CSC338. Homework 3

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Question 1

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -1 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}, b = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix},$$

a) $A = \begin{bmatrix} 1 & 0 & -2 \\ -1 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \xrightarrow{E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{bmatrix}$

$\xrightarrow{E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} -5 \\ 5 \\ 3 \end{bmatrix}$

b) $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

$U = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$LU = A$ □

Question 3

For this part prove , i use lemma state in website: https://www.statlect.com/matrix-algebra/matrix-inversion-lemmas

$$(A+uv^\top)^{-1} = A^{-1} - \frac{1}{1+v^\top A^{-1}u} A^{-1}uv^\top A^{-1}$$

Q3. a): $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Assume $A = LU$ $L: \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}$ $U = \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$

$$LU = \begin{bmatrix} L_{11}U_{11} & L_{11}U_{12} \\ L_{21}U_{11} & L_{21}U_{12} + L_{22}U_{22} \end{bmatrix}$$

$A_{11} = L_{11}U_{11} = 0$ then at least one of L_{11} and U_{11} must be zero, $\longrightarrow$ At least one of L and U is not invertible. This is contradiction that A is invertible [if A is invertible, LU are invertible]. then A cannot have a LU decomposition.

b) we have lemma:

u, v vector, A matrix

$$(A + uv^T)^{-1} = A^{-1} - \frac{1}{1+v^T A^{-1} u} A^{-1} u v^T A^{-1}$$

$$M_k = I - m_k e_k^T$$

$$(M_k)^{-1} = I^{-1} + \frac{1}{1 + e_k^T \cdot I^{-1} \cdot m_k} I^{-1} m_k e_k^T I^{-1} \quad [I^{-1} = I]$$

$$= I + \frac{1}{1 + e_k^T \cdot m_k} \cdot m_k e_k^T$$

$$= I + \frac{m_k e_k^T}{1 + e_k^T m_k} \quad [e_k^T m_k = 0]$$

$$= I + m_k e_k^T \text{ as needed}$$