CFD in a Pytho-cosm
Assignment 1: Incompressible Navier-Stokes
MECH6480: Computational Fluid Dynamics (S2, 2024)

BAILEY CARTER
S45805608

Task 1

Diffusion term: $\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2}$ and $\frac{\partial^2 \mathrm{T}}{\partial \mathrm{y}^2}$ can be approximated using Finite Volume Methods (FVM):

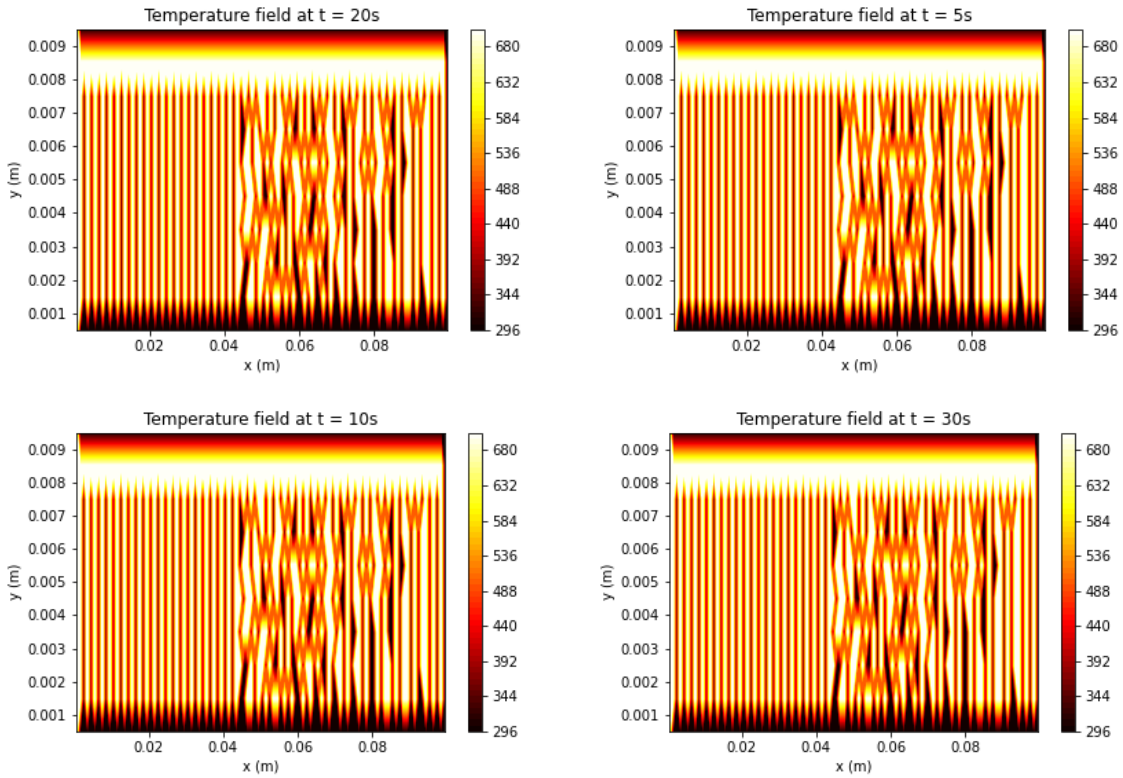
$$\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2} \approx \frac{\mathrm{T_{i+1,j}} - 2\mathrm{T_{i,j}} + \mathrm{T_{i-1,j}}}{dx^2}$$

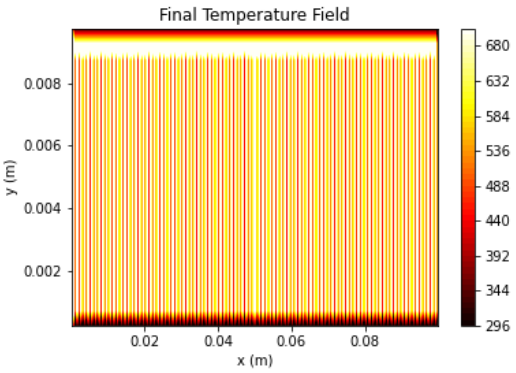
$$\frac{\partial^2 \mathrm{T}}{\partial \mathrm{y}^2} \approx \frac{\mathrm{T_{i,j+1}} - 2\mathrm{T_{i,j}} + \mathrm{T_{i,j-1}}}{dy^2}$$

