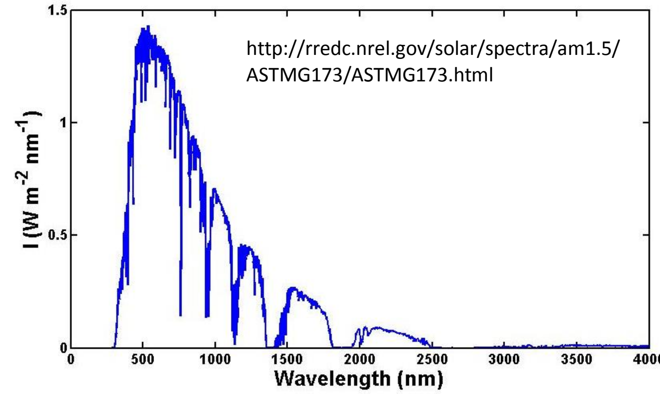
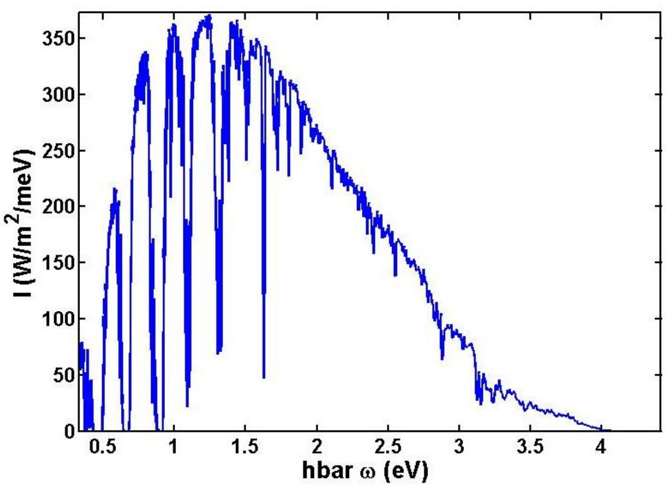
Homework 1, Due: Tue Feb 4, 2014

1. **Sunlight:**

Harvesting the energy from sunlight is a challenge to modern science and technology. In this problem, you will construct the EM field of sunlight using the knowledge of the first week of our lectures. We will do this problem for one component of electric field polarization, with the reasonable assumption that sunlight has both polarizations with random relative phases with respect to each other.





The spectrum of sunlight (as received on the surface of earth) is given to the right both as a function of wavelength and energy. Notice that the spectrum is in units of W/m^2 per dλ= 1nm on the top and per $\hbar d\omega$ = 1 meV in the bottom spectrum. It is also worth noting that since $\lambda=2\pi c/\omega$, the widths dω and dλ are not linearly related to each other, making the shape of the spectra looks different between the two cases. For this problem, it is most convenient to work with the spectrum as a function of $\hbar d\omega$, and the data as a function of ω can be found in the course website.

a)- From this spectrum prepare a **100 fs long temporal snapshot of the electric field** of sunlight with step size of 0.1 fs. **Appropriately convert to the correct units of electric field.** To do this problem, assume that the spectrum is composed of sinusoidal oscillations with a discrete set of frequencies that are dω=1 meV/$\hbar$ apart. The phases of these oscillations are random with respect to each other.

b)- Assume that the sunlight is passed through a **band pass Gaussian filter of fwhm=5 nm** centered at 500 nm (green light) before the lens in the last part. For simplicity, assume that the filter is Gaussian in the frequency axis (i.e. convert 500 nm to frequency and then find out how wide 5 nm is in units of frequency). Prepare a snapshot of electric field again and compare with part (a). Comment on the differences. What if the filter is 25 nm wide?

c)- For the 5 nm width spectrum of part b, assume that all photons have the energy of the central wavelength. How many photons are arriving at a leaf with an area of 5 cm^2 each second?

d)- Assume that 0.1 mole of a molecule occupies the 5 cm^2 area mentioned in the last part and absorbs 10% of the incoming photons. The excited state lifetime of the molecule is approximately 10 ns. What are the chances that a molecule will get doubly excited with the sunlight passing through the 5 nm filter?

c)- Now let us deviate from sunlight a bit. Assume that (by some magic that will be discussed much later in the semester) the phases of the sinusoidal electric field components in this problem are not randomized with respect each but are all the same (of course, this is not the case for sunlight). Plot the electric field for the cases of 5 nm and 25 nm filters from -100 fs to 100 fs and comment on your observations. (Note: Modelocked lasers can achieve such locking of relative phases). What is the difference between the 25 nm wide spectrum and the 5 nm wide spectrum?

