Introduction to Graph Theory
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Ch. 5 Solutions

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1 Draw all connected graphs that are regular of degree 1.

Solution: There's just one connected graph of degree one: two vertices connected to each other.

2 Satisfy yourself that every connected graph which is regular of degree 2 is a cyclic graph. Show by example that deleting the word "connected" results in a false statement.

Solution: An example is a graph that has multiple disconnected cyclic graphs.

3 Verify that there is only one platonic graph with $v = 6$ and $e = 12$ (there are five of them) and checking that only the octahedron is platonic.

Solution: I won't illustrate the graphs, but suffice to say it is probably easiest to check graphs whose complement has $v = 6$ and $e = 12$. In this way it is also easy to check that that there is only one graph whose complement can be regular.

4 Prove: there is no regular graph with $v = 6$ and $e = 10$.

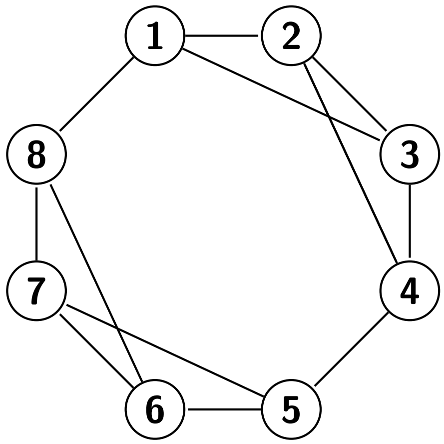
Solution: From Lemma 14, if G is regular then $e = dv/2$, so we have $10 = 3d$ which obviously is not solvable for an integer d. Therefore G is not regular.

5 Prove: if a graph has an odd number of vertices and is regular of degree d then d must be even.

Solution: From Lemma 14, we have $e = dv/2$. If d was odd then $dv/2$ would not be an integer, which is impossible. Therefore d (if it exists... we have done nothing to prove that must be the case) must be even.

6 Find a graph other than the cube that has $v = 8$ and is of degree 3.

Solution: Here is one such graph:



7 Omitted for size.

8 Omitted... I don't want to rob you of the fun of this one :)