population_2V_analysis

November 19, 2025

1 The Neural Population Oscillator

On the neural population level, the mesoscopic currents are modelled by the interaction of excitatory and inhibitory populations.

The predict oscillatory dynamics of e.g. the local field potential and the EEG.

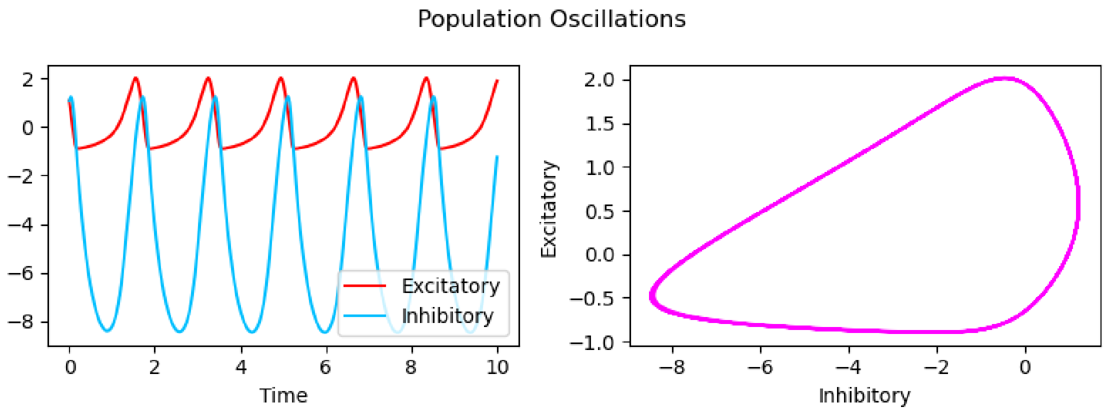
[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from numpy import linspace, tanh, around
from matplotlib.pyplot import subplots
from scipy.integrate import odeint
```

[2]:
```
def sigmoid(u):
    return tanh(u)

def oscillator(y, t, h_ex, h_in, pars, sr, time_stop):

    tau_ex, tau_in, c2, c4, c_EE, c_EI = pars

    dydt = (

        (h_ex - y[0] - c2*sigmoid(y[1]) + c_EE*sigmoid(y[0]))*tau_ex,

        (h_in - y[1] - c4*sigmoid(y[1]) + c_EI*sigmoid(y[0]))*tau_in

    )

    return dydt
```

[3]:
```
# Input parameter
h_ex_0 = -7.
h_in_0 = -4.0

# Oscillator parameters
pars = (1, 2.5, 10, 0, 5, 10) # Homoclinic
```

```
# Initial conditions
y_ini = (1.088, 1.071)

# Time array
time_stop = 10
sr        = 1000
time      = linspace(start=0, stop=time_stop, num=time_stop*sr)

pulse = []

# Simulation
y = odeint(func=oscillator, y0=y_ini, t=time,
          args=(h_ex_0, h_in_0, pars, sr, time_stop),
          hmax=0.1)

fig, ax = subplots(ncols=2, figsize=(8, 3))

ax[0].plot(time, y[:,0], color='red', label='Excitatory');
ax[0].plot(time, y[:,1], color='deepskyblue', label='Inhibitory');
ax[1].plot(y[:,1], y[:,0], color='magenta');
ax[0].legend(loc='lower right')
ax[0].set_xlabel('Time')
ax[1].set_xlabel('Inhibitory')
ax[1].set_ylabel('Excitatory')


chars = 'Population Oscillations'

fig.suptitle(chars)

fig.tight_layout()

