Coordinate Corrections to Fiber Maps for Projector Location

Assume a projector along the optical axis at some distance D from the front plane of a plate. If the coordinates of a hole is (x, y) in the front plane of the plate, then we can define this location's distance from the center as

$$r = (x^2 + y^2)^{1/2} \ ,$$

while the angle to the hole from the projector is given by

$$\tan\theta = \frac{r}{D + \delta} ,$$

where δ is defined as

$$\delta = (R^2 - r^2)^{1/2} - (R^2 - r_m^2)^{1/2} \ ,$$

and R is the radius of curvature of the front of the plate, and r_m is the maximum radius in this plane from the plate center to the end of its curved surface (the two parameters are R and r_m). The radial offset (inward, closer to the plate center) is therefore

$$\Delta r = \delta \tan\theta = \frac{\delta\, r}{D + \delta} \ .$$

Now, as viewing the plate from the projector, the (x, y) coordinates can be written as polar coordinates (r, φ) where the angle, φ, is measured clockwise from vertical. With this definition, $\tan\varphi = x/y$, and $r = (x^2 + y^2)^{1/2}$. Thus, we can define the new coordinates as

$$r' = r - \Delta r \ ,$$

$$x' = r' \sin\varphi \, ,$$

$$y' = r' \cos\varphi \, .$$

There is one final correction that aims to try to negate the distortion introduced by any residual errors in the keystone correction imposed by the projector itself (and observed directly in the stand at LCO). In this case, we define an additional parameter a such that

$$r'_{final} = r'(1 + a\cos\varphi) \, .$$

The final coordinates $(x', y')_{final}$ are then defined as

$$x'_{final} = r'_{final} \sin\varphi \, ,$$

$$y'_{final} = r'_{final} \cos\varphi \, ,$$

in analogy to (x', y') above. I anticipate that a will be fairly small (of order 0.01 or smaller) and likely negative. It might be necessary to add higher order terms to this, but for now it seems wisest to keep this simple form. The parameters R, r_m, and a need to be easy to modify while manipulating the Python GUIs to produce the fiber coordinate maps.

From design drawings, the values of the relevant parameters are:

$$R = 50.68 \text{ inches} = 1287.3 \text{ mm} \, ,$$

$$r_m = 13 \;\; 7/32 \text{ inches} \;\; = 335.8 \text{ mm} \, ,$$

$$D = 61 \text{ inches} = 1549.4 \text{ mm} \, ,$$

$$a = unknown\ at\ present\ but\ set\ to\ zero\ for\ initial\ guess.$$

Be sure these parameters remain easy to alter while producing plate maps.