* = H \frac{x \sin \theta + f \cos \theta}{-y \cos \theta + f \sin \theta} + d, \quad y^* = H \frac{y \sin \theta + f \cos \theta}{-y \cos \theta + f \sin \theta} + d, \text{ where } d = \left\lceil \frac{H(\sin \theta + f \cos \theta)}{f \sin \theta - \cos \theta} \right\rceil + 1$$



**Abbildung 2.3:** Procedures of IPM

### 2.3 Edge Detection

Edge detectors are essential parts of most computer vision systems. Edge detectors dramatically decrease the amount of data to be processed and extract the useful parts of images. They work by detecting discontinuities in brightness. In this project, the edge detector was used in order to detect the lanes and to exclude unnecessary information from images. There are different methods for edge detection, but they can be grouped into two categories. They are :

- **Gradient method :** This method searches for the maximum and minimum in the first derivative of the image and with that, the edges can be found. For this method, the first order derivative filter must be used. For example : Sobel-Operator.
- **Laplacian method :** This method searches for the zero crossing in the second derivative of the image and with that, the edges can be found. For this method, the second order derivative filter must be used. For example : Laplacian Filter.

According to [3],here are three steps of the edge detection algorithm. They are :

- **Filtering :** For edge detection, it is required to use a suitable smoothing filter. The filters sharpen the edges and ignore the unnecessary information. It is often utilized to improve the functioning of an edge detector against noise. The more filtering is applied, however, the greater the loss of edge strength.
- **Enhancement :** To be able to better detect edges, changes in the intensity in the area surrounding a point must be determined. Pixels in which a significant change in intensity occurs are emphasized by enhancement, which is usually applied by calculating the gradient magnitude.
- **Detection :** Though many points in an image have a nonzero value for the gradient, not all of these points are actually edges. Because only points with strong edge content are desired, a method must be applied to determine which points are actual edge points. Thresholding is often utilized to do so.

Well known smoothing filters are :

- Sobel-Operator
- Canny Edge Detector
- Laplacian-Filter
- Prewitt-Operator

In this master's thesis, the Sobel Operator was utilized, so it will be described in more detail.

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### 2.3.1 Sobel Operator

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The Sobel Operator, sometimes called the Sobel filter is one of the most used edge detectors in image processing and computer vision. The Sobel Operator uses vertical and horizontal masks. These masks used are odd-numbered square matrices and they are generally 3x3 matrices. Approximations of the derivatives for the horizontal changes and for the vertical changes are calculated by the operator by using two 3x3 kernels and convolving them with the original image. If  $A$  is defined as the source, if  $G_x$  is an image which contains the horizontal derivative approximations at each point, and if  $G_y$  is an image which contains the vertical derivative approximations at each point, then the calculations are:

$$G_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} * A \quad G_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ +1 & -2 & -1 \end{bmatrix} * A$$

where  $*$  here denotes the 2-dimensional signal processing convolution operation.

Since the Sobel kernels can be decomposed as the products of an averaging and a differentiation kernel, they compute the gradient with smoothing. For example,  $G_x$  can be written as

$$\begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} +1 & 0 & -1 \end{bmatrix}$$

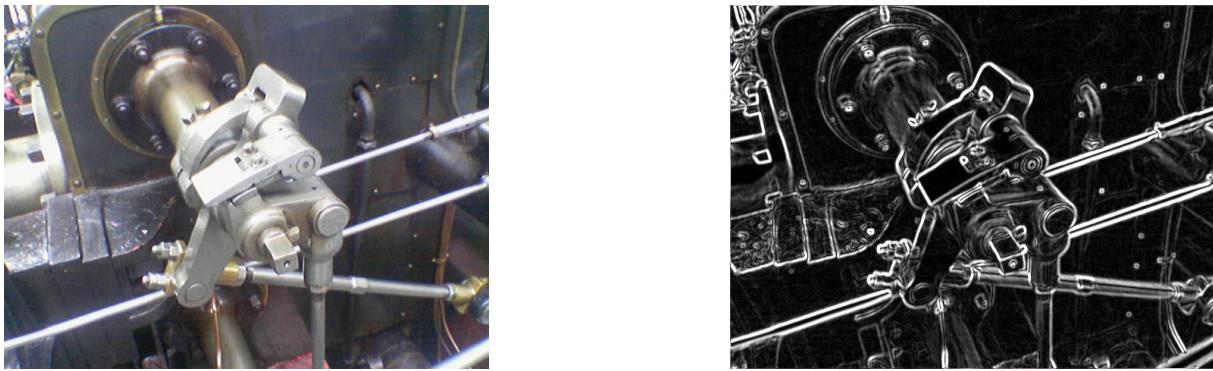
The x-coordinate is defined here as increasing in the 'right'-direction, and the y-coordinate is defined as increasing in the 'down'-direction. At each point in the image, the resulting gradient approximations can be combined to give the gradient magnitude, using:

$$G = \sqrt{G_x^2 + G_y^2}$$

Using this information, we can also calculate the gradient's direction:

$$\theta = \arctan\left(\frac{G_y}{G_x}\right)$$

where, for example,  $\theta$  is 0 for a vertical edge which is lighter on the right side.[4]



(a) Original Image

(b) Sobel Operator applied Image

**Abbildung 2.4: Sobel Operator[4]**

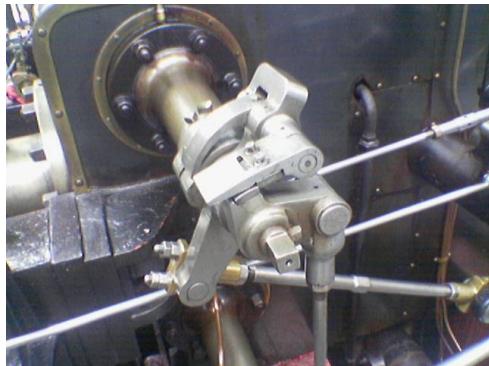
### 2.3.2 Canny Edge Detector

Canny edge detector was developed in 1986 and called with the name of its developer John F. Canny. Canny edge detector is also so popular edge detector like Sobel operator. Canny edge detector is a multi-stage algorithm and it can be analyzed in 5 different stages.[5]