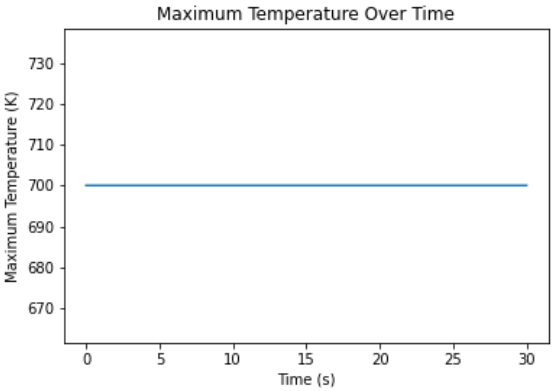
Convection term: For the convective term, a central difference scheme (linear interpolation) works as the flow is Poiseuille flow, which is stable with minimal numerical dissipation. In more complex or turbulent flows, an upwind scheme might be more accurate to handle sharp gradients or boundary layers, but for Poiseuille flow, linear interpolation is sufficient. In 2D:

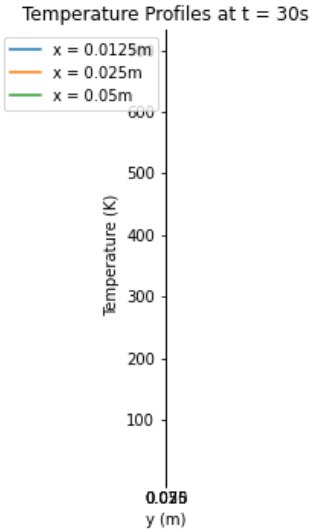
$$\mathrm{Convection}_x \approx \frac{\mathrm{u_{i+1,j}T_{i+1,j}} - \mathrm{u_{i-1,j}T_{i-1,j}}}{2\mathrm{dx}}$$

$$\mathrm{Convection}_y \approx \frac{\mathrm{u_{i,j+1}T_{i,j+1}} - \mathrm{u_{i,j-1}T_{i,j-1}}}{2\mathrm{dy}}$$

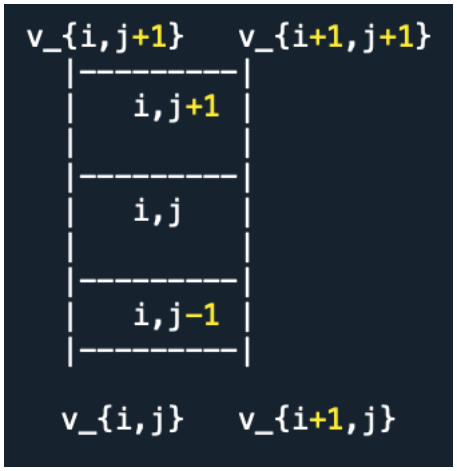



Final Temperature Field
y (m)
x (m)
0.002
0.004
0.006
0.008
0.02
0.04
0.06
0.08
680
632
584
536
488
440
392
344
296


Maximum Temperature Over Time
Maximum Temperature (K)
Time (s)
670
680
690
700
710
720
730
0
5
10
15
20
25
30


Temperature Profiles at t = 30s
x = 0.0125m
x = 0.025m
x = 0.05m
Temperature (K)
y (m)
100
200
300
400
500

Task 2



Velocity Components:

$v_{i,j}$ & $v_{i+1,j}$: Velocity components at the north and south faces of the control volume.

$u_{i,j}$ & $u_{i,j+1}$: Velocity components at the east and west faces of the control volume.

Pressure:

$p_{i,j}$: Pressure at the center of the control volume.

Discretisation of the y-Momentum Equation

The y-momentum equation is:

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}(\frac{\partial}{\partial x}(\mu\frac{\partial v}{\partial x}) + \frac{\partial}{y\partial}(\mu\frac{\partial v}{\partial y}))$$

2.1 Convection Term

Convection in the y-direction:

- **North Face**: The velocity v at i, j+1 and i, j is used.
- **South Face**: The velocity v at i, j and i, j−1 is used.

