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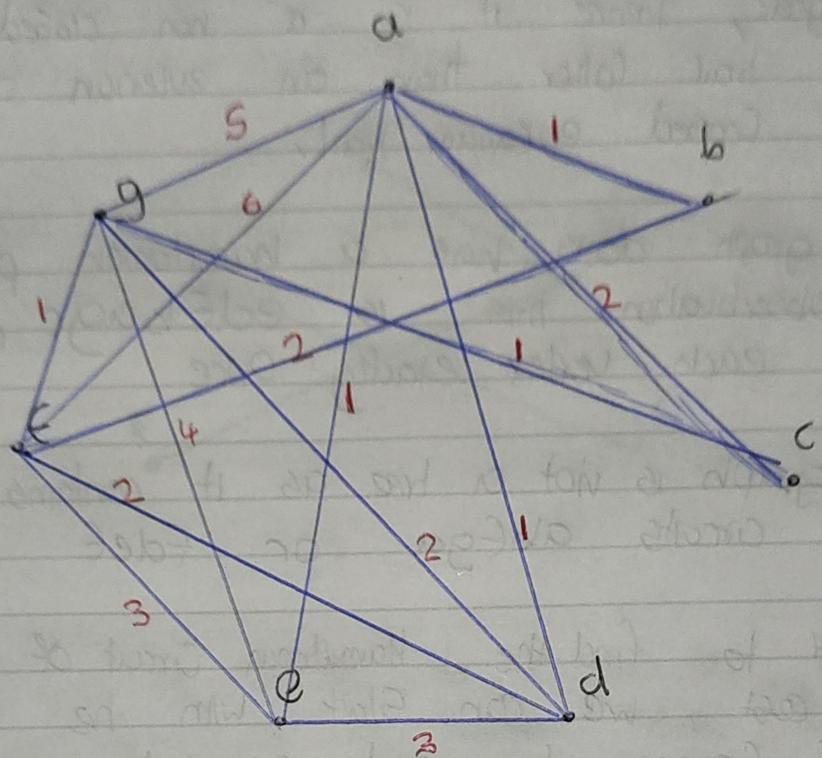
STUDENT NUMBER: 23373999

SIGNED:

DATE: 09-03-2025

Ma022000 Discrete Maths Assignment 3

Q1



(a) This graph is connected as at least one walk exists between each vertex of the graph

(b) Degrees of each vertex:

a: 6
b: 2
c: 2
d: 4
e: 4
f: 5
g: 5

We know a graph has an eulerian trail iff it has either 0 or 2 vertices of odd degree.

Here g and f are the only two odd, meaning the graph must have a non closed eulerian trail.

(c) This graph does not have an eulerian circuit, as it has two vertices with an odd degree, hence it is a non closed eulerian trail rather than an eulerian circuit, which is a closed eulerian trail,

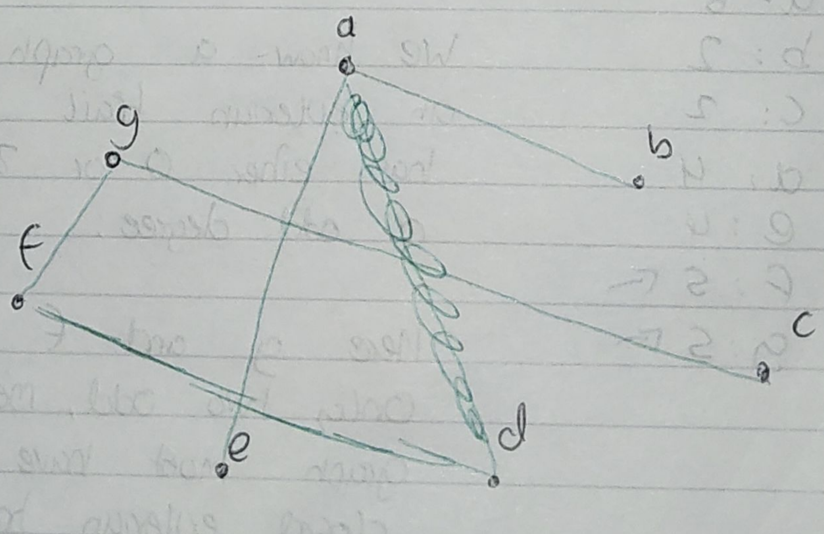
(d) This graph does have a hamiltonian path, by observation this is edfbagc, which visits each vertex exactly once

