Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science
School of Computer Science and Statistics

ALGORITHMS AND DATA STRUCTURES II
2022/23

Semester 2 **2023**

ALGORITHMS AND DATA STRUCTURES II

28 04 2023 **RDS** **14.00 – 16.00**

Dr Anthony Ventresque

Instructions to Candidates:
Answer ***four*** of the five questions. All questions carry equal marks. The paper is marked out of 100. No books or notes are permitted. No calculator permitted.

Instructions for invigilators:
No books or notes are permitted. No calculator permitted.

Question 1 **(25 Marks)**

a) What is the best case input for standard insertion sort? **(2 Marks)**
   a. Already sorted array
   b. Random array
   c. Array sorted in reverse order
   d. It does not matter – performance of insertion sort is the same for all inputs

b) What is the best case input for standard selection sort? **(2 Marks)**
   a. Already sorted array
   b. Random array
   c. Array sorted in reverse order
   d. It does not matter – performance of selection sort is the same for all inputs

c) How many passes are required for a selection sort algorithm on the array below? **(2 Marks)**

   a. 3
   b. 4
   c. 5
   d. 6

7	3	0	8	1

d) After pass 3, list the elements in their given order. **(2 Marks)**

e) What is the average case time complexity of bubble sort? **(2 Marks)**
   a. $O(n \log_2 n)$
   b. $O(n^2)$
   c. $O(n)$
   d. $O(n^3)$

f) There is no difference between the best and worst case efficiency for the bubble sort? **(2 Marks)**
   a. True
   b. False

g) The Quicksort algorithm uses recursion to sort both lower and upper sublists. In the recursive algorithm describe the stopping condition. **(3 Marks)**

h) Which situation represents the worst case for Quicksort? **(2 Marks)**

a. The elements in ascending order
b. The elements in descending order
c. The pivot is the smallest element all of the time.
d. The elements are in alternating small/large order.

i) Which situation represents the best case for Quicksort? **(2 Marks)**

a. The elements in ascending order
b. The pivot is the smallest element all of the time.
c. The elements are in random order
d. The elements are in alternating small/large order.

j) Fill the following table with the successive outputs of the mysterySort algorithm when the input if the array A: **(6 Marks)**

```
algorithm mysterySort
for i = 1 to n-1 do
x ← A[i]
j ← i
while j > 0 and A[j-1] >= x do
        A[j] ← A[j-1]
        j ← j – 1
endwhile
A[j] ← x
print A
endfor
```

A:

10	56	4	78	7	98	4	98

10	56	4	78	7	98	4	98

Question 2 **(25 Marks)**

Let G be an undirected graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below:

Vertex	Adjacent Vertices
1	(2, 3, 4)
2	(1, 3, 4)
3	(1, 2, 4)
4	(1, 2, 3, 6)
5	(6, 7, 8)
6	(4, 5, 7)
7	(5, 6, 8)
8	(5, 7)

a) Give the sequence of vertices of *G* visited using a DFS traversal starting at vertex 1. **(2 Marks)**

b) Give the sequence of vertices visited using a BFS traversal starting at vertex 1. **(2 Marks)**

c) Which famous traversal algorithm is this? **(2 Marks)**
   a. BFS
   b. DFS
   c. Dijkstra
   d. MapReduce

```
algorithm mysteryTraversal
Input: a Graph g and a node n
Output: the function explores every node from n flag n as visited
do something
for each neighbour n_c of n which is not visited do
        mysteryTraversal (n_c)
endfor
```

d) What data structure is used for breadth first traversal of a graph? **(2 Marks)**
   a. Queue
   b. Stack
   c. List
   d. None of the above

e) The adjacency matrix representation is usually preferred over adjacency lists, especially for storing sparse graphs compactly. **(2 Marks)**
   a. True
   b. False

f) If G is an unweighted directed graph with 20 vertices, how many boolean values will be needed to represent G using an adjacency matrix? **(2 Marks)**

a. 20
b. 40
c. 200
d. 400

g) What is the running time of DFS, as a function of |V | and |E|, if the input graph is represented by an adjacency matrix? **(2 Marks)**

a. $O(|V|^2)$
b. O(|E||V|)
c. O(log |V|)
d. O(|E| log |V|)

h) Dijkstra's algorithm can find shortest paths in a directed graph with negative weights, but no negative cycles. **(2 Marks)**

