

作业提交服务器: ftp://192.168.134.123

用户名: uploader

密码: stu1234

### ! 注意事项

1.

```
1  if 有多个文件:
2      打包成zip文件
3  else: #只有一个pdf
4      pass
5  用自己的学号将文件命名
6  上传至对应的文件夹中
```

2. 代码请上传源文件, 比如.py文件或.cpp文件, 不要上传工程文件
3. 非代码请上传pdf, 非pdf (比如.doc文件或.md文件) 请转换成pdf
4. 涉及公式的作业, 推荐使用markdown
5. 涉及伪代码的作业, 推荐使用在线 $LAT_EX$ 编辑器`Overleaf`完成, 提交编译生成的pdf文档即可。如果需要在overleaf中输入中文并能正确编译, 首先在文档中加入`package: \usepackage[UTF8]{ctex}`, 然后修改设置: 设置->修改Latex引擎->选择“XeLatex”

## 第二次作业

提交时间: 2022年11月3日上课之前

1. Let  $n$  be a positive integer. An **addition chain** with target  $n$  is a sequence of increasing integers  $x_0, \dots, x_l$  that satisfies the following properties: 1)  $x_0 = 1$ ; 2)  $x_l = n$ ; 3) for  $1 \leq k \leq l$ , we can express  $x_k$  in the form  $x_k = x_i + x_j$  for indices  $i$  and  $j$ , where  $0 \leq i \leq j \leq k-1$ . The **length** of the addition chain is  $l$  (but note that there are  $l+1$  integers in the sequence). For example, the following is an addition chain of length 8 with target 47: 1, 2, 3, 5, 10, 20, 23, 46, 47. Devise a backtracking algorithm to find all the addition chains with a given target  $n$  and thus determine the addition chain with target  $n$  that has the minimum length. Write a program to implement your algorithm and run it on the values  $n \in \{71, 127, 191, 379, 607\}$ .
2. Let  $n$  be a positive integer. Suppose the integers  $1, \dots, n$  are placed in a circle, with one integer in each position. We can think of this arrangement as a **cyclic permutation** of the set  $\{1, \dots, n\}$ . Among all the  $n$  possible sums of three cyclically consecutive values, find the maximum sum,  $S$ . We wish to minimize the value of  $S$ .  
  
For example, when  $n = 6$ , if we place the numbers  $1, \dots, 6$  in the cyclic order 5, 4, 1, 6, 3, 2, then the sums are 10, 11, 10, 11, 10, 11 and hence the maximum sum is  $S = 11$ .  
  
(a) Show that the average sum is  $\frac{3(n+1)}{2}$  and hence  $S \geq \lceil \frac{3(n+1)}{2} \rceil$ .

- (b) Given  $n$  and a target value  $T \geq \lceil \frac{3(n+1)}{2} \rceil$ , devise a backtracking algorithm to find all examples of cyclic permutations of  $\{1, \dots, n\}$  in which the maximum sum of three cyclically consecutive elements is at most  $T$ .
- (c) Write a program to implement your algorithm and run it with the following  $(n, T)$ -pairs:  $n = 12$  and  $T = 21$ ;  $n = 13$  and  $T = 23$ ;  $n = 14$  and  $T = 24$ ; and  $n = 15$  and  $T = 25$ .
3. Let  $k$  and  $v$  be positive integers. A  $(v, k)$ -**difference packing** is a set of  $k$  distinct elements of  $\mathbb{Z}_v$ , say  $D = \{x_1, \dots, x_k\}$ , such that the differences  $(x_i - x_j) \bmod v$  (for  $i \neq j$ ) are all distinct. For example,  $\{0, 1, 3, 9\}$  is a  $(13, 4)$ -difference packing. A necessary condition for the existence of  $(v, k)$ -difference packing is that  $v \geq k^2 - k + 1$ . A difference packing is also known as a **modular Golomb ruler**.
- (a) Design a backtracking algorithm to enumerate all  $(v, k)$ -difference packings for specified values of  $v$  and  $k$ .
- (b) Use your algorithm to find  $(21, 5)$ -,  $(31, 6)$ - and  $(48, 7)$ -difference packings.