(e) This graph is not a tree as it contains multiple circuits abfga or fdef

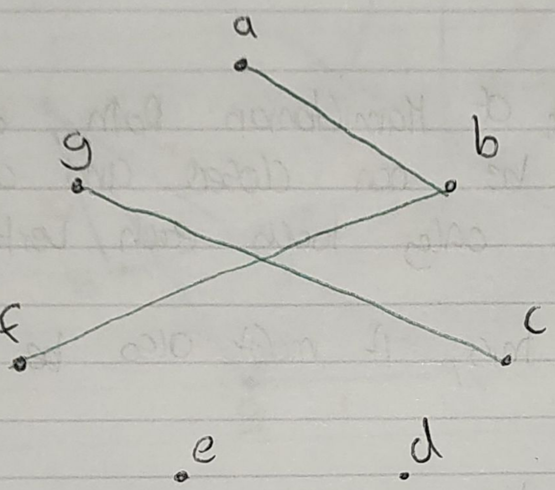
(f) In order to find the Hamiltonian circuit of at least cost, we can start with the minimal spanning tree and connect the neighbours of least cost to form the Hamiltonian circuit of least cost

Seeing as the edges in the question are already sorted, applying kruskals algorithm gives us:

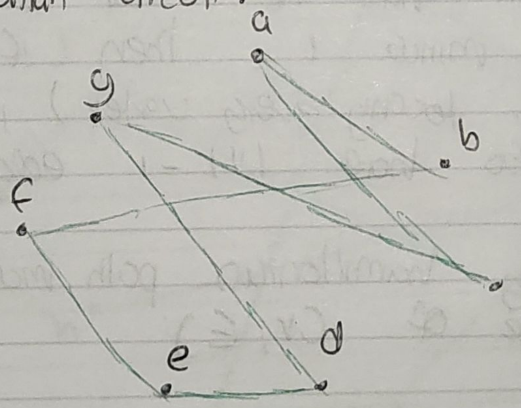
MST:



Following from a, we can choose either b or e as the nearest neighbour, however we take b as it has a lower connection to f



Next, g connects to e or d, but e -> d is shorter, hence this is the minimal hamiltonian circuit.



Q2 (V, E) is connected graph

Prove every Hamiltonian path in (V, E) is a spanning tree of (V, E)

Per the definition of Hamiltonian Path, a Hamiltonian Path must be non closed and acyclical as it must only touch each vertex once.

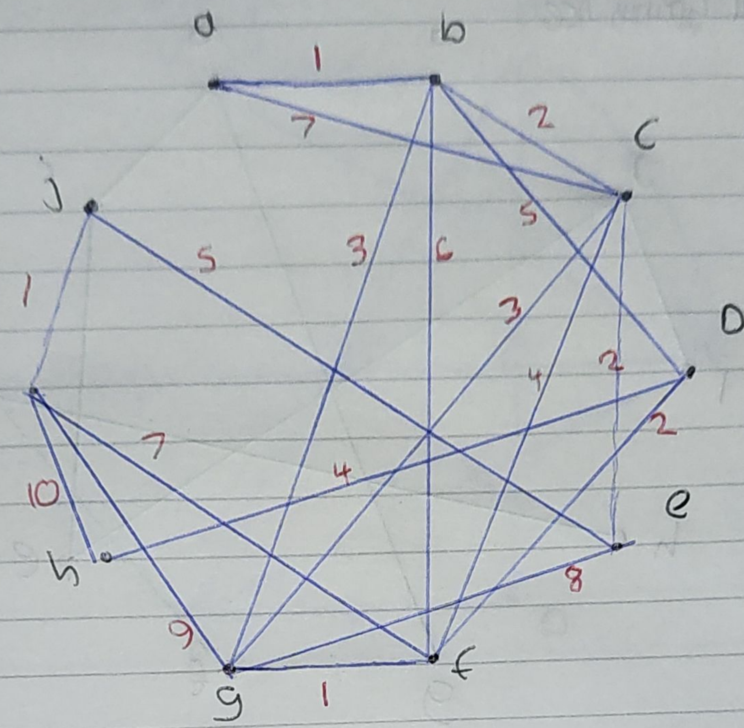
Because of this, it must also be connected.

A spanning tree must be acyclic, connected, and have $|V|-1$ ~~vertices~~ edges.

