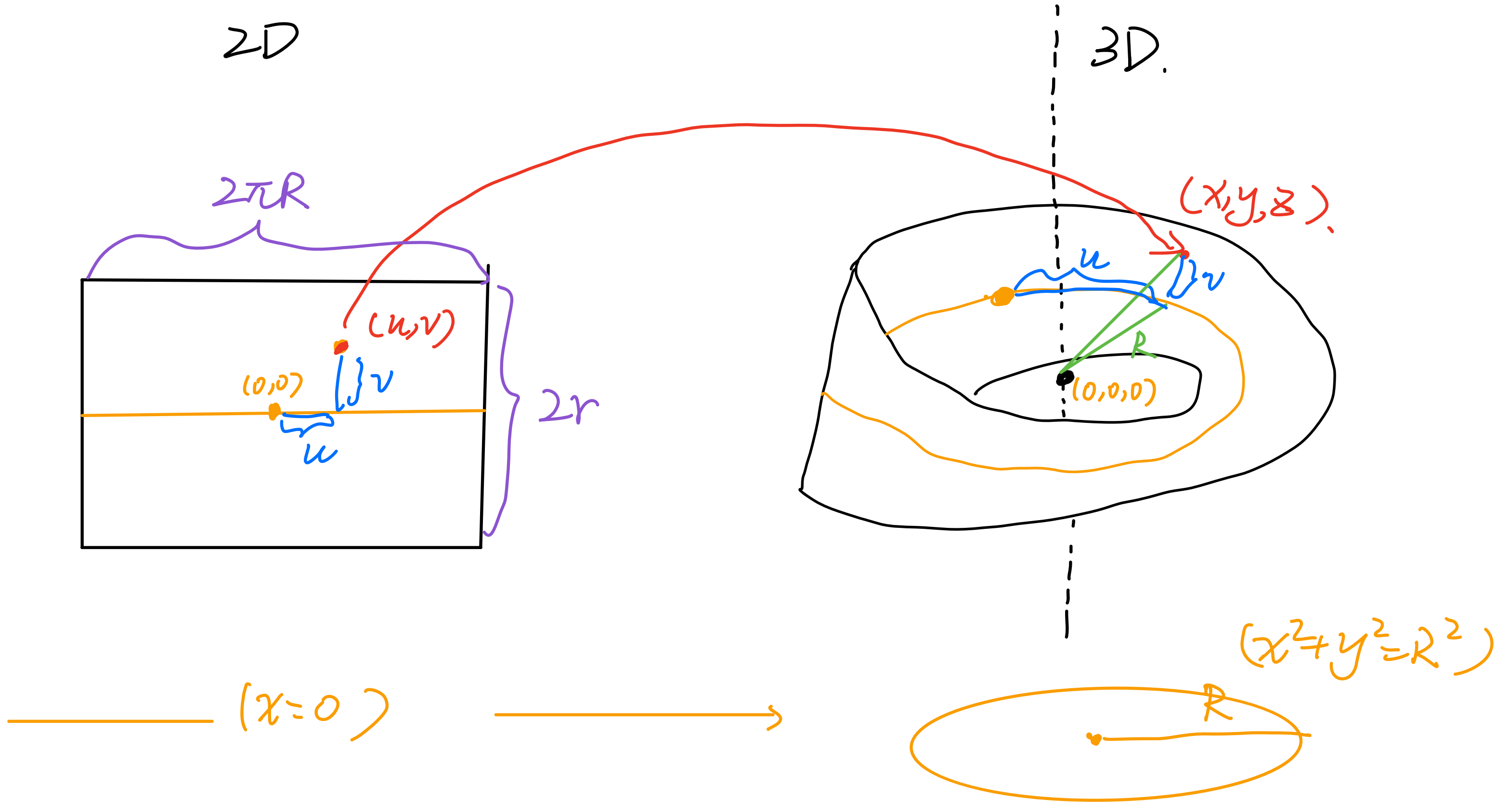
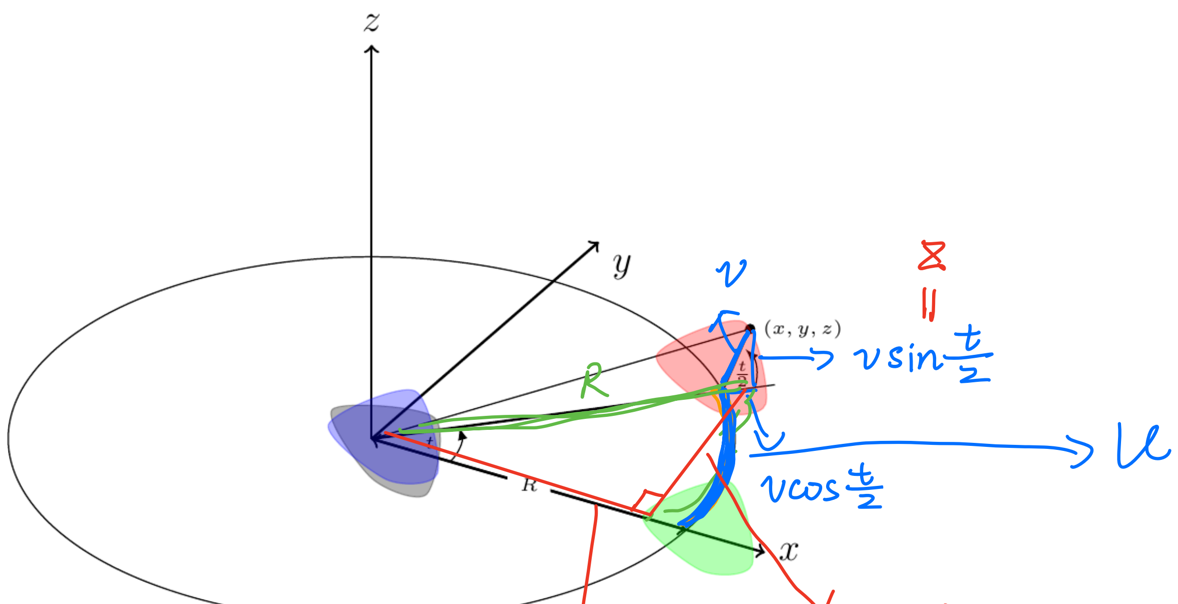




