THE ANIMAL FARM: A MATHEMATICAL MODEL FOR THE DISCUSSION OF SOCIAL STANDARDS FOR CONTROL OF THE ENVIRONMENT

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1. Old MacDonald has a farm; and on his farm he has a herd of CN ferthings. Each year he sells N mature animals and each year N young ferthings join the herd. Ferthings are worth most when they are C-years old. The profit from each animal sold is U groats.

2. Lately, however, because of a water-borne disease the farmer's profit has been reduced. It is less than NU. He must make a decision. He would like to have more groats.

3. The farmer's daughter, Honoria, who is taking a course in epidemiology, explains the etiology of the disease. Ferthing pathologists call it *hyfertitus*. It is carried by a small pathogen that lives in water. The animals contract the disease only from drinking the water. They do not get it from each other. Every pathogen ingested has the same likelihood of causing death, and this likelihood does not depend upon how many other pathogens have been ingested previously. The mortality:morbidity ratio is one. *Hyfertitus* is a very serious disease.

4. Honoria, who is also taking a course in probability, writes on a piece of paper as follows:

Let $Q(X)$ = the probability of any ferthing not dying from *hyfertitus* during the C-year period before sale, the pathogen density being X organisms in the water volume ingested per animal in this period. Then in accordance with the foresaid specifications as to the mechanism of the pathogenicity the probability of surviving if the density of pathogens were increased by a small amount δX would be the probability of surviving the first X micro-organisms, multiplied by one minus the probability of fatal infection by the incremental micro-organisms, or

$$Q(X + \delta X) = Q(X)[1 - K\delta X],$$

where K is an infectivity parameter measuring the virulence of the pathogens. Forming the differential quotient and letting $\delta X \to 0$ we obtain $dQ/dX = -KQ$. Integrating and adjusting the constant of integration so that $Q(0) = 1$ gives

$$Q(X) = e^{-KX}. \tag{1}$$

The larger X for a given K the smaller the proportion of surviving animals. If, for example, X happens to be equal to $0.69/K$ then only one-half of the ferthings will live. Honoria is on the honor roll in her course in probability.

5. A salesman named Young Sam comes to the farm. He is an honest and candid sales representative of an equipment company. Young Sam says that his company makes a remarkable water treatment unit called a "Disinfeclarminator" that will eliminate the disease 100 percent. It is easily installed and is completely effective in killing the pathogens of *hyfertitus* over all ranges of density X. It is simple to operate. There is nothing to do. And best of all, a treatment unit for a herd of size N costs only V groats!

6. MacDonald moves toward decision. To treat or not to treat. That is the question. First he makes a preliminary decision that he will try to maximize economic efficiency. This is his objective function.

(1) If the treatment unit is not installed his annual income or gain will be $NUQ(X)$. The present value of the future time stream of income will be

$$G_1 = \beta NUQ(X), \tag{2}$$

where

$$\beta = \frac{1 - (1 + r)^{-T}}{r}, \tag{3}$$

r = the discount rate for capital (opportunity cost of capital), and

T = the economic time horizon.

(2) If the treatment unit is installed, the disease will be eliminated and the gain will be

$$G_2 = \beta NU - V. \tag{4}$$

The break-even point in terms of pathogen density, X_c, is obtained by setting

$$G_1 = G_2. \tag{5}$$

Solving equation (5) for the quality criterion X_c by means of equations (1), (2) and (4), and using the approximate relation, $Q(X) = 1 - KX$, which is valid when KX is small,

$$X_c = \frac{V}{K\beta NU}. \tag{6}$$

7. MacDonald decides. He has numbers for all the factors on the right-hand side of equation (6). He calculates X_c and finds it to be smaller than the actual X in his water supply so that G_2 is larger than G_1. Therefore he installs the unit and obtains a greater income. Now he will have enough groats to pay for his daughter's

courses in probability and epidemiology. Young Sam makes a sale. Everybody gains! This is a nonzero sum model. Honoria takes equation (6) to her professor.

8. The professor is brave; he wishes to apply the model to people. He would like to dispel the mystique that often enshrouds the setting of quality standards for control of the environment. But he is not foolhardy and he speaks in the subjunctive mood.

9. The professor makes some inferences from equation (6). They are interesting if they pertain to ferthings; they are surprising if the model is considered as possibly applicable to human beings. It would appear that in a general way the setting of all criteria, standards, or rules for administration of man's environment might follow the rationale underlying equation (6).

(1) The tolerance level of the pathogen is not zero. A calculated risk is taken.

(2) The factor V appears in the numerator of equation (6). The critical concentration depends on the cost of treatment. If a technical innovation occurs that reduces the cost of treatment, in due course of time the tolerance level should be reduced. The stringency of a standard ought to be proportioned to cost.

(3) If as normally would be the case the per capita cost of treatment, V/N, decreases as N increases because of economies of scale, then big herds of animals should drink purer water than small herds. Equation (6) says this is true for a noncontagious disease. For a contagious disease the argument would have even greater force. It is pertinent to remark that in human populations the water quality as measured by coliform organism density is often found to be better in the large cities of a region than in the smaller towns. There are sizable economies of scale in rapid sand filtration plants.

(4) The tolerance concentration depends in part upon the economic parameters r and T. The equation (6) shows that if the interest rate, r, is large, X_c should be large; and if the economic life, T, is small, X_c should be large. A large r-value in a capital market indicates a high discounting of the future in favor of the present. Accordingly, if MacDonald's farm were in Brazil he would use a higher tolerance level in his decision process than he would if his farm were in New Zealand. Underdeveloped countries tend to have high capital discount rates relative to those in developed countries. Should people in Brazil drink less pure water than people in New Zealand? If the answer is not yes, and Brazilians are constrained to drink overpure water, then it must be asked whether such constraint does not deprive them of resources that might better go into

highways, schools or into other sectors of the public health budget where the prevailing discount rate is considered.

