CSC338. Homework 4

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Q1.d

d. they are not same

$$\left[\begin{array}{cc|c} 2^{-100} & 1 & 1+2^{-100} \\ 1 & 1 & 2 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2^{-100} & 1 & 1+2^{-100} \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1-2^{-100} & 2-2^{100}-1 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1-2^{-100} & 1-2^{100} \end{array}\right] \longrightarrow x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

the solution solved by pivoting is right.

Q2.B.C

b. we have triangle inequality:

$$|a+b| \le |a|+|b|$$

WTS: $\|A+B\| \le \|A\| + \|B\|$ A.B matrix

$$\|A+B\| = \max_j \sum_{i=1}^{n} |a_{ij}+b_{ij}| \le \max_j \sum_{i}^{n} \left[|a_{ij}| + |b_{ij}|\right]$$

$$= \max_j \sum_{i}^{n} |a_{ij}| + \max_j \sum_{i}^{n} |b_{ij}| = \|A\| + \|B\|$$ as needed

c. $\|v\|_\infty = \max x_i$

$\|v\|_1 = \sum_{i}^{n} |x_i|$

$\|v\|_\infty \le \|v\|_1$ is not always true

for example $v = |1\ 2\ 3|$

$\|v\|_\infty = 3$ $\|v\|_1 = 2.5$

$3 \nleq 2.5$

for matrix M. $\|M\|_\infty \le \|M\|_1$ is not always true

for example: $M = \begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix}$

$\|M\|_\infty = 3+3 = 6$ $\|M\|_1 = 3+1 = 4$

$6 \nleq 4$

Q3.C.D.E

c. it is ill - conditioned because the row1 and row2 are nearly parallell.

d. $\text{cond}(AB^{-1}) = \|AB^{-1}\| \, \|BA^{-1}\|$

$\leq \|A\| \, \|B^{-1}\| \, \|B\| \, \|A^{-1}\| = \text{cond}(A) \cdot \text{cond}(B)$

then $\text{cond}(AB^{-1})$ is bounded by $\text{cond}(A) \cdot \text{cond}(B)$

since A. B are well conditioned.

thus AB^{-1} is well conditioned.

e. $d^T B^T A^{-1} B d$

$(d^T B^T) = (Bd)^T$ so we can compute Bd then take transpose on that matrix. this is faster than $d^T B^T$