population_3V_display

November 19, 2025

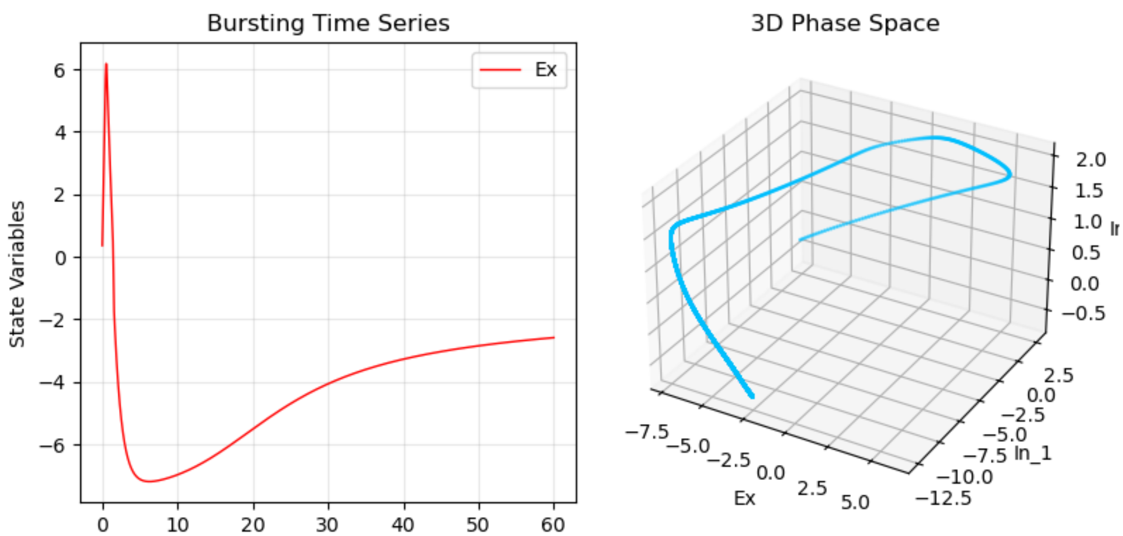
1 Neural Population Bursting

With inhibitory populations operating on different time scales, bursting activity is observed in neural population models. It consists of a main oscillation on which secondary fast oscillations are imposed.

[5]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
from numpy import tanh as sigmoid
```

[6]:
```
def bursting_system(t, y):
    h_ex, h_in_1, h_in_2, tau_ex, tau_in1, tau_in2, c1, c2, c3, c4, c5, c6, c7␣
 ↪= (
        -2.5, -4.5, 0.5, 1, 2.0, 0.01, 12, 10, 10, 2, 5, 5, 3)

    return [
        (h_ex   - y[0] + c1*sigmoid(y[0]) - c2*sigmoid(y[1]) -␣
 ↪c7*sigmoid(y[2])) * tau_ex,
        (h_in_1 - y[1] + c3*sigmoid(y[0]) - c4*sigmoid(y[1]))                    ␣
 ↪ * tau_in1,
        (h_in_2 - y[2] + c5*sigmoid(y[0]) - c6*sigmoid(y[2]))                    ␣
 ↪ * tau_in2
    ]

# Integrate the system
y_ini = [0.359, -12.325,  2]
# y_ini = [0.359, -12.325,  -2]

t_end  = 60
t_span = (0, t_end)
t_eval = np.linspace(0, t_end, 10000)
sol = solve_ivp(bursting_system, t_span, y_ini, t_eval=t_eval, method='RK45',␣
 ↪max_step=0.05)

# Create comprehensive visualization
fig = plt.figure(figsize=(8, 4))
```

```python
# 1. Time series - all variables
ax1 = plt.subplot(1, 2, 1)
plt.plot(sol.t, sol.y[0], 'red', label='Ex', linewidth=1)
# plt.plot(sol.t, sol.y[1], 'g-', label='Medium (y )', linewidth=1)
# plt.plot(sol.t, sol.y[2], 'b-', label='Slow (y )', linewidth=2)
plt.ylabel('State Variables')
plt.title('Bursting Time Series')
plt.legend()
plt.grid(True, alpha=0.3)


