Definitions

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1 Chapter 1

1.1 Maximum

Let $A \subset \mathbb{R}, m \in \mathbb{R}$. We say that the **maximum** of A is m if

1. for all $x \in A, m \geq x$
2. $m \in A$

1.2 Upper bound

Let A be a nonempty subset of $\mathbb{R}$, let $u \in \mathbb{R}$. u is an **upper bound** if for all $x \in A$ $x \leq u$

1.3 Supremum

Let A be a nenempty subset of $\mathbb{R}$, Let $l \in \mathbb{R}$. l is the **supremum** of A if for all $a \in A, a \leq l$ and for all $m \in \mathbb{R}$ if all $a \in A, a \leq m$ then $l \leq m$. l is the least upper bound.

1.4 Minium

Let $A \subset \mathbb{R}, m \in \mathbb{R}$. We say that the **minimum** of A is m if

1. for all $x \in A, m \leq x$
2. $m \in A$

1.5 Infimum

Let A be a nenempty subset of $\mathbb{R}$, Let $l \in \mathbb{R}$. l is the **infimum** of A if for all $a \in A, a \geq l$ and for all $m \in \mathbb{R}$ if all $a \in A, a \geq m$ then $l \geq m$. l is the least upper bound.

1.6 Bounded Above

A subset $A \subset \mathbb{R}$ is **bounded above** if there exists a supremum for A.

1.7 Bounded Below

A subset $A \subset \mathbb{R}$ is **bounded below** if there exists a infimum for A.

1.8 Bounded

A set is **bounded** if it is bounded below and bounded above

1.9 Bijection

A function f is a bijection if it is one to one and onto.

1.10 Card

Sets A, B have the same Card if there exists a bijection between them.

1.11 Finite

Let A be a set, A is finite if there exists an $n \in \mathbb{N}$ s.t.A has card n

1.12 Countable

A set is countable if there is a bijection between $\mathbb{N}$ and A.

1.13 Sequence

A sequences is a function whose domain is $\mathbb{N}$

1.14 Convergence

A sequence (a_n) converges to a real number a if for every positive number ϵ there exists an $N \in \mathbb{N}$ such that for all $n \geq N$, then $|a_n - a| < \epsilon$

1.15 ϵ-Neighborhood

$V_\epsilon(a) = \{x \in \mathbb{R} : |x - a| < \epsilon\}$

1.16 Topological Convergence

A sequence (a_n) converges to a if there exists an $n \in \mathbb{N}$ such that all terms after n are in an $\epsilon neighborhood$

1.17 Bounded

A sequence is bounded iff the set of all elements of the sequence is bounded.

1.18 Eventually

A sequence (a_n) is eventually in a set A if there exists an N s.t. $a_n \in A$ for all $n \geq N$

1.19 Frequently

A sequence (a_n) is eventually in a set A if for every N there exists an $n \geq N$ s.t. $a_n \in A$.

Chapter 2

2.4 The Monotone Convergence Theorem.

Increasing

A sequence is increasing if for all $n \in \mathbb{N}$, $a_n \leq a_{n+1}$

Decreasing

A sequence is decreasing if for all $n \in \mathbb{N}$, $a_n \geq a_{n+1}$

Monotone

A sequence is monotone if it is increasing or decreasing.

Infinite Series

An infinite series is a expression in the form of:

$$\sum_{n=1}^{\infty} b_n = b_1 + b_2 + ...$$

The sequence of partial sums (s_m) by $s_m = b_1 + b_2 + ... + b_m$. We say the series converges to B if the sequence of partial sums converges to B, denoted:

$$\sum_{n=1}^{\infty} b_n = B$$

2.5 Subsequences

Subsequence

Let (a_n) be a sequence of real numbers and let $n_1 < n_2 ...$ be an increasing sequence of natural numbers. Then the sequence $(a_{n_1}, a_{n_2}, ...)$ is a subsequence of (a_n) and is denoted by (a_{n_k}).

2.6 The Cauchy Criterion

Cauchy Sequence

A sequence (a_n) is called a Cauchy sequence if for every $\epsilon > 0$ there exists an $N \in \mathbb{N}$ such that whenever $m, n > N$ it follows that $|a_n - a_m| < \epsilon$

2.7 Series

Absolute and Conditional Convergence

if the series $\sum_{n=1}^{\infty} |a_n|$ converges we say the original series $\sum_{n=1}^{\infty} a_n$ converges absolutely. If the original series converges but the series of absolute values diverges, then we say the sequence converges conditionally.

Rearagements

Let $\sum_{n=1}^{\infty} a_n$ be a series. A series $\sum_{n=1}^{\infty} b_n$ is called a rearrangement of the original series if there exists a bijection between the the series.

2 Intro to Topology

3.2 Open and Closed Sets

Open

A set $O \subseteq \mathbb{R}$ is open if for all points $a \in O$ there exists an ϵ-neighborhood $V_\epsilon(a) \subseteq O$.

Limit Point

A point x is a limit point of of a set A if every ϵ-neighborhood $V_\epsilon(x)$ of x intersects A at some point other than x. Alternatively, x is a limit point if and only if $x = lim(a_n)$ for some sequence (a_n) contained in A satisfying $a_n \neq x$ for all $n \in \mathbb{N}$.

Isolated Point

A point $a \in A$ is an isolated point if it is not a limit point.

Closed Set

A set $F \subseteq \mathbb{R}$ is closed if it contains its limit points. Alternatively, a set is closed if and only if every Cauchy sequence contained in F has a limit that is also in F.