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Vector error correction models to measure connectedness of Bitcoin exchange markets

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Abstract

Bitcoins are traded on various exchange platforms and, therefore, prices may differ across trading venues. We aim to investigate return connectedness across eight of the major exchanges of Bitcoin, both from a static and a dynamic viewpoint. To this end, we employ an extension of the order-invariant forecast error variance decomposition proposed by Diebold and Yilmaz (2012) to a generalized vector error correction framework. Our results suggest that there is strong connectedness among the exchanges, as expected, although some of them behave dissimilarly. We identify Bitfinex and Coinbase as leading exchanges during the considered period, while Kraken as a follower exchange. We also obtain that connectedness across exchanges is strongly dynamic, as it evolves over time.

KEYWORDS

Bitcoin, forecast error variance decomposition, market linkages, market risk, spillovers, vector autoregression, vector error correction

1 | INTRODUCTION

Connectedness is gaining much importance in financial econometrics and risk management. The study of return connectedness is key to assess market risk and, in particular, to understand which are the market exchanges whose shocks in price are transmitted to the others; or which are those that receive shocks from the others and adjust their prices consequently. In other words, the study of connectedness across market exchanges is fundamental for price discovery purposes, that is, to determine the leader-follower relationships between markets. This becomes particularly interesting when analyzing nascent markets with peculiar features, such as the cryptocurrency one.

Several researches dealt with econometric connectedness measures development and interdependency measurement. Diebold and Yilmaz[1] propose measures for the total and directional volatility spillovers, based on forecast error variance decompositions from vector autoregressive models (VARs). Diebold and Yılmaz[2] relate the above said forecast error variance decompositions to a network topology representation, and apply it to measure the connectedness of financial firms. In addition, Billio et al[3] develop different econometric measures of connectedness and systemic risk, focusing in the finance and insurance sectors specifically. Ahelegbey et al[4] propose a Bayesian graph-based approach to solve the identification issue in Vector Autoregressive (VAR) models, as well as it contributes to the econometric literature on financial interconnectedness.

The available literature on price discovery on cryptocurrency (Bitcoin) exchanges is quite limited. The first researchers addressing this issue are Brandvold et al,[5] who found that Mt.Gox, a leading exchange that went bankrupt right after their analysis, and BTC-e were the leaders of price discovery. A more recent study by Pagnottoni and Dimpfl[6] makes use of the methodologies of Hasbrouck[7] and Gonzalo and Granger[8] taking into account also the impact of exchange rates. They find Chinese exchanges to be the ones leading price discovery during their analyzed period. More recently, Giudici and Abu-Hashish[9] employ a VAR model that embeds into its correlation structure the connectedness among eight Bitcoin exchanges. All previous papers have the merit of being the first ones in the field but, on the other hand, they are limited as they lack either a less restrictive modeling strategy (as is the case of the first two papers) or do not take into account important econometric aspects such as cointegration and stationarity of the considered series (as is the case with the third paper).

Recently, some studies that examine interconnectedness and spillovers in the cryptocurrency market arose. Koutmos[10] studied interconnectedness among 18 major cryptocurrencies and found, among others, a growing interdependence among them. Another noticeable example is given by Yi et al,[11] who investigate static and dynamic volatility connectedness among eight typical cryptocurrencies and build a volatility connectedness network that links 52 cryptocurrencies by using the LASSO-VAR for estimating high-dimensional VARs. Finally, Corbet et al[12] investigate the dynamic relationships, particularly volatility spillovers, between major cryptocurrencies (Bitcoin, Litecoin, and Ripple) and other financial assets through the generalized variance decomposition technique of Diebold and Yilmaz.[1]

A limitation of the existing literature on interconnectedness among cryptocurrencies is that all contributions employ a generalized VAR model, which does not take a potential cointegration structure of the series into account, a phenomena that is particularly evident when studying connectedness among market exchanges prices concerning the same asset, such as the Bitcoin. We aim to improve the latter contributions and suggest a model that, while fully grounded on an econometric approach, builds a comprehensive statistical model. To this aim, we rely on the order-invariant forecast error variance decomposition proposed by Diebold and Yilmaz.[1]

Indeed, Diebold and Yilmaz[1] develop measures of directional spillovers in a generalized VAR framework, which are suitable for several applications. However, when the time series under consideration are integrated of order one ($I(1)$), the VAR model is not suitable to model them in levels. Moreover, if the same time series additionally show a significant comovement around a common stochastic trend, ie, they are cointegrated, Engle and Granger[13] show that it is reasonable to model them as a vector error correction model (VECM), whose error correction term accounts for the common stochastic trend driving prices.

In line with the previous comment, here, we make use of an extension of the methodology of Diebold and Yilmaz[1] with a generalized VECM.* To the best of our knowledge, this is the first application of such a technique to measure connectedness of exchange platforms, particulary of Bitcoin. The methodology allows us to study market exchange connectedness at different levels: pairwise and system wide, as well as both from a static and time-varying point of view, accounting for the common stochastic trend driving the fundamental Bitcoin price.

We therefore contribute, from a methodological viewpoint to the econometric literature, particularly for what concerns concerning price discovery and connectedness of market exchanges, by employing an extension of the connectedness measure of Diebold and Yilmaz,[1] which relies on VECM rather than VAR models. The model allows to shed further light on price discovery in Bitcoin markets, extending the conclusions in the works of Pagnottoni and Dimpfl[6] and Giudici and Abu-Hashish[9] and, in particular, characterizing which are the leaders and followers in price formation among the considered exchanges, along time.

This paper proceeds as follows. Section 2 contains our methodological proposal. Section 3 presents the data analyzed and provide their preliminary analysis. In Section 4, we discuss the empirical results obtained. Section 5 concludes.

2 | PROPOSAL

We denote the Bitcoin price of an exchange i at time t as X_t^i, whereas its logarithm as X_t^i. We are in the situation in which the same asset is traded across different platforms. In such a framework, the law of one price prescribes that prices related to the same good should not deviate in the long run. Strictly speaking, the no-arbitrage condition implies, when Bitcoin prices are expressed in the same currency, that there exists linear combination of their (log-) prices yielding a stationary process.

The considerations from above make us expect there is a cointegration structure among our variables. Thus, the theoretical econometric framework delineated by Engle and Granger[13] suggests us to exploit the VECM. Namely, denoting the continuous returns for a generic exchange i at time t as

$$\Delta x_t^i = x_t^i - x_{t-1}^i, \tag{1}$$

where $i = 1, 2, \dots, n$ and n is the number of exchanges considered.

*This is in line with Pesaran and Shin,[14] who extend the generalized impulse analysis to VECMs.

Note that continuous returns are the first difference of the exchange prices in log levels. Defining $\Delta x_t = (\Delta x_t^1, \dots, \Delta x_t^i, \dots, \Delta x_t^n)'$ with $i = 1, 2, \dots, n$, the model assumes the following form:

$$\Delta x_t = \alpha\beta' x_{t-1} + \sum_{i=1}^{k-1} \zeta_i \Delta x_{t-i} + \varepsilon_t, \tag{2}$$

with α being the $(n \times h)$ adjustment coefficient matrix, β the $(n \times h)$ cointegrating matrix, ζ_i the $(n \times n)$ parameter matrices with $i = 1, \dots, n$, k the autoregressive order, and ε_t is a zero-mean white noise process having variance-covariance matrix Σ. We denote h as the cointegrating rank. In our case, the time series in levels should show one common stochastic trend, ie, economic theory suggests that the cointegrating rank of the system is $h = n - 1$.