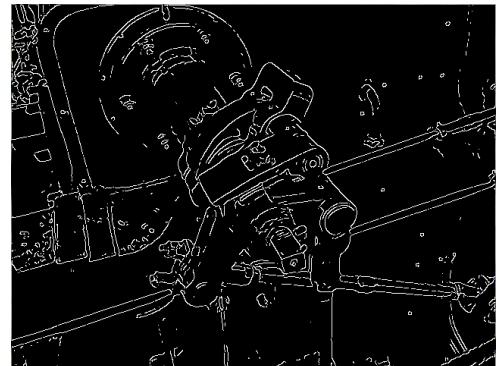
1. **Noise Reduction :** To get so stable lane detection results, we have to reduce/remove all noise from frames. Lane detection without filtering out the noise can cause false detection. Gaussian filter is used for removing noise in the frames. Gaussian filter blurs images and removes detail and noise. The size of Gaussian filter kernel must be  $(2k+1) \times (2k+1)$ . It is important to choose the size of Gaussian filter because if the size of kernel is larger, detector's sensitivity to noise is lower but on the other hand, with the increase in size of the Gaussian filter kernel, the localization error in the edge detection will also increase slightly. [6]
2. **Finding Intensity Gradient of the Image :** Essentially, the Canny algorithm locates edges in image where the grayscale intensity changes most starkly. In order to find these areas, the gradients of the image must be determined. In order to determine the gradients at each pixel in the smoothed image, the Sobel operator is applied. The Sobel operator has already been thoroughly discussed in section 2.3.1.
3. **Non-maximum Suppression :** Non-maximum suppression is an edge thinning technique which is used as an intermediate step in many computer vision algorithms. The image is scanned along the image gradient direction, and pixels that are not part of the local maxima are set to zero. This way, all image information that is not part of the local maxima is effectively suppressed.
4. **Double Thresholding :** The edge pixels remaining after applying non-maximum suppression provide a more accurate depiction of real edges in an image. Despite this, there are still some remaining edge pixels resulting from noise and color variation. Therefore, it is

necessary to filter out edge pixels with a weak gradient value while preserving edge pixels with a high gradient value. In order to do this, high and low threshold values must be selected. Edge pixels are marked as strong edge pixels when gradient values are higher than the high threshold value. They are marked as weak edge pixels when gradient values are lower than the high threshold value and higher than the low threshold value. They are suppressed when their values are lower than the low threshold value. The two threshold values are determined empirically and are dependent on the content of a given image.

**5. Hysteresis Thresholding :** Hysteresis Thresholding is the last part of Canny Edge Detector. Until this step, strong edge pixels are extracted from the true edges but there are also some weak edge pixels, some of them are extracted from true edges and some of them are extracted from some noise. So the weak edge pixels which are extracted from true edge, should be strong edge pixels and the weak edge pixels which are extracted from noises must be removed. If there is a weak edge pixel, 8 neighbour pixels of that weak edge pixel is checked and if at least, one pixel of these neighbour pixels is a strong edge pixel then, this weak edge pixels stay as edges in the end picture.



(a) Original Image



(b) Canny Edge Detector applied Image

**Abbildung 2.5:** Canny Edge Detector[6]

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## 2.4 Hough-Transformation

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### 2.4.1 Standart Hough-Transformation

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### 2.4.2 Probabilistic Hough-Transformation

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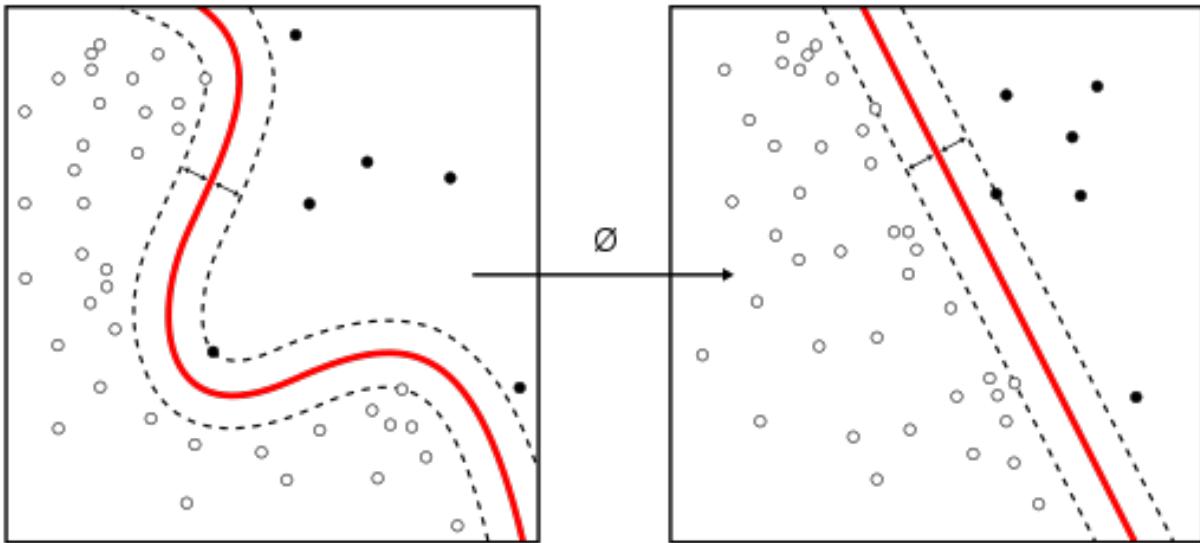
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## 2.5 K-Nearest Neighbors Algorithm

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K-Nearest Neighbor(KNN) is an non-parametric lazy learning algorithm. The non-parametric technique means that it doesn't make any assumptions on the underlaying data distribution. In

the definition of KNN, the term 'lazy learning algorithm' is used. It means it doesn't use the data training points to do any generalization. In other words, there is no explicit training phase or it is very minimal. It also means that the training phase is pretty fast. Most of the lazy algorithms - especially KNN - make decisions based on the entire training data set. On the other hand, KNN is one of the top 10 data mining algorithms[7].



**Abbildung 2.6:** K-Nearest-Neighbors Algorithm[8]

The K-Nearest Neighbors Algorithm has advantages and disadvantages. According to [9], the main advantages of KNN are simplicity, effectiveness, intuitiveness and competitive classification performance in many domains. On the other hand, KNN can have poor run-time performance when the training set is large. It is very sensitive to irrelevant or redundant features because all features contribute to the similarity and thus to the classification. The computation cost is also quite high because we need to compute distance of each query instance to all training samples.

## 2.6 Curve Fitting

*Curve fitting is used to find the 'best fit' line or curve for a series of data points. Curve fitting produces mostly mathematical equations that can be used to find points anywhere along the curve.[10]* They are several different types of curve fitting. Some of them are: linear, exponential, polynomial, exponential, power, logarithmic, etc. In this master thesis, polynomial curve fitting was used. Polynomial curve fitting differs from order of the polynomial. Polynomial curve fittings are called different names depending on their orders. First order polynomial curve fittings are called linear regression, second order as quadratic regression, and third order as cubic regression.

