MAU22C00: ASSIGNMENT 2
DUE BY FRIDAY, JAN. 24 BEFORE MIDNIGHT
UPLOAD SOLUTION ON BLACKBOARD

1) (10 points)

(a) Describe the formal language over the alphabet $\{a, b, c\}$ generated by the context-free grammar whose non-terminals are $\langle S\rangle$ and $\langle A\rangle$, whose start symbol is $\langle S\rangle$, and whose production rules are the following:
(1) $\langle S\rangle \to a\langle S\rangle a$
(2) $\langle S\rangle \to b\langle A\rangle$
(3) $\langle A\rangle \to b\langle A\rangle b$
(4) $\langle A\rangle \to c\langle A\rangle c$
(5) $\langle A\rangle \to c$
In other words, describe the structure of the strings generated by this grammar.

(b) Use the Pumping Lemma to prove that the language from part (a) is not regular.

2) (20 points) (Schol 2022-23) Let L be the language over the alphabet $A = \{0, 1\}$ consisting of every string of even length such that the number of occurrences of 0 in the string is odd. For example, $01 \in L$, but $0011 \notin L$.

(a) Draw a deterministic finite state acceptor that accepts the language L. Carefully label all the states including the starting state and the finishing states as well as all the transitions. Make sure you justify it accepts all strings in the language L and no others.

(b) Devise a regular grammar in normal form that generates the language L. Be sure to specify the start symbol, the non-terminals, and all the production rules. Make sure you justify it generates all strings in the language L and no others.

(c) Write L as the intersection of two regular languages and justify your answer.

(d) Since you were able to construct a finite state acceptor that recognises L in part (a), the language L is regular, so it must fulfill the conclusion of the Pumping Lemma. Consider $w = 010101$. Decompose w into x, u, and y such that $w = xuy$, $|xu| \leq 4$, $u \neq \epsilon$, and $xu^n y \in L\ \forall\, n \geq 0$. Justify your answer.

3) (10 points) Let A be a finite alphabet.

(a) (Schol 2022-23) Let L be a regular language over the alphabet A. Prove that L^*, the Kleene star of L, is also a regular language. (Hint: Think about the equivalent conditions characterising a regular language and figure out which one is easiest to check here.)

(b) Let L be a regular language over the alphabet A. Prove that $A^* \setminus L$, the complement of L in A^*, is also a regular language. (Hint: Same as for (a).)

4) (20 points) (Annual Exam 2024) Let (V, E) be the graph with vertices a, b, c, d, e, f, g, h, i, and j, and edges ab, ac, bc, bd, bf, df, ce, cg, eg, cf, bg, fg, fi, gi, dh, hi, ej, and ij.

(a) Draw this graph.

(b) Write down this graph's incidence table and its incidence matrix.

(c) Write down this graph's adjacency table and its adjacency matrix.

(d) Is this graph complete? Justify your answer.

(e) Is this graph bipartite? Justify your answer.

(f) Is this graph regular? Justify your answer.

(g) Does this graph have any regular subgraph? Justify your answer.

(h) What is the minimum number of edges you would have to remove to make this graph have exactly three connected components? Justify your answer.

(i) Give an example of an isomorphism φ from the graph (V, E) to itself satisfying that $\varphi(a) = a$.

(j) Is the isomorphism from part (i) unique or can you find another isomorphism ψ that is distinct from φ but also satisfies the condition that $\psi(a) = a$? Justify your answer.