Introduction to Graph Theory
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Ch. 6 Solutions

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1 Find X (chromatic number) for each of the following graphs: C_v, W_v, UG, Octahedron, dodecahedron, icosahedron.

Solution:
$X_{C_v} = 2$
$X_{W_v} = 3$
$X_{UG} = 2$
$X_{Oct} = 3$
$X_{Dod} = 3$
$X_{Ico} = 4$

2 Find X for each of the graphs in Figure 126.

Solution:

a) 3

b) 4

c) 3

3 Prove the "Two Color Theorem": the set of all graphs with $X = 2$ is equal to the set of all graphs having at least one edge and no odd cyclic subgraphs.

Solution: A solution is in the book.

4 K_3 has $X = 3$, so every supergraph of K_3 has $X \geq 3$. On the other hand, C_5 is a graph with $X = 3$ that is not a supergraph of K_3. Find a graph with $X = 4$ that is not a supergraph of K_3.

Solution: The Grtzsch graph is one such example.

5 If a graph G has chromatic number X and $\overline{G}$ has chromatic number $\overline{X}$, prove that $X\overline{X} \geq v$. Then use the fact that $(1/2)(m+n) \geq \sqrt{mn}$ whenever integers $m, n > 0$ to prove that $X + \overline{X} \geq 2\sqrt{v}$.

Solution: Suppose G has v vertices, and that G and $\overline{G}$ have chromatics numbers X and $\overline{X}$. Given the colorings of G and $\overline{G}$ we can color K_v using the ordered pairs $(X_{v_i}, \overline{X_{v_i}})$. Since $X_{K_v} = v$, we must have $X_{K_v} = v \leq X\overline{X}$.

The next part of the problem follows from $\sqrt{X\overline{X}} \geq \sqrt{v}$ and $(1/2)(X + \overline{X}) \geq \sqrt{X\overline{X}} \geq \sqrt{v}$.

6 Find a graph for which $X\overline{X} = v$ and for which a $X + \overline{X} = 2\sqrt{v}$. Could a single graph satisfy both equations?

Solution:

$K_3 = 3, \overline{K_3} = 1$

$C_4 = 2, \overline{C_4} = 2$

Yes, it is possible, take e.g. our second example: $v = 4$, and $X = \overline{X} = 2$.