Introduction to Graph Theory

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Ch. 8 Solutions

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1 Use Exercise 11 of Chapter 2 to prove that every graph has an even number of odd vertices.

Solution: Exercise 11 says that the sum of degrees of vertices in a graph is equal to $2e$. Since this number is always even, there is no possibility of hanging a graph with an odd number of odd vertices.

2 Find all integers $v \geq 2$ for which

a) K_v has an open euler walk.
Solution: K_2 is the only complete graph with only 2 vertices where $\deg(v)$ is odd.

b) K_v has an closed euler walk for $v \geq 3$.
Solution: K_v where $v \geq 3$ and v is odd.

c) K_v has an open hamilton walk.
Solution: All $v \geq 3$.

d) K_v has an closed hamilton walk.
Solution: All $v \geq 3$.

3 Explain why each drawing in Figure 155 are either bad or good puzzles.

a) **Solution**: Bad (insoluble). There are 4 vertices with odd degree, so no possible euler walks.

b) **Solution**: Bad (boring). Every vertex has even degree, so there is always a closed euler walk.

c) **Solution**: Good. There are two vertices with odd degree, so there are two possible euler walks (starting from the odd vertices).

4 Let C be a graph with $v = 64$, its vertices corresponding to the squares of a chess board. Let two vertices of C be joined by an edge whenever a knight can go from one of the corresponding squares to the other in one move. Does C have an euler walk? (You don't have to draw C to answer.)

Solution: No. For example, if the knight is on $A2$, $A7$, $H2$, $H7$ the vertices have odd degree, so there are more than 2 vertices with odd degree.

5 Prove that C from the previous exercise has a closed hamilton walk. Such a walk is called a "knight's tour" by puzzle enthusiasts.

Solution: Omitted, try drawing one.

6 Figure 157 depicts a system of bridges and land areas. Can you take a walk and cross each bridge exactly once? If so, where do you start and finish? Blow up the bridge from H to I and answer the same two questions.

Solution: Yes, there are exactly two vertices with odd degree: $F = 5$, $I = 3$, which are the vertices we would start and end at.

If we blow up bridge HI, we can still have a walk because there are still exactly two vertices with odd degree: $H = 3$, $F = 5$.