# Show final values of all variables
print('End of run:', around(y[-1,:],3))
print('')
```

```
End of run: [ 1.894 -1.238]
```



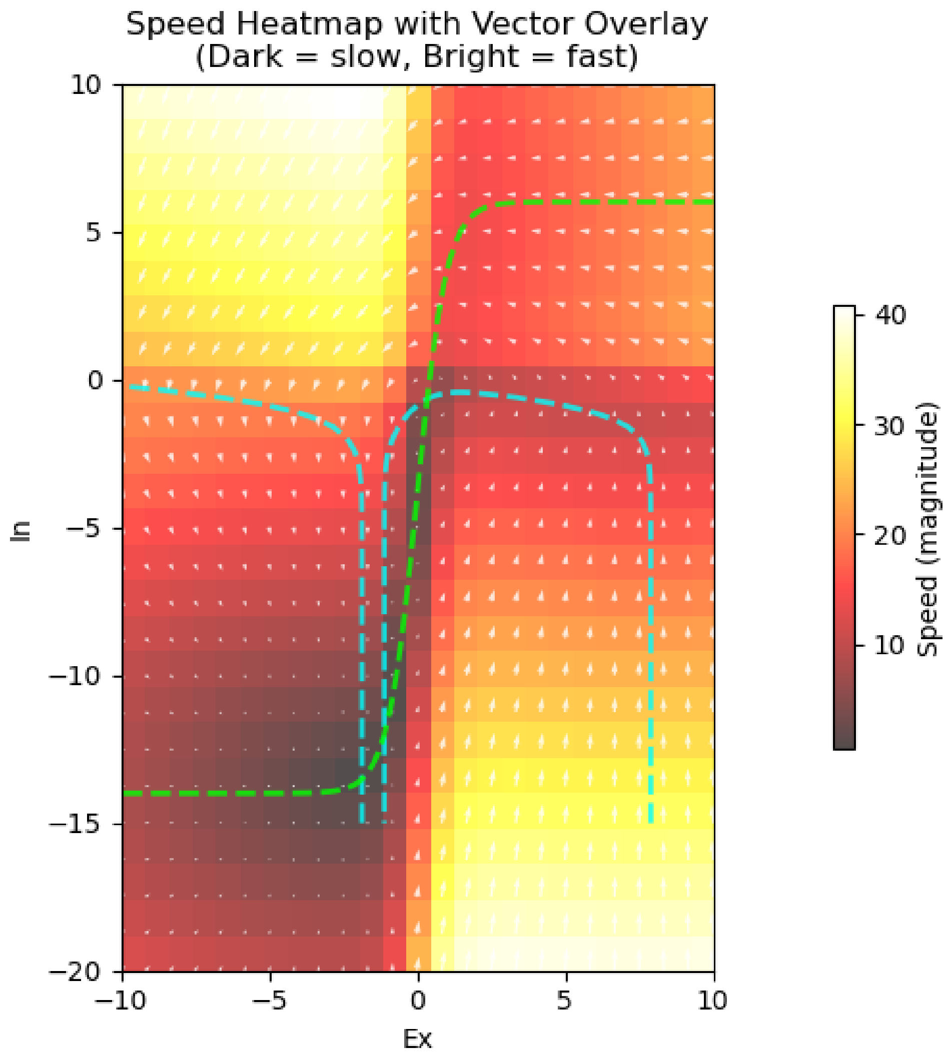
[ ]:

2 Quiver 2V analysis

[7]:
```
def dX_dt(X):
    """ Return the rates at all positions. """
    h_ex, h_in, tau_ex, tau_in, c2, c4, c_EE, c_EI = (-7.1, -4., 1, 1.5, 10, 0,␣
↪5, 10)  # SNIC

    return np.array([ (h_ex - X[0] - c2*tanh(X[1]) + c_EE*tanh(X[0]))*tau_ex,
                      (h_in - X[1] - c4*tanh(X[1]) + c_EI*tanh(X[0]))*tau_in
                    ])

# Create grid
x = np.linspace(-10, 10, 25)
y = np.linspace(-20, 10, 25)
X, Y = np.meshgrid(x, y)

# Calculate vector field and magnitudes
DX = np.zeros_like(X)
DY = np.zeros_like(Y)
magnitudes = np.zeros_like(X)

for i in range(X.shape[0]):
    for j in range(X.shape[1]):
        dX = dX_dt([X[i,j], Y[i,j]])
        DX[i,j] = dX[0]
        DY[i,j] = dX[1]
        magnitudes[i,j] = np.sqrt(dX[0]**2 + dX[1]**2)  # Calculate magnitude

# Add nullclines
```

```
x_nc = np.linspace(-10, 10, 100)
y_nc = np.linspace(-15, 10, 100)
X_nc, Y_nc = np.meshgrid(x_nc, y_nc)

# dx/dt = 0 nullcline
dX_null = np.zeros_like(X_nc)
for i in range(X_nc.shape[0]):
    for j in range(X_nc.shape[1]):
        dX_null[i,j] = dX_dt([X_nc[i,j], Y_nc[i,j]])[0]

# dy/dt = 0 nullcline
dY_null = np.zeros_like(Y_nc)
for i in range(Y_nc.shape[0]):
    for j in range(Y_nc.shape[1]):
        dY_null[i,j] = dX_dt([X_nc[i,j], Y_nc[i,j]])[1]

# Create plot
fig, ax1 = plt.subplots(figsize=(16, 6))

# The magnitude as a heatmap with arrows
im = ax1.imshow(magnitudes, extent=[-10, 10, -20, 10],
               origin='lower', cmap='hot', alpha=0.7)
q1 = ax1.quiver(X, Y, DX, DY, color='white',
               angles='xy', scale_units='xy', scale=50,
               width=0.004, alpha=0.8)

# Add colorbar for heatmap
cbar1 = plt.colorbar(im, ax=ax1, shrink=0.5)
cbar1.set_label('Speed (magnitude)')

# Add nullclines to second plot
ax1.contour(X_nc, Y_nc, dX_null, levels=[0], colors='cyan', linewidths=2,␣
 ↪linestyles='--', alpha=0.8)
ax1.contour(X_nc, Y_nc, dY_null, levels=[0], colors='lime', linewidths=2,␣
 ↪linestyles='--', alpha=0.8)

ax1.set_xlabel('Ex')
ax1.set_ylabel('In')
ax1.set_title('Speed Heatmap with Vector Overlay\n(Dark = slow, Bright = fast)')
ax1.set_xlim([-10, 10])
ax1.set_ylim([-20, 10])
ax1.set_aspect('equal')