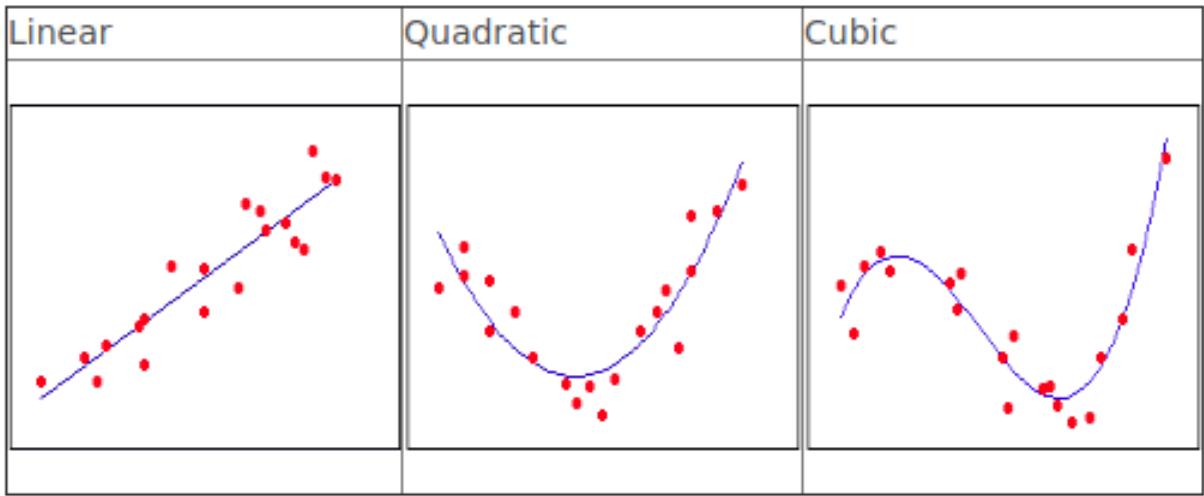


Abbildung 2.7: Types of Polynomial Curve Fitting[11]

In this master thesis, curve fitting is used for getting the best mathematical descriptions of lanes. Curve fitting uses as input the Hough points, which appear on the lanes and give as output a mathematical equation. First order polynomial curve fitting is more suitable for defining straight lines and second order polynomial curve fitting is more suitable for defining curves. There are straight and curve lanes so first order polynomial curve fitting wouldn't be enough for our project because of this reason, in this master thesis, second order polynomial curve fitting is used.

There are so many different methods for curve fitting. One of the most famous method is the least squares method. In this master thesis also the least squares method is used. Next, it will be described, how to generate a polynomial curve fitting with using the least squares method.

A data set can be mostly expressed the relationship between variable with an equation which is mostly represented with a  $k^{\text{th}}$  order polynomial. The general description of  $k^{\text{th}}$  is :

$$y = a_k x^k + \dots + a_1 x + a_0 + \epsilon$$

The general polynomial regression model with the error  $\epsilon$  provide typically an estimate rather than an implicit value of the dataset for any given value of  $x$ . A data set which has  $N$  data points, can be described with the maximum order of the polynomial which is  $k = N - 1$  but mostly the lowest possible order of polynomial is choosed.

The aim of the least squares' method is to minimise the variance between dataset values and estimated values from the polynomial equation.

For determining the coefficients of the polynomial regression model ( $a_k, a_{k-1}, \dots, a_1$ ) must be solved the following linear equations.

$$\begin{bmatrix} N & \sum_{i=1}^N x_i & \dots & \sum_{i=1}^N x_i^k \\ \sum_{i=1}^N x_i & \sum_{i=1}^N x_i^2 & \dots & \sum_{i=1}^N x_i^{k+1} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^N x_i^k & \sum_{i=1}^N x_i^{k+1} & \dots & \sum_{i=1}^N x_i^{2k} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N y_i \\ \sum_{i=1}^N x_i y_i \\ \vdots \\ \sum_{i=1}^N x_i^k y_i \end{bmatrix}$$

The equations, which in standard form of  $Ma = b$  can be solved with so many different methods. In this master thesis, Cramer's Rule was used for solving for the polynomial coefficients of curve fitting. With the help of the determinants of the square matrix  $M$  can be solved the linear system of equations for finding the coefficients. Each of the coefficients  $a_k$  may be determined using the following equation :

$$a_k = \frac{\det(M_i)}{\det(M)}$$

For finding coefficients of polynomial curve fitting, we have to use the equation shown above so the determinate of  $M_i$  must be divided by determinate of  $M$ . Above, the equation  $Ma = b$  were shown so the determinate of  $M$  matrix can be calculated but for the determinate of  $M_i$  matrix, the  $M$  matrix must be modified. For finding the  $M_i$  matrix, the  $i^{\text{th}}$  column must be replaced with the column vector  $b$ , which was used at the equation  $Ma = b$ . For example, if the  $M_0$  matrix wanted to be determined, the  $M$  matrix must be modified like at the following[12] :

$$M = \begin{bmatrix} N & \sum_{i=1}^N x_i & \dots & \sum_{i=1}^N x_i^k \\ \sum_{i=1}^N x_i & \sum_{i=1}^N x_i^2 & \dots & \sum_{i=1}^N x_i^{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^N x_i^k & \sum_{i=1}^N x_i^{k+1} & \dots & \sum_{i=1}^N x_i^{2k} \end{bmatrix} \quad M_0 = \begin{bmatrix} \sum_{i=1}^N y_i & \sum_{i=1}^N x_i & \dots & \sum_{i=1}^N x_i^k \\ \sum_{i=1}^N x_i y_i & \sum_{i=1}^N x_i^2 & \dots & \sum_{i=1}^N x_i^{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^N x_i^k y_i & \sum_{i=1}^N x_i^{k+1} & \dots & \sum_{i=1}^N x_i^{2k} \end{bmatrix}$$

In this master thesis, 2<sup>nd</sup> order polynomial equation is used so just the values of  $a_0, a_1$  and  $a_2$  must be calculated. For understanding curve fitting better, we are going to develop the 2<sup>nd</sup> order polynomial curve fit for the given dataset.

x	-3	-2	-1	-0.2	1	3
y	0.9	0.8	0.4	0.2	0.1	0

$$M = \begin{bmatrix} 6 & -2.2 & 24.04 \\ -2.2 & 24.04 & -8.008 \\ 24.04 & -8.008 & 180.0016 \end{bmatrix} \quad M_0 = \begin{bmatrix} 2.4 & -2.2 & 24.04 \\ -4.64 & 24.04 & -8.008 \\ 11.808 & -8.008 & 180.0016 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 6 & 2.4 & 24.04 \\ -2.2 & -4.64 & -8.008 \\ 24.04 & 11.808 & 180.0016 \end{bmatrix} \quad M_2 = \begin{bmatrix} 6 & -2.2 & 2.4 \\ -2.2 & 24.04 & -4.64 \\ 24.04 & -8.008 & 11.808 \end{bmatrix}$$

$$a_0 = \frac{\det(M_0)}{\det(M)} \implies a_0 = \frac{2671.1962}{11661.2736} = 0.2291$$

$$a_1 = \frac{\det(M_1)}{\det(M)} \implies a_1 = \frac{-1898.4602}{11661.2736} = -0.1628$$

$$a_2 = \frac{\det(M_2)}{\det(M)} \implies a_2 = \frac{323.7632}{11661.2736} = 0.0278$$

In this case, the fitted curve function is :

$$y = 0.0278x^2 - 0.1628x + 0.2291$$



Abbildung 2.8: Curve Fitting[13]

## 2.7 Rectangle Method



# 3 Implementation

## 3.1 Test Track

As previously mentioned, the medium-term goal of this master thesis is attending the Carola-Cup at Braunschweig University, so the test truck was prepared according to the Carola-Cup criteria by Nicolas Acero Sepulveda, who also did his bachelor's thesis with this model automobile. For this test truck, two black PVC floor carpets were used and on these floor carpets, the lanes of the track were made by using white electrical tape. The straight part of the track was made on one of these PVC floor carpets and the curved part of track was made on the second PVC floor carpet. The straight part of the track is approximately 2 meters long and the curve radius of the curved part of test track is approximately 1 meter. This curve is the tightest curve at Carola-Cup, so with this, the test track can be tested in the worst case scenario. In the Carola-Cup competition, the track is much larger; however, for the purposes of this master thesis, a larger test track in not needed.