2. Polarization:

a) The green light (533 nm) from a laser pointer is passed through a waveplate that shifts the relative phase of the x and y components of the electric field by π/4. Plot a line (in a 3D plot) that corresponds to the tip of the electric field vector for duration of 15 fs.
b) Now assume that the phase shift is π/3, repeat part (a).
c) Assume that the phase shift is π/4 but the amplitudes of the x and y components are different from each other with the y component 4 times larger than the x component.

3. Relating Beer's Law to the Complex Dielectric Constant:

In this problem we will relate our familiar Beer's law to the complex dielectric constant through the following steps.

a)- The μ and ϵ of a linear medium can be related to μ_0 and ϵ_0 via $\mu = \mu_r\mu_0$ and $\epsilon = \epsilon_r\epsilon_0$. Write the expression for the speed of light v and refractive index $n = c/v$ in this medium. This will obviously affect the propagation wavevector k of a plane wave as $k = \frac{n\omega}{c}$. As in most optical spectroscopy, we will assume that $\mu_r = 1$. Show that $\epsilon_r = n^2$.

b)- One may generalize n so that it is complex $\tilde{n} = n_r + in_i$. The wavevector k will correspondingly have a real and an imaginary component. What is the influence of the imaginary component on the propagation of a plane wave? The Beer's law coefficient α is the exponential decay factor for the intensity of light through a medium $I = I_0e^{-\alpha x}$. How is α related to n_i?

c)- Similar to the final result of part (a), we are inspired to define a complex dielectric constant $\tilde{\epsilon_r}=\epsilon_1 + i\epsilon_2$ and relate it to $\tilde{n}$ via $\tilde{\epsilon}_r = \tilde{n}^2$. Based on this show that $\epsilon_1 = {n_r}^2 - {n_i}^2$ and $\epsilon_2 = 2n_in_r$ and:

$$n_r = \frac{1}{\sqrt{2}}\left(\epsilon_1 + ({\epsilon_1}^2 + {\epsilon_2}^2)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$n_i = \frac{1}{\sqrt{2}}(-\epsilon_1 + ({\epsilon_1}^2 + {\epsilon_2}^2)^{\frac{1}{2}})^{\frac{1}{2}}$$

and when the medium is not absorbing strongly: $n_r = \sqrt{\epsilon_1}$ and $n_i = \epsilon_2/2n_r$.

This shows us that for a weakly absorbing medium, the refractive index is mainly determined by the real part and the absorption is mainly determined by the imaginary part of the dielectric constant. In the general case, as shown by the equations above, both the real and imaginary components of the dielectric constant contribute to both the refractive index and the absorption coefficient.

4. Lorentz Oscillator:

a)- The Lorentz model is a classical damped, driven harmonic oscillator model of the electrons bound to nuclei with a spring constant $k = m_0 {\omega_0}^2$ and driven by a classical sinusoidal electric field. Following the damped, driven harmonic oscillator model discussed in class, show that the amplitude of the displacement of the electrons is:

$$|x| = \frac{-eE/m_0}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

b)- Relate the time varying displacement of the electron to the molecular dipole p and total macroscopic polarization $P = Np$, where N is the number of atoms per unit volume.

c)- The electric displacement is related to the electric field and the induced polarization via:

$$D = \epsilon_0 \tilde{\epsilon}_r E = \epsilon_0 E + P_{background} + P_{Lorentz}$$

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d)- Write an expression for $\tilde{\epsilon}_r$ and separate out its real and imaginary parts. Write $\tilde{\epsilon}_r$ in the limit of $\omega \to 0$ as $\tilde{\epsilon}_{r\,static}$ and in the limit of $\omega \to \infty$ as $\tilde{\epsilon}_r(\infty)$.

e)- Using your results from problem 3, plot the Beer's law coefficient and dispersion (refractive index) as a function of frequency for a transition centered at 530 nm with a $\gamma = 5 \times 10^{12} s^{-1}$ and $\tilde{\epsilon}_{r\,static} = 12.1$ and $\tilde{\epsilon}_r(\infty) = 10$.