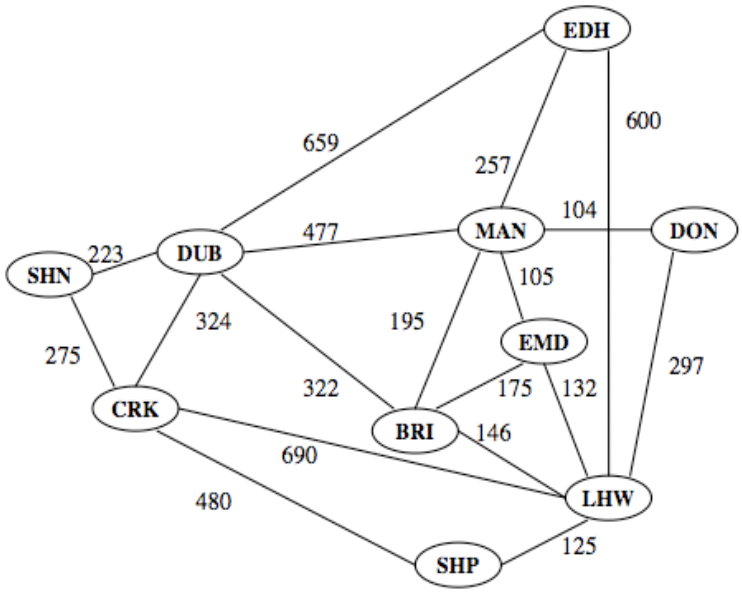
a. True
b. False

i) Explain in your own words what a minimum spanning tree is. **(3 Marks)**

j) Which of the following is true? **(2 Marks)**

a. Prim's algorithm initialises with a vertex
b. Prim's algorithm initialises with an edge
c. Prim's algorithm initialises with a vertex which has smallest edge
d. Prim's algorithm initialises with a forest

k) Draw the minimum spanning tree from the graph below: **(4 Marks)**



Question 3 **(25 Marks)**

a) Let $e = v \rightarrow w$ be an edge with weight 17.0. Suppose that during the generic shortest paths algorithm, $distTo[v] = \infty$ and $distTo[w] = 15.0$. What will $distTo[w]$ be after relaxing e ? **(2 Marks)**

   a. The program will crash
   b. 15.0
   c. 17.0
   d. ∞

b) Explain in your own words what a topological sort is. **(4 Marks)**

c) Consider the directed graph where vertices are reachable tic-tac-toe board positions and edges represent valid moves (cf. Practical number 3). What are the in-degree and out-degree of the following vertex? (It is O's turn.) **(2 Marks)**

X	O	X
	O	
	X	

   a. In-degree is 1, out-degree is 3
   b. In-degree is 3, out-degree is 1
   c. In-degree is 3, out-degree is 4
   d. In-degree is 4, out-degree is 3

d) Bellmann ford algorithm provides solution for what problems? **(2 Marks)**

   a. All pair shortest path
   b. Sorting
   c. Network flow
   d. Single source shortest path

