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CSCI567 Homework 5

Problem 1.1

$$\alpha_t(j) = P(x_t|Z_t = s_j) \sum_i a_{ij}\alpha_{t-1}(i)$$

$$\alpha_1(j) = P(X_1 = A|Z_1 = s_j)P(Z_1 = s_j) = \pi_j P(X_1 = A|Z_1 = s_j)$$

$$\alpha_1(1) = \pi_1 P(X_1 = A|Z_1 = s_1) = 0.7 * 0.4 = 0.28$$

$$\alpha_1(2) = 0.3 * 0.2 = 0.06$$

$$\alpha_2(1) = b_{1G} * \left(a_{11} * \alpha_1(1) + a_{21}\alpha_1(2)\right) = 0.4 * (0.8 * 0.28 + 0.4 * 0.06) = 0.0992$$

$$\alpha_2(2) = 0.2 * (0.2 * 0.28 + 0.6 * 0.06) = 0.0184$$

$$\alpha_3(1) = b_{1C} * \left(a_{11} * \alpha_2(1) + a_{21}\alpha_2(2)\right) = 0.1 * (0.8 * 0.0992 + 0.4 * 0.0184) = 0.008672$$

$$\alpha_3(2) = 0.3 * (0.2 * 0.0992 + 0.6 * 0.0184) = 0.009264$$

$$\alpha_4(1) = 0.4 * (0.8 * 0.008672 + 0.4 * 0.009264) = 0.00425728$$

$$\alpha_4(2) = 0.2 * (0.2 * 0.008672 + 0.6 * 0.009264) = 0.00145856$$

$$\alpha_5(1) = 0.1 * (0.8 * 0.00425728 + 0.4 * 0.00145856) = 0.0003989248$$

$$\alpha_5(2) = 0.3 * (0.2 * 0.00425728 + 0.6 * 0.00145856) = 0.0005179776$$

$$\alpha_6(1) = 0.4 * (0.8 * 0.0003989248 + 0.4 * 0.0005179776) = 0.00021053235$$

$$\alpha_6(2) = 0.2 * (0.2 * 0.0003989248 + 0.6 * 0.0005179776) = 0.0000781143$$

$$P(\{A, G, C, G, T, A\}) = \sum_j \alpha_6(j) = 0.00021053235 + 0.0000781143 = 0.00028864665$$

Problem 1.2

$$\delta_t(j) = \max_i \delta_{t-1}(i)a_{ij}P(x_t|Z_t = s_j)$$

$$\delta_1(1) = 0.7 * 0.4 = 0.28$$

$$\delta_1(2) = 0.3 * 0.2 = 0.06$$

$$\delta_2(1) = \max(\delta_1(1) * a_{11} * b_{1G}, \delta_1(2) * a_{21} * b_{1G})$$

1

$$\delta_2(1) = \max(0.28 * 0.8 * 0.4, 0.06 * 0.4 * 0.4) = \max(0.0896, 0.0096) = 0.0896$$

$$1 \to 1$$

$$\delta_2(2) = \max(0.28 * 0.2 * 0.2, 0.06 * 0.6 * 0.2) = \max(0.0112, 0.0072) = 0.0112$$

$$1 \to 2$$

$$\delta_3(1) = \max(0.0896 * 0.8 * 0.1, 0.0112 * 0.4 * 0.1) = \max(0.007168, 0.000448)$$
$$= 0.007168$$

$$1 \to 1 \to 1$$

$$\delta_3(2) = \max(0.0896 * 0.2 * 0.3, 0.0112 * 0.6 * 0.3) = \max(0.005376, 0.002016)$$
$$= 0.005376$$

$$1 \to 1 \to 2$$

$$\delta_4(1) = \max(0.007168 * 0.8 * 0.4, 0.005376 * 0.4 * 0.4) = \max(0.00229376, 0.00086016)$$
$$= 0.00229376$$

