CSC338. Homework 5

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q1b

(b) $\|x\|_A = (x^TAx)^{\frac{1}{2}}$

(I) $\forall v \in V, \|v\| \geq 0$ and $\|v\| = 0$ if and only if $v = 0$

(II) $\forall v \in V, \forall a \in \mathbb{R}, \|av\| = |a|\|v\|$

(III) $\forall v, w \in V, \|v + w\| \leq \|v\| + \|w\|$

(I) $\because$ A is positive difinite $\therefore \forall \vec{v} \in \mathbb{R}^n$. $\vec{v} \neq 0$ $vAv^T > 0$

$\therefore$ $x^TAx \geq 0$, $(x^TAx)^{\frac{1}{2}} \geq 0$,

if $x = 0 \rightarrow (x^TAx)^{\frac{1}{2}} = 0$

if $(x^TAx)^{\frac{1}{2}} = 0 \rightarrow x^TAx = 0$. because A is positive difinite

$\rightarrow x = 0$

So (I) holds

(II) WTS: $\forall a \in \mathbb{R}$ $\|ax\|_A = a\|x\|_A$

$\rightarrow (ax^TAax)^{\frac{1}{2}} = a(x^TAx)^{\frac{1}{2}}$

$(ax^TAax)^{\frac{1}{2}} = (a^2x^TAx)^{\frac{1}{2}} = a(x^TAx)^{\frac{1}{2}}$ as needed

So (II) holds

(III) WTS: $\|x+y\|_A \leq \|x\|_A + \|y\|_A$

Let $f(x) = x^TAx$, A is positive difinite matrix

WTS: f(x) is convex: $f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$ $\lambda \in [0,1]$

$\lambda x^TAx + (1-\lambda)y^TAy$

$f(\lambda x + (1-\lambda)y) = (\lambda x + (1-\lambda)y)^TA(\lambda x + (1-\lambda)y)$

$= (\lambda x + (1-\lambda)y)^TA\lambda x + (\lambda x + (1-\lambda)y)^TA(1-\lambda)y$

don't know....

q2b

Q2. b.

because the $1-v^TAu$ is nearlly 0, then the bottom part of Sherman-Morrison formular we used in solve_rank_update() is 0.

q3a

Q3 (a)

$$(A-UV^T)^{-1} = A^{-1} + A^{-1}U(I-V^TA^{-1}U)^{-1}V^TA^{-1}$$

WTS: $\underline{(A-UV^T)(A^{-1}+A^{-1}U(I-V^TA^{-1}U)^{-1}V^TA^{-1}} = I$

$$I + I\cdot U(I-V^TA^{-1}U)^{-1}V^TA^{-1} - UV^TA^{-1} + UV^TA^{-1}U(I-V^TA^{-1}U)^{-1}V^TA^{-1}$$

$$= [U(I-V^TA^{-1}U)^{-1} - U + UV^TA^{-1}U(I-V^TA^{-1}U)^{-1}]V^TA^{-1} + I$$

$$= U[(I-V^TA^{-1}U)^{-1} - I + V^TA^{-1}U(I-V^TA^{-1}U)^{-1}]V^TA^{-1} + I$$

$$= U[I - (I-V^TA^{-1}U)^{-1} + (I-V^TA^{-1}U)^{-1}](I-V^TA^{-1}U)^{-1}V^TA^{-1} + I$$

$$= U[0](I-V^TA^{-1}U)^{-1}V^TA^{-1} + I$$

$$= I$$

q3b

上午11:13 6月15日周一 VPN 29%

(b) ① $A - B = xy^T \Longleftrightarrow$ ② $A^{-1} - B^{-1} = uv^T$

x, y, u, v are column vectors, $A - B$ are non-singular

(I) ① $\rightarrow$ ②

WTS: $A - B = xy^T \longrightarrow A^{-1} - B^{-1} = uv^T$

$A - B = xy^T \Rightarrow A - xy^T = B \Rightarrow (A - xy^T)^{-1} = B^{-1}$

$\Rightarrow A^{-1} + A^{-1}x(1 - y^TA^{-1}x)^{-1}y^TA^{-1} = B^{-1}$

$\Rightarrow A^{-1}x(1 - y^TA^{-1}x)^{-1}y^TA^{-1} = B^{-1} - A^{-1}$

$\Rightarrow -A^{-1}x\underbrace{(1 - y^TA^{-1}x)^{-1}}_{\text{constant}}\underbrace{y^TA^{-1}}_{} = A^{-1} - B^{-1}$

$A^{-1}x$: Assme A^{-1} is a $m \times n$ matrix
then $A^{-1}x$ is a column vector denote $\underline{\underline{u}}$

y^TA^{-1} : is a row vector : denot $\underline{\underline{v^T}}$

$(1 - y^TA^{-1}x)$: is a constant denote as $\underline{\underline{a}}$

then $-A^{-1}x(1 - y^TA^{-1}x)^{-1}y^TA^{-1} = A^{-1} - B^{-1}$

$\Rightarrow A^{-1} - B^{-1} = -u \cdot av^T$

the $A^{-1} - B^{-1}$ can be written as a column vector dot a row vector, then $A^{-1} - B^{-1}$ is rank one. as needed □

next page .

(II) ② → ①

WTS: $A^{-1} - B^{-1} = uv^T \longrightarrow A - B = xy^T$

$A^{-1} - B^{-1} = uv^T \longrightarrow (A^{-1} - (uv^T))^{-1} = B^{-1}$

$\rightarrow A + Au(1 - v^T A u)^{-1} v^T A = B$

$\rightarrow \underbrace{-Au}_{\text{a column vector}} \underbrace{(1 - v^T A u)^{-1}}_{\text{constant}} \underbrace{v^T A}_{\text{row vector}} = A - B$

then $A - B$ can be written as a column vector dot a row vector, then $A^{-1} - B^{-1}$ is rank one. as needed □