CS5854 Homework1

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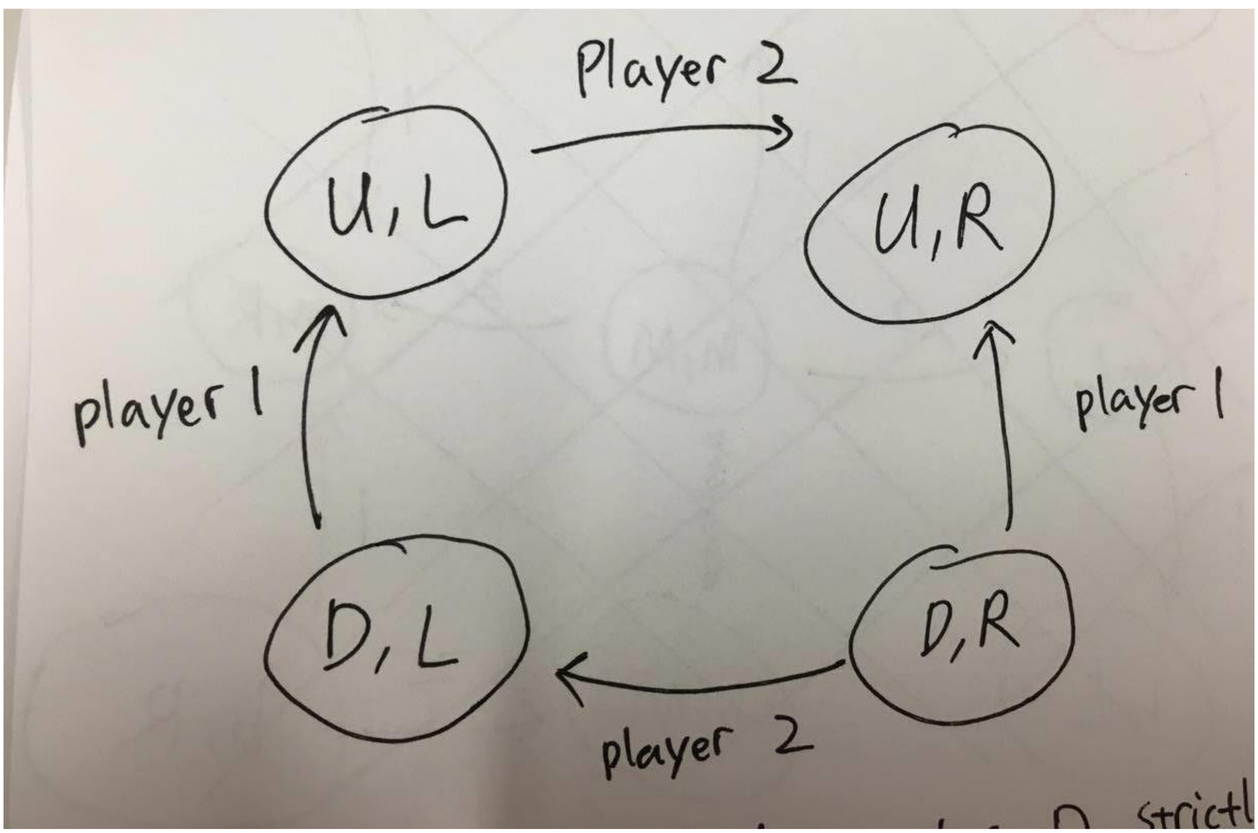
Part 1 Game Theory

Q1

(i) all strictly dominant strategies, (ii) the action pro les which survive iterative removal of strictly dominated strategies, and (iii) all pure-strategy Nash equilibria.

Use graph representation of game to solve this problem. In each graph, a node represents an action profile and there is an edge between action profiles whenever we can go from the first action profile to the second in one step of BRD. In my graph, I wrote utilities for both players in the node and labelled which player would take an action to deviate.

(a)

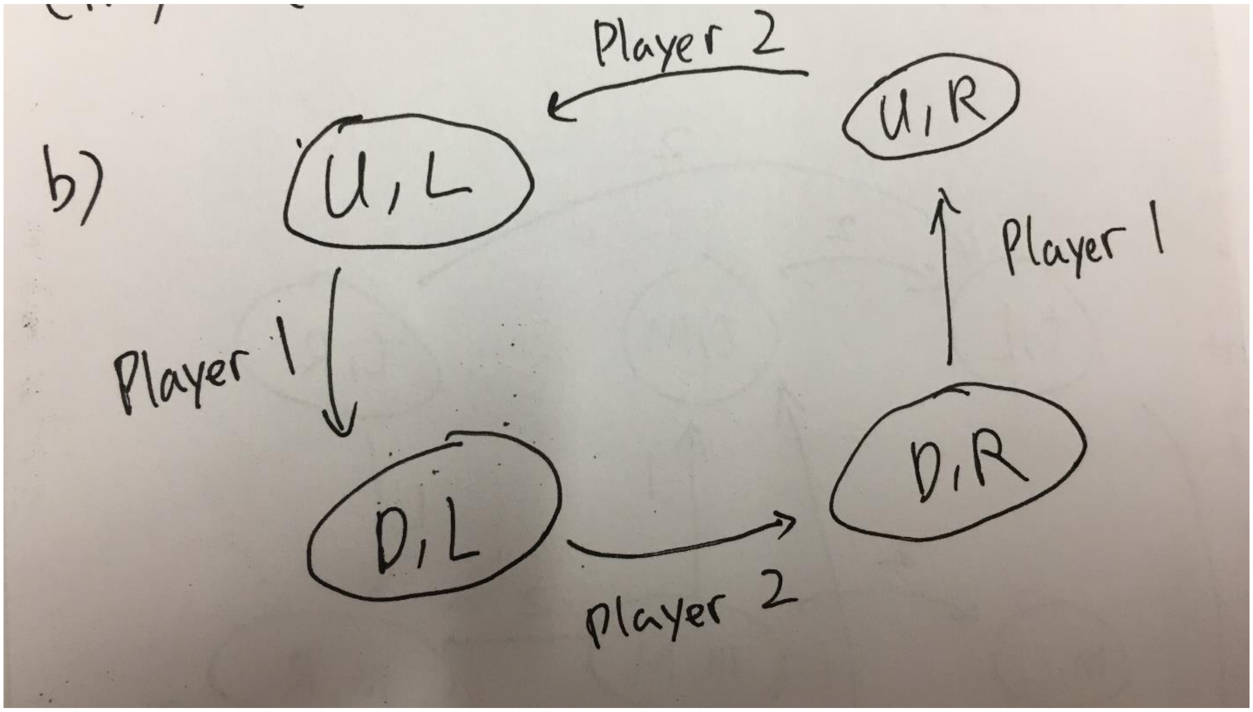


(i) For player 1, U strictly dominates D because player 1 deviates to (U,*) whenever (D,*) situation he is in.

(ii) (U,R) survive iterative removal. We remove (D,*) as D is dominated by U, then (U,L) is removed because player 2 wants to deviate to (U,R). At last, we have (U,R) left.

(iii) (U, R) is PNE since it's the only node survived ISD.

(b)

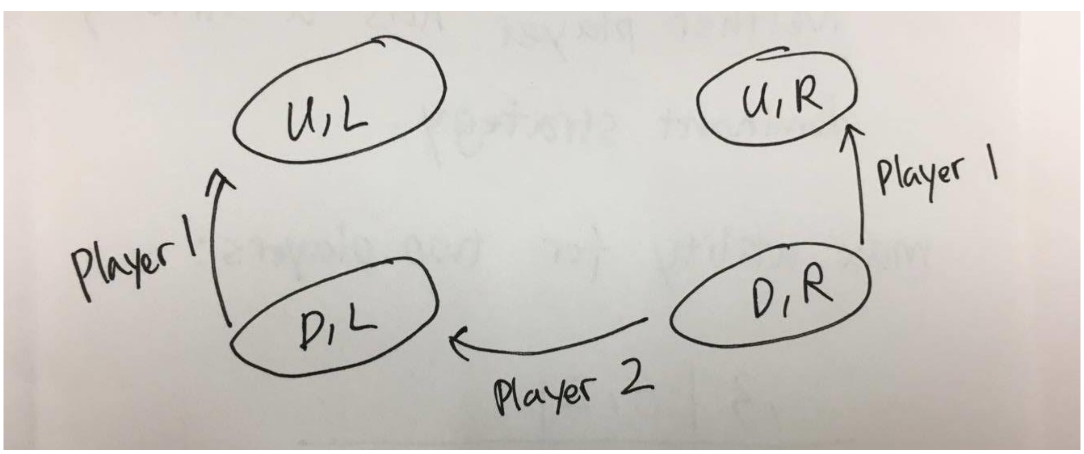


(i) There is no strictly dominate strategy because all nodes form a cycle in the graph;

(ii) All action profiles survive iterative removal as each of them is pointed to;

(iii) There is no PNE because there is no sink in the graph.

(c)



(i) For player 1, U strictly dominates D because he deviates to U no matter which (D,*) he is;

(ii) (U,L) and (U,R) survive iterative removal, while (D,L) and (D,R) are eliminated since D is dominated by U;

(ii) (U,L) and (U,R) are PNE because they are sinks in the graph.

Q2

(a) Neither player has a strictly dominant strategy. We need to change at least 2 entries to have a strictly dominant strategy.

max utility for two players:

,3	6,2	
,3		7,3
5,	,2	

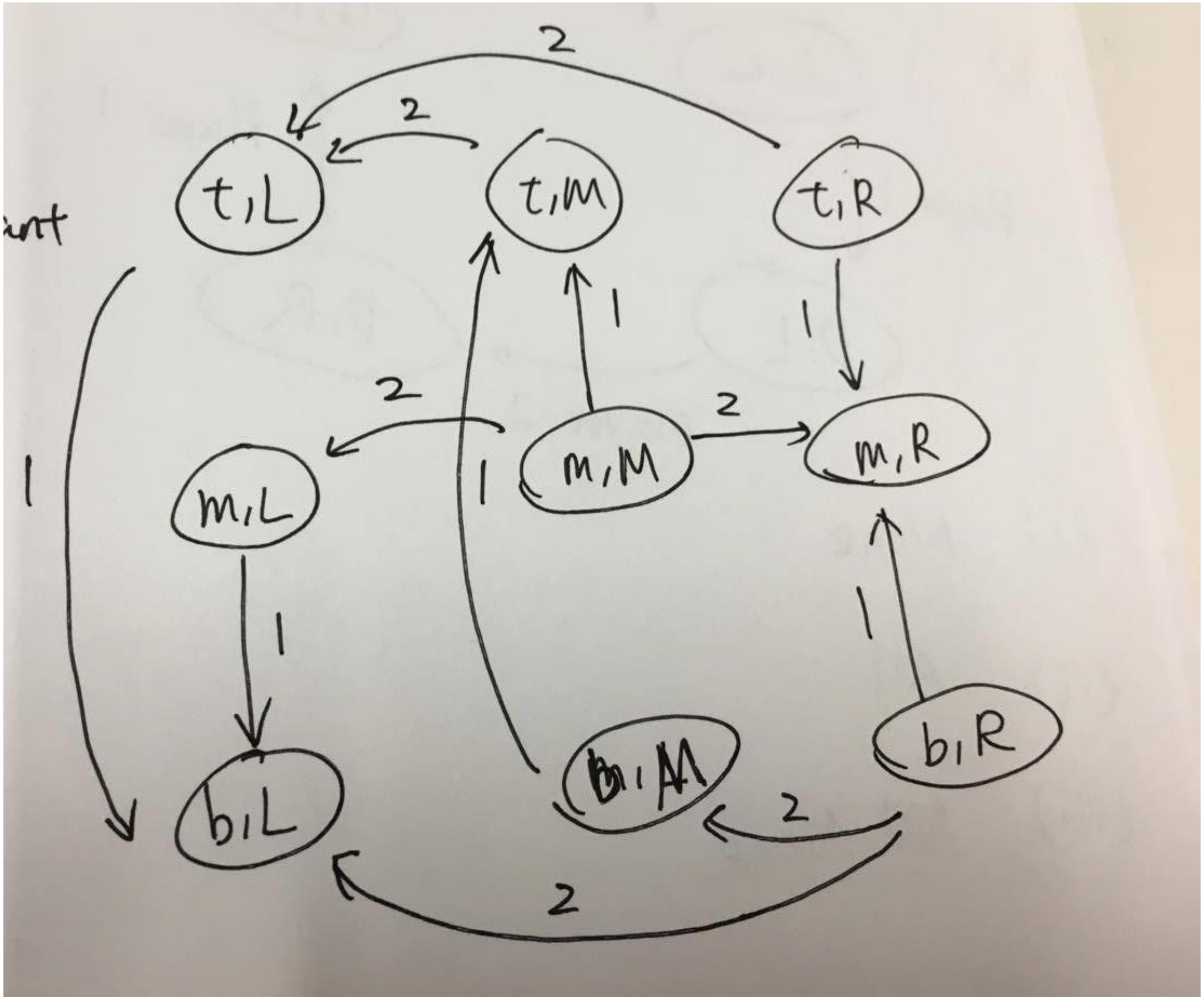
Need to change at least 2 entries to have a strictly dominant strategy.

e.g. (m, L) → (2,4)
(b, L) → (5,3)

then L will be a strictly dominant strategy

(b) Player 1 deviates to ‘b’ since (5,2) is better than (2,3); Player 2 won’t deviate since utility 3 is the best he can get given first player plays ‘m’.

(c)



As the BRD graph shown above, we find the sinks in the graph are (b,L) and (m,R), which are the PNE by theorem. Follow arrows on the graph, we find BRD converges to PNEs.

Q3

(a) By definition of dominant strategy, player 1 has no other actions that improves his utility if he plays $s1$. Likewise, when player 2 plays the best response to $s1$, he has no other actions that improves his utility. By definition of PNE, no player can increase their own utility by unilaterally deviating and thus we say $(s1, s2)$ achieves PNE.

(b) No. For example, there could be $s2' \neq s2 \in BR(s1)$. Then $(s1, s2')$ is also a PNE and a different one from $(s1, s2)$.

(c) Yes. For example, there could be $s1' \neq s1$ where $s1'$ is also a dominant strategy. Then $(s1', BR(s1')\ )$ is also a PNE.

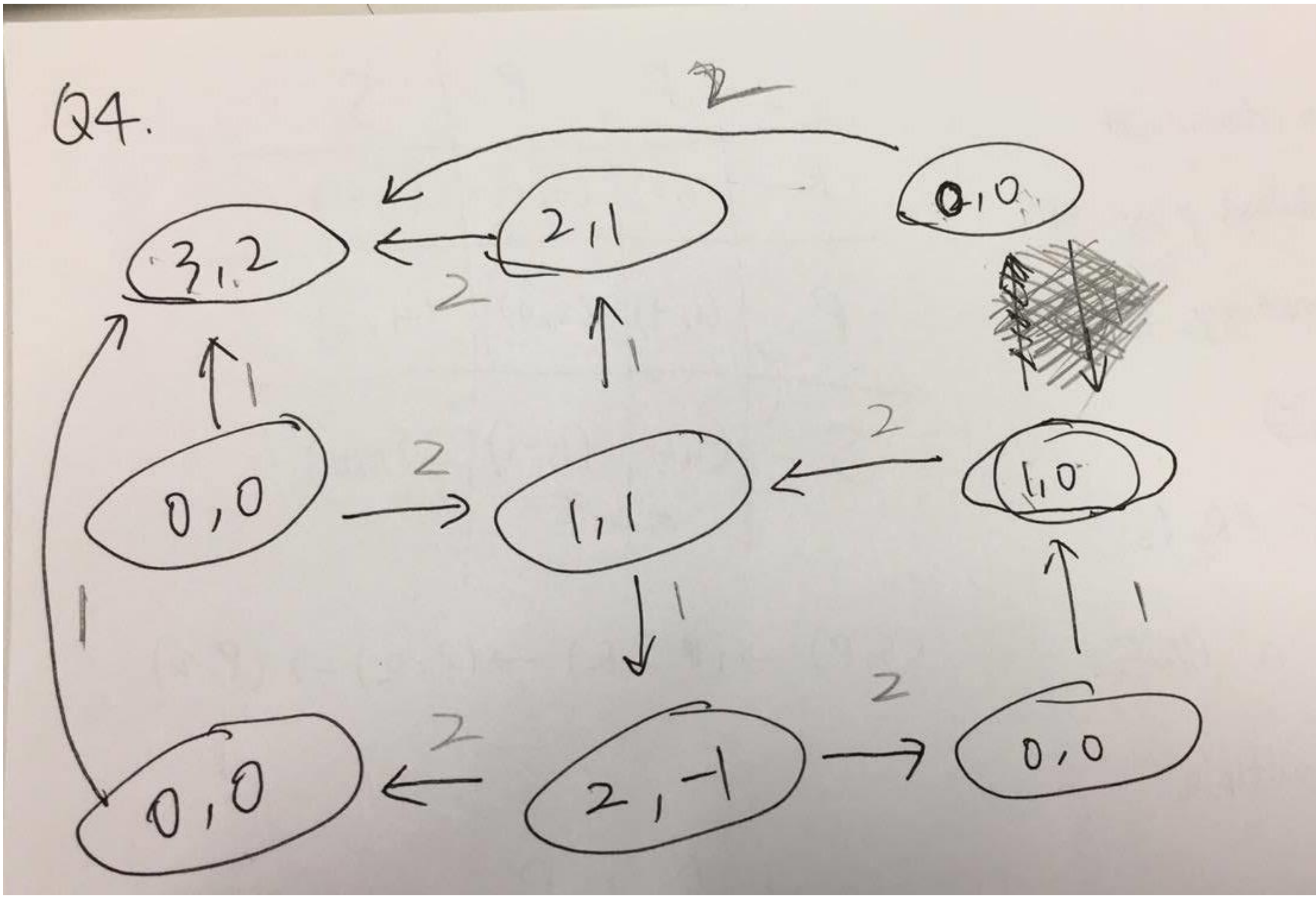
(d) strictly dominant

(a) Same proof: By definition of dominant strategy, player 1 has no other actions that improves his utility if he plays $s1$. Likewise, when player 2 plays the best response to $s1$, he has no other actions that improves his utility. By definition of PNE, no player can increase their own utility by unilaterally deviating and thus we say $(s1, s2)$ achieves PNE.

(b) No. . For example, there could be $s2' \neq s2 \in BR(s1)$. Then $(s1, s2')$ is also a PNE and a different one from $(s1, s2)$.

(c) No. If player don't play $s1$ and we are given $s1$ is a strictly dominant strategy, then player 1 can deviate to $s1$ for a greater utility. Therefore, there is not PNE where player 1 does not play $s1$.

Q4



We can construct a Rock, Paper, Scissor game represented by the graph above. Here is the action profile utility map:

Player 1, Player 2	R, *	P, *	S,*
R, *	3,2	2,1	0,0
P, *	0,0	1,1	1,0
S, *	0,0	2,-1	0,0

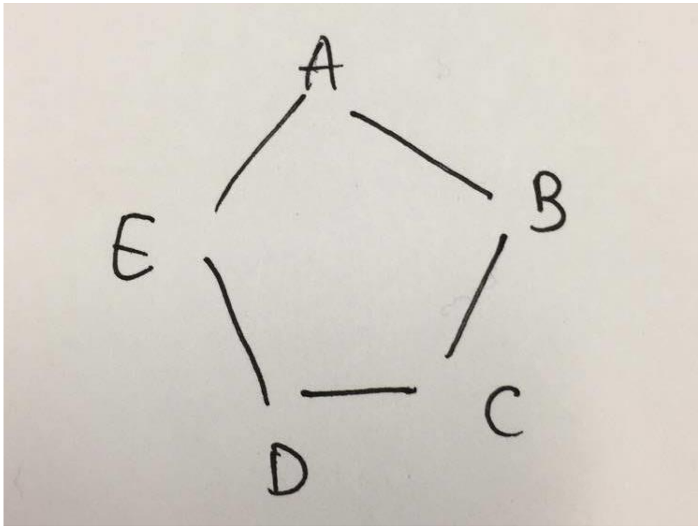
(R,R) is the only sink in the graph, so it is the unique PNE.

There is a circle in the lower-right corner of the graph, therefore BRD does not converge.

Part 2 Graph Theory

Q5

(a)



A is pivotal for (B,E);

B is pivotal for (A,C);

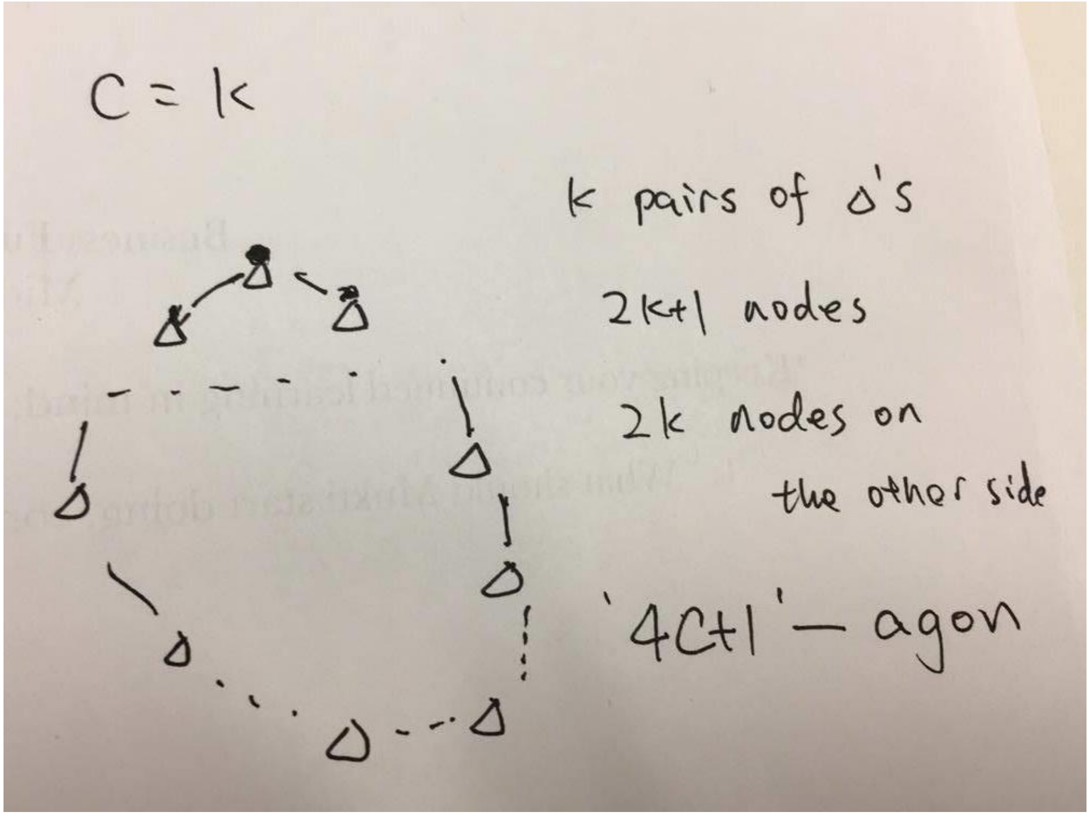
C is pivotal for (B,D);

D is pivotal for (C,E);

E is pivotal for (A,D);

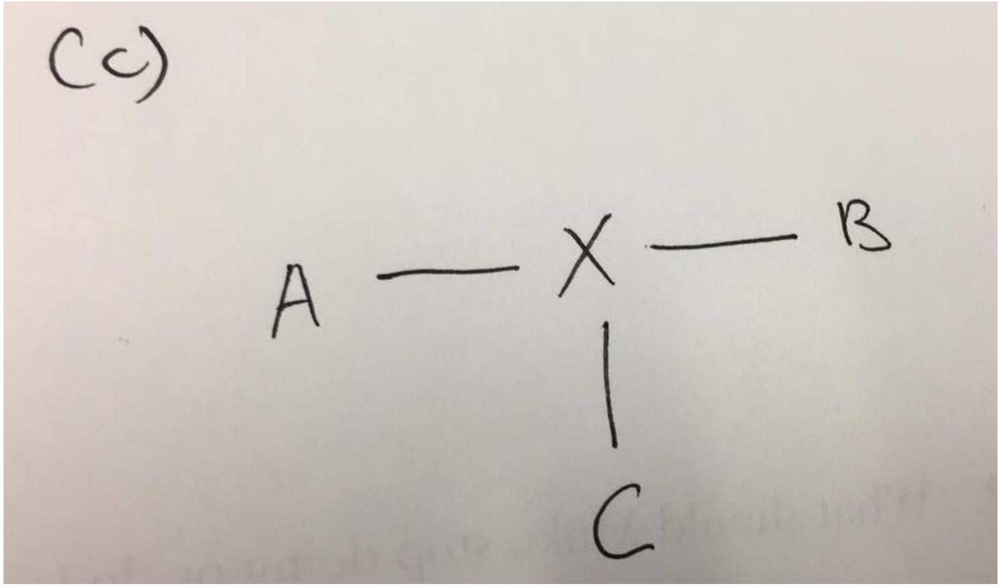
Therefore, the graph above is the graph wanted.

(b)



First, we construct the top node. Then we construct c nodes on each side of the top node, so that the top node is pivotal for at least c different pair of nodes. To ensure that top node lies in every shortest path of those c different pair of nodes, we need to add nodes on the other side. Adding 2c more nodes should suffice. Then we have a '4c+1'-agon where each node only connects with adjacent two nodes and meets the acquirement.

(c)



X is pivotal for (A, B);

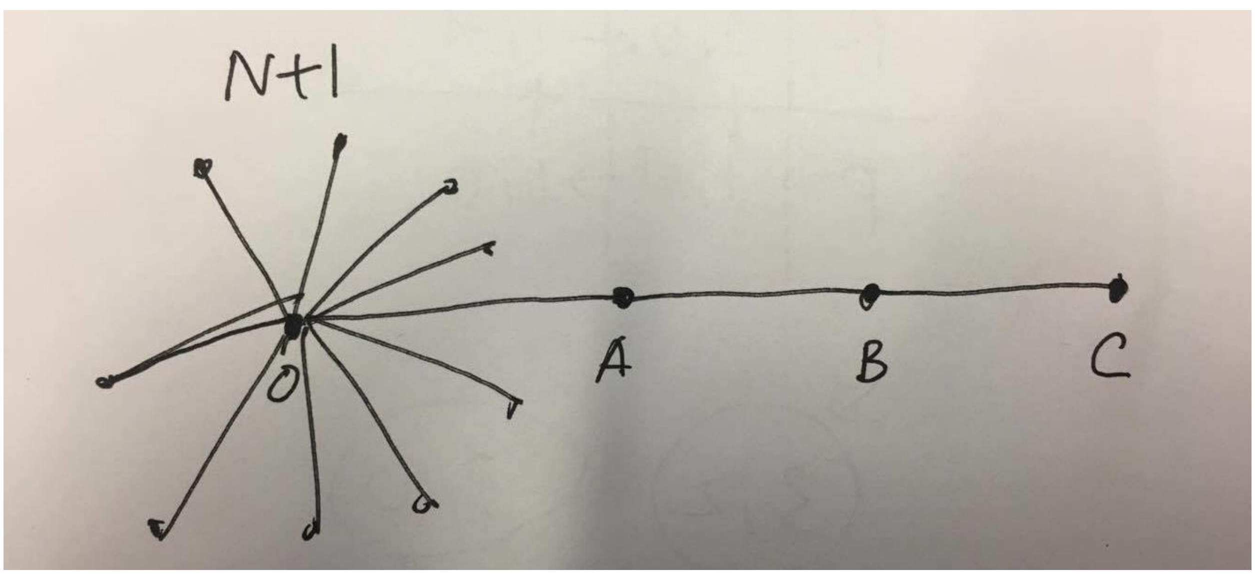
X is pivotal for (A, C);

X is pivotal for (B, C);

Therefore, X is pivotal for every pair of nodes.

Q6

(a)



In the above graph, diameter = 4. Left side is a complete subgraph with N+1 nodes and a center node we labelled O. Each pair of nodes in the N+1 subgraph is connected.

Avg Dist = Total distance/ total number of pair of nodes

Total distance is computed by the chart below

Node	Number of edges	Total Distance for a node
i (in N)	N+3	N-1+10
O	N+3	N+6
A	N+3	2N+4
B	N+3	3N+4
C	N+3	4N+6
Total	(N+4)(N+3)/2	(N^2+19N+20)/2

So we need $4 > 3 * \frac{N^2+19N+20}{N^2+7N+12}$

Plug in $\mathrm{N} = 1000$, we find $4 > 3 * 1.011$.

(b)

Similar to part a, we construct a complete graph with N nodes and stick out c nodes.

Then we can have a diameter c+1

Node	Number of edges	Total Distance for a node, counted once
i (in N-1)	N+c-1	(N-1)+2+…+c+1
O	N+c-1	1+2+…+c
A_1	N+c-1	1+2+..+c-1
A_2	N+c-1	1+2+…+c-2
…	…	…
A_{C-1}	N+c-1	1
A_C	N+c-1	0
Total	(c+N)(c+N-1)/2	N(N-1)/2+(N-1)c(c+3)/2+c(c+1)(c+2)/6

By Tetrahedral number(https://en.wikipedia.org/wiki/Tetrahedral_number), $(1 + \cdots + \mathrm{c}) + (1 + \cdots + \mathrm{c} - 1) + (1 + \cdots + \mathrm{c} - 2) + \cdots + 1 = \frac{c(c+1)(c+2)}{6}$

$$\mathrm{c} + 1 > \mathrm{c}\frac{(\mathrm{N}-1)\mathrm{N}/2 + (\mathrm{N}-1)\mathrm{c}(\mathrm{c}+3)/2 + \mathrm{c}(\mathrm{c}+1)(\mathrm{c}+2)/6}{(\mathrm{c}+\mathrm{N})(\mathrm{c}+\mathrm{N}-1)/2}$$

Solved by online program (https://www.symbolab.com/solver/solve-for-equation-calculator/solve%20for%20n%2C%20%5Cleft(n%2Bc%5Cright)%5Cleft(n%2Bc-1%5Cright)%5Cleft(c%2B1%5Cright)%3Dc%5Cleft(%5Cfrac%7Bc%5Cleft(c%2B1%5Cright)%5Cleft(c%2B2%5Cright)%7D%7B3%7D%2Bn%5Cleft(n-1%5Cright)%2Bc%5Cleft(c%2B3%5Cright)%5Cleft(n-1%5Cright)%5Cright))

$$n = \frac{c^3 + c^2 - 2c + 1 + \sqrt{c^6 + 2c^5 - 3c^4 - 6c^3 - 6c^2 - \frac{4\left(-c^4 - 2c^2\right)}{3} + 1}}{2},$$

When N is significantly large (greater than the critical value given above), the inequality will be easily achieved.

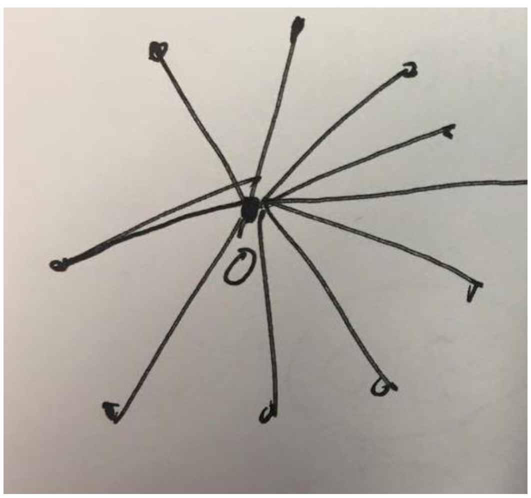
Q7

(a) If there are n nodes in the graph, there should be at least n-1 edges such that the graph is connected. A graph of n nodes on a line is an easy example.

(b) n(n-1)/2. Since any two nodes have a shortest path of 1, any two nodes must be connected. Since there are n nodes in total, then at least we need [n choose 2] = n(n-1)/2 edges. Such graph satisfy the requirement so n(n-1)/2 is the minimum.

(c) Construct the graph such that there is a central node, and (n-1) outer nodes which only connects with the central node. Then number of edges we need is (n-1) and we have a graph such that any two nodes have a shortest path length of at most 2.

To satisfy the requirement, the graph must be connected. According to Q7a, the graph should have at least n-1 edges. Since we can achieve n-1 edges, the minimum is guaranteed.



hw1

September 13, 2017

1 Part 3 Coding: Shortest Path

2 Q8

```
In [1]: import numpy as np
        import random
        import matplotlib.pyplot as plt
        import networkx as nx
```

2.1 Q8a

A procedure that produces a graph (represented by a matrix) of N nodes, each pair connected with probability p input: number of nodes N, probability p output: a graph represented by a matrix, start node, end node

```
In [2]: def generate_graph(N, p):
            graph = (np.random.rand(N, N) < p).astype(int)
            # make the graph undirected/ matrix symmetric
            i_lower = np.tril_indices(N, -1)
            graph[i_lower] = graph.T[i_lower]
            np.fill_diagonal(graph, 0)
            return graph
```

```
In [3]: graph = generate_graph(10, 0.5)
        print (graph)

        # Print AdjacencyList representation of graph
        def converGraph(graph, N):
            result = [[] for i in range(N)]
            for i in range(N):
                for j in range(i, N):
                    if graph[i,j] == 1:
                        result[i].append(j)
                        result[j].append(i)
            return result

        print (converGraph(graph, 10))
```

```
[[0 1 0 1 1 1 0 0 1 1]
 [1 0 1 1 0 1 1 0 0 1]
 [0 1 0 1 1 0 0 0 1 0]
 [1 1 1 0 0 1 0 0 1 0]
 [1 0 1 0 0 1 1 1 1 0]
 [1 1 0 1 1 0 0 1 0 1]
 [0 1 0 0 1 0 0 1 0 1]
 [0 0 0 0 1 1 1 0 1 1]
 [1 0 1 1 1 0 0 1 0 1]
 [1 1 0 0 0 1 1 1 1 0]]
[[1, 3, 4, 5, 8, 9], [0, 2, 3, 5, 6, 9], [1, 3, 4, 8], [0, 1, 2, 5, 8], [0, 2, 5, 6, 7, 8], [0,
```

2.2 Q8b

Generalized Shortest Path Algorithm: BFS (Breadth First Search) Input: a graph represented by a matrix, start node, end node Output: list of nodes leading from start node to end node

```
In [4]: def bfs(graph, start, end):
            # maintain a queue of paths
            queue = []
            explored = []
            # push the first path into the queue
            queue.append([start])
            while queue:
                # get the first path from the queue
                path = queue.pop(0)
                node = path[-1]
                if node not in explored:
                    explored.append(node)
                    if node == end:
                        return path
                    for adjacent in graph[node]:
                        new_path = list(path)
                        new_path.append(adjacent)
                        queue.append(new_path)
```

2.3 Q8c

Generate a graph w/ p = 0.1; Pick any two nodes and compute their shortest dist 10000 times; 100 sample output is printed; Average distance is 1.9

```
In [154]: N = 1000
          p = 0.1
          maxIter = 10000
          total = 0;
          pCount = 100;
          graph = generate_graph(N, p)
```

```
        myGraph = converGraph(graph, N)
        numDC = 0;
        for i in range(maxIter):
            start,end = random.sample(range(N), 2)
            path = bfs(myGraph, start, end)
            if (path == "infinity"):
                dist = sys.maxint
                numDC = numDC + 1
            else:
                dist = len(path)-1
                total = total + dist
            if pCount > 0:
                print ("(node A " + repr(start) + ", node B "+ repr(end) + "): path length " +
                pCount = pCount - 1

        print ("Average distance is " + repr(float(total/(maxIter-numDC))))
```

```
(node A 988, node B 882): path length 2
(node A 753, node B 30): path length 2
(node A 792, node B 288): path length 2
(node A 990, node B 473): path length 2
(node A 78, node B 46): path length 2
(node A 401, node B 500): path length 1
(node A 978, node B 210): path length 2
(node A 379, node B 780): path length 2
(node A 391, node B 267): path length 2
(node A 30, node B 919): path length 2
(node A 673, node B 342): path length 2
(node A 516, node B 162): path length 2
(node A 336, node B 163): path length 2
(node A 803, node B 59): path length 2
(node A 436, node B 991): path length 2
(node A 565, node B 784): path length 2
(node A 641, node B 216): path length 2
(node A 621, node B 708): path length 2
(node A 450, node B 615): path length 2
(node A 121, node B 970): path length 2
(node A 416, node B 661): path length 2
(node A 114, node B 197): path length 2
(node A 137, node B 200): path length 2
(node A 452, node B 319): path length 2
(node A 777, node B 835): path length 2
(node A 748, node B 879): path length 2
(node A 408, node B 786): path length 2
(node A 196, node B 558): path length 1
(node A 80, node B 821): path length 2
(node A 651, node B 573): path length 1
(node A 625, node B 108): path length 2
```

```
(node A 59, node B 775): path length 2
(node A 190, node B 186): path length 2
(node A 334, node B 520): path length 1
(node A 520, node B 387): path length 1
(node A 435, node B 6): path length 2
(node A 425, node B 718): path length 2
(node A 36, node B 816): path length 2
(node A 986, node B 553): path length 2
(node A 360, node B 842): path length 2
(node A 952, node B 25): path length 2
(node A 150, node B 488): path length 2
(node A 701, node B 841): path length 2
(node A 106, node B 28): path length 2
(node A 991, node B 725): path length 2
(node A 521, node B 285): path length 2
(node A 790, node B 932): path length 2
(node A 683, node B 791): path length 2
(node A 79, node B 247): path length 2
(node A 719, node B 42): path length 2
(node A 279, node B 103): path length 1
(node A 194, node B 86): path length 1
(node A 620, node B 449): path length 2
(node A 668, node B 993): path length 2
(node A 586, node B 35): path length 2
(node A 645, node B 849): path length 2
(node A 653, node B 694): path length 2
(node A 359, node B 795): path length 2
(node A 883, node B 990): path length 2
(node A 256, node B 968): path length 2
(node A 215, node B 934): path length 2
(node A 281, node B 478): path length 2
(node A 160, node B 643): path length 2
(node A 983, node B 135): path length 2
(node A 211, node B 944): path length 2
(node A 189, node B 884): path length 2
(node A 691, node B 295): path length 2
(node A 883, node B 101): path length 2
(node A 27, node B 198): path length 2
(node A 219, node B 381): path length 1
(node A 606, node B 479): path length 2
(node A 418, node B 104): path length 2
(node A 834, node B 371): path length 2
(node A 82, node B 434): path length 2
(node A 450, node B 892): path length 2
(node A 213, node B 113): path length 2
(node A 34, node B 22): path length 2
(node A 645, node B 763): path length 2
(node A 175, node B 357): path length 2
```

```
(node A 438, node B 797): path length 2
(node A 82, node B 928): path length 2
(node A 396, node B 338): path length 2
(node A 611, node B 283): path length 2
(node A 521, node B 688): path length 2
(node A 818, node B 600): path length 2
(node A 215, node B 603): path length 2
(node A 872, node B 212): path length 2
(node A 911, node B 527): path length 2
(node A 370, node B 238): path length 2
(node A 980, node B 112): path length 2
(node A 129, node B 787): path length 2
(node A 100, node B 448): path length 2
(node A 965, node B 827): path length 2
(node A 434, node B 209): path length 2
(node A 882, node B 899): path length 2
(node A 160, node B 23): path length 2
(node A 54, node B 92): path length 2
(node A 62, node B 838): path length 1
(node A 995, node B 2): path length 2
(node A 450, node B 830): path length 2
Average distance is 1.9027
```

2.4 Q8d

Run the shortest-path algorithm on data sets constructed with many values of p Numerical data: [3.244, 2.637, 2.388, 2.14, 2.031, 1.901, 1.852, 1.801, 1.752, 1.689, 1.634, 1.585, 1.544, 1.502] for p values: [0.01, 0.02, 0.03, 0.04, 0.05, 0.1, 0.15000000000000002, 0.2, 0.25, 0.3, 0.35000000000000003, 0.4, 0.45, 0.5] Relation plotted below

```
In [5]: def avgDist(N, p, maxIter):
            total = 0;
            graph = generate_graph(N, p)
            myGraph = converGraph(graph, N)
            numDC = 0;
            for i in range(maxIter):
                start,end = random.sample(range(N), 2)
                path = bfs(myGraph, start, end)
                if (path == "infinity"):
                    numDC = numDC + 1
                else:
                    dist = len(path) - 1
                total = total + dist
            return float(total/(maxIter-numDC))
```

```
In [168]: p = [0.01, 0.02, 0.03, 0.04]
          for i in range(10):
```

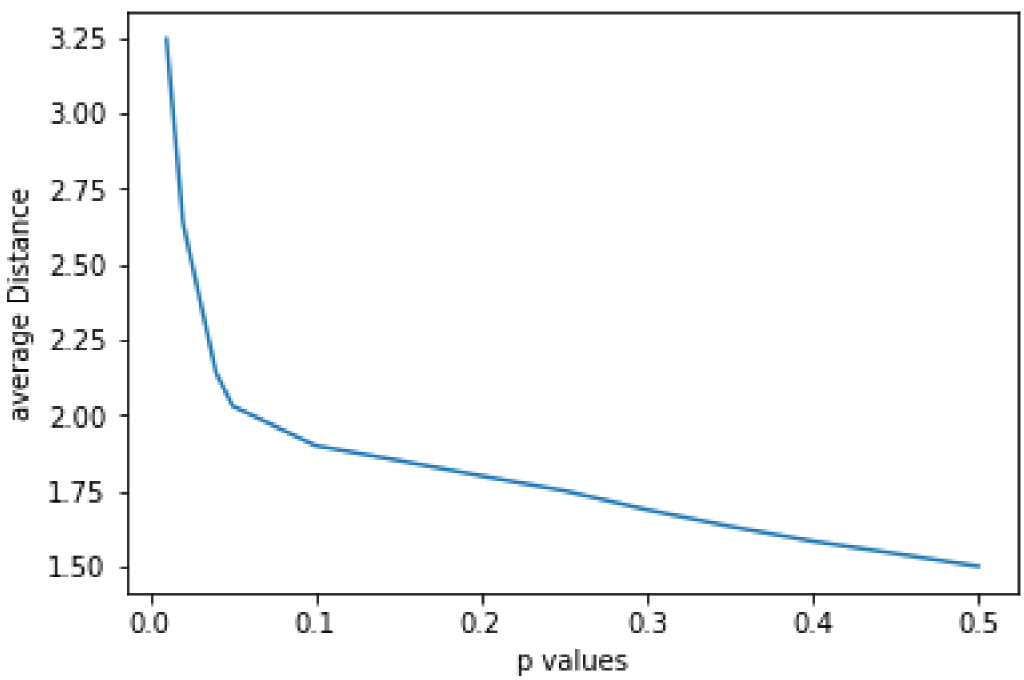
```
        p.append(0.05+0.05*i)
    distList = []
    for pvalue in p:
        distList.append(avgDist(1000, pvalue, 1000))
    print (distList)
```

```
[3.244, 2.637, 2.388, 2.14, 2.031, 1.901, 1.852, 1.801, 1.752, 1.689, 1.634, 1.585, 1.544, 1.502
```

```
In [169]: print (p)
```

```
[0.01, 0.02, 0.03, 0.04, 0.05, 0.1, 0.15000000000000002, 0.2, 0.25, 0.3, 0.35000000000000003, 0.
```

```
In [170]: plt.plot(p, distList)
          plt.xlabel("p values")
          plt.ylabel("average Distance")
          plt.show()
```



2.5 Q8e

As p increases, the average distance decreases. Intuitively, the more likely two nodes are connected, the shorter their shortest distance becomes. As p gets closer to 1, the shortest distance should converge to 1 as well because two nodes are highly like to be connected directly. The asymptotic line is "avg Dist = 1".

3 Q9

Load data and convert graph into adjacency list representation

```
In [136]: # Q9
          # Load fb data
          fb =  np.loadtxt(fname = 'facebook_combined.txt', delimiter = ' ', dtype = 'int')
```

```
In [141]: # Convert fb data to graph
          N = 4039
          fb_graph = np.zeros((N, N))
          for i in range(fb.shape[0]):
              fb_graph[fb[i,0], fb[i,1]] = 1
              fb_graph[fb[i,1], fb[i,0]] = 1
          print (fb_graph)
```

```
[[ 0.  1.  1. ...,  0.  0.  0.]
 [ 1.  0.  0. ...,  0.  0.  0.]
 [ 1.  0.  0. ...,  0.  0.  0.]
 ...,
 [ 0.  0.  0. ...,  0.  0.  0.]
 [ 0.  0.  0. ...,  0.  0.  0.]
 [ 0.  0.  0. ...,  0.  0.  0.]]
```

```
In [147]: myfb = converGraph(fb_graph, N)
          print (bfs(myfb, 0, 1))
```

```
[0, 1]
```

3.1 Q9a

Below is the code for simulating Q8c on facebook graph; 100 sample results are printed; Average distance is 3.64

```
In [157]: # Q9a
          N = 4039
          maxIter = 100
          total = 0;
          pCount = 100;
          numDC = 0;
          myfb = converGraph(fb_graph, N)
          for i in range(maxIter):
              start,end = random.sample(range(N), 2)
              path = bfs(myfb, start, end)
              if (path == "infinity"):
                  dist = sys.maxint
                  numDC = numDC + 1
```

```
            else:
                dist = len(path)-1
                total = total + dist
            if pCount > 0:
                print ("(node A " + repr(start) + ", node B "+ repr(end) + "): path length " +
                pCount = pCount - 1

        print ("Average distance is " + repr(float(total/(maxIter-numDC))))
```

```
1
(node A 517, node B 873): path length 5
(node A 2208, node B 940): path length 4
(node A 1599, node B 2273): path length 3
(node A 701, node B 1048): path length 6
(node A 2076, node B 1255): path length 4
(node A 697, node B 729): path length 2
(node A 261, node B 1536): path length 3
(node A 1068, node B 3615): path length 4
(node A 3639, node B 3203): path length 5
(node A 2172, node B 939): path length 4
(node A 1980, node B 1638): path length 4
(node A 2767, node B 2782): path length 2
(node A 1960, node B 293): path length 4
(node A 1557, node B 552): path length 3
(node A 3793, node B 3421): path length 5
(node A 443, node B 2668): path length 4
(node A 1618, node B 1336): path length 2
(node A 3465, node B 91): path length 5
(node A 2460, node B 3469): path length 5
(node A 1024, node B 3600): path length 4
(node A 2763, node B 1964): path length 4
(node A 2431, node B 544): path length 3
(node A 2193, node B 3468): path length 5
(node A 3143, node B 2125): path length 4
(node A 1514, node B 2877): path length 3
(node A 3964, node B 3821): path length 2
(node A 1015, node B 1411): path length 2
(node A 1950, node B 1231): path length 4
(node A 3903, node B 1678): path length 4
(node A 1893, node B 2812): path length 3
(node A 3727, node B 1139): path length 4
(node A 136, node B 1869): path length 3
(node A 2333, node B 207): path length 4
(node A 4037, node B 2533): path length 5
(node A 3251, node B 2236): path length 4
(node A 2164, node B 1088): path length 4
(node A 4033, node B 234): path length 6
(node A 1686, node B 2982): path length 3
```

```
(node A 3426, node B 1361): path length 3
(node A 1290, node B 1467): path length 2
(node A 2480, node B 1301): path length 4
(node A 1507, node B 816): path length 6
(node A 871, node B 134): path length 6
(node A 2123, node B 545): path length 3
(node A 3135, node B 1046): path length 3
(node A 1023, node B 3675): path length 4
(node A 2464, node B 284): path length 4
(node A 1389, node B 3184): path length 3
(node A 3622, node B 3553): path length 2
(node A 2249, node B 401): path length 4
(node A 2312, node B 2032): path length 2
(node A 3636, node B 3398): path length 5
(node A 1324, node B 278): path length 3
(node A 3501, node B 3250): path length 4
(node A 3580, node B 3410): path length 5
(node A 245, node B 164): path length 2
(node A 1524, node B 113): path length 3
(node A 3536, node B 3013): path length 5
(node A 3294, node B 3281): path length 2
(node A 1702, node B 719): path length 5
(node A 3309, node B 1220): path length 3
(node A 977, node B 3121): path length 3
(node A 2378, node B 3993): path length 5
(node A 1450, node B 3797): path length 3
(node A 2559, node B 2304): path length 2
(node A 4001, node B 1919): path length 5
(node A 3937, node B 3565): path length 2
(node A 3551, node B 1390): path length 4
(node A 3866, node B 2470): path length 5
(node A 1271, node B 3012): path length 3
(node A 1593, node B 1831): path length 2
(node A 3391, node B 2399): path length 4
(node A 1960, node B 2322): path length 1
(node A 1677, node B 636): path length 3
(node A 1858, node B 1753): path length 2
(node A 107, node B 2538): path length 3
(node A 1070, node B 604): path length 2
(node A 2703, node B 520): path length 4
(node A 2281, node B 2312): path length 2
(node A 3416, node B 3912): path length 5
(node A 225, node B 1697): path length 3
(node A 3104, node B 3473): path length 5
(node A 264, node B 1807): path length 3
(node A 196, node B 409): path length 4
(node A 792, node B 2036): path length 7
(node A 3828, node B 2148): path length 5
```

```
(node A 426, node B 2277): path length 3
(node A 1048, node B 85): path length 3
(node A 976, node B 130): path length 3
(node A 3699, node B 345): path length 5
(node A 3374, node B 2345): path length 4
(node A 2987, node B 3213): path length 2
(node A 1987, node B 2106): path length 2
(node A 2548, node B 3855): path length 5
(node A 1405, node B 2584): path length 4
(node A 2112, node B 2396): path length 2
(node A 204, node B 2779): path length 4
(node A 63, node B 3666): path length 5
(node A 2813, node B 1362): path length 3
(node A 1986, node B 2688): path length 4
Average distance is 3.64
```

3.2 Q9b

p = number of edges/ total possible edges = 0.0108

```
In [172]: # Q9b
          p = fb.shape[0]/ ((N*(N-1))/2)
          print ("Facebook data has p value: " + repr(p))
```

```
Facebook data has p value: 0.010819963503439287
```

3.3 Q9C

Average shortest path from Facebook data is about 3.6; Average shortest path from constructed data is about 2.6, which is smaller than the actual data. The reason may be: in the constructed data, the graph is uniformly generated with probability p. Each person has the same expected number of friends. All are equal. However, in the real world, the distribution of number of friends is not uniform. Some people tend to have more friends, some less. The distribution should be more like gaussian. In the graph view, some nodes in the actual Facebook data is more dense than others. That's why the average shortest path is larger in the real data set.

```
In [7]: # Q9c
        avgDist(4039, 0.01081996, 1000)
```

```
Out[7]: 2.58
```