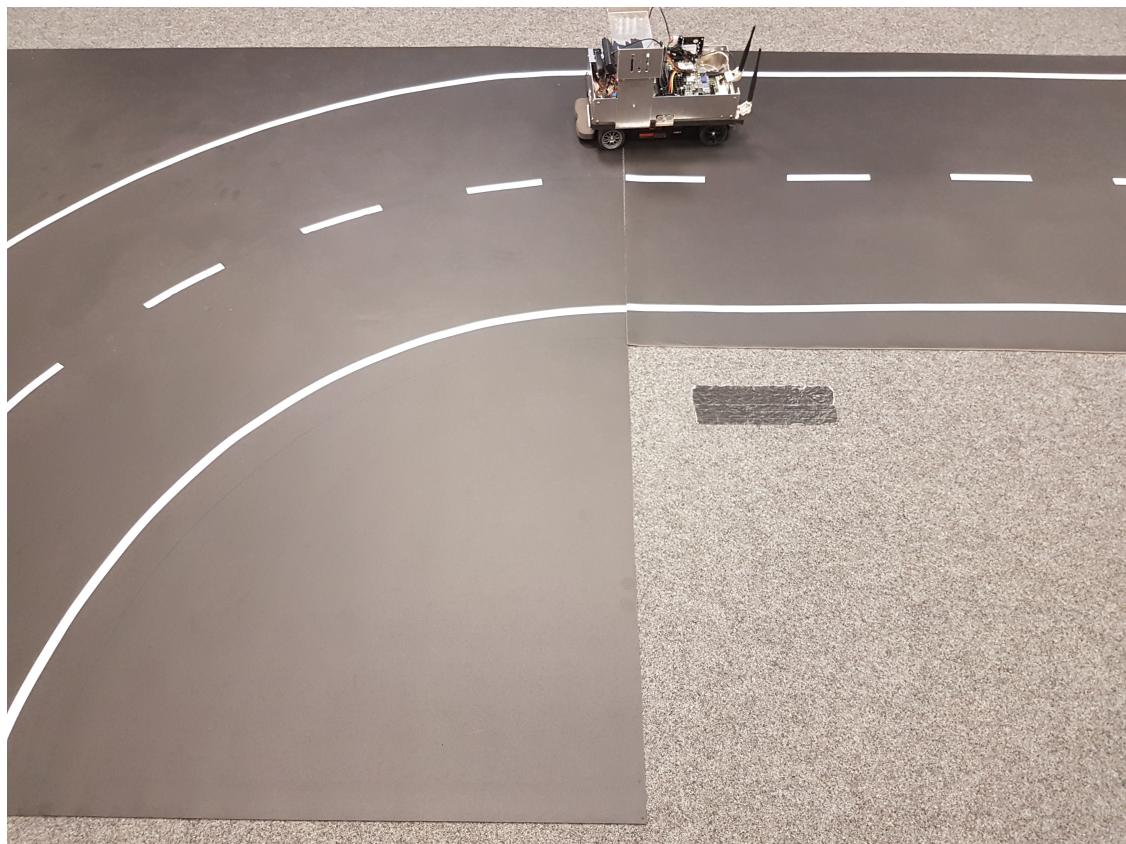


Abbildung 3.1: Test Truck

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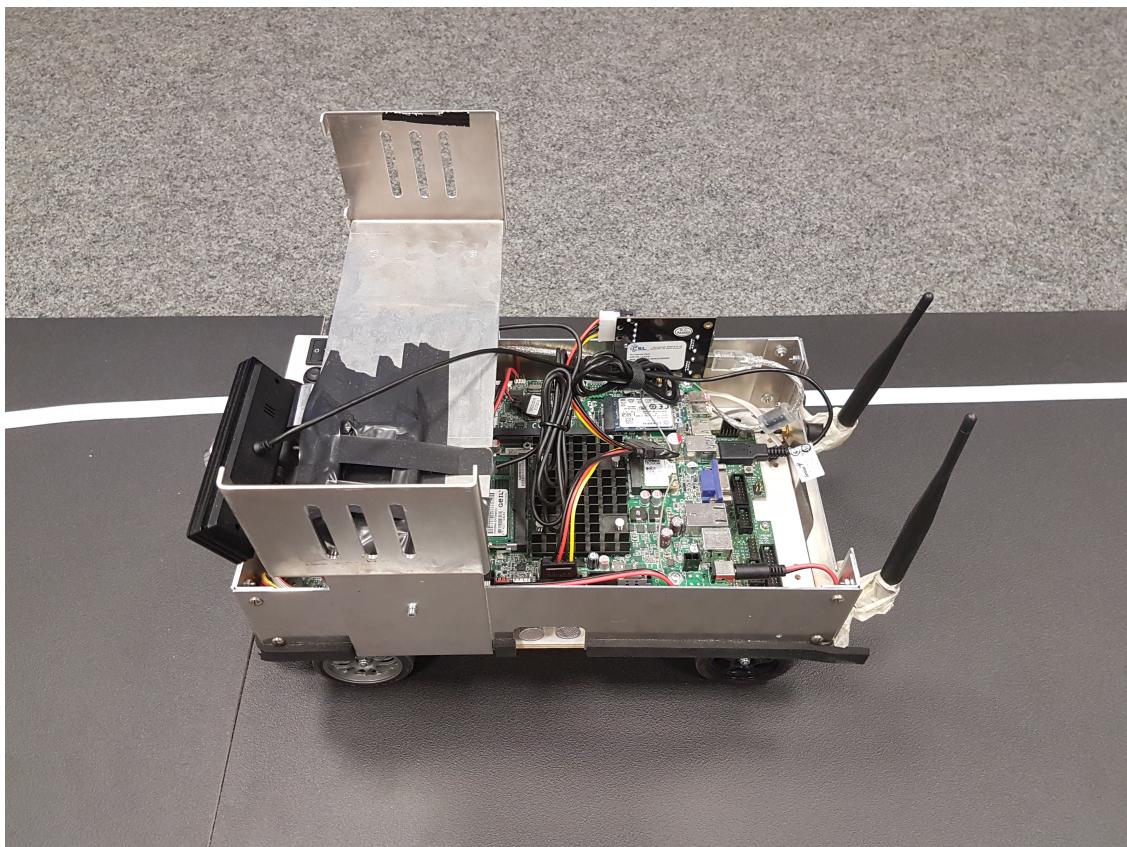
## 3.2 Hardware

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### 3.2.1 Model Auto

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During the course of this master's thesis, a model automobile was being used which was prepared for the Projectseminar Echtzeitsysteme at Technical University of Darmstadt. The chassis, steering mechanism, power train, and engine control were derived from the model-building of a Japanese company, Tamiya. The maximum velocity of the model automobile is approximately 1 m/s and the minimum steering radius is around 90 cm.