Approximating $\frac{\partial v}{\partial y}$:

$$\frac{\partial v}{\partial y} \approx \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y}$$

The convection term is:

$$Convection\ term = [v_{i,j} \cdot \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y}]\Delta x$$

Here, $v_{i,j}$ is used to approximate $\frac{\partial v}{\partial y}$

2.2 Diffusion Term

Diffusion in the y-direction:

Approximating the second derivatives:

- **In x-direction**:

$$\frac{\partial^2 v}{\partial x^2} \approx \frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{\Delta x^2}$$

- **In y-direction**:

$$\frac{\partial^2 v}{\partial y^2} \approx \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{\Delta y^2}$$

The diffusion term is:

$$Diffusion\ term = \frac{\mu}{\rho}\left[\frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{\Delta x^2} + \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{\Delta y^2}\right]$$

Task 3

Boundary Conditions Implementation

No-slip Boundary Condition: The velocity components at the walls (north, south, east, and west) are set to make sure there is no slip at the boundaries. Setting the velocities at these boundaries to match the opposite of their neighbouring cells achieves this.

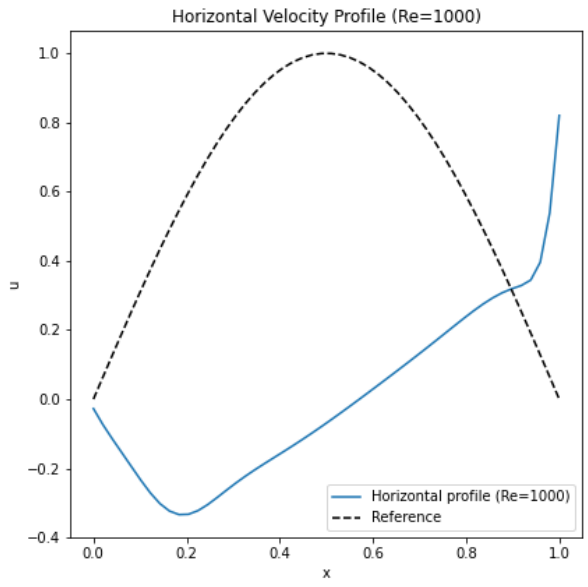
Driven Lid Boundary Condition: The velocity at the top boundary (north) is set to 1.0 m/s. This represents the moving lid, while the velocities at the other boundaries are put to zero.

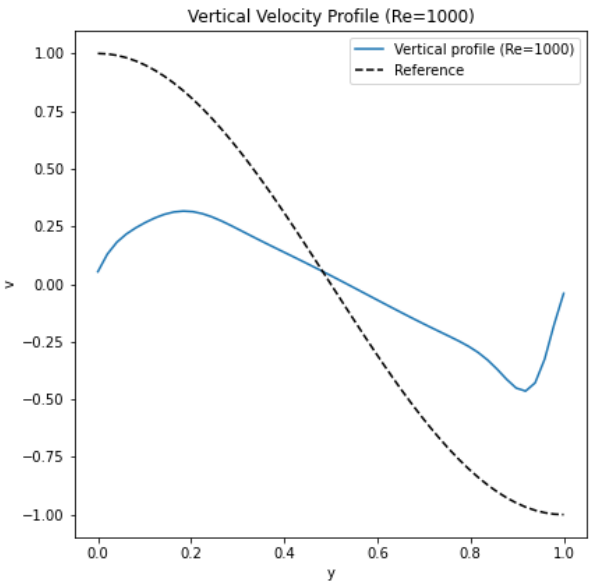
Justification for Discretization Parameters