$$1 \to 1 \to 1 \to 1$$

$$\delta_4(2) = \max(0.007168 * 0.2 * 0.2, 0.005376 * 0.6 * 0.2) = \max(0.00028672, 0.00064512)$$
$$= 0.00064512$$

$$1 \to 1 \to 2 \to 2$$

$$\delta_5(1) = \max(0.00229376 * 0.8 * 0.1, 0.00064512 * 0.4 * 0.1)$$
$$= \max(0.0001835008, 0.0000258048) = 0.0001835008$$

$$1 \to 1 \to 1 \to 1 \to 1$$

$$\delta_5(2) = \max(0.00229376 * 0.2 * 0.3, 0.00064512 * 0.6 * 0.3)$$
$$= \max(0.0001376256, 0.0001161216) = 0.0001376256$$

$$1 \to 1 \to 1 \to 1 \to 2$$

$$\delta_6(1) = \max(0.0001835008 * 0.8 * 0.4, 0.0001376256 * 0.4 * 0.4)$$
$$= \max(0.00005872025, 0.00002202009) = 0.00005872025$$

$$1 \to 1 \to 1 \to 1 \to 1 \to 1$$

$$\delta_6(2) = \max(0.0001835008 * 0.2 * 0.2, 0.0001376256 * 0.6 * 0.2)$$
$$= \max(0.00000734003, 0.00001651507) = 0.00001651507$$

$$1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 2 \rightarrow 2$$

$$\delta_6(1) > \delta_6(2)$$

The most likely path is:

$$1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1$$

Problem 1.3

$$x^* = \arg\max_{x} P(X_7 = x | X_{1:6} = O_{1:6}; \theta) = \arg\max_{x} \frac{P(X_7 = x, X_{1:6} = O_{1:6}; \theta)}{P(X_{1:6} = O_{1:6})}$$

$$= \arg\max_{x} P(X_7 = x, X_{1:6} = O_{1:6}; \theta)$$

$$P(\{A, G, C, G, T, A, A\})$$

$$\alpha_7(1) = 0.4 * (0.8 * 0.00021053235 + 0.4 * 0.0000781143) = 0.00007986864$$

$$\alpha_7(2) = 0.2 * (0.2 * 0.00021053235 + 0.6 * 0.0000781143) = 0.00001779501$$

$$P(\{A, G, C, G, T, A, A\}) = 0.00007986864 + 0.00001779501 = 0.00009766365$$

$$P(\{A, G, C, G, T, A, C\})$$

$$\alpha_7(1) = 0.1 * (0.8 * 0.00021053235 + 0.4 * 0.0000781143) = 0.00001996716$$

$$\alpha_7(2) = 0.3 * (0.2 * 0.00021053235 + 0.6 * 0.0000781143) = 0.00002669251$$

$$P(\{A, G, C, G, T, A, C\}) = 0.00001996716 + 0.00002669251 = 0.00004665967$$

$$P(\{A, G, C, G, T, A, G\})$$

$$\alpha_7(1) = 0.4 * (0.8 * 0.00021053235 + 0.4 * 0.0000781143) = 0.00007986864$$

$$\alpha_7(2) = 0.2 * (0.2 * 0.00021053235 + 0.6 * 0.0000781143) = 0.00001779501$$

$$P(\{A, G, C, G, T, A, G\}) = 0.00007986864 + 0.00001779501 = 0.00009766365$$

$$P(\{A, G, C, G, T, A, T\})$$

$$\alpha_7(1) = 0.1 * (0.8 * 0.00021053235 + 0.4 * 0.0000781143) = 0.00001996716$$

$$\alpha_7(2) = 0.3 * (0.2 * 0.00021053235 + 0.6 * 0.0000781143) = 0.00002669251$$

$$P(\{A, G, C, G, T, A, T\}) = 0.00001996716 + 0.00002669251 = 0.00004665967$$

$$x^* = A \ or \ G$$