ASGN 1

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Abstract

Hey! This is my first time using latex, so it probably won't look great. I also don't really know how to use latex, as you can see from this being in the abstract.

1. Determine if the following are fields.

   (a) $\mathbb{Q}$ is a field.

   (b) $\mathbb{Z}$ is not a field.

   - 7. $\forall x \in \mathbb{Z}$ does not have an inverse. For example $x = 7 : x^{-1} = \frac{1}{7} \notin \mathbb{Z}$

   (c) $\{a \in \mathbb{R} : 0 \leq a\}$

   - 1. Fails multiplication. $-1 * -1 = 1 > 0$
   - 5. There is no element 1 in the set.
   - 6. There is no multiplictive inverse.

   (d) $\{a \in \mathbb{R} : -1 \leq a \leq 1\}$

   - 1. $1 + 1 > 1$

   (e) $\{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$

   - 1. Any nonzero value for b produces an irrational number.

   (f) $\{0, 1\}$ with $0 + 0 = 0, 1 + 0 = 0 + 1 = 1$, and $1 + 1 = 0$: Field. I think this is actually a field.

   (g) {0,1,2} with operations defined in the instructions. I think this is a Galois field?

   (h) $\{0, 1, 2, 3\}$ Field!

   (i) the set of 2x2 matrices with entries in $\mathbb{C}$ is the same as $\mathbb{C}^2$, and after reading the textbook and the listening to the lecture in class, F^n is a field. Thus C^2 is a field.

2. let $a \neq 0$ be an element of a field $\mathbb{F}$ Prove that the element a^{-1} in 7 is unique. Prove that $(a^{-1})^{-1} = a$.

   (a) let $a, b, c \in \mathbb{F}$ such that $a \cdot b = a \cdot c = 1$

$$1 = a \cdot b$$
$$1 = 1 \cdot a \cdot b$$
$$1 = 1 \cdot a \cdot b$$
$$1 = a \cdot c \cdot a \cdot b$$
$$1 = a \cdot c \cdot 1$$
$$1 = a \cdot c$$

(b) let $(a^{-1})^{-1} = b$

$$\begin{aligned}
(a^{-1})^{-1} &= b \\
1 \cdot (a^{-1})^{-1} &= b \\
a \cdot a^{-1}(a^{-1})^{-1} &= b \\
a \cdot 1 &= b \\
a &= b
\end{aligned}$$