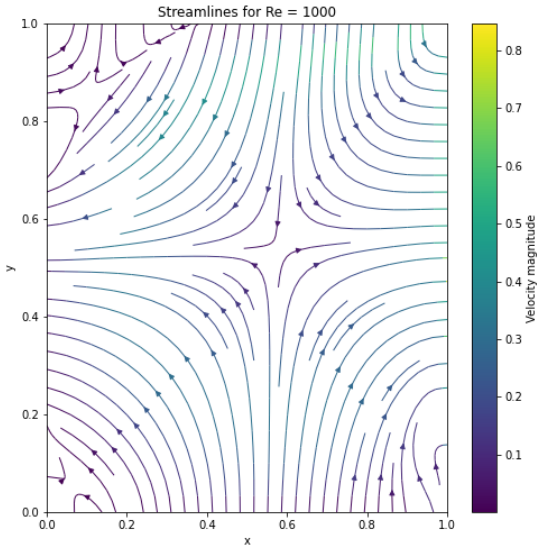
Δx and Δy: The spatial discretisation parameters are determined by dividing the domain length L by the number of grid points N in each direction. This gives $\Delta x=\Delta y=L/N$ and for the given grid (NX=50, NY=50), the result is $\Delta x=\Delta y=0.02$ meters. This discretisation gives accuracy and computational efficiency. A finer grid would increase accuracy however is not necessary as it increases computational cost.

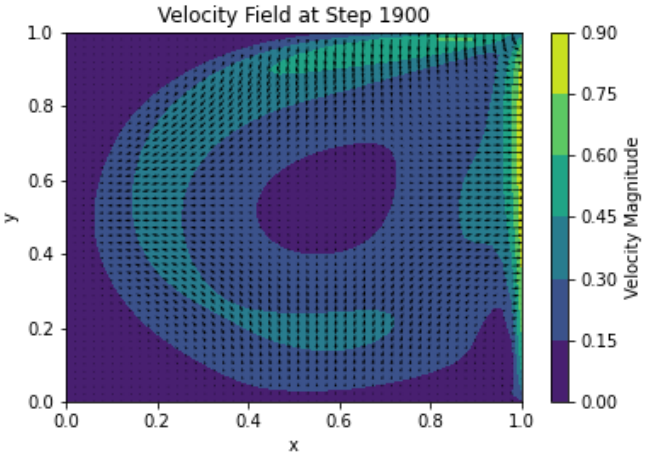
Δt: The time step Δt is chosen as 0.01 seconds. This value is small enough for numerical stability and convergence of the simulation. The time step should be sufficiently small to capture the dynamics of the flow while not encountering high computational cost.

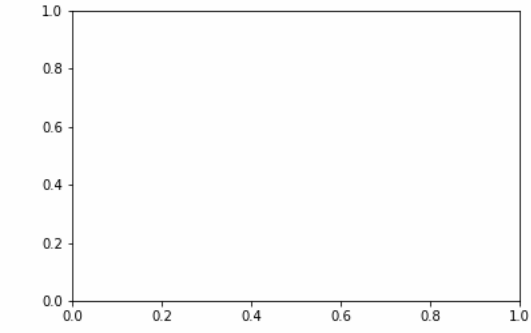
Steady-State Determination: Steady-state is determined by checking whether the max change in the velocity fields, u and v, between successive iterations is below the threshold 1e-4. This chosen threshold makes sure that changes are minimal, and the system has converged to a steady state. The time to reach steady-state changes depending on the Re and grid resolution. Typically it takes several thousand iterations to reach steady-state.