# 3. 3D Phase Space
ax2 = plt.subplot(1, 2, 2, projection='3d')
ax2.scatter(sol.y[0], sol.y[1], sol.y[2], c='deepskyblue', s=1, alpha=0.6)
ax2.set_xlabel('Ex')
ax2.set_ylabel('In_1')
ax2.set_zlabel('In_2')
ax2.set_title('3D Phase Space')

plt.tight_layout()
plt.show()
```



[7]:
```python
from numpy import around
around(sol.y[:, -1], 3)
```

[7]: array([-2.588, -12.39 , -0.692])

2 3D Interactive Plot

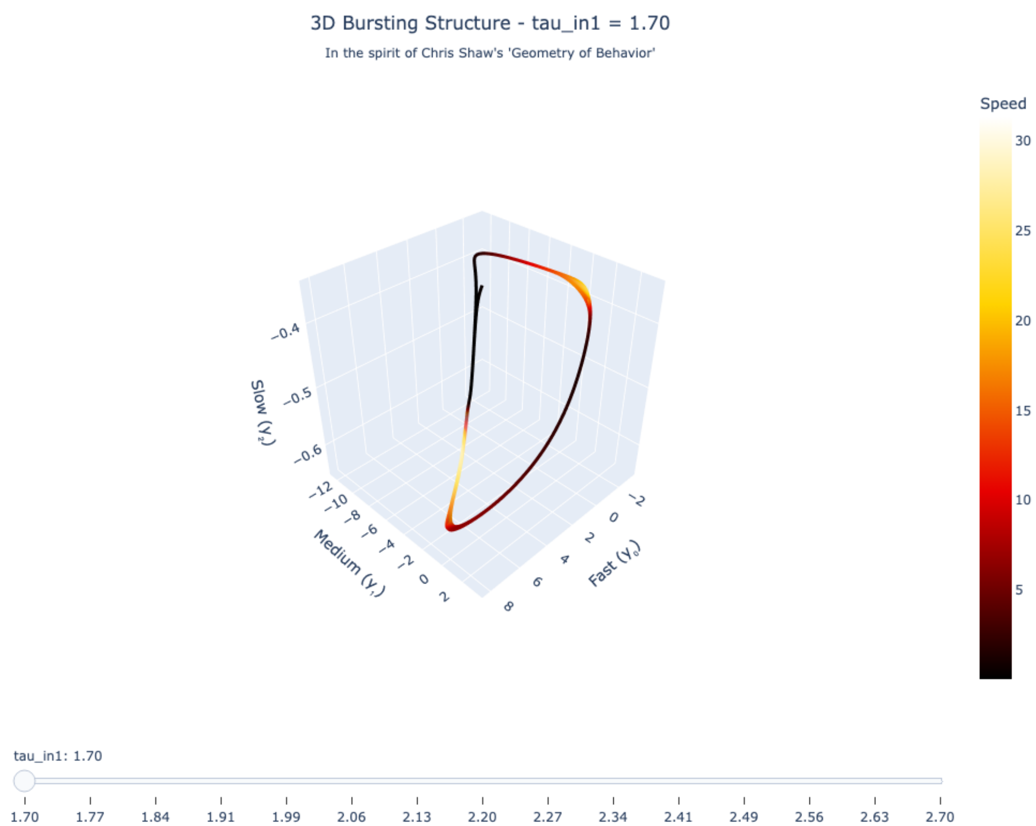
[4]:

```python
import plotly.graph_objects as go
import plotly.express as px  # This line was missing!
from plotly.subplots import make_subplots
import numpy as np
from scipy.integrate import solve_ivp
from numpy import tanh as sigmoid

def bursting_system(t, y, tau_in1):
    """Bursting system with tau_in1 as parameter"""
    h_ex, h_in_1, h_in_2, tau_ex, tau_in2, c1, c2, c3, c4, c5, c6, c7 = (
        -2.0, -4.5, 0.5, 1, 0.01, 12, 10, 10, 2, 5, 5, 3)

    return [
        (h_ex   - y[0] + c1*sigmoid(y[0]) - c2*sigmoid(y[1]) -␣
 ↪c7*sigmoid(y[2])) * tau_ex,
        (h_in_1 - y[1] + c3*sigmoid(y[0]) - c4*sigmoid(y[1])) * tau_in1,
        (h_in_2 - y[2] + c5*sigmoid(y[0]) - c6*sigmoid(y[2])) * tau_in2
    ]

# Precompute trajectories for different tau_in1 values
tau_in1_values = np.linspace(1.7, 2.7, 15)
trajectories = []
speeds_data = []

print("Precomputing trajectories for different tau_in1 values...")