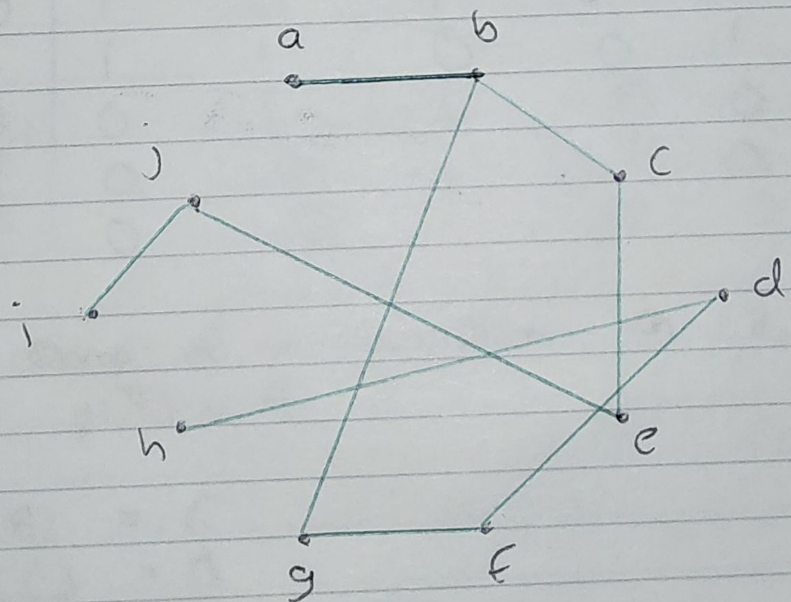
Since a path has the number of edges equal to vertices minus 1, then if a Hamiltonian path (i.e. touching every vertex) is used, we also have $|V|-1$ edges.

Hence every hamiltonian path must be a spanning tree of (V, E)

Q3 (a)



(b) Minimum Spanning tree:



Edges added:

1 ab
2 fg
3 ij
4 ce
5 df
6 bc
7 bg
8 dh
9 ej

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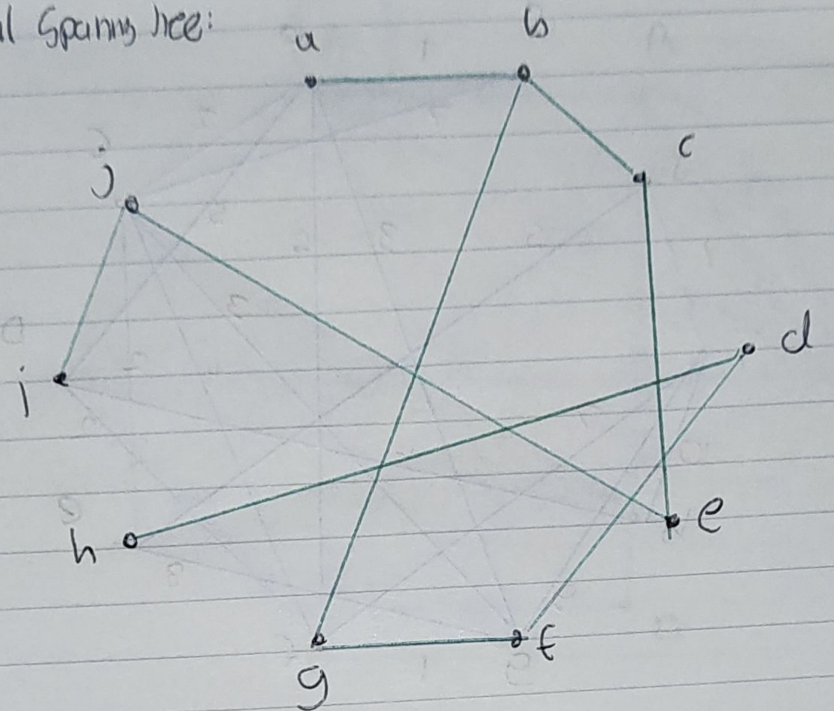
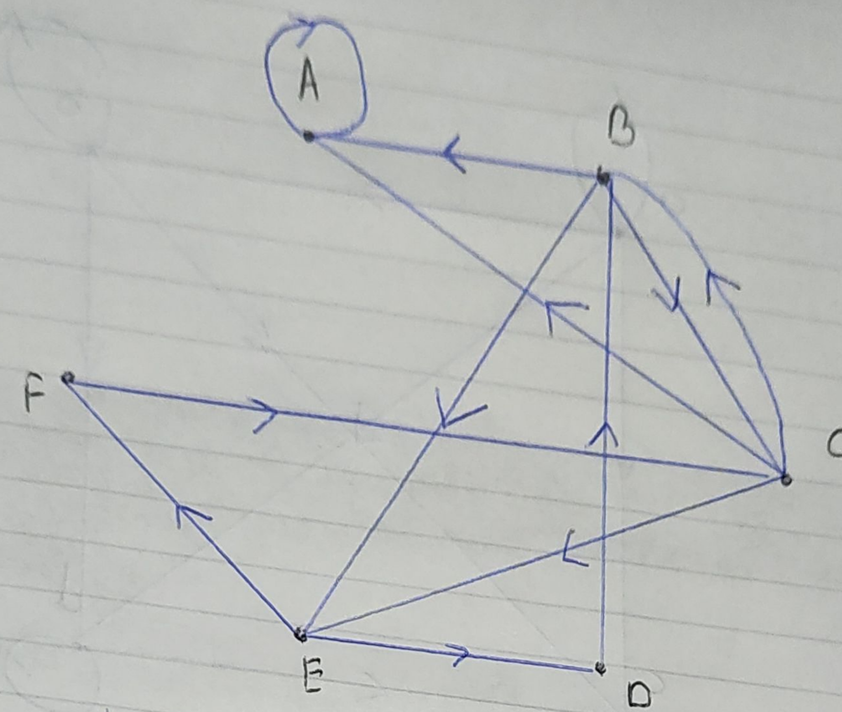
(c) Minimal Spanning tree:



Edges:

1 ce
2 bc
3 ba
4 bg
5 gf
6 fd
7 dh
8 ej
9 ji

(Q4 (a)



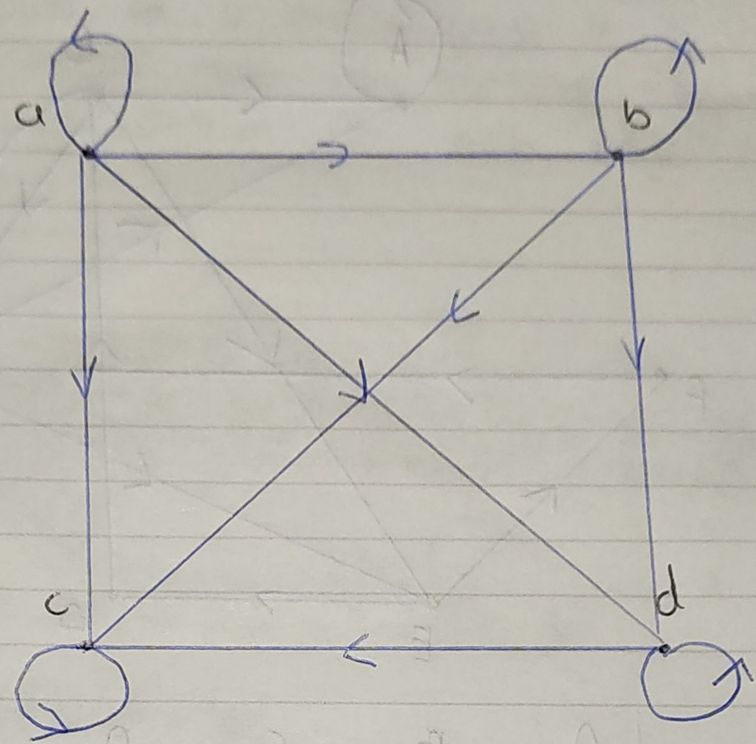
(b)

	A	B	C	D	E	F
A	1	0	0	0	0	0
B	1	0	1	0	1	0
C	1	1	0	0	1	0
D	0	1	0	0	0	0
E	0	0	0	1	0	1
F	0	0	1	0	0	0

(c) An example of an isomorphism on (V, E) such that $\varphi(A) = A$ is

$\varphi(B) = C$
$\varphi(C) = B$
$\varphi(D) = F$
$\varphi(F) = D$
$\varphi(E) = E$

Q5 (a)



(b) R is not an equivalence relation, because if you follow any path to c you are stuck here, hence this is not a Symmetric relation which is a requirement to be an equivalence relation

(c) To make this an equivalence relation the following ordered pairs must be added:

(c, a)

Q6a)

In order for a directed graph to be connected, all vertices must be reachable by travelling in he direction of each edge repeatedly.

(b)

~~def connected?(graph) do~~

~~start_node = hd(Map.keys(graph))~~

Eulir Cile Pseudocode:

(b)
```
def Connected(graph) do
    Start = graph.Start_node
    is_reachabee?(graph, Start)      and
    is_reachable?(reversed(graph), Start)
end

def is_reachable?(graph, node) do
    visited_nodes = depth_first_search(graph, node)

    size(visited) = size(graph)
end
```

The graph can be considered connected if all nodes can be reached if we search to the end of each path, then reverse the graph and do the same

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