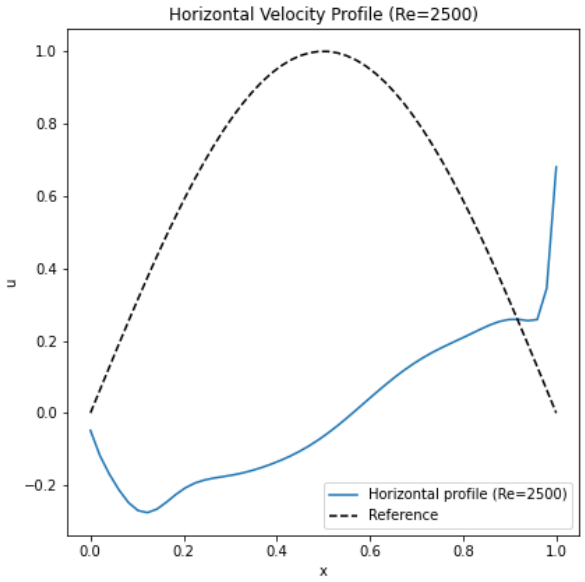

Horizontal Velocity Profile (Re=1000)
u
x
Horizontal profile (Re=1000)
Reference

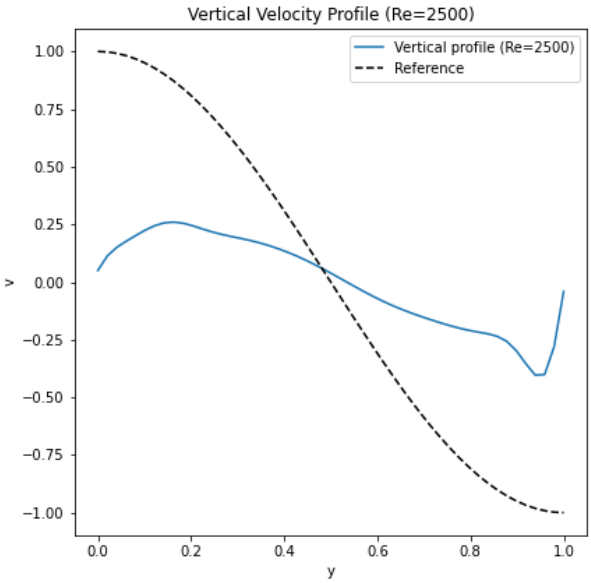

Vertical Velocity Profile (Re=1000)
v
y
Vertical profile (Re=1000)
Reference

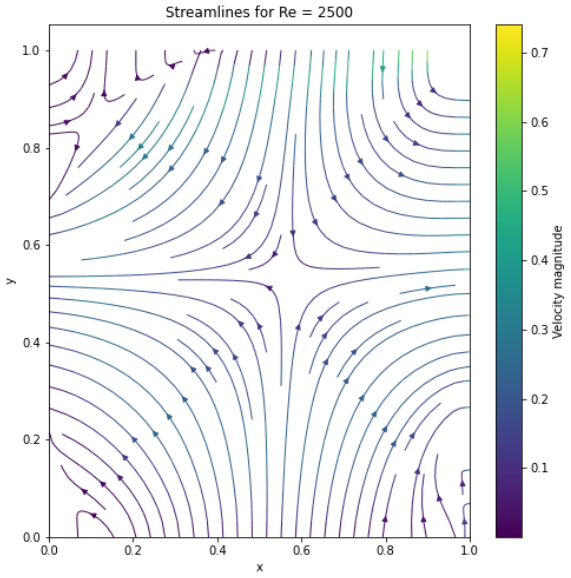

Streamlines for Re = 1000
y
x
Velocity magnitude

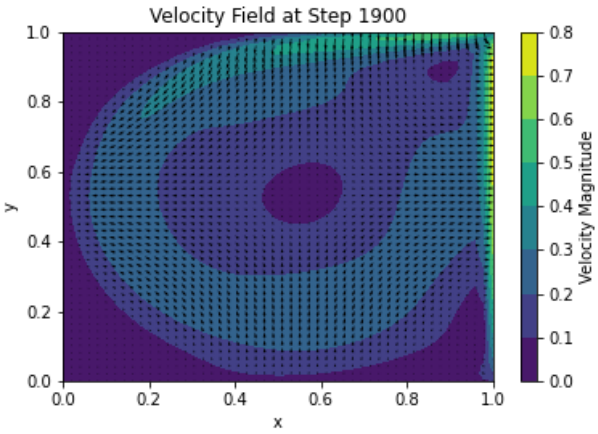

Velocity Field at Step 1900
y
x
Velocity Magnitude




Horizontal Velocity Profile (Re=2500)
u
x
Horizontal profile (Re=2500)
Reference


Vertical Velocity Profile (Re=2500)
v
y
Vertical profile (Re=2500)
Reference


Streamlines for Re = 2500
y
x
Velocity magnitude


Velocity Field at Step 1900
y
x
Velocity Magnitude