**Abbildung 3.2:** Model Auto

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### 3.2.2 Microcontroller and Main Board

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In this model automobile, there is a microcontroller and a main board. The microcontroller is used for controlling steering and receiving the measurements from ultrasonic sensors and hall effect sensors. The 16-bit microcontroller is from MB96300 series from Fujitsu company.

The main board on the model car is from PD10BI-MT ThinMini-ITX series from MiTAC company. This main board communicates with the microcontroller over via UART interface through USB connection. On this main board, there is an Intel Quadcore-Processor and an Intel HD Graphics card. Furthermore, there is an 8 GB DDR3-1600 RAM and 1Gbit/s Ethernet, VGA, HDMI,

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USB 2.0/3.0, SATA ports and an Intel Dual Band Wireless AC 7260 Network adapter, which is connected to two external WLAN antennas. A 60 GB Kingston SSD-Harddisk is connected over an integrated PCI-Express Port. A 3200 mAh Li-Fe battery is used as a power supply.

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### 3.2.3 Camera

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The camera is one of the main components of lane detection and accordingly, autonomous driving. For this thesis, I had to research the most suitable camera because all cameras have different properties.

At the beginning of the Projectseminar Echtzeitsysteme, the Logitech C270 HD Webcam was being used. The resolution of the camera is 1280x960 pixels and the Frame per Second (FPS) value is 30 Hertz (Hz) at a 640x480 pixel resolution. The field of View (FOV) is just 60 degrees. The problem with this camera is that if there is a curve, the camera cannot see all of the lanes, and thus is not very suitable for lane detection. When I started my master's thesis, there was a Kinect v2 camera on the model car. The Kinect v2 camera was developed by Microsoft and released in 2013. This camera has a depth sensor with a resolution of 512x424 pixels and its FOV is 70x60 degrees. The FPS value is 30 Hz at a 512x424 pixel resolution. This camera also has a color camera with resolution of 1920x1080 pixels and a FOV of 84.1x53.8 degrees. The FPS value is 30 Hz at a 1920x1080 pixel resolution. This camera had two main disadvantages for this master's thesis. The first disadvantage is the FOV value of camera. This value is better than the value of Logitech C270 camera but it is still not enough for curve lane detection. The second main disadvantage is the location of the color camera. The color camera of this camera is not in the middle of camera, but rather, on the right. This is a disadvantage for us because when there are curves going left as opposed to right, the camera is unable to see the left and even perhaps the middle lane of the truck. Thus, this is problematic for lane detection.

Due to these reasons, I had to choose a camera which has a sufficiently high FOV value. After doing research, I decided that the Genius WideCam F100 camera is the best choice for this master's thesis because this camera has a FOV value of 120 degrees and it can also be used with the Linux Operating System. The resolution of this camera is 1920x1080 pixels and the FOV value is 120 degrees. The FPS is 30 Hz at a 1920x1080 pixel resolution. With this camera, it is possible to detect most if not all lanes, including when there are curves.



**Abbildung 3.3:** Genius 120-degree Ultra Wide Angle Full HD Conference Webcam(WideCam F100)

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### 3.3 Software

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In this chapter, the software algorithms defined in this master's thesis will be focused on. With the aid of program flow charts and explanations of all their steps, the algorithms will themselves be better explained. In order to find the best solution, five different source code versions(?variants/?methods) were generated. For all these source codes, the computing times were calculated and compared in terms of which solution can detect the lanes better. In the following pages, there are detailed explanations of the versions utilized (?variants/?methods). The development environment and the software utilized in this master's thesis will be also described.

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#### 3.3.1 Development Environment and Related Softwares

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As also mentioned at subsection 3.2.2, in this project the previously introduced main board was utilized. One of the compact and fast versions of the Linux 16.04 operating system, *Lubuntu* was installed in this main board.

The version *Kinetic* of ROS was used for implementation of this master's thesis. ROS is the abbreviation of Robotic Operating System, which is a robotics middleware (i.e. collection of software frameworks for robot software development). On the ROS wiki page[14], ROS is defined as an open-source, meta-operating system for your robot. It provides the services you would expect from an operating system, including hardware abstraction, low-level device control, implementation of commonly-used functionality, message-passing between processes, and

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package management. It also provides tools and libraries for obtaining, building, writing, and running code across multiple computers.

For using prewritten image processing functions, an open-source computer vision and machine learning software library called OpenCV was used. According to the OpenCV website[?], there are more than 2500 optimized algorithms in the OpenCV library and OpenCV has a user community of more than 47 thousand people.

ROS can be programmed with Python, C++ or Lisp programming languages and OpenCV can be programmed with Python or C++ programming languages. In this master's thesis, C++ was used.

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### 3.3.2 Preprocessing :

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There are many different possibilities for lane detection algorithms. Of course, each has its own advantages and disadvantages. In this master's thesis, some methods were defined and in this chapter, these methods will be explained in detail.

In all of these methods, some processes are common, and this is called the preprocessing phase. At the beginning, *the frames are obtained from the camera via ROS-Topic*. ROS uses different image formats than OpenCV, which uses the image format Matrix(Mat). The frame obtained via ROS-Topic must be converted from the ROS image data type to a Mat object. In order to convert the frame, a ROS-Package *cvbridge*[15] was used. cvbridge converts the ROS image format to a Mat Object, which is the OpenCV Image Format.

Mat is a class which has two parts. The parts are a matrix header and a pointer to the matrix containing the pixel values. The matrix header contains information like the size of matrix, storing method, etc. It always has a fixed size but the size of matrix is variable from image to image.

There are so many methods which can store the pixel values to the Mat object. In this case, the color space and the data type utilized can be chosen. For gray images, it is easy to choose the color space because there are just two colors : black and white. By changing the density of colors(black and white), it is possible to create many shades of gray. There are more methods for color images. Color images generally have three or four channels. These three channels are used for RGB color values. The RGB colors are based on the colors red, green, and blue, which can all be detected by the human eye. For the transparency of a color, a fourth channel called alpha(A) can be used.

There are also other color formats, which have some advantages[16].

- The RGB format is quite similar to the human eye, but the OpenCV display system uses the BGR format, which uses another row of colors.
- The HSV and the HLS formats are more natural ways to display colors. They decompose colors into their hue, saturation, and value/luminance components. Another advantage of

the HSV and the HLS formats is that they are less sensitive to the light conditions of the input image.

- In JPEG image formats the YCrCb format is used.
- if the distance of a given color to another color is to be measured, CIE L\*a\*b\* format is more suitable than others.

After the frame from Camera via ROS-Topic was received, the color frame had to be converted to a grayscale frame. For detection lanes, Hough Transformation is used and for Hough Transformation, the grayscale format of the input image is needed. Converting a frame from BGR format to grayscale format has some advantages, the main advantage being the processing time. Normally color frame matrix content has three or four channels, but grayscale frame matrix content has just one channel, so grayscale frame matrix size is much smaller compared to color frame matrix size. Because of this, the image processing time is much more less for the grayscale format compared to BGR format. In order to covert the BGR formatted frame to a grayscale formatted frame, the *cvtColor* function from OpenCV is used.

