ASGN 1

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1. (a) $\{x_1, x_2, x_3 \in \mathbb{F}^3 : x_1 + 2x_2 + 3x_3 = 0\}$ Subspace
   (b) $\{x_1, x_2, x_3 \in \mathbb{F}^3 : x_1 + 2x_2 + 3x_3 = 4\}$ Not a subspace
   (c) $\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 \cdot x_2 \cdot x_3\}$ Not a subspace
   (d) $\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 = 5x_3\}$ Subspace

3. Show differentiable real-valued functions f on (-4, 4), such that $f'(-1) = 3f(2)$ is a subspace of $\mathbb{R}^{(-4,4)}$

   - let $f'(-1) = 3f(2)$ and $g'(-1) = 3g(2)$

$$\begin{aligned} &= 3f(2) + 3g(2) \\ &= 3(f(2) + g(2)). \end{aligned}$$

$$\begin{aligned} cf'(1) &= c(3f(2)) \\ &= 3(cf(2)). \end{aligned}$$

   I think that's right at least...

7. Give a nonempty subset U of $\mathbb{R}^2$ st U is closed under addition and under taking additive inverses, but U is not a subspace.

   - $\{x \in \mathbb{R} : 2|x\}$ Closed under addition, any two even numbers added produces an even number, but not cloded under multiplication by non integer or odd numbers.

8. Give an example of U of $\mathbb{R}^2$ st U is closed under scalar multiplication but not additon.

   - $[0, x]$ and $[x, 0]$ both of which can be extended forever but when added they leave their sets. I think this can be any line through the origin but I am not sure.

15. $U + U$: U, addition on subspaces is defined by $u_1 + v_1 + u_2 + v_2$ which can be rewritten as $2U$, and since U is closed under scalar multiplication. $U + U = U$

16. Is addition on subspaces comutive? I think this follows from the axioms. $u + v = v + u$, and since addition is defined $u_1 + v_1 + u_2 + v_2$... so $v_1 + u_1$ follows from associativity.

1. Show that $U = \{cu : c \in \mathbb{F}\}$ and $W = \{cw : c \in \mathbb{F}\}$ where no $u = cw$

   - $cu + cu = 2cu$ but $2cu$ is the same as cu where c is a different constant. By definition, this in U.
   - $kcu = cu$ for another $c \in \mathbb{F}$. This is because scaling a $c \in \mathbb{F}$ results in another value in $\mathbb{F}$ Thus U is a subspace. The same arguments follow for $w \in W$
   - Since the sets are disjoint, a direct sum follows by theorem.

2. Give a nontrivial subspace of R^{R^R}

   - The set of functions $f : A \to B$ s.t. their derivtive exists.