From the VECM$(k-1)$ in (2), one can derive the equivalent n−variable VAR(k) representation, that is,

$$x_t = \sum_{i=1}^{k} \Phi_i x_{t-i} + \varepsilon_t, \tag{3}$$

where $\Phi_1, \Phi_2, \dots, \Phi_k$ with $i = 1, \dots, n$ are the $(n \times n)$ autoregressive parameter matrices. This is done recalling that $\alpha\beta' = \sum_{i=1}^{k} \Phi_i - I_n$ and $\Psi_i = -\sum_{j=i+1}^{k} \Phi_j$.

Equation (3) is the starting point of the approach developed by Diebold and Yilmaz.[1] Indeed, to retrieve the impact of shocks on the system variables to others, we rewrite the VAR model in (3) into its vector moving average (VMA) representation

$$x_t = \varepsilon_t + \Psi_1 \varepsilon_{t-1} + \Psi_2 \varepsilon_{t-2} + \cdots, \tag{4}$$

where we denote as $\Psi_1, \Psi_2, \dots$ the $(n \times n)$ matrices containing the VMA coefficients. The VMA coefficients are such that the recursion $\Psi_i = \Phi_1 \Psi_{i-1} + \Phi_2 \Psi_{i-2} + \cdots + \Phi_i \Psi_1$ holds true, with $\Psi_i = 0 \; \forall i < 0$ and $\Psi_1 = I_n$.

The VMA representation of the system is fundamental to evaluate the effect of a shock in one system variable on the others due to the impulse response functions and variance decomposition tools. In particular, the variance decomposition allows to decompose the H-step-ahead error variance in predicting x_i due to shocks to x_j, $\forall j \neq i$ and $\forall i = 1, \dots, n$.

In our paper, we rely, as in Diebold and Yilmaz,[1] on Koop-Pesaran-Potter-Shin[15,16] (KPPS) H-step-ahead forecast errors. They have the advantage to be invariant to the variable ordering, unlike the popular although restrictive Cholesky factorization, which would require an ordering of the Bitcoin exchange prices *a priori* with regard to the influence of shocks across the system variables.

Taking two generic variables x_i and x_j, Diebold and Yilmaz[1] define the own variance shares as the proportion of the H-step-ahead error variance in forecasting x_i due to shocks in x_i itself, $\forall i = 1, \dots, n$, whereas the cross variance shares (spillovers) are defined as the H-step-ahead error variance in predicting x_i due to shocks in x_j, $\forall i = 1, \dots, n$ with $j \neq i$.

That said, using $\theta_{ij}^{g}(H)$ to denote the KPSS H-step-ahead forecast error variance decompositions, with $h = 1, \dots, H$, we have

$$\theta_{ij}^{g}(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} \left(e_i' \Psi_h \Sigma e_j\right)^2}{\sum_{h=0}^{H-1} \left(e_i' \Psi_h \Sigma \Psi_h' e_j\right)}, \tag{5}$$

where σ_{jj} is the standard deviation of the innovation for equation j and e_i represents the selection vector with one as element i and zeros elsewhere.

However, we have that $\sum_{h=0}^{H-1} \theta_{ij}^{g}(H) \neq 1$, ie, the sum of the row elements of the generalized variance decomposition is not equal to 1. That is the reason why in the calculation of the spillover indexes, Diebold and Yilmaz[1] proposed to normalize the entries of the variance decomposition matrix by the row sum, that is,

$$\tilde{\theta}_{ij}^{g}(H) = \frac{\theta_{ij}^{g}(H)}{\sum_{j=1}^{n} \theta_{ij}^{g}(H)}. \tag{6}$$

By construction, we have that $\sum_{j=1}^{n} \tilde{\theta}_{ij}^{g}(H) = 1$ as well as $\sum_{j,i=1}^{n} \tilde{\theta}_{ij}^{g}(H) = n$.

From the use of the total contributions to the forecast error variance decomposition, we estimate the Total Spillover Index (TSI) as

$$TSI(H) = \frac{\sum_{\substack{j=1\\ j\neq i}}^{n} \tilde{\theta}_{ij}^{g}(H)}{\sum_{j,i=1}^{n} \tilde{\theta}_{ij}^{g}(H)} \cdot 100 = \frac{\sum_{\substack{j=1\\ j\neq i}}^{n} \tilde{\theta}_{ij}^{g}(H)}{n} \cdot 100. \tag{7}$$

Moreover, we also get the Directional Spillovers Indexes (DSI) to measure respectively through Equations (8) and (10) the spillover from exchange i to all exchanges J, as well as the spillover from all exchanges J to exchange i as

$$DSI_{J\leftarrow i}(H) = \frac{\sum_{\substack{j=1\\ j\neq i}}^{n} \tilde{\theta}_{ji}^{g}(H)}{\sum_{j,i=1}^{n} \tilde{\theta}_{ij}^{g}(H)} \cdot 100 \tag{8}$$

$$DSI_{i\leftarrow J}(H) = \frac{\sum_{\substack{j=1\\ j\neq i}}^{n} \tilde{\theta}_{ij}^{g}(H)}{\sum_{j,i=1}^{n} \tilde{\theta}_{ij}^{g}(H)} \cdot 100. \tag{9}$$

We also derive the Net Spillover Index (NSI) from market i to all other markets J as

$$NSI_{J\leftarrow i}(H) = DSI_{i\leftarrow J}(H) - DSI_{J\leftarrow i}(H). \tag{10}$$

Finally, we derive the Net Pairwise Spillovers (NPS) to measure the difference between the gross shocks transmitted from market i to j and gross shocks transmitted from j to i.

$$PNS_{ij}(H) = \left(\frac{\tilde{\theta}_{ij}^{g}(H)}{\sum_{q=1}^{n} \tilde{\theta}_{iq}^{g}(H)} - \frac{\tilde{\theta}_{ji}^{g}(H)}{\sum_{q=1}^{n} \tilde{\theta}_{jq}^{g}(H)} \right) \cdot 100. \tag{11}$$

By means of the variance decompositions, these measures allow to analyze exchange spillovers both from a system wide and a net pairwise viewpoint. Outcomes are presented in the results section.

3 | DATA

We consider for our empirical analysis what is arguably the most relevant cryptocurrency nowadays existing: Bitcoin. We indeed examine Bitcoin exchange prices denominated in USD on a daily basis during a timeframe from 18 May 2016 and 30 April 2018, as in the work of Giudici and Abu-Hashish.[9] Data were collected from https://www.investing.com/crypto/bitcoin and through the CryptoCompare API. With the aim to study system-wise connectedness as well as the pairwise one

TABLE 1 Bitcoin daily trading volume shares

Exchange	Daily volume share
Bitfinex	27.37%
Coinbase	12.45%
Bitstamp	11.05%
Kraken	9.97%
HitBTC	8.56%
Gemini	3.08%
Bittrex	1.84%
itBit	1.61%
Total	75.93%

Note: The table from above shows daily trading volume shares for the considered exchanges at the end of the sample period considered (30 April 2018). We synthesized data retrieved from https://data.bitcoinity.org/markets.

