Morphology as Algorithm: Deriving Reward Prediction Error from the Physics of Dendritic Geometry

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Abstract

*Brains lack dedicated subtraction circuits for $\delta = R - V$, yet error learning is foundational. We propose **neuronal geometry as RPE hardware code.** Using an active dendrite model, we show RPE is physically enforced by Ohm's law between soma (prediction) and distal dendrites (reality). Axial voltage gradients encode "surprise" and "disappointment" into currents driving Hebbian updates. This analog mechanism, requiring no digital circuits, enables emergent "epiphany" and "risk-aversion." Code: https://github.com/bailan137/neuron .*

1 Introduction

Silicon uses ALUs for subtraction; biological "wetware" RPE ($\delta = R - V$) faces physical constraints. We propose a first-principle approach: **Morphology transforms "subtraction" into "gradient equilibrium."** The neuron acts as a physical comparator: (1) **Soma** integrates history, its membrane potential representing the **Prediction (V)**; (2) **Dendrites** receive environmental inputs, their local depolarization representing the **Reward/Reality (R)**; (3) **Morphology** connects them via **Axial Resistance** (R_a). When $R \neq V$, potential differences generate axial currents representing physical RPE, driving learning.

2 Related Work

Since London [7], views on dendritic computation shifted from perceptrons [4] to morphology.

1. Single Neurons as DNNs: Cortical neurons match 5-8 layer DNNs [2]. Masoli et al. [8] attribute this to morphological complexity. We ground RPE in this structure.

2. Dendritic Logic: Branches are functional units [3]. Beyond XOR [5] and binding [11], we prove morphology performs analog "subtraction."

3. Isolation & Feedback: Subtraction needs isolation and feedback. Compartmentalization provides isolation [1]; bAPs provide feedback [10]. Combining stochastic growth [9], our model uses R_a and bAPs to physically reproduce dual-mechanism coding [6].

3 Methods and Model Construction

We use the NEURON simulation environment to build a multi-compartment model that accurately captures spatiotemporal ion dynamics.

1. Morphology: We adopt a classic "Ball-and-Stick" topology.

- **Soma:** A sphere with both diameter and length of $20\mu m$, serving as the integration hub for the "prediction" signal (V).
- **Dendrite:** A $500\mu m$ long, $2\mu m$ diameter cylinder, divided into 51 segments ('nseg=51') for fine-grained simulation.
- **Core Parameters:** Axial resistance R_a is set to $150\,\Omega \cdot cm$ and membrane capacitance c_m to $1\,\mu F/cm^2$ for all sections.

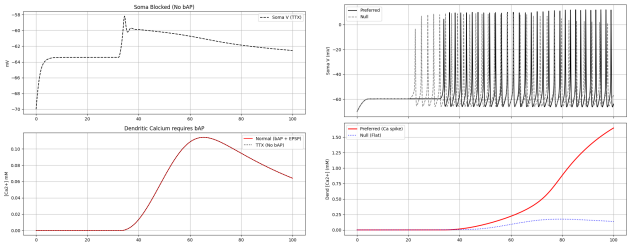
The long dendrite with high axial resistance is key to "Ohmic subtraction," decoupling distal reality (R) from proximal prediction (V) to create meaningful voltage gradients.

2. Ion Channel Gradients: We implement heterogeneous Hodgkin-Huxley (HH) type channel densities.

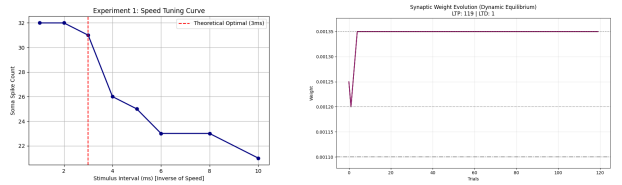
- **Soma:** High density of fast Na^+ ($\bar{g}_{Na,hh} = 0.3\,S/cm^2$) and K^+ ($\bar{g}_{K,hh} = 0.036\,S/cm^2$) channels to ensure reliable action potential (AP) generation.
- **Dendrite:** A moderate Na^+ channel density ($\bar{g}_{Na,hh} = 0.052\,S/cm^2$) supports back-propagating APs (bAPs) but not initiation. High-threshold Ca^{2+} channels ($\bar{g}_{Ca\_HVA} = 0.01\,S/cm^2$) and a calcium pump ('cad') with a decay time of $\tau_{cad} = 50ms$ are located distally ($> 250\mu m$ from soma).

3. Synapses:

- **Excitatory:** NMDA-like synapses ('Exp2Syn') with $\tau_{rise} = 1ms$, $\tau_{decay} = 30ms$, and $E_{rev} = 0mV$. Spatially distributed synapses incorporate intrinsic conduction delays from 0 to $12ms$.
- **Inhibitory (Risk):** A GABAa-like synapse at the proximal dendrite ('dend(0.1)') with $E_{rev} = -80mV$ produces powerful shunting inhibition.



(a) Whole-cell Synergy **(b)** Direction Selectivity



(c) Speed Tuning (3ms opt) **(d)** Self-organized Evol.

Figure 1: Morphology as a calculator. (a) Coincidence of bAP and EPSP triggers a dendritic calcium plateau. (b) This enables direction selection. (c) The structure resonates with specific input timings. (d) It guides weights to a stable representation.