For stable lane detection, the light conditions must be considered. Because of this, after converting the frame with BGR format to grayscale format, the lightest and darkest pixels have to be searched for. After finding the lightest and darkest pixels, it is possible to estimate the lighting conditions. These values are then used in the next step. After this processing, a filter is applied to all frames, transforming images into binary images by transforming each pixel according to whether it is inside or outside a specified range. The user chooses a threshold value to process. If a pixel is greater than this value, it is assigned an 'inside' value; otherwise, it is assigned an 'outside' value. Depending on the lightest and darkest pixel values, the threshold value changes. Through this dynamic parameter, the lanes are more able to be more clearly detected and noise can be cancelled more successfully.

After the threshold filter, an edge detection filter must be applied. In this master's thesis, the Sobel-Operator is used. This is explained in Chapter 2 in detail.

This preprocessing part is common for all cases but after this process, the cases diverge.

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### 3.3.3 Method 1/Case 1 : Hough Transformation + Rectangle Method + Curve Fitting + IPM (v0.9.2)

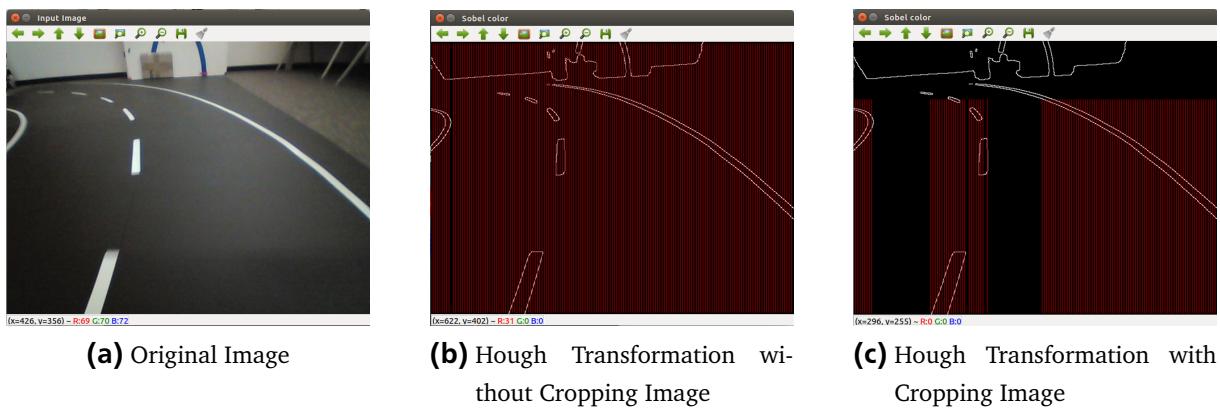
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When the preprocessing part is over, the lanes must be detected. In order to find the position of lanes, the Standart Hough Transformation is used. But the Standart Hough Transformation does not utilize all of the frame; rather, the frame is cropped. There is an advantage of cropping the frame, which will be explained in detail.

The frames obtained from the camera are at a resolution of 640x480 pixels, which is the default value. The resolution of the camera can be increased by adjusting its settings, but it is not possible to decrease it this way. In order to decrease the resolution of the camera, there is

another method. This method is used in another case, and will be explained there. In this case, the minimum resolution the camera allows is used. There are some reasons for this, the most important being that if the resolution is increased, there are also more pixels, and thus the computing time increases as well. High computing time is of course undesirable.

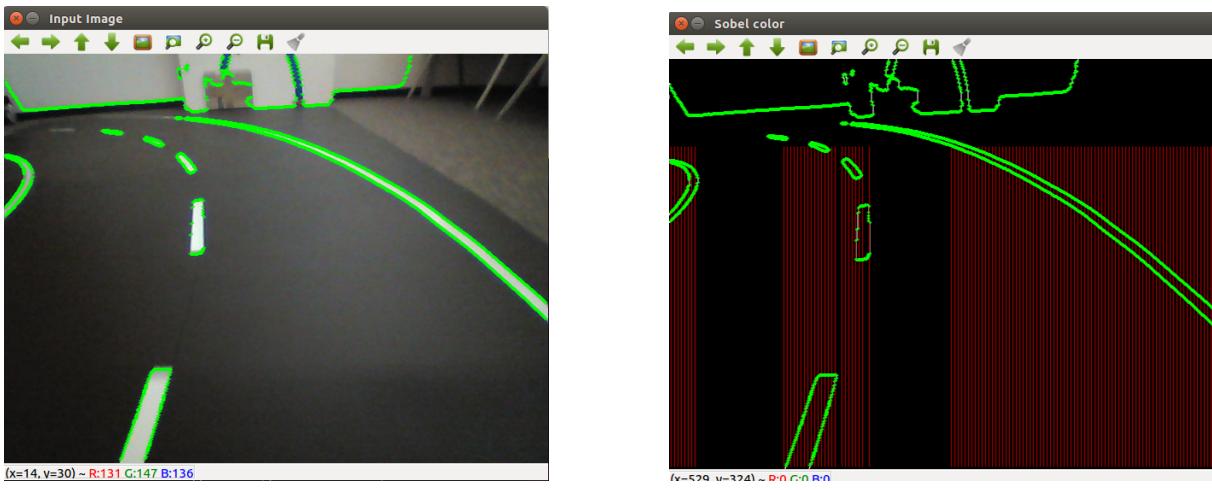
The original height of the frame was 640 pixels, but as previously mentioned, the frame was cropped. As seen in Figure 3.4b, when there is a curve, the camera also detects points that do not belong to the track. When the frame is not cropped, the detection of irrelevant points would result in the undesired production of red Standard Hough Transformation lines. As a result, first 100 pixels of the frame height were removed and the last 540 pixels were used. As seen at Figure 3.4c, the red lines are shown only in the last 540 pixels of frame height and do not cover the irrelevant parts of the frame. If there is a lane, the red lines are very close each other, but if there is no lane, there is a distance of at least 50 pixels between the red lines. In Figure 3.4c, there are three different groups of red lines. In each group, there is a lane. Thanks to the Standard Hough Transformation, we know which lanes are between which pixel columns.



**Abbildung 3.4:** Detecting Lane Positions

After this processing, we have to find the starting points of lanes and the all pixels which are on the lanes. For that, we have to use Probabilistic Hough Transformation. Probabilistic Hough Transformation is a bit different than Standart Hough Transformation. Standart Hough Transformation is more suitable for straight lanes but if there is a curve, Standart Hough Transformation doesn't work enough good. As before mentioned, a line can be represented as  $y = mx + c$  or in parametric form, as  $\rho = x \cos \theta + y \sin \theta$  where  $\rho$  is the perpendicular distance from origin to the line, and  $\theta$  is the angle formed by this perpendicular line and horizontal axis measured in counter-clockwise. But it is different at Probabilistic Hough Transformation. A line is represented by two or more points. If Probabilistic Hough Transformation finds at least two points from the same lane, it represents beginning and ending points of these lines.