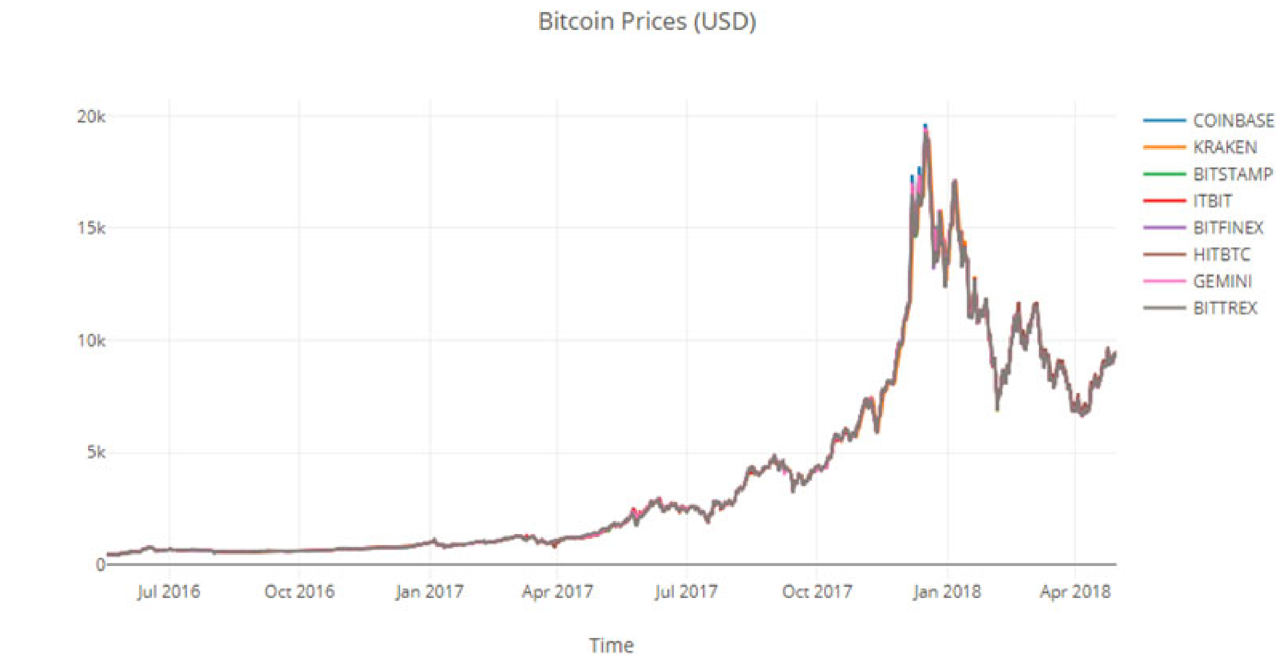

FIGURE 1 Bitcoin exchange price evolution. *Note:* The figure from above illustrates the eight Bitcoin exchange price series related to the full sample period [Colour figure can be viewed at wileyonlinelibrary.com]

among Bitcoin trading venues, we consider eight Bitcoin exchanges, ie, Bitfinex, Coinbase, Bitstamp, HitBTC, Gemini, itBit, Kraken, and Bittrex. We remark that the investigated exchanges are geographically widespread, with their daily trading volumes summing up to more than 75% at the ending date of the sample, as illustrated in Table 1.

The Bitcoin price dynamics of the considered exchanges are illustrated in Figure 1.

From Figure 1, note that prices related to the eight Bitcoin exchanges follow a common pattern, a result in line with economic expectations, as we are investigating the same asset traded on different venues. Furthermore, given the dynamics of Bitcoin prices over time, it is reasonable to expect that, in our case, we deal with nonstationary time series, arguably $I(1)$.

However, we also provide a plot of the difference between the return of each series and the average return across exchanges in Figure 2.

What emerges from a visual inspection of Figure 2 is that prices (and returns) may vary quite consistently across platforms. Specifically, there are some dates in which the misalignment across exchanges prices is particularly marked. The summary statistics in Table 2 gives an overview of the features of the dataset and confirms the previous claims.

As a preliminary analysis, we compute the pairwise correlation existing among the exchange returns and test for their significance. Results are contained in Table 3.

Table 3 confirms that, as we are investigating the same asset traded on different platforms, returns exhibit high pairwise correlations. However, some exchanges present lower pairwise correlations with the others, meaning that their dynamics has a weaker link with that of the other analyzed exchanges. In particular, we find that Kraken has a weak link with the other exchanges during the analyzed timespan. As a matter of fact, considering that we are analyzing prices related to the same asset, correlation involving Kraken is low on a relative basis, as it is also proved by the values of correlations between other platforms. Note that all correlations are tested and found to be significant at all conventional significance levels, with p-values well below 1%.

Moreover, to control for the effect of the other exchanges in the pairwise relationship, we compute the partial correlations of the Bitcoin exchange returns. Results are shown in Table 4.

First of all, partial correlations further support our previous considerations. Indeed, Kraken shows a negative partial correlation with as much as four of the other exchanges out of seven, further confirming its dissimilar behavior. Secondly, partial correlations already suggest that the interconnectedness among returns of different exchanges is of heterogeneous nature. Indeed, while most of the exchanges show positive partial correlations, some of which of a relatively high magnitude (being the highest +0.5843 for Coinbase and Gemini), a few of them show negative partial correlation (being the lowest −0.0694 for Bitfinex and Kraken).