for i, tau_in1 in enumerate(tau_in1_values):
    print(f"Computing {i+1}/{len(tau_in1_values)}: tau_in1 = {tau_in1:.2f}")

    # Integrate system
    rng = np.random.default_rng(12345)
    y_ini = rng.uniform(size=3)
    y_ini = [-3.13127628, -12.46379042,  -0.45491688]
    t_span = (0, 1500)
    t_eval = np.linspace(0, 1500, 8000)

    sol = solve_ivp(bursting_system, t_span, y_ini, t_eval=t_eval,
                    method='RK45', args=(tau_in1,), max_step=0.05)

    # Calculate speeds
    speeds = np.zeros(len(sol.t))
    for j in range(len(sol.t)):
        dydt = bursting_system(sol.t[j], sol.y[:, j], tau_in1)
        speeds[j] = np.sqrt(dydt[0]**2 + dydt[1]**2 + dydt[2]**2)
```

```
    trajectories.append(sol)
    speeds_data.append(speeds)

# Create interactive 3D plot with slider
fig_3d = go.Figure()

for i, (tau_in1, sol, speeds) in enumerate(zip(tau_in1_values, trajectories,␣
 ↪speeds_data)):
    visible = (i == 0)

    fig_3d.add_trace(go.Scatter3d(
        x=sol.y[0], y=sol.y[1], z=sol.y[2],
        mode='lines',
        line=dict(
            color=speeds,
            colorscale='Hot',
            width=6,
            showscale=True,
            colorbar=dict(title="Speed")
        ),
        name=f'tau_in1 = {tau_in1:.2f}',
        visible=visible
    ))

# Create slider steps for 3D plot
steps_3d = []
for i, tau_in1 in enumerate(tau_in1_values):
    step = dict(
        method="update",
        args=[{"visible": [False] * len(trajectories)},
              {"title": f"3D Bursting Structure - tau_in1 = {tau_in1:.2f}<br>"
                        f"<sub>G.O.S.H.-approved visualization!</sub>"}],  #␣
 ↪Credit to the Ghost Of Shaw!
        label=f"{tau_in1:.2f}"
    )
    step["args"][0]["visible"][i] = True
    steps_3d.append(step)

sliders_3d = [dict(
    active=0,
    currentvalue={"prefix": "tau_in1: "},
    pad={"t": 50},
    steps=steps_3d
)]

fig_3d.update_layout(
    title=dict(
```

```
        text=f"3D Bursting Structure - tau_in1 = {tau_in1_values[0]:.2f}<br>"
             "<sub>In the spirit of Chris Shaw's 'Geometry of Behavior'</sub>",
        x=0.5
    ),
    scene=dict(
        xaxis_title='Fast (y )',
        yaxis_title='Medium (y )',
        zaxis_title='Slow (y )',
        camera=dict(
            eye=dict(x=1.8, y=1.8, z=1.8)
        )
    ),
    sliders=sliders_3d,
    width=1000,
    height=800
)

fig_3d.show()


print("\n  G.O.S.H. APPROVED!")
print("Chris Shaw's geometric spirit lives on in these interactive␣
 ↪visualizations!")
print("Use the sliders to watch the torus winding dance - pure mathematical␣
 ↪poetry!  ")
```

```
Precomputing trajectories for different tau_in1 values…
Computing 1/15: tau_in1 = 1.70
Computing 2/15: tau_in1 = 1.77
Computing 3/15: tau_in1 = 1.84
Computing 4/15: tau_in1 = 1.91
Computing 5/15: tau_in1 = 1.99
Computing 6/15: tau_in1 = 2.06
Computing 7/15: tau_in1 = 2.13
Computing 8/15: tau_in1 = 2.20
Computing 9/15: tau_in1 = 2.27
Computing 10/15: tau_in1 = 2.34
Computing 11/15: tau_in1 = 2.41
Computing 12/15: tau_in1 = 2.49
Computing 13/15: tau_in1 = 2.56
Computing 14/15: tau_in1 = 2.63
Computing 15/15: tau_in1 = 2.70
```




```
 G.O.S.H. APPROVED!
Chris Shaw's geometric spirit lives on in these interactive visualizations!
Use the sliders to watch the torus winding dance - pure mathematical poetry!
```

[ ]: