

WordRank: Learning Word Embeddings via Robust Ranking

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Abstract

Embedding words in a vector space has gained a lot of attention in recent years. While state-of-the-art methods provide efficient computation of word similarities via a low-dimensional matrix embedding, their motivation is often left unclear. In this paper, we argue that word embedding can be naturally viewed as a *ranking* problem due to the ranking nature of the evaluation metrics. Then, based on this insight, we propose a novel framework *WordRank* that efficiently estimates word representations via robust ranking, in which the attention mechanism and robustness to noise are readily achieved via the DCG-like ranking losses. The performance of *WordRank* is measured in word similarity and word analogy benchmarks, and the results are compared to the state-of-the-art word embedding techniques. Our algorithm is very competitive to the state-of-the-art on large corpora, while outperforms them by a significant margin when the training set is limited (i.e., sparse and noisy). With 17 million tokens, *WordRank* performs almost as well as existing methods using 7.2 billion tokens on a popular word similarity benchmark. Our multi-machine distributed implementation of *WordRank* is open sourced for general usage.

由于评估指标的排序性质，将词嵌入视为一个排序问题。

论文提出了一种新型框架WordRank：通过robust ranking来评估单词表示，其中通过类似DCG的排序损失可以容易获得注意力机制和对噪声的鲁棒性。

WordRank的性能以词相似度和词类比为基准测量

将单词嵌入向量空间使得语义和语法规则能够保留下来，是许多NLP应用的重要子任务。

1 Introduction

Embedding words into a vector space such that *semantic* and *syntactic* regularities between words are preserved, is an important sub-task for many applications of natural language processing. Mikolov et al. [22] generated considerable excitement in the machine learning and natural language processing communities by introducing a neural network based model, which they call *word2vec*. It was shown that *word2vec* produces state-of-the-art performance on both word similarity as well as word analogy tasks. The *word similarity task* is to retrieve words that are similar to a given word. On the other hand, *word analogy* requires answering queries of the form *a:b:c:?*, where *a*, *b*, and *c* are words from the vocabulary, and the answer to the query must be semantically related to *c* in the same way as *b* is related to *a*. This is best illustrated with a concrete example: Given the query *king:queen:man:?* we expect the model to output *woman*.

Word Similarity: 单词相似性任务是检索与给定单词相似的单词。

Word analogy: 词类比任务就是，如果知道a与b语义相关，依这种相关性，那么c与?相关

Word2vec的一种解释是近似最大化正例的点互信息PMI，并且直接优化它可以获得好的结果。

The impressive performance of *word2vec* led to a flurry of papers, which tried to explain and improve the performance of *word2vec* both theoretically [2] and empirically [16]. One interpretation of *word2vec* is that it is approximately maximizing the positive pointwise mutual information (PMI), and Levy and Goldberg [16] showed that directly optimizing this gives good results. On the other

pointwise pairwise listwise是learning to rank中的三大方法

hand, Pennington et al. [25] showed performance comparable to *word2vec* by using a modified matrix factorization model, which optimizes a log loss.

Somewhat surprisingly, Levy et al. [17] showed that much of the performance gains of these new word embedding methods are due to certain hyperparameter optimizations and system-design choices. In other words, if one sets up careful experiments, then existing word embedding models more or less perform comparably to each other. We conjecture that this is because, at a high level, all these methods are based on the following template: From a large text corpus eliminate infrequent words, and compute a $|\mathcal{W}| \times |\mathcal{C}|$ word-context co-occurrence count matrix; a context is a word which appears less than d distance away from a given word in the text, where d is a tunable parameter. Let $w \in \mathcal{W}$ be a word and $c \in \mathcal{C}$ be a context, and let $X_{w,c}$ be the (potentially normalized) co-occurrence count. One learns a function $f(w, c)$ which approximates a transformed version of $X_{w,c}$. Different methods differ essentially in the transformation function they use and the parametric form of f [17]. For example, *GloVe* [25] uses $f(w, c) = \langle \mathbf{u}_w, \mathbf{v}_c \rangle$ where \mathbf{u}_w and \mathbf{v}_c are k dimensional vectors, $\langle \cdot, \cdot \rangle$ denotes the Euclidean dot product, and one approximates $f(w, c) \approx \log X_{w,c}$. On the other hand, as Levy and Goldberg [16] show, *word2vec* can be seen as using the same $f(w, c)$ as *GloVe* but trying to approximate $f(w, c) \approx \text{PMI}(X_{w,c}) - \log n$, where $\text{PMI}(\cdot)$ is the pairwise mutual information [8] and n is the number of negative samples.

In this paper, we approach the word embedding task from a different perspective by formulating it as a ranking problem. That is, given a word w , we aim to output an ordered list (c_1, c_2, \dots) of context words from \mathcal{C} such that words that co-occur with w appear at the top of the list. If $\text{rank}(w, c)$ denotes the rank of c in the list, then typical ranking losses optimize the following objective: $\sum_{(w,c) \in \Omega} \rho(\text{rank}(w, c))$, where $\Omega \subset \mathcal{W} \times \mathcal{C}$ is the set of word-context pairs that co-occur in the corpus, and $\rho(\cdot)$ is a ranking loss function that is monotonically increasing and concave (see Sec. 2 for a justification). Casting word embedding as ranking has two distinctive advantages. First, our method is *discriminative* rather than *generative*; in other words, instead of modeling (a transformation of) $X_{w,c}$ directly, we only aim to model the relative order of $X_{w,\cdot}$ values in each row. This formulation fits naturally to popular word embedding tasks such as word similarity/analogy since instead of the likelihood of each word, we are interested in finding the most relevant words in a given context¹. Second, casting word embedding as a ranking problem enables us to design models robust to noise [33] and focusing more on differentiating top relevant words, a kind of attention mechanism that has been proved very useful in deep learning [3, 14, 24]. Both issues are very critical in the domain of word embedding since (1) the co-occurrence matrix might be noisy due to grammatical errors or unconventional use of language, i.e., certain words might co-occur purely by chance, a phenomenon more acute in smaller document corpora collected from diverse sources; and (2) it's very challenging to sort out a few most relevant words from a very large vocabulary, thus some kind of attention mechanism that can trade off the resolution on most relevant words with the resolution on less relevant words is needed. We will show in the experiments that our method can mitigate some of these issues; with 17 million tokens our method performs almost as well as existing methods using 7.2 billion tokens on a popular word similarity benchmark.

2 Word Embedding via Ranking

2.1 Notation

We use w to denote a word and c to denote a context. The set of all words, that is, the vocabulary is denoted as \mathcal{W} and the set of all context words is denoted \mathcal{C} . We will use $\Omega \subset \mathcal{W} \times \mathcal{C}$ to denote the set of all word-context pairs that were observed in the data, Ω_w to denote the set of contexts that co-occurred with a given word w , and similarly Ω_c to denote the words that co-occurred with a given context c . The size of a set is denoted as $|\cdot|$. The inner product between vectors is denoted as $\langle \cdot, \cdot \rangle$.

¹Roughly speaking, this difference in viewpoint is analogous to the difference between pointwise loss function vs listwise loss function used in ranking [15].

粗略地说, 这种观点上的差异类似于在排序中, pointwise损失函数和listwise损失函数之间的差异

2.2 Ranking Model

Let \mathbf{u}_w denote the k -dimensional embedding of a word w , and \mathbf{v}_c denote that of a context c . For convenience, we collect embedding parameters for words and contexts as $\mathbf{U} := \{\mathbf{u}_w\}_{w \in \mathcal{W}}$, and $\mathbf{V} := \{\mathbf{v}_c\}_{c \in \mathcal{C}}$.

We aim to capture the relevance of context c for word w by the inner product between their embedding vectors, $\langle \mathbf{u}_w, \mathbf{v}_c \rangle$; the more relevant the context is, the larger we want their inner product to be. We achieve this by learning a ranking model that is parametrized by \mathbf{U} and \mathbf{V} . If we sort the set of contexts \mathcal{C} for a given word w in terms of each context's inner product score with the word, the rank of a specific context c in this list can be written as [28]:

$$\text{rank}(w, c) = \sum_{c' \in \mathcal{C} \setminus \{c\}} I(\langle \mathbf{u}_w, \mathbf{v}_c \rangle - \langle \mathbf{u}_w, \mathbf{v}_{c'} \rangle \leq 0) = \sum_{c' \in \mathcal{C} \setminus \{c\}} I(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle \leq 0), \quad (1)$$

where $I(x \leq 0)$ is a 0-1 loss function which is 1 if $x \leq 0$ and 0 otherwise. Since $I(x \leq 0)$ is a discontinuous function, we follow the popular strategy in machine learning which replaces the 0-1 loss by its convex upper bound $\ell(\cdot)$, where $\ell(\cdot)$ can be any popular loss function for binary classification such as the hinge loss $\ell(x) = \max(0, 1 - x)$ or the logistic loss $\ell(x) = \log_2(1 + 2^{-x})$ [4]. This enables us to construct the following convex upper bound on the rank:

$$\text{rank}(w, c) \leq \overline{\text{rank}}(w, c) = \sum_{c' \in \mathcal{C} \setminus \{c\}} \ell(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle). \quad (2)$$

It is certainly desirable that the ranking model positions relevant contexts at the top of the list; this motivates us to write the objective function to minimize as:

$$J(\mathbf{U}, \mathbf{V}) := \sum_{w \in \mathcal{W}} \sum_{c \in \Omega_w} r_{w,c} \cdot \rho\left(\frac{\overline{\text{rank}}(w, c) + \beta}{\alpha}\right), \quad (3)$$

where $r_{w,c}$ is the weight between word w and context c quantifying the association between them, $\rho(\cdot)$ is a monotonically increasing and concave ranking loss function that measures goodness of a rank, and $\alpha > 0$, $\beta > 0$ are the hyperparameters of the model whose role will be discussed later. Following Pennington et al. [25], we use

$$r_{w,c} = \begin{cases} (X_{w,c}/x_{max})^\epsilon & \text{if } X_{w,c} < x_{max} \\ 1 & \text{otherwise,} \end{cases} \quad (4)$$

where we set $x_{max} = 100$ and $\epsilon = 0.75$ in our experiments. That is, we assign larger weights (with a saturation) to contexts that appear more often with the word of interest, and vice-versa. For the ranking loss function $\rho(\cdot)$, on the other hand, we consider the class of monotonically increasing and concave functions. While monotonicity is a natural requirement, we argue that concavity is also important so that the derivative of ρ is always non-increasing; this implies that the ranking loss ρ be the most sensitive at the top of the list (where the rank is small) and becomes less sensitive at the lower end of the list (where the rank is high). Intuitively this is desirable, because we are interested in a small number of relevant contexts which frequently co-occur with a given word, and thus are willing to tolerate errors on infrequent contexts². Meanwhile, this insensitivity at the bottom of the list makes the model robust to noise in the data either due to grammatical errors or unconventional use of language. Therefore, a single ranking loss function $\rho(\cdot)$ serves two different purposes at two ends of the curve (see the example plots of ρ in Figure 1); while the left hand side of the curve encourages “high resolution” on most relevant words, the right hand side becomes less sensitive (with “low resolution”) to infrequent and possibly noisy words³. As we will demonstrate in our experiments, this is a fundamental attribute (in addition to the ranking nature) of our method that contributes its superior performance as compared to the state-of-the-arts when the training set is limited (*i.e.*, sparse and noisy).

²This is similar to the attention mechanism found in human visual system that is able to focus on a certain region of an image with “high resolution” while perceiving the surrounding image in “low resolution” [14, 24].

³Due to the linearity of $\rho_0(x) = x$, this ranking loss doesn't have the benefit of attention mechanism and robustness to noise since it treats all ranking errors equally.

对于 $\rho(x)=x$ 的线性性，排序损失函数并没有从注意力机制和对噪声的鲁棒性中获取任何好处，因为它平等对待所有排序错误。

这个类似于人类视觉系统中的注意力机制，它能够以高分辨率关注图像中的特定区域，而对图像周围保持低分辨率。

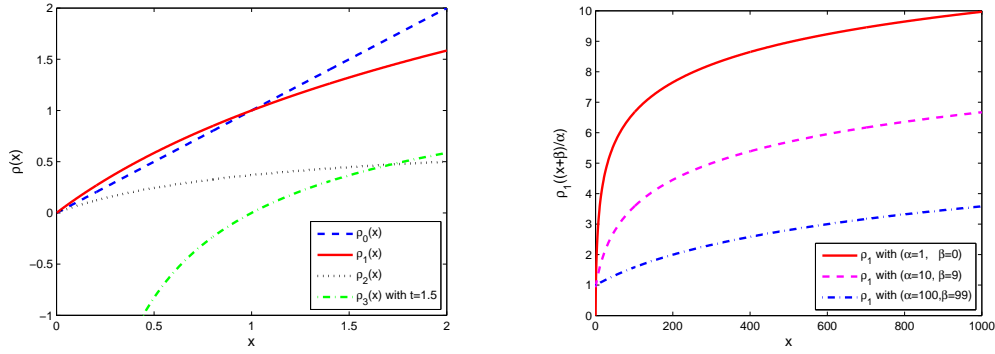


Figure 1: (a) Visualizing different ranking loss functions $\rho(x)$ as defined in Eqs. (5–8); the lower part of $\rho_3(x)$ is truncated in order to visualize the other functions better. (b) Visualizing $\rho_1((x + \beta)/\alpha)$ with different α and β .

What are interesting loss functions that can be used for $\rho(\cdot)$? Here are four possible alternatives, all of which have a natural interpretation (see the plots of all four ρ functions in Figure 1(a) and the related work section for references and a discussion).

$$\rho(x) = \rho_0(x) := x \quad (\text{identity}) \quad (5)$$

$$\rho(x) = \rho_1(x) := \log_2(1 + x) \quad (\text{logarithm}) \quad (6)$$

$$\rho(x) = \rho_2(x) := 1 - \frac{1}{\log_2(2 + x)} \quad (\text{negative DCG}) \quad (7)$$

$$\rho(x) = \rho_3(x) := \frac{x^{1-t} - 1}{1 - t} \quad (\log_t \text{ with } t \neq 1) \quad (8)$$

We will explore the performance of each of these variants in our experiments. For now, we turn our attention to efficient stochastic optimization of the objective function (3).

2.3 Stochastic Optimization

Plugging (2) into (3), and replacing $\sum_{w \in \mathcal{W}} \sum_{c \in \Omega_w}$ by $\sum_{(w,c) \in \Omega}$, the **objective function** becomes:

$$J(\mathbf{U}, \mathbf{V}) = \sum_{(w,c) \in \Omega} r_{w,c} \cdot \rho \left(\frac{\sum_{c' \in \mathcal{C} \setminus \{c\}} \ell(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) + \beta}{\alpha} \right). \quad (9)$$

This function contains summations over Ω and \mathcal{C} , both of which are expensive to compute for a large corpus. Although stochastic gradient descent (SGD) [5] can be used to replace the summation over Ω by random sampling, the summation over \mathcal{C} cannot be avoided unless $\rho(\cdot)$ is a linear function. To work around this problem, we propose to optimize a linearized upper bound of the objective function obtained through **a first-order Taylor approximation**. Observe that due to the concavity of $\rho(\cdot)$, we have

$$\rho(x) \leq \rho(\xi^{-1}) + \rho'(\xi^{-1}) \cdot (x - \xi^{-1}) \quad (10)$$

for any x and $\xi \neq 0$. Moreover, the bound is tight when $\xi = x^{-1}$. This motivates us to introduce a set of auxiliary parameters $\Xi := \{\xi_{w,c}\}_{(w,c) \in \Omega}$ and define the following **upper bound of $J(\mathbf{U}, \mathbf{V})$** :

$$\bar{J}(\mathbf{U}, \mathbf{V}, \Xi) := \sum_{(w,c) \in \Omega} r_{w,c} \cdot \left\{ \rho(\xi_{w,c}^{-1}) + \rho'(\xi_{w,c}^{-1}) \cdot \left(\alpha^{-1} \beta + \alpha^{-1} \sum_{c' \in \mathcal{C} \setminus \{c\}} \ell(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) - \xi_{w,c}^{-1} \right) \right\}. \quad (11)$$

Note that $J(\mathbf{U}, \mathbf{V}) \leq \bar{J}(\mathbf{U}, \mathbf{V}, \Xi)$ for any Ξ , due to (10)⁴. Also, minimizing (11) yields the same \mathbf{U} and \mathbf{V} as minimizing (9). To see this, suppose $\hat{\mathbf{U}} := \{\hat{\mathbf{u}}_w\}_{w \in \mathcal{W}}$ and $\hat{\mathbf{V}} := \{\hat{\mathbf{v}}_c\}_{c \in \mathcal{C}}$ minimizes

⁴When $\rho = \rho_0$, one can simply set the auxiliary variables $\xi_{w,c} = 1$ because ρ_0 is already a linear function.

(9). Then, by letting $\hat{\Xi} := \{\hat{\xi}_{w,c}\}_{(w,c) \in \Omega}$ where

$$\hat{\xi}_{w,c} = \frac{\alpha}{\sum_{c' \in \mathcal{C} \setminus \{c\}} \ell(\langle \hat{\mathbf{u}}_w, \hat{\mathbf{v}}_c - \hat{\mathbf{v}}_{c'} \rangle) + \beta}, \quad (12)$$

we have $\bar{J}(\hat{\mathbf{U}}, \hat{\mathbf{V}}, \hat{\Xi}) = J(\hat{\mathbf{U}}, \hat{\mathbf{V}})$. Therefore, it suffices to optimize (11). However, unlike (9), (11) admits an efficient SGD algorithm. To see this, rewrite (11) as

$$\bar{J}(\mathbf{U}, \mathbf{V}, \Xi) := \sum_{(w,c,c')} r_{w,c} \cdot \left(\frac{\rho(\xi_{w,c}^{-1}) + \rho'(\xi_{w,c}^{-1}) \cdot (\alpha^{-1}\beta - \xi_{w,c}^{-1})}{|\mathcal{C}| - 1} + \frac{1}{\alpha} \rho'(\xi_{w,c}^{-1}) \cdot \ell(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) \right), \quad (13)$$

where $(w, c, c') \in \Omega \times (\mathcal{C} \setminus \{c\})$. Then, it can be seen that if we sample uniformly from $(w, c) \in \Omega$ and $c' \in \mathcal{C} \setminus \{c\}$, then $j(w, c, c') :=$

$$|\Omega| \cdot (|\mathcal{C}| - 1) \cdot r_{w,c} \cdot \left(\frac{\rho(\xi_{w,c}^{-1}) + \rho'(\xi_{w,c}^{-1}) \cdot (\alpha^{-1}\beta - \xi_{w,c}^{-1})}{|\mathcal{C}| - 1} + \frac{1}{\alpha} \rho'(\xi_{w,c}^{-1}) \cdot \ell(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) \right), \quad (14)$$

which does not contain any expensive summations and is an unbiased estimator of (13), i.e., $\mathbb{E}[j(w, c, c')] = \bar{J}(\mathbf{U}, \mathbf{V}, \Xi)$. On the other hand, one can optimize $\xi_{w,c}$ exactly by using (12). Putting everything together yields a stochastic optimization algorithm *WordRank*, which can be specialized to a variety of ranking loss functions $\rho(\cdot)$ with weights $r_{w,c}$ (e.g., DCG is one of many possible instantiations). Algorithm 1 contains detailed pseudo-code. It can be seen that the **algorithm is divided into two stages**: a stage that updates (\mathbf{U}, \mathbf{V}) and **another** that updates Ξ . Note that the time complexity of the first stage is $\mathcal{O}(|\Omega|)$ since the cost of each update in Lines 8–10 is independent of the size of the corpus. On the other hand, the time complexity of updating Ξ in Line 15 is $\mathcal{O}(|\Omega||\mathcal{C}|)$, which can be expensive. To amortize this cost, we employ two tricks: we only update Ξ after a few iterations of \mathbf{U} and \mathbf{V} update, and we exploit the fact that the most computationally expensive operation in (12) involves a matrix and matrix multiplication which can be calculated efficiently via the SGEMM routine in BLAS [10].

Algorithm 1 *WordRank* algorithm.

```

1:  $\eta$ : step size
2: repeat
3:   // Stage 1: Update  $\mathbf{U}$  and  $\mathbf{V}$ 
4:   repeat
5:     Sample  $(w, c)$  uniformly from  $\Omega$ 
6:     Sample  $c'$  uniformly from  $\mathcal{C} \setminus \{c\}$ 
7:     // following three updates are executed simultaneously
8:      $\mathbf{u}_w \leftarrow \mathbf{u}_w - \eta \cdot r_{w,c} \cdot \rho'(\xi_{w,c}^{-1}) \cdot \ell'(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) \cdot (\mathbf{v}_c - \mathbf{v}_{c'})$ 
9:      $\mathbf{v}_c \leftarrow \mathbf{v}_c - \eta \cdot r_{w,c} \cdot \rho'(\xi_{w,c}^{-1}) \cdot \ell'(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) \cdot \mathbf{u}_w$ 
10:     $\mathbf{v}_{c'} \leftarrow \mathbf{v}_{c'} + \eta \cdot r_{w,c} \cdot \rho'(\xi_{w,c}^{-1}) \cdot \ell'(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) \cdot \mathbf{u}_w$ 
11:   until  $\mathbf{U}$  and  $\mathbf{V}$  are converged
12:   // Stage 2: Update  $\Xi$ 
13:   for  $w \in \mathcal{W}$  do
14:     for  $c \in \mathcal{C}$  do
15:        $\xi_{w,c} = \alpha / \left( \sum_{c' \in \mathcal{C} \setminus \{c\}} \ell(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) + \beta \right)$ 
16:     end for
17:   end for
18: until  $\mathbf{U}$ ,  $\mathbf{V}$  and  $\Xi$  are converged

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2.4 Parallelization

The updates in Lines 8–10 have one remarkable property: To update \mathbf{u}_w , \mathbf{v}_c and $\mathbf{v}_{c'}$, we only need to *read* the variables \mathbf{u}_w , \mathbf{v}_c , $\mathbf{v}_{c'}$ and $\xi_{w,c}$. What this means is that updates to another triplet of variables $\mathbf{u}_{\hat{w}}$, $\mathbf{v}_{\hat{c}}$ and $\mathbf{v}_{\hat{c}'}$ can be performed independently. This observation is the key to developing a parallel optimization strategy, by distributing the computation of the updates among multiple processors. Due to lack of space, details including pseudo-code are relegated to Appendix A.

2.5 Interpreting α and β

The update (12) indicates that $\xi_{w,c}^{-1}$ is proportional to $\overline{\text{rank}}(w, c)$. On the other hand, one can observe that the loss function $\ell(\cdot)$ in (14) is weighted by a $\rho'(\xi_{w,c}^{-1})$ term. Since $\rho(\cdot)$ is concave, its gradient $\rho'(\cdot)$ is monotonically non-increasing [27]. Consequently, when $\overline{\text{rank}}(w, c)$ and hence $\xi_{w,c}^{-1}$ is large, $\rho'(\xi_{w,c}^{-1})$ is small. In other words, the loss function “gives up” on contexts with high ranks in order to focus its attention on top of the list. The rate at which the algorithm gives up is determined by the hyperparameters α and β . For the illustration of this effect, see the example plots of ρ_1 with different α and β in Figure 1(b). Intuitively, α can be viewed as a *scale parameter* while β can be viewed as an *offset parameter*. An equivalent interpretation is that by choosing different values of α and β one can modify the behavior of the ranking loss $\rho(\cdot)$ in a problem dependent fashion. In our experiments, we found that a common setting of $\alpha = 1$ and $\beta = 0$ often yields uncompetitive performance, while setting $\alpha = 100$ and $\beta = 99$ generally gives good results.

3 Related Work

Our work sits at the intersection of *word embedding* and *ranking optimization*. As we discussed above, it’s also related to the *attention mechanism* widely used in deep learning. We therefore review the related work along these three threads.

Word Embedding. We already discussed some related work (*word2vec* and *GloVe*) on word embedding in the introduction. Essentially, *word2vec* and *GloVe* derive word representations by modeling a transformation (PMI or log) of $X_{w,c}$ directly, while *WordRank* learns word representations via robust ranking. Besides these state-of-the-art techniques, a few ranking-based approaches have been proposed for word embedding recently (e.g., [7, 18, 29]). However, all of them adopt a pair-wise binary classification approach with a linear ranking loss ρ_0 . For example, [7, 29] employ a hinge loss on positive/negative word pairs to learn word representations and ρ_0 is used implicitly to evaluate ranking losses. As we discussed above, ρ_0 has no benefit of the attention mechanism and robustness to noise since its linearity treats all the ranking errors uniformly; empirically, sub-optimal performances are often observed with ρ_0 in our experiments. More recently, by extending the Skip-Gram model of *word2vec*, Liu et al. [18] incorporates additional pair-wise constraints induced from 3rd-party knowledge bases, such as WordNet, and learns word representations jointly. In contrast, *WordRank* is a fully ranking-based approach without using any additional data source for training. Furthermore, all the aforementioned word embedding methods are online algorithms, while *WordRank* and *GloVe* are not.

word2vec 和 glove 通过直接对单词和语境context矩阵的变体建模来得到单词向量表示；
wordRank通过robust ranking 来学习单词向量表示。

Robust Ranking. The second line of work that is very relevant to *WordRank* is that of ranking objective (3). The use of score functions $\langle \mathbf{u}_w, \mathbf{v}_c \rangle$ for ranking is inspired by the latent collaborative retrieval framework of Weston et al. [31]. Writing the rank as a sum of indicator functions (1), and upper bounding it via a convex loss (2) is due to [28]. Using $\rho_0(\cdot)$ (5) corresponds to the well-known pairwise ranking loss (see e.g., [15]). On the other hand, Yun et al. [33] observed that if they set $\rho = \rho_2$ as in (7), then $-J(\mathbf{U}, \mathbf{V})$ corresponds to the DCG (Discounted Cumulative Gain) [21], one of the most popular ranking metrics used in web search ranking. In their RobiRank algorithm they proposed the use of $\rho = \rho_1$ (6), which they considered to be a special function for which one can derive an efficient stochastic optimization procedure. However, as we showed in this paper, the general class of monotonically increasing concave functions can be handled efficiently. Another important difference of our approach is the hyperparameters α and β , which we use to modify the behavior of ρ , and which we find are critical to achieve good empirical results. Ding and Vishwanathan [9] proposed the use of $\rho = \log_t$ in the context of robust binary classification, while here we are concerned with ranking, and our formulation is very general and applies to a variety of ranking losses $\rho(\cdot)$ with weights $r_{w,c}$. Optimizing over \mathbf{U} and \mathbf{V} by distributing the computation across processors is inspired by work on distributed stochastic gradient for matrix factorization [12].

Attention. Attention is one of the most important advancements in deep learning in recent years [14], and is now widely used in state-of-the-art image recognition and machine translation systems [3, 24]. Recently, attention has also been applied to the domain of word embedding. For example, under the intuition that not all contexts are created equal, Wang et al. [30] assign an importance

直观上看，并不是单词所有的语境contexts都是等同的，Wang给每一个语境位置上的单词分配了一个重要性权重，学习基于注意力机制的连续词袋模型CBOW。类似，在排序框架下，wordRank通过引入辅助变量来表达语境的重要性，这个变量会在语境contexts高rank值(值越大，越不相关)时丢弃该context，从而将注意力集中在低rank值上的contexts。

weight to each word type at each context position and learn an attention-based Continuous Bag-Of-Words (CBOW) model. Similarly, within a ranking framework, *WordRank* expresses the context importance by introducing the auxiliary variable $\xi_{w,c}$, which “gives up” on contexts with high ranks in order to focus its attention on top of the list.

4 Experiments

In our experiments, we first evaluate the impact of the weight $r_{w,c}$ and the ranking loss function $\rho(\cdot)$ on the test performance using a small dataset. We then pick the best performing model and compare it against *word2vec* [23] and *GloVe* [25]. We closely follow the framework of Levy et al. [17] to set up a careful and fair comparison of the three methods. Our code and experiment scripts are open sourced for general usage at <https://bitbucket.org/shihaoji/wordrank>.

Training Corpus Models are trained on a combined corpus of 7.2 billion tokens, which consists of the 2015 Wikipedia dump with 1.6 billion tokens, the WMT14 News Crawl⁵ with 1.7 billion tokens, the “One Billion Word Language Modeling Benchmark”⁶ with almost 1 billion tokens, and UMBC webbase corpus⁷ with around 3 billion tokens. The Wikipedia dump is in XML format and the article content needs to be parsed from wiki markups, while the other corpora are already in plain text format. The pre-processing pipeline breaks the paragraphs into sentences, tokenizes and lowercases each corpus with the Stanford tokenizer. We further clean up the dataset by removing non-ASCII characters and punctuation, and discard sentences that are shorter than 3 tokens or longer than 500 tokens. In the end, we obtain a dataset with 7.2 billion tokens, with the first 1.6 billion tokens from Wikipedia. When we want to experiment with a smaller corpus, we extract a subset which contains the specified number of tokens.

Co-occurrence matrix construction We use the *GloVe* code to construct the co-occurrence matrix X , and the same matrix is used to train *GloVe* and *WordRank* models. When constructing X , we must choose the size of the vocabulary, the context window and whether to distinguish left context from right context. We follow the findings and design choices of *GloVe* and use a symmetric window of size win with a decreasing weighting function, so that word pairs that are d words apart contribute $1/d$ to the total count. Specifically, when the corpus is small (e.g., 17M, 32M, 64M) we let $win = 15$ and for larger corpora we let $win = 10$. The larger window size alleviates the data sparsity issue for small corpus at the expense of adding more noise to X . The parameter settings used in our experiments are summarized in Table 1.

Table 1: Parameter settings used in the experiments.

Corpus Size	17M*	32M	64M	128M	256M	512M	1.0B	1.6B	7.2B
Vocabulary Size $ \mathcal{V} $	71K	100K	100K	200K	200K	300K	300K	400K	620K
Window Size win	15	15	15	10	10	10	10	10	10
Dimension k	100	100	100	200	200	300	300	300	300

* This is the Text8 dataset from <http://mattmahoney.net/dc/text8.zip>, which is widely used for word embedding demo.

Using the trained model It has been shown by Pennington et al. [25] that combining the \mathbf{u}_w and \mathbf{v}_c vectors with equal weights gives a small boost in performance. This vector combination was originally motivated as an ensemble method [25], and later Levy et al. [17] provided a different interpretation of its effect on the cosine similarity function, and show that adding context vectors effectively adds first-order similarity terms to the second-order similarity function. In our experiments, we find that vector combination boosts the performance in word analogy task when training set is small, but when dataset is large enough (e.g., 7.2 billion tokens), vector combination doesn’t help anymore. More interestingly, for the word similarity task, we find that vector combination is

⁵<http://www.statmt.org/wmt14/translation-task.html>

⁶<http://www.statmt.org/lm-benchmark>

⁷<http://ebiquity.umbc.edu/resource/html/id/351>

detrimental in all the cases, sometimes even substantially⁸. Therefore, we will always use \mathbf{u}_w on word similarity task, and use $\mathbf{u}_w + \mathbf{v}_c$ on word analogy task unless otherwise noted.

4.1 Evaluation

Word Similarity We use six datasets to evaluate word similarity: WS-353 [11] partitioned into two subsets: WordSim Similarity and WordSim Relatedness [1]; MEN [6]; Mechanical Turk [26]; Rare words [19]; and SimLex-999 [13]. They contain word pairs together with human-assigned similarity judgments. The word representations are evaluated by ranking the pairs according to their cosine similarities, and measuring the Spearman’s rank correlation coefficient with the human judgments.

Word Analogies For this task, we use the Google analogy dataset [22]. It contains 19544 word analogy questions, partitioned into 8869 semantic and 10675 syntactic questions. The semantic questions contain five types of semantic analogies, such as capital cities (*Paris:France;Tokyo:?*), currency (*USA:dollar;India:?*) or people (*king:queen;man:?*). The syntactic questions contain nine types of analogies, such as plural nouns, opposite, or comparative, for example *good:better;smart:?*. A question is correctly answered only if the algorithm selects the word that is exactly the same as the correct word in the question: synonyms are thus counted as mistakes. There are two ways to answer these questions, namely, by using 3CosAdd or 3CosMul (see [16] for details). We will report scores by using 3CosAdd by default, and indicate when 3CosMul gives better performance.

Handling questions with out-of-vocabulary words Some papers (e.g., [17]) filter out questions with out-of-vocabulary words when reporting performance. By contrast, in our experiments if any word of a question is out of vocabulary, the corresponding question will be marked as unanswerable and will get a score of zero. This decision is made so that when the size of vocabulary increases, the model performance is still comparable across different experiments.

4.2 The impact of $r_{w,c}$ and $\rho(\cdot)$

In Sec. 2.2 we argued the need for adding weight $r_{w,c}$ to ranking objective (3), and we also presented our framework which can deal with a variety of ranking loss functions ρ . We now study the utility of these two ideas. We report results on the 17 million token dataset in Table 2. For the similarity task, we use the WS-353 test set and for the analogy task we use the Google analogy test set. The best scores for each task are underlined. We set $t = 1.5$ for ρ_3 . “Off” means that we used uniform weight $r_{w,c} = 1$, and “on” means that $r_{w,c}$ was set as in (4). For comparison, we also include the results using RobiRank [33]⁹.

Table 2: Performance of different ρ functions on Text8 dataset with 17M tokens.

Task	Robi	ρ_0		ρ_1		ρ_2		ρ_3	
		off	on	off	on	off	on	off	on
Similarity	41.2	69.0	<u>71.0</u>	66.7	70.4	66.8	70.8	68.1	68.0
Analogy	22.7	24.9	31.9	34.3	<u>44.5</u>	32.3	40.4	33.6	42.9

It can be seen from Table 2 that adding the weight $r_{w,c}$ improves performance in all the cases, especially on the word analogy task. Among the four ρ functions, ρ_0 performs the best on the word similarity task but suffers notably on the analogy task, while $\rho_1 = \log$ performs the best over all. Given these observations, which are consistent with the results on large scale datasets, in the experiments that follow we only consider *WordRank* with the best configuration, i.e., using ρ_1 with the weight $r_{w,c}$ as defined in (4).

⁸This is possible since we optimize a ranking loss: the absolute scores don’t matter as long as they yield an ordered list correctly. Thus, *WordRank*’s \mathbf{u}_w and \mathbf{v}_c are less comparable to each other than those generated by *GloVe*, which employs a point-wise L_2 loss.

⁹We used the code provided by the authors at <https://bitbucket.org/dijkstra/robirank>. Although related to an unweighted version of *WordRank* with the ρ_1 ranking loss function, we attribute the difference in performance to the use of the α and β hyperparameters in our algorithm and many implementation details.

4.3 Comparison to state-of-the-arts

In this section we compare the performance of *WordRank* with *word2vec*¹⁰ and *GloVe*¹¹, by using the code provided by the respective authors. For a fair comparison, *GloVe* and *WordRank* are given as input the same co-occurrence matrix X ; this eliminates differences in performance due to window size and other such artifacts, and the same parameters are used to *word2vec*. Moreover, the embedding dimensions used for each of the three methods is the same (see Table 1). With *word2vec*, we train the Skip-Gram with Negative Sampling (SGNS) model since it produces state-of-the-art performance, and is widely used in the NLP community [23]. For *GloVe*, we use the default parameters as suggested by [25]. The results are provided in Figure 2 (also see Table 4 in Appendix B for additional details).

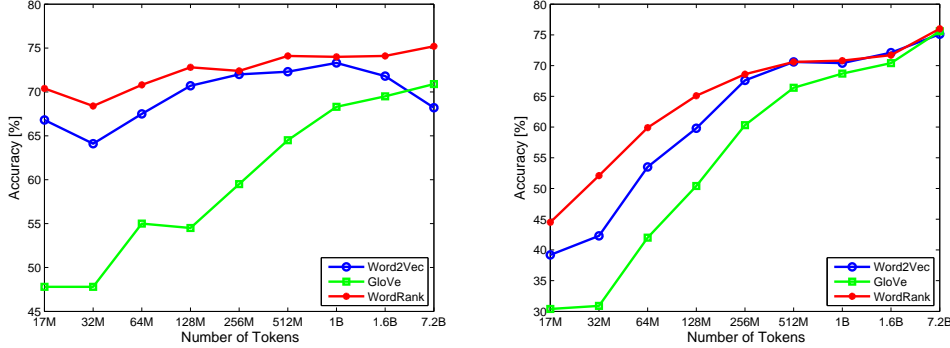


Figure 2: Performance evolution as a function of corpus size (a) on WS-353 word similarity benchmark; (b) on Google word analogy benchmark.

As can be seen, when the size of corpus increases, in general all three algorithms improve their prediction accuracy on both tasks. This is to be expected since a larger corpus typically produces better statistics and less noise in the co-occurrence matrix X . When the corpus size is small (e.g., 17M, 32M, 64M, 128M), *WordRank* yields the best performance with significant margins among three, followed by *word2vec* and *GloVe*; when the size of corpus increases further, on the word analogy task *word2vec* and *GloVe* become very competitive to *WordRank*, and eventually perform neck-to-neck to each other (Figure 2(b)). This is consistent with the findings of [17] indicating that when the number of tokens is large even simple algorithms can perform well. On the other hand, *WordRank* is dominant on the word similarity task for all the cases (Figure 2(a)) since it optimizes a ranking loss *explicitly*, which aligns more naturally with the objective of word similarity than the other methods; with 17 million tokens our method performs almost as well as existing methods using 7.2 billion tokens on the word similarity benchmark.

As a side note, on a similar 1.6-billion-token Wikipedia corpus, our *word2vec* and *GloVe* performance scores are somewhat better than or close to the results reported by Pennington et al. [25]; and our *word2vec* and *GloVe* scores on the 7.2-billion-token dataset are close to what they reported on a 42-billion-token dataset. We believe this discrepancy is primary due to the extra attention we paid to pre-process the Wikipedia and other corpora.

To further evaluate the model performance on the word similarity/analogy tasks, we use the best performing models trained on the 7.2-billion-token corpus to predict on the six word similarity datasets described in Sec. 4.1. Moreover, we breakdown the performance of the models on the Google word analogy dataset into the semantic and syntactic subtasks. Results are listed in Table 3. As can be seen, *WordRank* outperforms *word2vec* and *GloVe* on 5 of 6 similarity tasks, and 1 of 2 Google analogy subtasks.

5 Conclusion

We proposed *WordRank*, a ranking-based approach, to learn word representations from large scale textual corpora. The most prominent difference between our method and the state-of-the-art tech-

¹⁰<https://code.google.com/p/word2vec/>

¹¹<http://nlp.stanford.edu/projects/glove>

Table 3: Performance of the best *word2vec*, *GloVe* and *WordRank* models, learned from 7.2 billion tokens, on six similarity tasks and Google semantic and syntactic subtasks.

Model	Word Similarity						Word Analogy	
	WordSim Similarity	WordSim Relatedness	Bruni et al. MEN	Radinsky et al. MT	Luong et al. RW	Hill et al. SimLex	Goog Sem.	Goog Syn.
<i>word2vec</i>	73.9	60.9	75.4	66.4	45.5	36.6	78.8	72.0
<i>GloVe</i>	75.7	67.5	78.8	69.7	43.6	41.6	80.9	71.1
<i>WordRank</i>	79.4	70.5	78.1	73.5	47.4	43.5	78.4	74.7

niques, such as *word2vec* and *GloVe*, is that **WordRank** learns word representations via a robust ranking model, while *word2vec* and *GloVe* typically model a **transformation of co-occurrence count** $X_{w,c}$ directly. Moreover, through the monotonically increasing and concave ranking loss function $\rho(\cdot)$, *WordRank* achieves its attention mechanism and robustness to noise naturally, which are usually lacking in other ranking-based approaches. These attributes significantly boost the performance of *WordRank* in the cases where **training data are sparse and noisy**, from which the other competitive methods often suffer. We open sourced our multi-machine distributed implementation of *WordRank* to facilitate future research in this direction.

wordRank通过robust ranking model 学习单词的向量表示，word2vec和glove则是直接对共生计数矩阵的变体建模来学习。进一步看，通过排序损失函数的单调递增和凹性，wordRank很自然地拥有了注意力机制和对噪声的鲁棒性，而这恰恰是其他排序方法所缺乏的。当训练集是稀疏和具有噪声时，这些属性非常重要地提升了模型的性能，而其他方法却无法避免。

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WordRank: Learning Word Embeddings via Robust Ranking

(Supplementary Material)

A Parallel WordRank

Given number of p workers, we partition words \mathcal{W} into p parts $\{\mathcal{W}^{(1)}, \mathcal{W}^{(2)}, \dots, \mathcal{W}^{(p)}\}$ such that they are mutually exclusive, exhaustive and approximately equal-sized. This partition on \mathcal{W} induces a partition on \mathbf{U} , Ω and Ξ as follows: $\mathbf{U}^{(q)} := \{\mathbf{u}_w\}_{w \in \mathcal{W}^{(q)}}$, $\Omega^{(q)} := \{(w, c) \in \Omega\}_{w \in \mathcal{W}^{(q)}}$, and $\Xi^{(q)} := \{\xi_{(w,c)}\}_{(w,c) \in \Omega^{(q)}}$ for $1 \leq q \leq p$. When the algorithm starts, $\mathbf{U}^{(q)}$, $\Omega^{(q)}$ and $\Xi^{(q)}$ are distributed to worker q .

At the beginning of each outer iteration, an approximately equal-sized partition $\{\mathcal{C}^{(1)}, \mathcal{C}^{(2)}, \dots, \mathcal{C}^{(p)}\}$ on the context set \mathcal{C} is sampled; note that this is independent of the partition on words \mathcal{W} . This induces a partition on context vectors $\mathbf{V}^{(1)}, \mathbf{V}^{(2)}, \dots, \mathbf{V}^{(p)}$ defined as follows: $\mathbf{V}^{(q)} := \{\mathbf{v}_c\}_{c \in \mathcal{C}^{(q)}}$ for each q . Then, each $\mathbf{V}^{(q)}$ is distributed to each worker q . Now we define

$$\bar{J}^{(q)}(\mathbf{U}^{(q)}, \mathbf{V}^{(q)}, \Xi^{(q)}) := \sum_{(w,c) \in \Omega \cap (\mathcal{W}^{(q)} \times \mathcal{C}^{(q)})} \sum_{c' \in \mathcal{C}^{(q)} \setminus \{c\}} j(w, c, c'), \quad (15)$$

where $j(w, c, c')$ was defined in (14). Note that $j(w, c, c')$ in the above equation only accesses \mathbf{u}_w , \mathbf{v}_c and $\mathbf{v}_{c'}$ which belong to no sets other than $\mathbf{U}^{(q)}$ and $\mathbf{V}^{(q)}$, therefore worker q can run stochastic gradient descent updates on (15) for a predefined amount of time without having to communicate with other workers. The pseudo-code is illustrated in Algorithm 2.

Considering that the scope of each worker is always confined to a rather narrow set of observations $\Omega \cap (\mathcal{W}^{(q)} \times \mathcal{C}^{(q)})$, it is somewhat surprising that Gemulla et al. [12] proved that such an optimization scheme, which they call stratified stochastic gradient descent (SSGD), converges to the same local optimum a vanilla SGD would converge to. This is due to the fact that

$$\mathbb{E} \left[\bar{J}^{(1)}(\mathbf{U}^{(1)}, \mathbf{V}^{(1)}, \Xi^{(1)}) + \bar{J}^{(2)}(\mathbf{U}^{(2)}, \mathbf{V}^{(2)}, \Xi^{(2)}) + \dots + \bar{J}^{(p)}(\mathbf{U}^{(p)}, \mathbf{V}^{(p)}, \Xi^{(p)}) \right] \approx \bar{J}(\mathbf{U}, \mathbf{V}, \Xi), \quad (16)$$

if the expectation is taken over the sampling of the partitions of \mathcal{C} . This implies that the bias in each iteration due to narrowness of the scope will be washed out in a long run; this observation leads to the proof of convergence in Gemulla et al. [12] using standard theoretical results from Yin and Kushner [32].

B Additional Experimental Details

Table 4 is the tabular view of the data plotted in Figure 2 to provide additional experimental details.

C Visualizing the results

To understand whether *WordRank* produces *syntactically* and *semantically* meaningful vector space, we did the following experiment: we use the best performing model produced using 7.2 billion tokens, and compute the nearest neighbors of the word “cat”. We then visualize the words in two dimensions by using t-SNE, a well-known technique for dimensionality reduction [20]. As can be seen in Figure 3, our ranking-based model is indeed capable of capturing both semantic (*e.g.*, cat, feline, kitten, tabby) and syntactic (*e.g.*, leash, leashes, leashed) regularities of the English language.

Algorithm 2 Distributed *WordRank* algorithm.