In this master thesis, the lines are not shown because we don't see the Probabilistic Hough Lines. We need just these points(pixels), which are on the lanes.



**(a)** Original Image with Probabilistic Hough Transformation

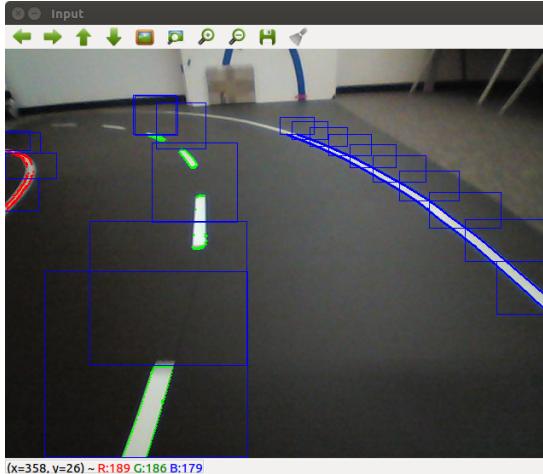
**(b)** A Image with Sobel Operator and Probabilistic Hough Transformation

**Abbildung 3.5:** Probabilistic Hough Transformation Points

At Figure 3.5a and Figure 3.5b, Probabilistic Hough Transformation Points(green pixels) are shown. They are so many pixels, which are found by Probabilistic Hough Transformation. There are some parameters at this function in OpenCV. With changing the parameters of Probabilistic Hough Transformation in OpenCV, less Hough Points on the lanes can be found but finding too few Hough points can cause some problems while detecting the lanes. On the other hand, finding so many Hough points need also more computing time, this is also a situation which we don't want to have. So in this case, the parameters of Probabilistic Hough Transformation fuction in OpenCV must be optimized. Thanks of optimization, the best solution (less computing time and good lane detection) is found.

Now, we know, that between which pixel columns stand lanes so we can find the start pixels of lanes. For that, we have to find the Hough pixels for all lanes(right, middle and left lane) at the lowest part of the frame. Each Hough points which are on a lane, can be compared with each other so the Hough point of the lane, which is the lowest part of the frame can be found. It is the starting point of the lane. This process must be done for all lanes which can be seen on the frame.

Next part of the project is getting the Hough points which are relevant the lanes. For each lane, the Hough points must be grouped. For getting Hough points which are relevant with lanes, 'rectangle' method is used, which is named by me. At rectangle method, a rectangle is drawn and save coordinates of all Hough points in that rectangle. All Hough points are saved in a vector and the uppest Hough point at the rectangle must be found. From that point, another rectangle must be drawn but the size of rectangles are going to progressively smaller. Because the objects are going to seem smaller when they are far away from the camera. There is an exception at middle lane. Middle lane has dashed lines so the rectangles at middle lane must be bigger than at the left and right lanes.



**Abbildung 3.6:** Rectangle Method

Last part of this case is curve fitting. Curve Fitting is an algorithm which gives a mathematical description and this mathematical description has the best fit to a series of data points. But in this master thesis, usage of curve fitting is changed a bit. At curve fitting, the coordinates of Hough points are used but we changed the x and y axes of these Hough points because y-axes of these coordinates have more range than x-axes so this swapping raises the stability of the curve fitting.

After using the Hough points as input, three different mathematical equation are produced. One of these equation is for the left lane, the other one is for the middle lane and the last equation is for the right lane. While producing of these equations, of course just relevant Hough points are used. For example, for left lane curve equation, the Hough points from the left lane were used.

Next part of this case is plotting the curves which are produced by curve fitting function. We start from 0th pixel to 480th pixel vertically and the output values of equation for each vertically pixels are found. For each lane, the curves are plotted in different color.

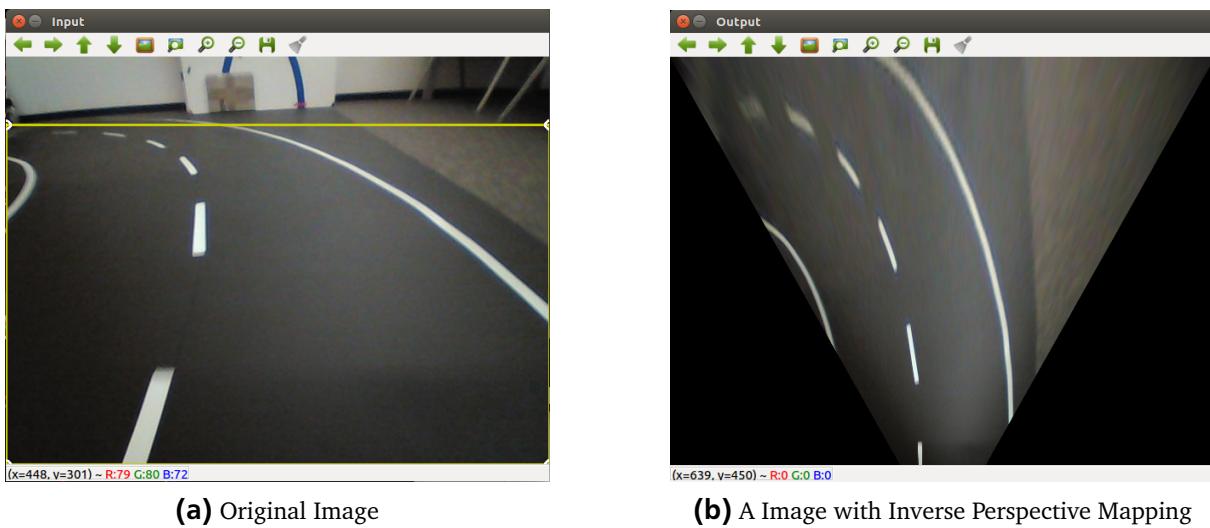
At the last part of this case is Inverse Perspective Mapping. End of the project, we want to get mathematical description of lanes from bird's-eye view. So the perspective of lanes must be changed from the camera side to the top of the truck side. For that, we have to use 'findHomography' function from OpenCV. Thanks this function, all curve points can be converted to perspective of the top of the truck side. All pixels could also be converted from camera perspective to the top of the truck perspective but in that case, we had to convert 640x480 pixels, respectively 307200 pixels. But in this case, we convert just 480 pixels for each lane, it means, we convert totally 1440 pixels. Because of this reason, this case is efficienter than to convert all pixels.

*Here must be a picture of curve fitting*

all of the pixels in the 100th column

### 3.3.4 Method 2/Case 2 : IPM + Hough Transformation + KNN + Curve Fitting(v0.4.2)

As we before mentioned, preprocessing part is common for all cases. In this case, frames which are taken from camera, are convert directly from camera perspective to top of the truck perspective. At Section 3.3.3 (Case 1), we convert the just Curve Fitting pixels from the camera perspective to the top of the truck perspective. To convert 640x480 pixels, respectively 307200 pixels take a bit longer time compare to the Case 1. Original frame which can be seen at Figure 3.7a is converted to the frame which can be see at Figure 3.7b by OpenCV 'findHomography' function.



**Abbildung 3.7: Inverse Perspective Mapping**

Next step of this case is that finding between which vertically pixels stand the lanes. At Case 1(Section 3.3.3 we had to apply Standart Hough Transformation to the frame whose the first 100 pixels of frame vertically were took off(380x640 pixels). After IPM implementation, we don't need to apply Standart Hough Transformation to the so big part of the frame. Just last 160 vertically pixels (160x640 pixels) is enough to find all positions of lanes.

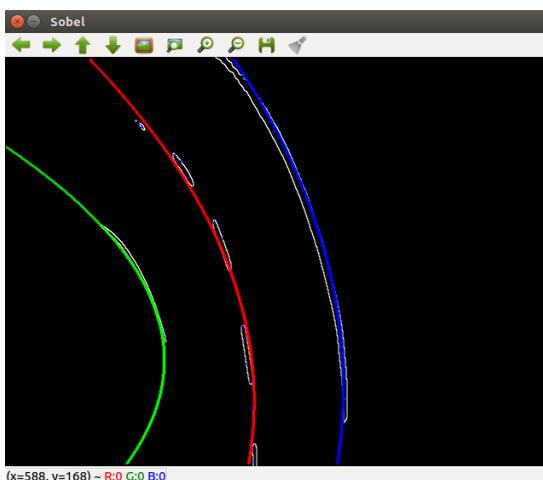
After finding between which pixel columns stand the lanes, the Probabilistic Hough Transformation was used. Thanks to Probabilistic Hough Transformation, on the lanes appear Hough points. As metioned in Case 1 at Section 3.3.3, there are some parameters at Probabilistic Hough Transformations, so number of Hough points can be descreased or increased. Decreasing the number of Hough Points descrease also the computing time but, so much decreasing the number of Hough Points can cause the loose stability of detecting lanes. So the parameters must be setted in the most efficient way.

We have already found out between which pixel columns stand the lanes. So we have to group Hough points according to lanes. If the camera can see all there lanes, then we have to Hough points to the three groups but if there is a curve, the camera can see just the right and middle

lane so the Hough points must be in this case to the two groups. For each lane must be found the starting points of the lanes. For that, all Hough points in a group must be compared with each other then the Hough point which are the lowest points of the frame, are the starting points.

The next step of this case is k-nearest neighbors algorithm (KNN). KNN is a learning algorithm. Here KNN is used instead of rectangle method which is used in Case 1 at Section 3.3.3. For KNN are used the functions from OpenCV. There are some parameters at these functions. For example, we can set, that 'how many neighbour points will the functions find?'. The parameters must be setted in the most efficient way. It means, we want to get stabil and fast results.

The latest step of this case is curve fitting. End of the KNN algoritm, we don't get so many Hough points like in rectangle method for curve fitting algorithm. It means, the curve fitting algorithm uses less points as input so it produces the mathematical equation of lanes much more faster. Here also x and y coordinates of Hough points were swapped for getting more stable curves. After getting the mathematical description of lanes, for each vertical pixels, the equation result is calculated and plotted on the frame. So the lanes can be detected, mathematically described and plotted on the frame.



**Abbildung 3.8:** Output Image

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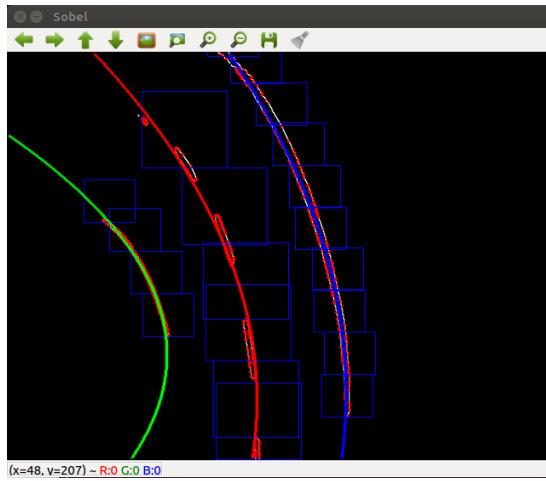
### 3.3.5 Method 3/Case 3 : IPM + Hough Transformation + Rectangle Method + Curve Fitting(v0.5.1)

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This case is so similar to case 2 at Section 3.3.4. Like in Case 2, at the beginning of this case, Preprocessing Part is used, then Inverse Perspective Mapping for all pixels implemented. After that, for 160x640 pixels, Standard Hough Transformation is used and then for all pixels Probabilistic Hough Transformation is implemented. Until now, the implementation was totally same with Case 2. In Case 2, there were used k-nearest neighbours (KNN) method was used but in this case, the rectangle method is used which was used in Case 1 at Section 3.3.3. But at the usage of rectangle method, there is a difference than the rectangle method which was used at Case 1. At Case 1, the size of rectangles were going to progressively smaller but in this case,

the size of rectangles are always same because the Inverse Perspective Method(IPM) was used at the beginning of case so the lanes are shown on the top of track. But the size of rectangles at middle lane is bigger than the size of rectangle at left and right lanes. Because of the dashed lines, we had to use bigger sized rectangles at the middle lane.

After the rectangle method was used, the curve fitting method is used, like in other cases, x and y coordinates of Probabilistic Hough points should be swapped after rectangle methods. Thanks these swapping, the curve fitting works more stable. After producing 3 different equations for three different lanes, output values for all pixel columns are calculated and plotted at the frame.



**Abbildung 3.9:** Output Image

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### 3.3.6 Method 4/Case 4 : Resize + IPM + Hough Transformation + KNN + Curve Fitting(v0.7.1)

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This case is also so similar to case 2. There is just one difference between Case 2 at 3.3.4 and this case. The only difference is resizing the frame at the beginning of the case. The frame comes from the camera with 64x480 pixels resolution but we resize this frame from 640x480 pixels resolution to 320x240 pixels resolution. For resizing, a 'resize' function from OpenCV were used because we can't get less resolution than 640x480 pixels from the camera so we had to use a function for resizing. Because of resizing, we had to use sometimes different parameters than Case 2. We had to find the optimal parameters for efficient and good lane detection.

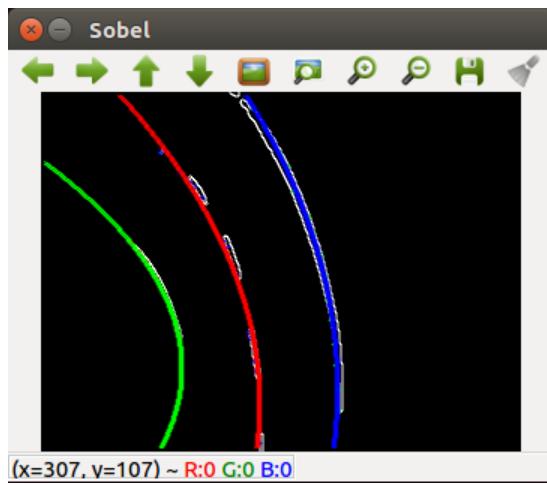


Abbildung 3.10: Output Image(320x240 pixels resolution)



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## **4 Evaluation and Discussion**



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## 5 Related Works



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## **6 Conclusion**



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