4 Experiments and Analysis

4.1 Phase 1: Geometric Perception

Dendritic geometry shapes how the neuron filters spatiotemporal patterns.

(1) **Coincidence Detection.** A dendritic calcium plateau emerges only when a somatic bAP meets a distal EPSP (Fig. 1a). Silencing somatic Na^+ channels removes the plateau, confirming its bAP dependence. This interaction directly produces direction selectivity (Fig. 1b).

(2) **Speed Tuning.** Acting as a physical delay line, the dendrite shows a resonance peak when inputs arrive $\sim$3ms apart, revealing a preferred stimulus speed (Fig. 1c).

(3) **Self-Organization.** A calcium-threshold plasticity rule (LTP if $[\mathrm{Ca}^{2+}]_{peak} > 0.107\,\mu M$; LTD otherwise) drives synapses toward a stable pattern under mixed-direction stimulation (Fig. 1d).

4.2 Phase 2: Physical Derivation of RPE

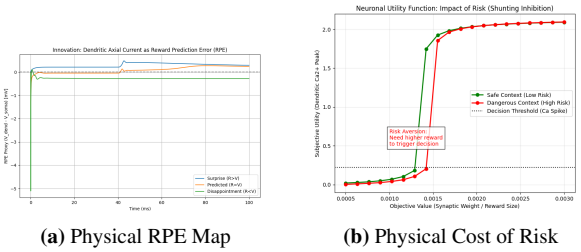
We compute RPE by directly reading out the axial voltage gradient. In the model, the agent's confidence sets a somatic clamp V_{soma}, while feedback (`is_correct`) controls distal synaptic drive.

$$I_{\text{axial}} = \frac{V_{\text{dend}} - V_{\text{soma}}}{R_{\text{axial}}} \quad \propto \quad \text{RPE} \tag{1}$$

Fig. 2a shows a clean mapping between voltage gradients and behavioral outcomes:

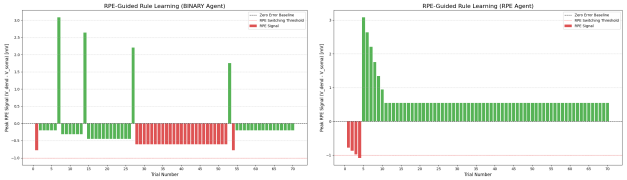
Surprise ($R > V$). Low confidence paired with reward produces a positive gradient ($V_{\text{dend}} > V_{\text{soma}}$), generating inward current that supports LTP.

Disappointment ($R < V$). High confidence without reward reverses the gradient, producing outward current consistent with LTD.In effect, the neuron implements RPE through a simple Ohmic voltage difference—turning subtraction into physics.



(a) Physical RPE Map **(b)** Physical Cost of Risk

Figure 2: Physical mechanisms of learning and choice. (a) A positive voltage gradient (Surprise, blue) drives inward current for LTP; a negative gradient (Disappointment, green) drives outward current for LTD. (b) Proximal shunting inhibition leaks current, increasing the threshold for action, physically encoding risk aversion.



(a) Binary Feedback Agent **(b)** Morphological RPE Agent

Figure 3: The Epiphany Moment. (a) A binary agent oscillates, struggling to find the correct rule. (b) The morphological agent experiences a large, continuous RPE signal, leading to a decisive, one-shot "epiphany" and stable learning.

4.3 Phase 3: Emergent Behavior: Epiphany & Risk Aversion

1. Epiphany vs. Oscillation: We compared two learning agents in a cognitive loop. A control agent using binary "correct/incorrect" feedback oscillates when faced with a difficult rule (Fig. 3a). In contrast, our agent uses the **continuous, signed, analog RPE signal** from the neuron. A massive negative RPE signal (e.g., ¡ -1.0mV), representing an "extreme surprise," physically forces a decisive, one-shot switch in the agent's hypothesis. This emerges as an "Epiphany" moment in the learning curve (Fig. 3b), a behavior impossible with simple digital feedback.

2. The Physics of Risk Aversion: Modeling risk as proximal shunting inhibition (g_{inh}) shifts the neuron's input-output curve to the right (Fig. 2b). Physically, the inhibitory conductance acts as a "leak" in the dendritic cable, shunting excitatory currents away before they reach the soma. Consequently, a much stronger synaptic input (a higher "reward") is required to overcome this leak and reach the decision threshold (e.g., Ca^{2+} spike). *I*Risk aversion is not an abstract calculation but the direct physical energy cost required to overcome shunting inhibition.

5 Conclusion

Our simulations identify neuronal geometry as the hardware for reinforcement learning: (1) Dendrites are spatiotemporal filters, (2) Axial resistance is a physical subtractor, and (3)

Voltage gradients are the error signal. Intelligence arises from structure; geometry downscales high-level "subtraction" to low-level physical "flow," offering a blueprint for novel, physics-based computing architectures.

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A Author Contributions

Yonghui Niu Conceptualization, initial model implementation, manuscript revision and supervision. 50% contribution.

Weicheng Tang Initial manuscript draft, experimental design and execution, code organization and finalization. 50% contribution.