```
1:  $\eta$ : step size
2: repeat
3:   // Start outer iteration
4:   Sample a partition over contexts  $\{\mathcal{C}^{(1)}, \dots, \mathcal{C}^{(q)}\}$ 
5:   // Stage 1: Update  $\mathbf{U}$  and  $\mathbf{V}$  in parallel
   for all machine  $q \in \{1, 2, \dots, p\}$  do in parallel
6:     Fetch all  $\mathbf{v}_c \in \mathbf{V}^{(q)}$ 
7:     repeat
8:       Sample  $(w, c)$  uniformly from  $\Omega^{(q)} \cap (\mathcal{W}^{(q)} \times \mathcal{C}^{(q)})$ 
9:       Sample  $c'$  uniformly from  $\mathcal{C}^{(q)} \setminus \{c\}$ 
10:      // following three updates are done simultaneously
11:       $\mathbf{u}_w \leftarrow \mathbf{u}_w - \eta \cdot r_{w,c} \cdot \rho'(\xi_{w,c}^{-1}) \cdot \ell'(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) \cdot (\mathbf{v}_c - \mathbf{v}_{c'})$ 
12:       $\mathbf{v}_c \leftarrow \mathbf{v}_c - \eta \cdot r_{w,c} \cdot \rho'(\xi_{w,c}^{-1}) \cdot \ell'(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) \cdot \mathbf{u}_w$ 
13:       $\mathbf{v}_{c'} \leftarrow \mathbf{v}_{c'} + \eta \cdot r_{w,c} \cdot \rho'(\xi_{w,c}^{-1}) \cdot \ell'(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) \cdot \mathbf{u}_w$ 
14:    until predefined time limit is exceeded
15:  end for
16:  // Stage 2: Update  $\Xi$  in parallel
  for all machine  $q \in \{1, 2, \dots, p\}$  do in parallel
17:    Fetch all  $\mathbf{v}_c \in \mathbf{V}$ 
18:    for  $w \in \mathcal{W}^{(q)}$  do
19:      for  $c \in \mathcal{C}$  do
20:         $\xi_{w,c} = \alpha / \left( \sum_{c' \in \mathcal{C} \setminus \{c\}} \ell'(\langle \mathbf{u}_w, \mathbf{v}_c - \mathbf{v}_{c'} \rangle) + \beta \right)$ 
21:      end for
22:    end for
23:  end for
24: until  $\mathbf{U}$ ,  $\mathbf{V}$  and  $\Xi$  are converged
```

Table 4: Performance of *word2vec*, *GloVe* and *WordRank* on datasets with increasing sizes; evaluated on WS-353 word similarity benchmark and Google word analogy benchmark.

Corpus Size	WS-353 (Word Similarity)			Google (Word Analogy)		
	<i>word2vec</i>	<i>GloVe</i>	<i>WordRank</i>	<i>word2vec</i>	<i>GloVe</i>	<i>WordRank</i>
17M	66.8	47.8	70.4	39.2	30.4	44.5
32M	64.1	47.8	68.4	42.3	30.9	52.1
64M	67.5	55.0	70.8	53.5	42.0	59.9
128M	70.7	54.5	72.8	59.8	50.4	65.1
256M	72.0	59.5	72.4	67.6	60.3	68.6
512M	72.3	64.5	74.1	70.6	66.4	70.6
1.0B	73.3	68.3	74.0	70.4	68.7	70.8
1.6B	71.8	69.5	74.1	72.1	70.4	71.7
7.2B	68.2	70.9	75.2/77.4 ¹	75.1 ²	75.6 ²	76.0 ^{2,3}

¹ When ρ_0 is used, corresponding to $\xi = 1.0$ in the training and no ξ update² Use 3CosMul instead of regular 3CosAdd for evaluation³ Use \mathbf{u}_w instead of default $\mathbf{u}_w + \mathbf{v}_c$ as word representation for evaluation

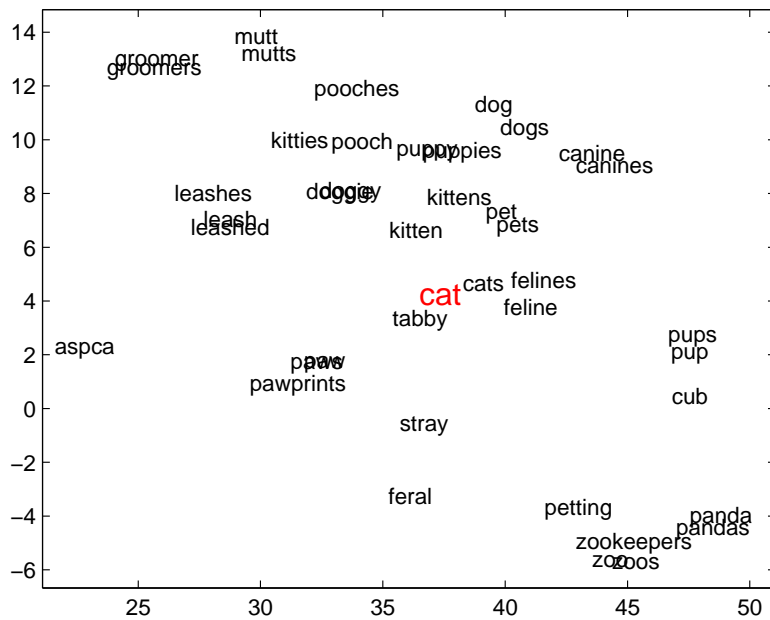


Figure 3: Nearest neighbors of “cat” found by projecting a 300d word embedding learned from *WordRank* onto a 2d space.