# plt.tight_layout()
plt.show()

# Print some statistics about the speeds
```

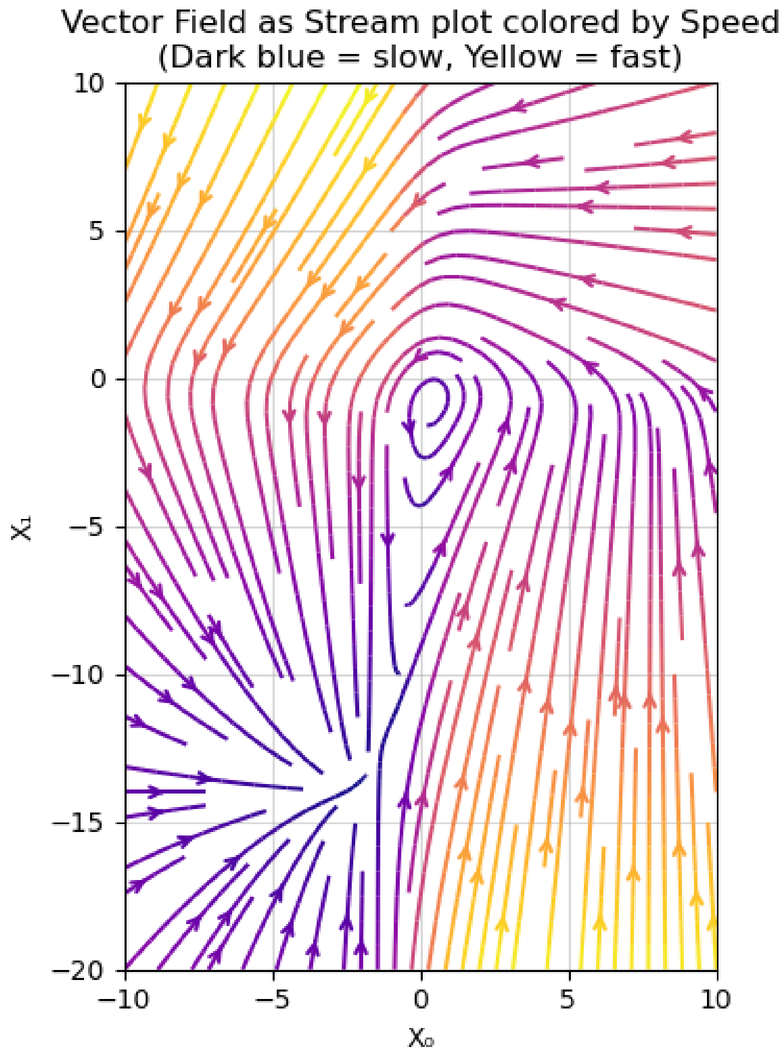
```python
print(f"Speed statistics:")
print(f"Min speed: {magnitudes.min():.4f}")
print(f"Max speed: {magnitudes.max():.4f}")
print(f"Mean speed: {magnitudes.mean():.4f}")
print(f"Areas with speed < {magnitudes.mean()/2:.4f} represent slow dynamics")
```




```
Speed statistics:
Min speed: 0.4984
Max speed: 40.7814
Mean speed: 19.6661
Areas with speed < 9.8331 represent slow dynamics
```

[ ]:

[8]:
```python
def dX_dt(X):
    """ Return the rates at all positions. """
    h_ex, h_in, tau_ex, tau_in, c2, c4, c_EE, c_EI = (-7.1, -4., 1, 1.5, 10, 0, 5, 10)

    return np.array([ (h_ex - X[0] - c2*tanh(X[1]) + c_EE*tanh(X[0]))*tau_ex,
                       (h_in - X[1] - c4*tanh(X[1]) + c_EI*tanh(X[0]))*tau_in
                    ])

# Grid
x = np.linspace(-10, 10, 25)
y = np.linspace(-20, 10, 25)
X, Y = np.meshgrid(x, y)

# Vector field and magnitudes
DX = np.zeros_like(X)
DY = np.zeros_like(Y)
magnitudes = np.zeros_like(X)

for i in range(X.shape[0]):
    for j in range(X.shape[1]):
        dX = dX_dt([X[i,j], Y[i,j]])
        DX[i,j] = dX[0]
        DY[i,j] = dX[1]
        magnitudes[i,j] = np.sqrt(dX[0]**2 + dX[1]**2)

# Plot
fig, ax1 = plt.subplots(figsize=(16, 6))

# Streamlines with color mapping to magnitude
strm = ax1.streamplot(X, Y, DX, DY, color=magnitudes, cmap='plasma',
                      linewidth=1.5, density=1.2, arrowstyle='->')

ax1.set_xlabel('X ')
ax1.set_ylabel('X ')
ax1.set_title('Vector Field as Stream plot colored by Speed\n(Dark blue = slow, Yellow = fast)')
ax1.set_xlim([-10, 10])
ax1.set_ylim([-20, 10])
ax1.grid(True, alpha=0.5)
ax1.set_aspect('equal')
```



3 Current Winds

https://earth.nullschool.net/#current/wind/surface/level/orthographic=4.77,48.88,1230

[ ]:

```

```