$y = (R + v\cos(\frac{t}{2}))\sin(t)$

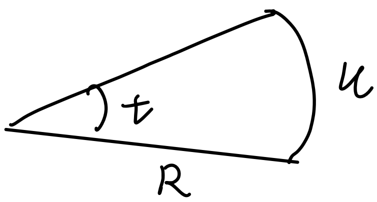
$x = (R + v\cos(\frac{t}{2}))\cos(t)$

$t \in [0, 2\pi]$ $\quad v \in [-r, r]$.

Parametrization

$$X(t,v) = \left(\left(R + v\cos\left(\tfrac{t}{2}\right)\right)\cos(t),\ \left(R + v\cos\left(\tfrac{t}{2}\right)\right)\sin(t),\ v\sin\left(\tfrac{t}{2}\right) \right)$$

t, u relationship:



in radian ($360° = 2\pi$)

$$tR = u$$

$$t = \frac{u}{R}$$

$$X(u,v) = \left(\left(R + v\cos\left(\frac{u}{2R}\right)\right)\cos\left(\frac{u}{R}\right),\ \left(R + v\cos\left(\frac{u}{2R}\right)\right)\sin\left(\frac{u}{R}\right),\ v\sin\left(\frac{u}{2R}\right) \right)$$

Inverse Mapping

$$x = \left(R + v\cos\left(\frac{u}{2R}\right)\right)\cos\left(\frac{u}{R}\right)$$

$$y = \left(R + v\cos\left(\frac{u}{2R}\right)\right)\sin\left(\frac{u}{R}\right)$$

$$z = v\sin\left(\frac{u}{2R}\right)$$

$$\frac{y}{x} = \tan\left(\frac{u}{R}\right) \Rightarrow u = R\arctan\left(\frac{y}{x}\right)$$

$$z = v\sin\left(\frac{u}{2R}\right) \Rightarrow v = \frac{z}{\sin\left(\frac{u}{2R}\right)} = \frac{z}{\sin\left(\frac{\arctan\left(\frac{y}{R}\right)}{2}\right)}$$

① 2D → 3D:

$$x = \left(R + v\cos\left(\frac{u}{2R}\right)\right)\cos\left(\frac{u}{R}\right)$$

$$y = \left(R + v\cos\left(\frac{u}{2R}\right)\right)\sin\left(\frac{u}{R}\right)$$

$$z = v\sin\left(\frac{u}{2R}\right)$$

② 3D → 2D:

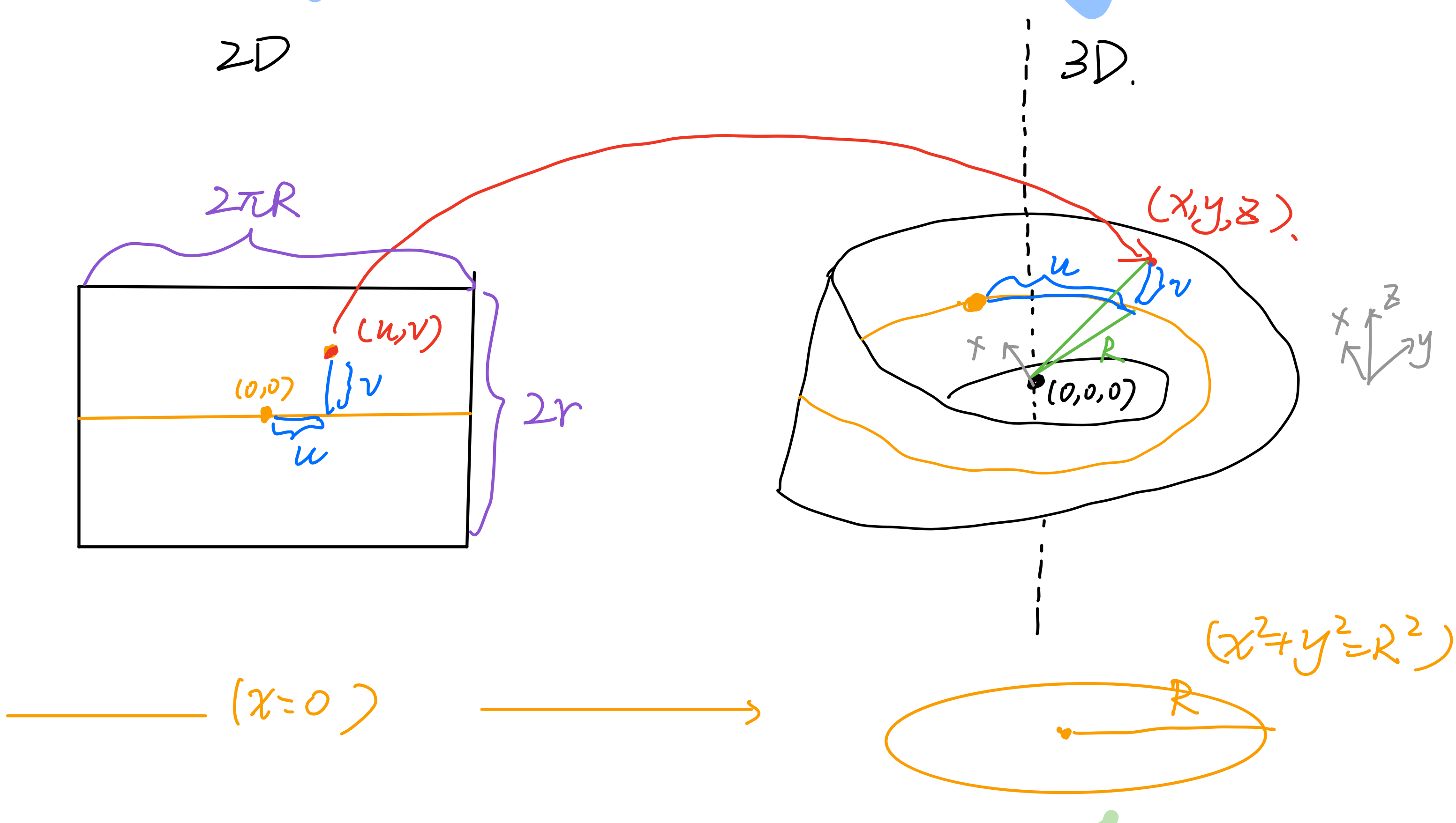
$$u = R\arctan\left(\frac{y}{x}\right)$$

$$v = \frac{z}{\sin\left(\frac{\arctan\left(\frac{y}{R}\right)}{2}\right)}$$

$$x = \left(R + v\cos\left(\frac{u}{2R}\right)\right)\cos\left(\frac{u}{R}\right)$$

$$y = \left(R + v\cos\left(\frac{u}{2R}\right)\right)\sin\left(\frac{u}{R}\right)$$

$$z = v\sin\left(\frac{u}{2R}\right)$$



$$u = R\arctan\left(\frac{y}{x}\right)$$

$$v = \frac{z}{\sin\left(\frac{\arctan\left(\frac{y}{R}\right)}{2}\right)}$$