(5) In equation (6) it is evident that the tolerance level, X_c, in the ferthing problem depends upon two physical parameters (N and K) and four economic factors (r, T, U and V). MacDonald had numbers for all of these and could compute X_c.

If we now consider potable water for human beings (or air, or food, or comfort and safety) we must ask which, if any, of these parameters becomes fuzzy? The answer, of course, is loud and quick — we do not usually assign a utility to a human being in groats or dollars. The parameter U is not known numerically. The remaining parameters do not become fuzzy with the switch.

(6) But for many years decisions have been made as to quality criteria and standards. In the United States drinking water should have an average coliform concentration not greater than about one organism per 100 milliliters. In British practice Class 3 water (coliform density 3 to 10 per 100 milliliters, presumptive) is rated as suspicious. The upper limit for concentration of boron in irrigation water classed as "good" is 0.67 milligram per liter if sensitive plants are to be grown. The World Health Organization specifies that cyanide (as CN) has a maximum allowable concentration of 0.01 milligram per liter. The Standards for Protection Against Radiation adopted by the U. S. Atomic Energy Commission require that the gross quantity of radioactive material released into the waste water disposal system at any licensed plant or institution be less than one curie per year. There are recommended limiting concentrations in the atmosphere of such pollutants as ozone, lead and hydrogen sulfide. All civilized countries have legislated on the question of preservatives in food. At the First Opium Convention at The Hague in 1911 regulatory criteria were promulgated. The maximum permissible speed on the New York Thruway is 65 miles per hour.

These criteria and many others like them represent socioeconomic decisions and reflect in each case the inherent tension between human desires and human capability. The criteria of quality represent a balance of costs and benefits. In equation (6) it is seen that the tolerance level, X_c, is proportional to a cost:benefit ratio. In a general way using more elaborate and realistic mathematical models it may be shown that without exception *every quality criterion or rule whether it pertains to health, to aesthetics or to property damage is always equal to a function of a cost:benefit ratio.*

(7) The setting of any quality criterion or standard relating to health and well-being inevitably entails making an implicit estimate of a cost:benefit ratio based on whatever data or other factors are available for judgment. In situations where the costs are known within reasonable limits, the establishment of a standard amounts to assigning a certain utility to human life, health or well-being. For example, if in the simple decision model for MacDonald's farm we are able to set down numerical values for the factors of cost, infectivity, discount rate, and economic life (V, K, r, and T), then an assignment of a numerical value for the water quality criterion is tantamount to assignment of a numerical value for the benefit parameter U. The setting of a standard is in effect an imputation of a certain finite level of utility or benefit accruing to life, to health, or to well-being.

(8) A close connection always obtains between quality criteria for water, air, food, safety, and aesthetics and a relevant, implicit, estimated cost:benefit ratio. In the mathematical model of the farm that has been discussed, the relation is one of simple proportion. The constant of proportionality is equal to the infectivity parameter K. More elaborate models are needed to take into account more complicated features of reality such as physical and economic uncertainty, errors in data, measurement difficulties, more complex patterns of epidemiology, meteorology, hydrology, technology, human psychology and economic behavior that enter into the decision process.

The modern large electronic digital computer has made it possible to construct elaborate models and to apply them in simulating the problems of environmental science so as to elucidate the effect of various factors and interactions that are germane to the establishment of quality criteria. But despite the greater complexity of these models, the principle stated in section (7) retains its central thrust: the fixing or setting of quality criteria for control of the environment always involves a value judgment; and this value judgment always has the form of a cost:benefit ratio. *To set a criterion is to impute a cost:benefit ratio.*

(9) The professor proposes an arduous, long-range program to strengthen the rational base of decision-making in the management of the environment. He says that the following steps should be taken:

(i) Identify and classify the problems of environmental control on the basis of (a) the mathematical structure; and (b) the type of utility or disutility pertaining to people such as longevity, health, aesthetics, well-being and safety. This classification would differ markedly from previous problem-classifying schemes that have been used, such as, for example, the classification based upon the dominant branch of science involved (fluid mechanics, microbiology, etc.).

(ii) For each class construct an appropriate generalized mathematical model that relates the quality criterion with the corresponding estimated cost:benefit ratio. The relation should make explicit the effect and importance of all relevant physical and economic factors.

(iii) Apply the generalized model for each class to those areas of environmental management where decisions have already been made and quality criteria have been established. Compute from the model that value of the utility parameter (benefit vector) which has been implicitly assigned in the assignment of a definite numerical value (or range of values) for the quality criterion. In the decision model for MacDonald's farm, equation (6) would be solved for U, all other factors being known.

(iv) Compare these utilities within classes and establish norms and ranges. Within each class a wide range of results may be expected. Large inconsistencies will appear. Some classes will yield more consistent and reliable estimates than others. Particularly useful perhaps would be an analysis of the technological function relating highway speed and highway death (or other disutility) with existing highway speed control legislation and enforcement. A large segment of the public has active participation with officialdom in the balancing of costs and benefits and the setting of norms in this activity. In other enterprises the direct feedback of public response may be impeded or obscured by technical complexity. In these cases bureaucratic bias and institutional constraints may develop that hinder or retard the equilibrating process.

(v) Use these norms and ranges as rational guidelines in setting new quality criteria and in the re-evaluation of old standards. In this way the entire structure of regulatory codes for control of the environment can be systemized. Internal consistency will beget external viability.

(10) The professor concludes. Not with a Q.E.D. but with an E.I.E.I.O.

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