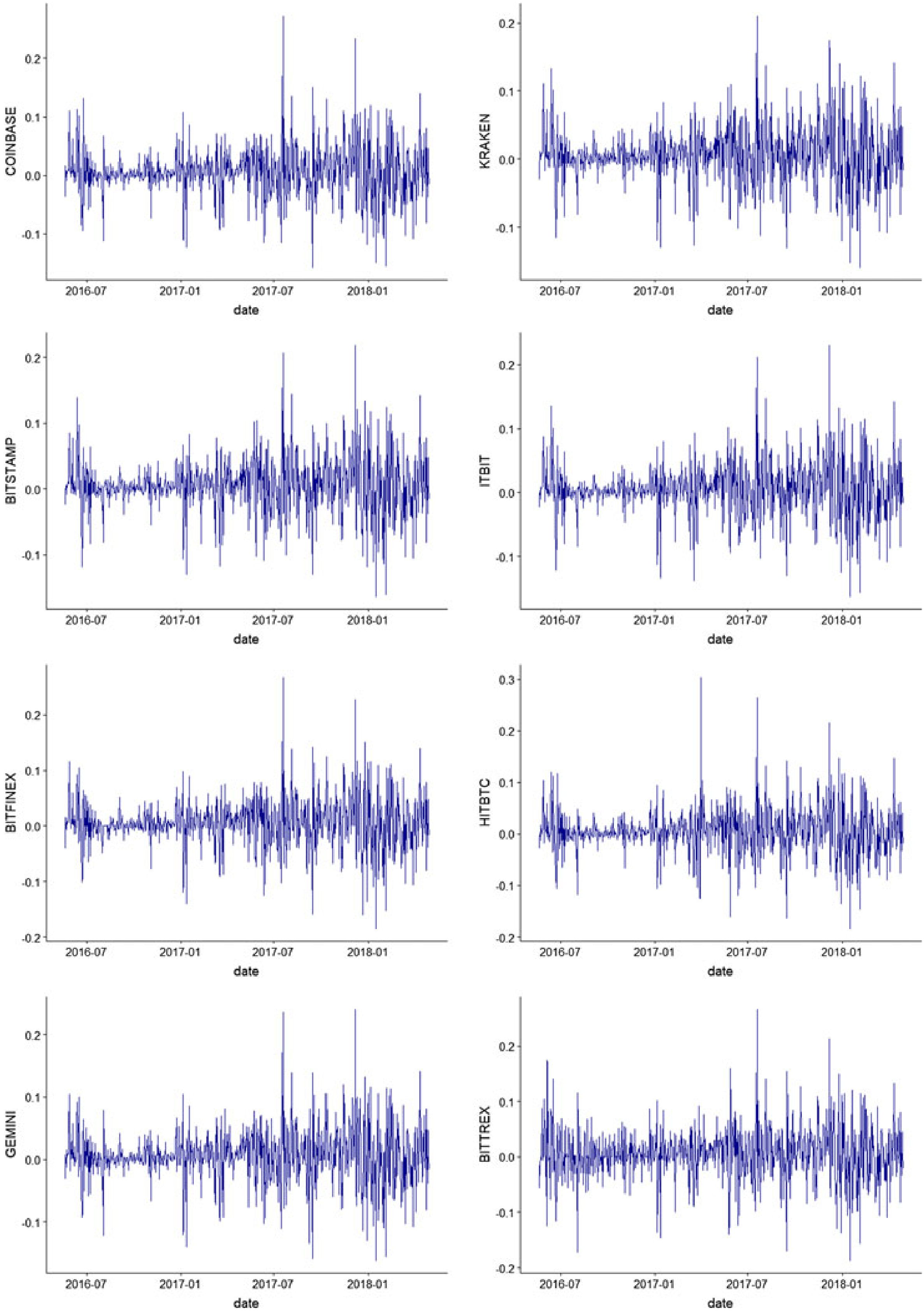

FIGURE 2 Exchange return deviation from daily average. *Note:* The figure shows the return deviation of each exchange from the daily return average of the exchanges. The plot refers to the full sample period [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 2 Summary statistics of returns

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
Mean	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042
Median	0.0042	0.0049	0.0041	0.0040	0.0042	0.0036	0.0033	0.0028
Maximum	0.2406	0.1906	0.1976	0.2073	0.2372	0.2653	0.2154	0.2364
Minimum	−0.1707	−0.1728	−0.1782	−0.1770	−0.2038	−0.2023	−0.1764	−0.2069
Std. Dev.	0.0449	0.0442	0.0440	0.0440	0.0458	0.0473	0.0453	0.0500
Skewness	0.0471	−0.2035	−0.1763	−0.1685	−0.1743	0.0527	−0.1779	−0.1906
Kurtosis	6.2512	5.0798	5.5469	5.7150	6.2570	6.7983	5.9265	5.5252

Note: The table includes relevant summary statistics for returns related to the analyzed exchanges considering the entire sample period. All correlations are significant at a 1% significance level.

TABLE 3 Returns correlation matrix

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
Coinbase	1.0000	0.7737	0.9459	0.9155	0.9674	0.9263	0.9784	0.8858
Kraken	0.7737	1.0000	0.8224	0.7896	0.7670	0.7298	0.7703	0.7211
Bitstamp	0.9459	0.8224	1.0000	0.9609	0.9504	0.9030	0.9499	0.8807
itBit	0.9155	0.7896	0.9609	1.0000	0.9161	0.8722	0.9200	0.8525
Bitfinex	0.9674	0.7670	0.9504	0.9161	1.0000	0.9277	0.9682	0.8948
HitBTC	0.9263	0.7298	0.9030	0.8722	0.9277	1.0000	0.9215	0.8542
Gemini	0.9784	0.7703	0.9499	0.9200	0.9682	0.9215	1.0000	0.8922
Bittrex	0.8858	0.7211	0.8807	0.8525	0.8948	0.8542	0.8922	1.0000

Note: The table illustrates the return correlation matrix related to the full sample. Significance tests show that correlations are all significant at 1% significance level.

TABLE 4 Returns partial correlation matrix

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
Coinbase	1.0000	0.0656	0.0373	0.0209	0.2652	0.1830	0.5843	0.0014
Kraken	0.0656	1.0000	0.3316	−0.0022	−0.0694	−0.0236	−0.0455	0.0251
Bitstamp	0.0373	0.3316	1.0000	0.6380	0.2366	0.0529	0.1232	0.0701
itBit	0.0209	−0.0022	0.6380	1.0000	−0.0407	0.0016	0.0500	0.0240
Bitfinex	0.2652	−0.0694	0.2366	−0.0407	1.0000	0.2249	0.2571	0.1668
HitBTC	0.1830	−0.0236	0.0529	0.0016	0.2249	1.0000	0.0378	0.0774
Gemini	0.5843	−0.0455	0.1232	0.0500	0.2571	0.0378	1.0000	0.1164
Bittrex	0.0014	0.0251	0.0701	0.0240	0.1668	0.0774	0.1164	1.0000

Note: The table illustrates the return partial correlation matrix related to the full sample.

TABLE 5 Stationarity tests

	ADF tests							
	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
$log(p_t)$	0.8622	0.8653	0.8660	0.8646	0.8589	0.8598	0.8613	0.8547
$\Delta log(p_t)$	<0.0010	<0.0010	<0.0010	<0.0010	<0.0010	<0.0010	<0.0010	<0.0010
	KPSS tests							
	Coinbase	**Kraken**	**Bitstamp**	**itBit**	**Bitfinex**	**HitBTC**	**Gemini**	**Bittrex**
$log(p_t)$	<0.0100	<0.0100	<0.0100	<0.0100	<0.0100	<0.0100	<0.0100	<0.0100
$\Delta log(p_t)$	>0.1000	>0.1000	>0.1000	>0.1000	>0.1000	>0.1000	>0.1000	>0.1000

Note: The table above illustrates the resulting p-values for the Augmented Dickey-Fuller (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests for the entire sample period. The tests in levels are executed including a constant but no time trend, as well as the KPSS ones do not include trends. Both tests are conducted using an optimal lag length determined according to the Bayes-Schwarz information criterion, as well as on a 5% significance level. The minimum p-value reported is 0.001 for the ADF and 0.01 KPSS tests, whereas the maximum p-value reported for the KPSS test is 0.1.

As a further preliminary analysis, we perform stationarity and cointegration tests to ensure that the series we analyze meet the features we expect.

To test for (non)stationarity, we conduct the Augmented Dickey-Fuller (ADF)[17] and Kwiatkowski-Phillips-Schmidt-Shin (KPSS)[18] tests on prices, expressed in log levels. The results from the tests are reported in Table 5.

TABLE 6 Cointegration (Max Eigenvalue test)

h	Statistics	Critical value	p-value	eigVal
0	215.0009	48.8789	< 0.0010	0.2613
1	171.1194	42.7706	< 0.0010	0.2142
2	167.6692	36.6291	< 0.0010	0.2103
3	136.1324	30.4392	< 0.0010	0.1745
4	122.6643	24.1605	< 0.0010	0.1587
5	57.1105	17.7966	< 0.0010	0.0773
6	32.6407	11.2252	< 0.0010	0.0449
7	4.9952	4.1302	0.0302	0.0070

Note: The table above illustrates the statistics, critical values, p-values end eigenvalues for the Johansen Maximum Eigenvalue test for cointegration for the full sample period. The test does not include any constant or time trend, neither in the model specification nor in the cointegrating relationship. The minimum p-value reported is 0.001.