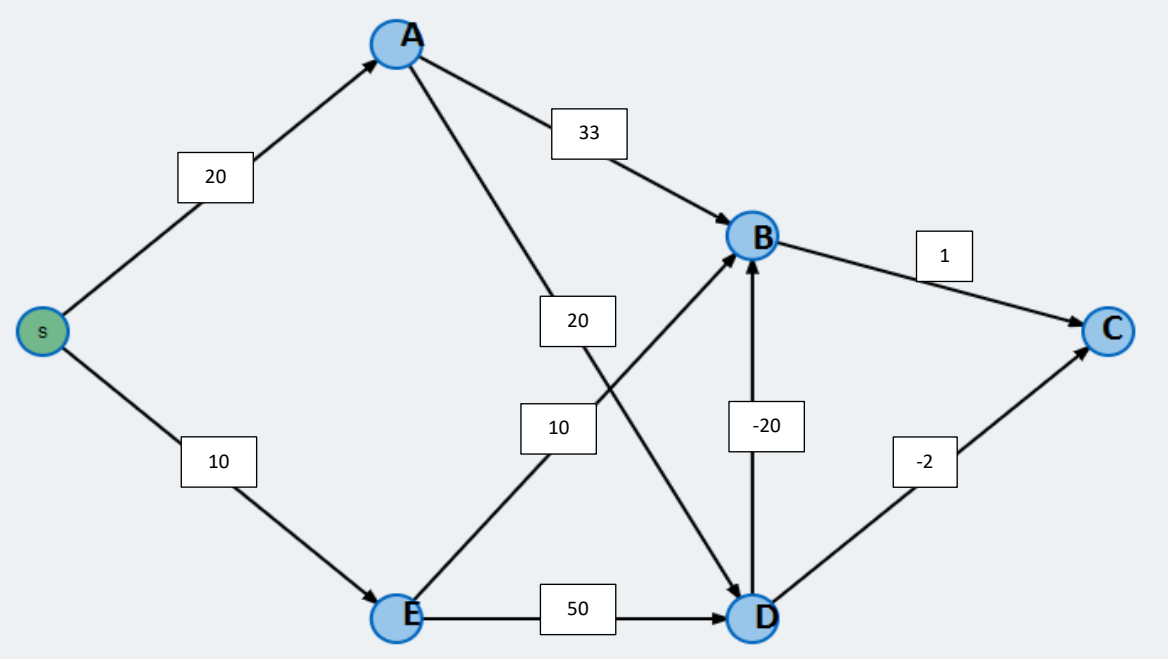
e) Floyd Warshall's Algorithm is used for solving **(2 Marks)**

   a. All pair shortest path problems
   b. Single Source shortest path problems
   c. Network flow problems
   d. Sorting problems

f) Floyd Warshall's Algorithm can be applied on **(2 Marks)**

   a. Undirected and unweighted graphs
   b. Undirected graphs
   c. Directed graphs
   d. Acyclic graphs

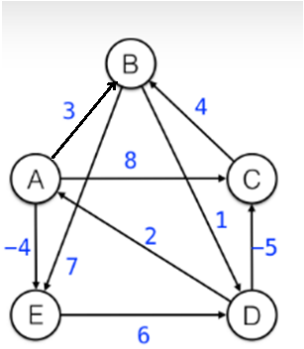
Given the following graph:



g) Fill-in the following matrix with the different values at different steps of the Bellman-Ford algorithm: **(5 Marks)**

	S	A	B	C	D	E
0	0	∞	∞	∞	∞	∞
1						
2						
3						
4						
5						

Given the following graph, use Floyd-Warshall to answer the next two questions:



h) Initialise the following matrix (step 0 of Floyd-Warshall) **(2 Marks)**

	A	B	C	D	E
A					
B					
C					
D					
E					

i) Give the value of the matrix at the last step of Floyd-Warshall **(4 Marks)**

	A	B	C	D	E
A					
B					
C					
D					
E					

Question 4 **(25 Marks)**

a) Explain the following approaches to algorithm design, giving an example of a well known algorithm for each. Explain why the algorithm you select fits into that category, using pseudo-code or diagrams to illustrate your explanations.
   a. Brute force
   b. Divide and conquer **(6 Marks)**

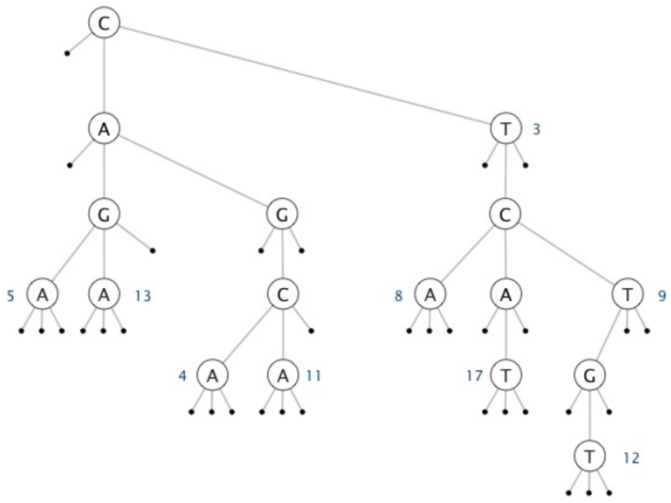
b) Suppose that you insert a set of *n* strings into a multiway trie. What determines the shape of the trie? **(2 Marks)**
   a. The set of strings that you insert.
   b. The order in which you insert the set of strings.
   c. Both a and b.
   d. Neither a nor b.

