

作业提交服务器: ftp://192.168.134.123

用户名: uploader

密码: stu1234

! 注意事项

1.

```
1  if 有多个文件:
2      打包成zip文件
3  else: #只有一个pdf
4      pass
5  用自己的学号将文件命名
6  上传至对应的文件夹中
```

2. 代码请上传源文件, 比如.py文件或.cpp文件, 不要上传工程文件
3. 非代码请上传pdf, 非pdf (比如.doc文件或.md文件) 请转换成pdf
4. 涉及公式的作业, 推荐使用markdown
5. 涉及伪代码的作业, 推荐使用在线 $LaTeX$ 编辑器`Overleaf`完成, 提交编译生成的pdf文档即可。如果需要在overleaf中输入中文并能正确编译, 首先在文档中加入`package: \usepackage[UTF8]{ctex}`, 然后修改设置: 设置->修改Latex引擎->选择“XeLatex”

第四次作业

提交时间: 2022年12月8日上课之前

1. Let's consider a long, quiet country road with houses scattered very sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road, so that every house is within four miles of one of the base stations.
Give an efficient algorithm that achieves this goal, using as few base stations as possible.
提示: 贪心法, 需要给出正确性证明。
2. Yuckdonald's is considering opening a series of restaurants along Quaint Valley Highway (QVH). The n possible locations are along a straight line, and the distances of these locations from the start of QVH are, in miles and in increasing order, m_1, m_2, \dots, m_n . The constraints are as follows:
(a) At each location, Yuckdonald's may open at most one restaurant. The expected profit from opening a restaurant at location i is p_i , where $p_i > 0$ and $i = 1, 2, \dots, n$.
(b) Any two restaurants should be at least k miles apart, where k is a positive integer.
Give an efficient algorithm to compute the maximum expected total profit subject to the given constraints.
提示: 动态规划, 给出子问题的定义, 递归关系, 算法的时间复杂度和空间复杂度。

3. Let us define a multiplication operation on three symbols a, b, c according to the following table; thus $ab = b, ba = c$, and so on. Notice that the multiplication operation defined by the table is neither associative nor commutative.

	a	b	c
a	b	b	a
b	c	b	a
c	a	c	c

Find an efficient algorithm that examines a string of these symbols, say $bbbbac$, and decides whether or not it is possible to parenthesize the string in such a way that the value of the resulting expression is a . For example, on input $bbbbac$ your algorithm should return yes because $((b(bb))(ba))c = a$.

提示：动态规划，给出子问题的定义，递归关系，算法的时间复杂度和空间复杂度。

4. You are given a rectangular piece of cloth with dimensions $X \times Y$, where X and Y are positive integers, and a list of n products that can be made using the cloth. For each product $i \in [1, n]$ you know that a rectangle of cloth of dimensions $a_i \times b_i$ is needed and that the final selling price of the product is c_i . Assume the a_i, b_i , and c_i are all positive integers. You have a machine that can cut any rectangular piece of cloth into two pieces either horizontally or vertically. Design an algorithm that determines the best return on the $X \times Y$ piece of cloth, that is, a strategy for cutting the cloth so that the products made from the resulting pieces give the maximum sum of selling prices. You are free to make as many copies of a given product as you wish, or none if desired.

提示：动态规划，给出子问题的定义，递归关系，算法的时间复杂度和空间复杂度。