TABLE 7 Cointegration (Trace test)

h	Statistics	Critical value	p-value	eigVal
0	907.3326	154.8020	< 0.0010	0.2613
1	692.3317	121.7464	< 0.0010	0.2142
2	521.2123	92.7173	< 0.0010	0.2103
3	353.5431	67.6430	< 0.0010	0.1745
4	217.4107	46.5743	< 0.0010	0.1587
5	94.7464	29.5103	< 0.0010	0.0773
6	37.6359	16.3589	< 0.0010	0.0449
7	4.9952	6.9399	0.0302	0.0070

Note: The table above illustrates the statistics, critical values, p-values end eigenvalues for the Johansen Trace test for cointegration related to the full sample period. The minimum p-value reported is 0.001.

From Table 5, note that both tests point toward nonstationarity of the Bitcoin log prices, whereas when first differencing the time series, the tests provide support toward stationarity, for all conventional significance level (10%, 5%, 1%). We may then argue that the series are found to be $I(1)$.

As far as cointegration is concerned, we conduct the Johansen Maximum Eigenvalue and the Trace tests.[19] As mentioned in Section 2, we expect the cointegrating rank of the system to be $h = n - g = 8 - 1 = 7$, given that we have the same asset (Bitcoin) traded on different platforms and we therefore reasonably expect the series to be driven by $g = 1$ common stochastic trend. The results of the Maximum Eigenvalue and Trace test are presented in Table 6 and Table 7, respectively.

Tables 6 and 7 show that, although the evidence is not particularly strong, the p-values associated to the tests are low enough to reject their null hypotheses at a 1% significance level, when compared to the alternative $h = 7$, meaning that tests point to a cointegrating rank of 7, which is consistent with the fundamental economic law of "one underlying asset, one price".

4 | EMPIRICAL FINDINGS

As already anticipated, we study connectedness and lead-lag relationships across Bitcoin exchanges both from a static point of view and from a dynamic one. In Section 4.1, we will present the full sample results. In Section 4.2, we split the sample to investigate connectedness and leaderships before and after a period of arguable structural break. In Section 4.3, we perform a dynamic analysis to study the evolution of connectedness and price discovery on Bitcoin exchanges.

4.1 | Full sample results

Here, we present our results on the spillover analysis related to the full sample period that is from 18 May 2016 to 30 April 2018. All results are based on a VECM of order 2, where the order is determined using the Bayes-Schwarz Information Criterion (BIC), and a generalized variance decomposition of the $H = 10$ step-ahead forecast errors.[†]

[†]The choice of $H = 10$ step-ahead forecast errors for the generalized variance decomposition is made for the sake of consistency and comparability with that of the work of Diebold and Yilmaz.[1]

TABLE 8 Directional Spillover Indexes

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
FROM	10.7430	11.6435	11.0222	11.0059	10.6811	10.8392	10.7684	10.6276
TO	12.1979	5.9122	10.2804	10.3914	12.5541	11.1419	12.0487	12.8043
NET	**1.4549**	**−5.7313**	**−0.7418**	**−0.6145**	**1.8730**	**0.3027**	**1.2803**	**2.1767**

Note: The table shows the directional return spillover indexes "from" others (FROM), "to" others (TO), as well as the net ones (NET) for the full sample period. Values are expressed in percentage terms.

TABLE 9 Net Pairwise Spillovers

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
Coinbase	-	−7.21	−2.23	−2.12	0.48	−1.1	−0.19	0.73
Kraken	7.21	-	4.97	5.12	7.61	5.99	7.03	7.92
Bitstamp	2.23	−4.97	-	0.12	2.57	1.06	2.04	2.88
itBit	2.12	−5.12	−0.12	-	2.44	0.94	1.91	2.75
Bitfinex	−0.48	−7.61	−2.57	−2.44	-	−1.58	−0.61	0.31
HitBTC	1.1	−5.99	−1.06	−0.94	1.58	-	0.93	1.95
Gemini	0.19	−7.03	−2.04	−1.91	0.61	−0.93	-	0.86
Bittrex	−0.73	−7.92	−2.88	−2.75	−0.31	−1.95	−0.86	-

Note: The table contains the net pairwise return spillovers for the full sample period. Values are expressed in percentage terms.

The TSI based on the full sample is 87.33%. The high value of the index shows that Bitcoin exchange prices, as we expect, are highly interconnected among each other. In other words, a consistent portion of the forecast error variance of the system is due to contributions among exchanges, rather than on the own contribution of single exchanges.

We then provide outcomes for the "from", "to", and "net" Directional Spillover Indexes (DSI) in Table 8.

Looking at the results in Table 8, we may argue that the directional spillovers have quite similar magnitude across all exchanges, with Kraken being the most influenced when shocks in other venues occur and with Bittrex being the least influenced exchange. When looking at the directional spillovers to others, however, the picture slightly changes. Exchanges still show quite similar magnitudes regarding spillovers to others, with Bittrex surprisingly resulting as the one giving more spillover to others, followed by Bitfinex and Coinbase. This is likely due to a high peak that is registered in one single data point, which influences our results. This explanation is further clarified in the dynamic analysis, and it is related to the anomalous return misalignment discussed in the previous section. On the other hand, the exchange having the smallest impact toward the others in the system is Kraken, which counts only about 5.91% of directional spillover to others, a relatively low value when compared to the rest of the exchanges analyzed.

The interpretation of the net directional spillovers in Table 8 is immediate, given that they represent the difference between the gross shocks transmitted to and received from other platforms. We can see that the exchange showing the strongest positive net contribution is Bittrex (+2.18%), consistently with what commented before, followed by Bitfinex (+1.87%) and Coinbase (+1.45%). Gemini (+1.28%) and HitBTC (+0.30%) also show a positive net contribution to others, despite a lower magnitude. itBit (−0.61%), Bitstamp (−0.74%), and Kraken (−5.73%), instead, show a negative net return spillover, meaning that the return shocks they receive is greater than those transmitted to all other exchanges. This is particularly true for Kraken, that is, the exchange resulting more sensitive to return shocks occurring in other platforms.

To provide a wider picture of connectedness, we also investigate Net Pairwise (return) Spillovers (NPS) between exchanges, whose results are presented in a tabular fashion in Table 9. In this way, we are able to assess the pairwise net contribution to return shocks of each exchange with the remaining ones, investigating pairwise connectedness.

From Table 8, the net pairwise spillover outcomes are in line with what observed before. On one hand, Bittrex transmits return spillovers to all other exchanges, with the biggest contribution being the one toward Kraken. On the other hand, Kraken is the most influenced exchange from a price settlement point of view, receiving return spillovers from all other platforms. We may additionally notice that the magnitude of spillovers toward Kraken are relatively high.

The analysis of net pairwise spillovers helps in discriminating between the leader and follower Bitcoin exchanges, in terms of transmitting information about price changes to others. Besides the already mentioned Bittrex, we can identify Bitfinex as a leading exchange, given its influence exerted to all other markets except for Bittrex. We also identify Coinbase as a leading exchange, whereas Bitstamp and itBit are followers. The remaining exchanges show both positive and negative pairwise spillovers, meaning that their behavior is dissimilar with respect to the exchanges analyzed.

TABLE 10 Directional Spillover Indexes - Subsample 1

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
FROM	12.44	5.85	9.74	9.63	12.38	11.62	12.29	13.35
TO	10.71	11.65	11.10	11.11	10.70	10.77	10.73	10.53
NET	**1.73**	**−5.80**	**−1.36**	**−1.48**	**1.68**	**0.85**	**1.56**	**2.82**

Note: The table shows the directional return spillover indexes “from” others (FROM), “to” others (TO), as well as the net ones (NET) for the subsample 1 (18 May 2016 -15 December 2017). Values are expressed in percentage terms.

TABLE 11 Directional Spillover Indexes - Subsample 2

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
FROM	11.23	7.59	11.53	11.62	11.28	11.56	11.36	11.28
TO	10.89	11.40	10.85	10.84	10.89	10.81	10.88	10.89
NET	**0.33**	**−3.82**	**0.68**	**0.78**	**0.40**	**0.75**	**0.49**	**0.39**

Note: The table shows the directional return spillover indexes “from” others (FROM), “to” others (TO), as well as the net ones (NET) for the subsample 2 (16 December 2017 - 30 April 2018). Values are expressed in percentage terms.

TABLE 12 Net Pairwise Spillovers - Subsample 1

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
Coinbase	-	−7.60	−3.14	−3.29	0.04	−0.80	−0.20	1.12
Kraken	7.60	-	4.46	4.36	7.44	6.54	7.42	8.56
Bitstamp	3.14	−4.46	-	−0.13	3.00	2.23	2.94	4.15
itBit	3.29	−4.36	0.13	-	3.12	2.36	3.06	4.26
Bitfinex	−0.04	−7.44	−3.00	−3.12	-	−0.84	−0.15	1.18
HitBTC	0.80	−6.54	−2.23	−2.36	0.84	-	0.64	2.06
Gemini	0.20	−7.42	−2.94	−3.06	0.15	−0.64	-	1.22
Bittrex	−1.12	−8.56	−4.15	−4.26	−1.18	−2.06	−1.22	-

Note: The table contains the net pairwise return spillovers for the subsample 1 (18 May 2016 - 15 December 2017). Values are expressed in percentage terms.

We remark that, overall, the size of return spillovers appears quite linked to the trading volume sizes of the exchanges themselves. In other words, exchanges whose trading volume is large generally show positive net contributions to return spillovers, whereas for smaller ones the same quantity is mostly negative.

4.2 | Subsample results

It could be inferred from Figure 1 that the Bitcoin price series exhibits a possible structural break. Indeed, the series has a positive trend, building up to mid December 2017, immediately followed by a downward trend. This phenomenon has attracted a lot of attention in the community and it should be strictly linked to the leadership evolution of the exchanges and their interconnectedness, which deserve to be investigated.

The previous setup motivates our approach, which consists of splitting the analysis into two subsamples, denoted as subsample 1 (18 May 2016 - 15 December 2017) and subsample 2 (16 December 2017 - 30 April 2018). We then conduct the same analysis as described before and analyze connectedness and price discovery in the two periods of interest. All results refer to VECMs of order determined through the BIC and $H = 10$ step-ahead forecast errors for the generalized variance decomposition.

From Tables 10 and 11, our expectations are confirmed. Overall, connectedness seems to be quite stable in magnitude. Despite that, the size of net return spillovers among Bitcoin exchanges shrinks in the period after the price surge, which occurred until the end of 2017. Besides different magnitudes, the exchanges additionally show different directions of the net contribution in terms of return spillover. To illustrate, Bitstamp and itBit turn from showing a negative net spillover in the first phase, whereas they exhibit a positive net contribution for the second phase, ie, the Bitcoin price decline. Furthermore, the exchange that in the previous section showed the strongest positive net return spillover to others reveal to be highly weaker in its magnitude of contribution when considering the Bitcoin price decline timespan.

To complete the overview, we investigate the net pairwise spillovers related to the two subsamples, which are presented in a tabular fashion in Tables 12 and 13.

TABLE 13 Net Pairwise Spillovers - Subsample 2

	Coinbase	Kraken	Bitstamp	itBit	Bitfinex	HitBTC	Gemini	Bittrex
Coinbase	-	−4.04	0.33	0.42	0.04	0.40	0.15	0.03
Kraken	4.04	-	4.44	4.57	4.31	4.60	4.23	4.34
Bitstamp	−0.33	−4.44	-	0.10	−0.30	0.05	−0.18	−0.31
itBit	−0.42	−4.57	−0.10	-	−0.42	−0.05	−0.27	−0.43
Bitfinex	−0.04	−4.31	0.30	0.42	-	0.36	0.10	−0.01
HitBTC	−0.40	−4.60	−0.05	0.05	−0.36	-	−0.25	−0.36
Gemini	−0.15	−4.23	0.18	0.27	−0.10	0.25	-	−0.12
Bittrex	−0.03	−4.34	0.31	0.43	0.01	0.36	0.12	-

Note: The table contains the net pairwise return spillovers for the subsample 2 (16 December 2017 - 30 April 2018). Values are expressed in percentage terms.

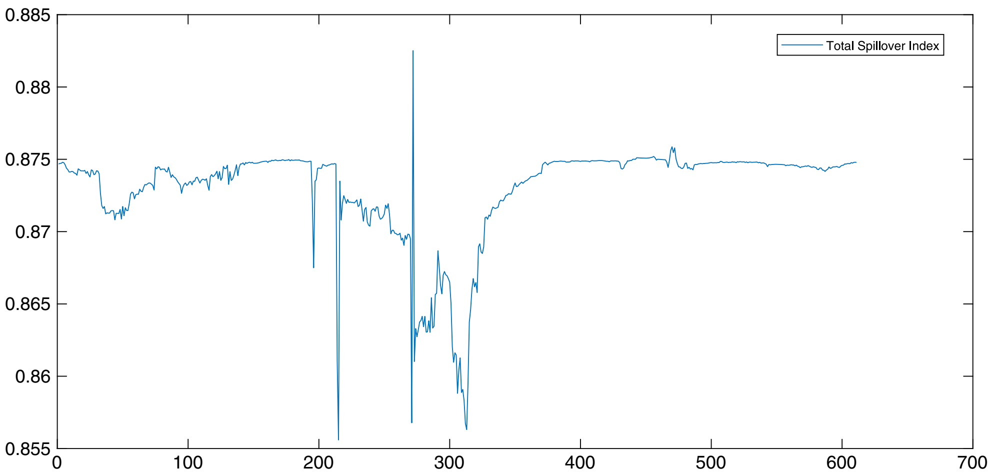

FIGURE 3 Total Spillover Index. *Note:* The plot contains the dynamic TSI for the period 22 September 2016 - 30 April 2018. The rolling window set for the estimations is $w = 125$ days [Colour figure can be viewed at wileyonlinelibrary.com]

Pairwise results further confirm our expectations regarding the potential change in exchange connectedness between the two analyzed subperiods. In line with the considerations from above, pairwise net return spillovers lose their original size when considering the second subsample.

In particular, we can notice that Kraken behaves as a follower throughout the whole analyzed period. Indeed, even though magnitudes of spillovers are indisputably bigger when considering the first subsample, it keeps receiving price change spillovers from other exchanges during the second timeframe in a considerable manner. In contrast, as already suggested by the net spillovers, some of the exchanges change their behavior from one phase to the other. This is particularly true for Bittrex, showing most of its pairwise net contributions changing not only magnitude but also sign from one period to the other. We may therefore conclude that the leadership composition of the exchanges also varies accordingly over time. This is in line with what found in the price discovery studies by Brandvold et al[5] and Pagnottoni and Dimpfl,[6] which rely on the information shares by Hasbrouck[7] and on the common factor weights approaches by Gonzalo and Granger.[8] Both researches find that information shares are dynamic and significantly evolve over time.

The latter findings highlight the importance of tuning the right estimation timespans to get meaningful insights. This further motivates our dynamic analysis, which is performed in Section 4.3.

4.3 | Dynamic results

As noticed by Diebold and Yilmaz,[1] the full sample outcomes provide information about the "average" or "unconditional" features of connectedness. However, in most applications, it is of interest to examine the exchange connectedness dynamics over time. This is the reason why in this section, we perform a dynamic analysis by means of rolling window estimations. In detail, we fix a rolling estimation window of 125 day and a 10-step-ahead forecast horizon for the variance decomposition.[‡] As already anticipated, the dynamic analysis should be also able to better explain the outcomes of the "unconditional" connectedness measures derived before.

At first, we examine the TSI, whose plot is illustrated in Figure 3.

[‡]Repeating the analysis with different choices of the rolling estimation window and forecast horizon steps, ie, increasing and decreasing them up to 50% of the fixed choices, shows that results do not change appreciably.

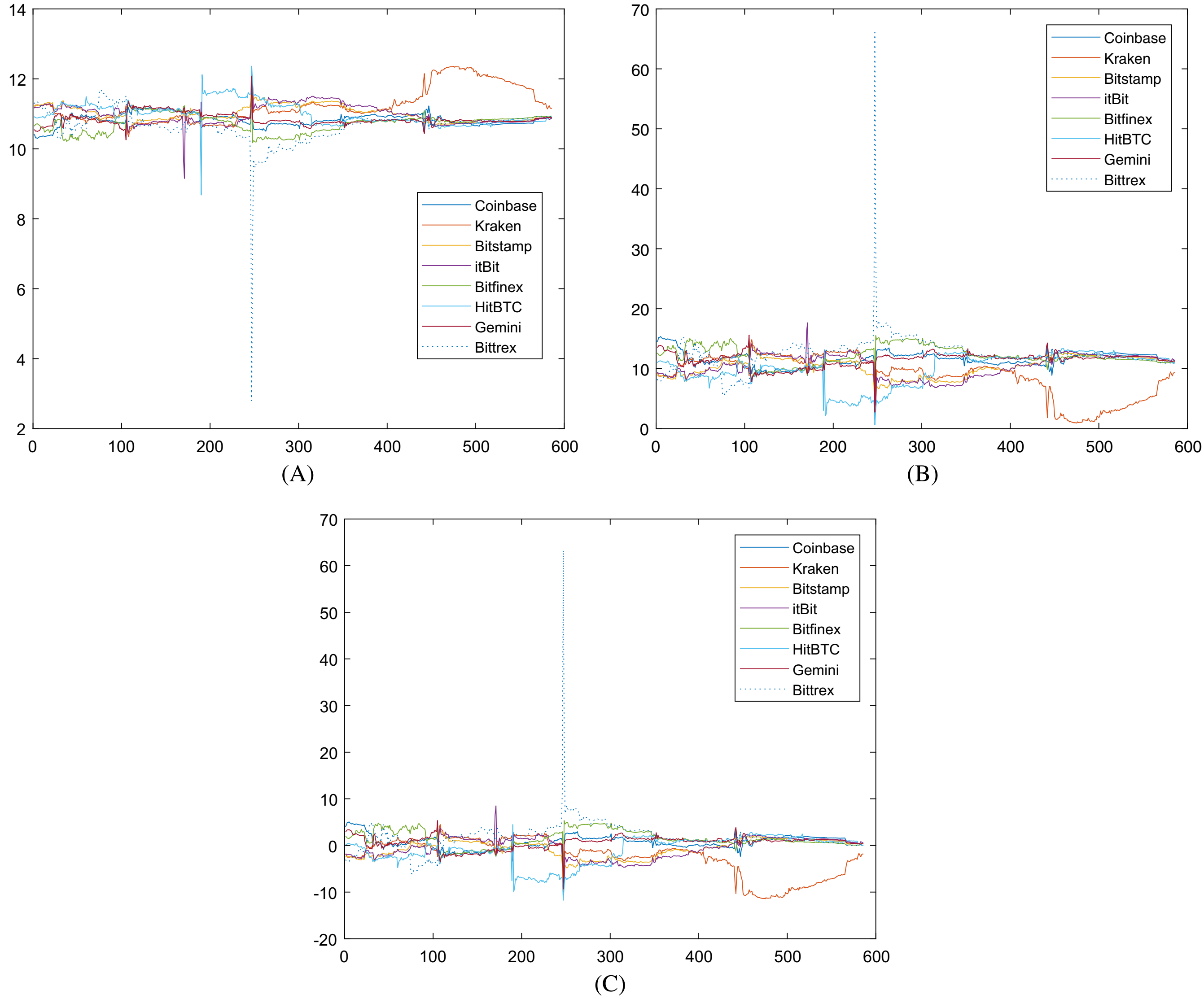

FIGURE 4 Directional and Net Spillover Indexes. A, Directional Spillover Index "from"; B, Directional Spillover Index "to"; C, Net Spillover Index. *Note:* The figure from above illustrates the dynamic “from”, “to”, and “net” return spillover indexes for the period 22 September 2016 - 30 April 2018. The rolling window set for the estimations is w = 125 days. Values are expressed in percentage terms [Colour figure can be viewed at wileyonlinelibrary.com]

We may notice two different cycles in the Bitcoin return spillover evolution across the exchanges. After a first period of relative stability for the index, from observation 171 (4 November 2016) to roughly 371 (25 May 2017), we can notice some turbulences. Indeed, the index becomes more volatile during this second phase, with exchanges weakening their overall connectedness, behaving more dissimilarly than before. This is particularly true for some of the exchanges. However, from the end of May 2017 onwards, the TSI goes back toward its initial values and remains quite stable. The most relevant peaks correspond to the dates we have discussed in the data section, ie, the points in time in which we have a strong misalignment in Bitcoin prices of the analyzed exchanges. Those misalignment are likely due to exits and entrances of big players in the market.

We then analyze the DSI “from” and “to” others, as well as the NSI, which are illustrated in Figure 4.

From Figure 4, note that peaks in the TSI are mainly caused by single exchange returns, whose misalignment create turbulences in the spillover indexes. The clearest example is the spillover from Bittrex to others, whose influence on 24 May 2017 becomes suddenly high and almost immediately levels out. This occurrence has likely influenced our full sample results about Bittrex being a return spillover transmitter.

However, what we are interested in is the dynamic leader-follower relationship among exchanges, which can be visually inspected by jointly examining the three figures. In this regard, results are again in line with those obtained in the full

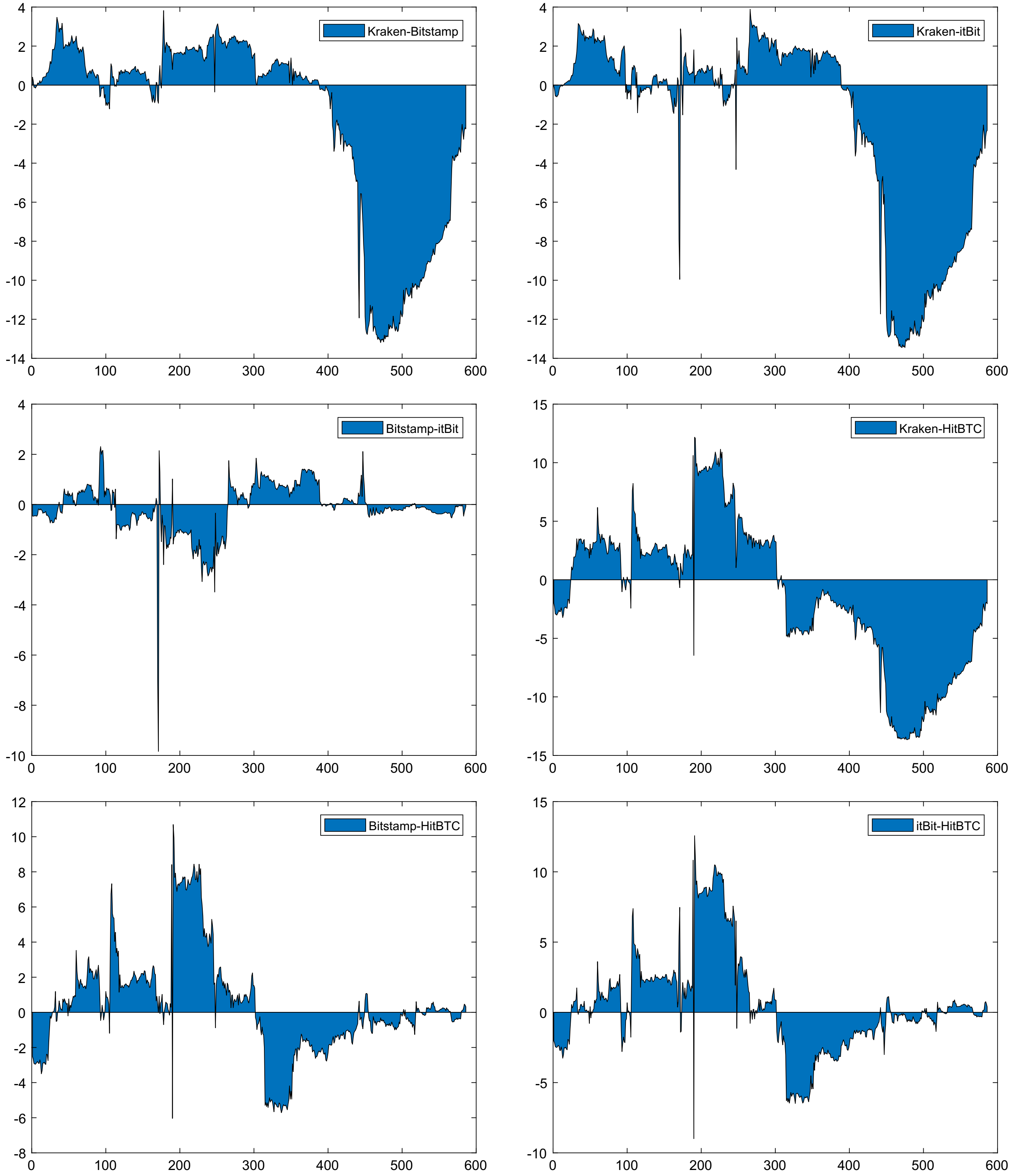

FIGURE 5 Net Pairwise Spillovers. *Note:* The figures from above illustrate the net pairwise return spillovers between the four selected exchanges, that is, Kraken, Bitstamp, itBit, and HitBTC, for the period 22 September 2016 - 30 April 2018. The rolling window set for the estimations is $w = 125$ days. Values are expressed in percentage terms [Colour figure can be viewed at wileyonlinelibrary.com]

sample analysis: besides Bittrex, Bitfinex appears to be the exchange receiving the least and contributing the most to others in terms of return spillovers over time, immediately followed by Coinbase. In both cases, the magnitude of their influence varies over time. Notice also that Kraken is the exchange being the most influenced from others. From the beginning of

May 2017, its return spillover contribution to others starts declining, whereas the one transmitted by others begins to rise. It is interesting to notice that Kraken's follower behavior begins with the surge in Bitcoin prices, a day in which exchanges connectedness are arguably expected to experience some changes. This marks the beginning of a “follower phase” for Kraken, which lasts until the end of the sample, where we see its net contribution converging to its previous values.

As a final step, we investigate the evolution of the net pairwise spillovers between exchanges over time. We focus on the net pairwise spillovers between the four platforms showing the weakest net spillovers in the full sample analysis, namely, Kraken, Bitstamp, itBit, and HitBTC, whose dynamics are shown in Figure 5. Although Kraken mostly dominates small exchanges during the first period of analysis (roughly until May 2015), it starts losing its positive influence by then and receives return spillovers. It is clear that spillovers show some kind of cyclicity. However, taking the Bitstamp-HitBTC plot as an example, we may argue that major exchanges (in terms of trading volumes) generally show larger magnitudes of transmitted shocks, with respect to received shocks, as expected.

In general, the dynamic analysis gives insights about the dynamic nature of return spillovers. Indeed, the composition of leader-follower exchanges is time-varying and may consistently change over time. Again, this is in line with the stream of literature about price discovery on Bitcoin exchanges, see the works of Brandvold et al[5] and Pagnottoni and Dimpfl.[6]

5 | CONCLUSION

In this paper, we employ an extension of the Diebold and Yilmaz[1] forecast error variance decomposition, which relies on a cointegrated vector error correction framework, to estimate return spillovers across Bitcoin exchanges. We believe that our proposal can be extended, without loss of generality, to other cryptocurrencies, as well as to traditional markets.

From a methodological point of view, we adapt the generalized variance decomposition technique introduced by Diebold and Yilmaz.[1] While Diebold and Yilmaz[1] derive measures for the directional spillovers across markets within a generalized VAR framework, we apply their methodology to the case in which the same asset (Bitcoin) is traded on multiple exchanges. Since we deal with $I(1)$ price series related to a unique asset, ie, arguably cointegrated, we rely on a generalized VECM framework to derive directional spillovers.

From an empirical viewpoint, our results show that Bitfinex and Coinbase are the leader exchanges in the price formation process, transmitting a significant portion of return spillover to other exchanges. Moreover, we find Kraken among the follower exchanges. This is in line with the fact that the exchanges in which most of the trading volumes lie are generally also the ones giving substantial contribution to other markets from a price discovery point of view.

In addition, our results show that return spillovers across Bitcoin exchanges are dynamic and sensibly evolve over time. In other words, the leader-follower compositions are not constant in time and may consistently evolve. This is in line with what observed by Brandvold et al[5] and Pagnottoni and Dimpfl,[6] who concluded that information shares are dynamic and may consistently evolve over time.

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