c) It is possible to replace key-index counting to sort digits in the radix sort algorithms (LSD and MSD) with any valid sorting algorithm and radix sort will still sort correctly. Is this true or false? Explain. **(3 Marks)**

d) Construct a Trie with the following key-value pairs. **(5 Marks)**

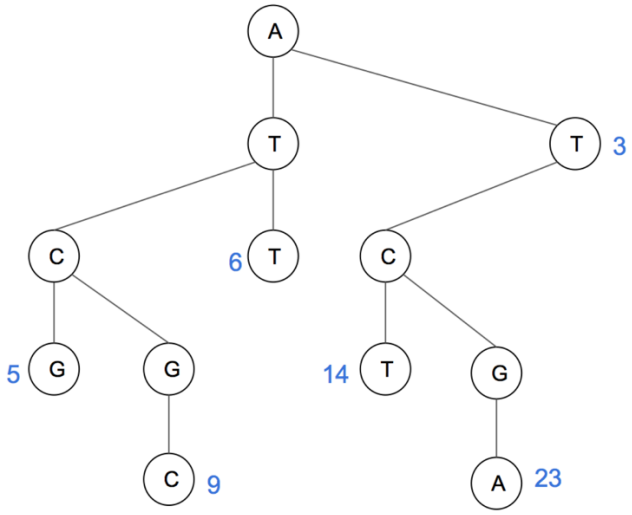
Key	Value
Bed	50
Better	3
Backend	30
Backup	1
Auto	1

e) Which of the following are NOT key–value pairs in the Ternary Search Trie (TST) shown below? **(2 Marks)**
   a. CAGA–13
   b. CGA–4
   c. CGCA–11
   d. TTGT–12



f) We would like to construct the below TST by inserting six strings into an empty TST. Which of the sequences below can produce the above TST. There may be multiple correct answers. **(4 Marks)**

a. Sequence 1: ATT ACG T CT AGC GA
b. Sequence 2: ATT T CT ACG GA AGC
c. Sequence 3: ATT T GA ACG CT AGC
d. Sequence 4: ATT T AGC ACG CT GA
e. Sequence 5: ATT ACG AGC T CT GA
f. Sequence 6: ATT T AGC GA ACG CT
g. Sequence 7: ATT ACG T CT GA AGC



j) Insert the three strings CA, AGA, and GAC into the TST with the associated values 0, 18, and 29, respectively. Update the figure above to reflect the changes. **(3 Marks)**

Question 5 **(25 Marks)**

a) Explain in your own words what a constraint solver does. **(3 Marks)**

b) What is the problem with hill-climbing algorithms? **(3 Marks)**

c) Which algorithm would you use to sort one million 32-bit integers – and why? **(3 Marks)**

d) Provide the trace of sorting the array of strings given in the table below using LSD sort, providing an equivalent table for each of the 3 passes of the algorithm. **(4 Marks)**

D	A	B
A	D	D
C	A	B
F	A	D
F	E	E
B	A	D
D	A	D
B	E	E

e) Provide the trace of sorting the array of strings given in the table below using MSD sort, providing an equivalent table for each of the 3 passes of the algorithm. **(4 Marks)**

D	A	B
A	D	D
C	A	B
F	A	D
F	E	E
B	A	D
D	A	D
B	E	E

f) Consider the following method that implements LSD radix sort algorithm . Write the missing 4 lines (`//TODO`). **(8 Marks)**

```
/**
 * Rearranges the array of w-character strings in ascending order.
 *
 * @param a the array to be sorted
 * @param w the number of characters per string
 */
public static void sort(String[] a, int w) {
    int n = a.length;
    int R = 256;   // extend ASCII alphabet size
    String[] aux = new String[n];

    for (int d = w-1; d >= 0; d--) {
        // sort by key-indexed counting on dth character

        // compute frequency counts
        int[] count = new int[R+1];
        for (int i = 0; i < n; i++)
            //TODO

        // compute cumulates
        for (int r = 0; r < R; r++)
            //TODO

        // move data
        for (int i = 0; i < n; i++)
            //TODO

        // copy back
        for (int i = 0; i < n; i++)
            //TODO
    }
```

[oOo]