Introduction to Graph Theory

by Richard Trudeau

Ch. 4 Solutions

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1 N_1 is certainly planar, and we proved that it is connected. Prove now that it is polygonal by proving that the statement "every edge of N_1 borders on two different faces" is true.

Solution: This is vacuously true because there are no edges in N_1.

2 Omitted. There are many lengthy discussions about fake induction proofs online.

3 Believe it or not, the graph of Figure 104a is planar. Find its number of faces.

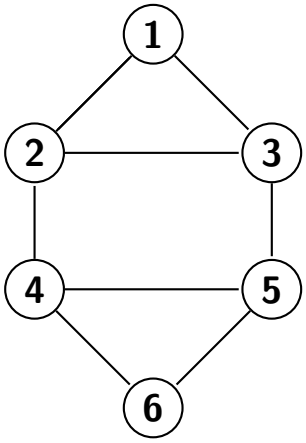
Solution: $v = 9$, $e = 20$, so $v + f - e = 2 \implies f = 13$

4 Imitate the proof of Corollary 12 to construct a proof that the graph in 104b is nonplanar.

Solution: Suppose it is planar. By inspection, the figure is not a supergraph of K_3, so by Theorem 12, we have $e \leq 2v - 4$. But, inspecting the graph, we see that $v = 8$ and $e = 16$ which implies $16 \leq 12$, a contradiction. Therefore, it must not be planar.

5 Find a polygonal graph G having a face bordering the infinite face which, if removed, results in a subgraph H which is not polygonal.

Solution: Removing the face determined by $(2, 3, 5, 4)$ below would leave us with a graph that is not polygonal, since it would be disconnected.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