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Price discovery on Bitcoin exchanges

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ABSTRACT

Bitcoin is an open source peer-to-peer electronic money and payment system. It is traded at several exchanges and high-frequency trade data are publicly available. We study the contributions of Bitcoin exchanges to price discovery. Our results show that Mt.Gox and BTC-e are the market leaders with the highest information share. Our analysis further suggests that information share is dynamic and evolves significantly over time.

1. Introduction

Bitcoin is a decentralized peer-to-peer crypto-currency protocol first outlined in a paper by Nakamoto (2008). Since first going online in 2009, Bitcoin has grown from an experimental commodity traded between enthusiasts, to a booming economy receiving substantial media attention. The Bitcoin user base is becoming increasingly global and diversified, and so is the currency exchanges. The market capitalization of Bitcoin reached USD 10 billion recently, and the transaction volume keeps growing. Bitcoin is traded on a myriad of exchanges that support different currencies and are based in countries all over the globe. The days when a single exchange completely dominated the market are gone, and the question of where the value of Bitcoin is decided emerges. We attempt answering this question.

An introduction to the key concepts of Bitcoin can be found in Becker et al. (2013), Segendorf (2014), Dwyer (2014) or Bitcoin (2015). Discussion of Bitcoin in the context of other alternative monetary systems can be found in Rogojanu and Badea (2014) and Shubik (2014). We therefore introduce Bitcoin only briefly. Unlike fiat currencies, the total amount of Bitcoins which were or will ever created is capped. Bitcoins are created in a process called mining. The economics of Bitcoin mining is analyzed in Kroll et al. (2013). Another specific feature of Bitcoin is that instead of trusting that the central bank is guaranteeing the value of your money, as is common for fiat currencies, you trust that the cryptographic proofs provided by the network is correct. In most western countries lack of trust in the central bank has not traditionally been a problem, but with the recent euro crisis and ongoing financial uncertainty around the world, this started to change. After the Cypriotic bank crisis for instance, Bitcoin gained a lot of publicity and surged in value, see Cox (2013). Because of the decentralized structure, no central authority has direct control over the Bitcoin exchange rate. However, regulatory issues still influence the exchange rate between Bitcoin and other currencies. One example is the process of withdrawing and depositing – the easier and cheaper this process is at a particular exchange, the more users it gains and the price at this exchange changes accordingly. This can lead to different prices at different exchanges, an effect we will discuss later.

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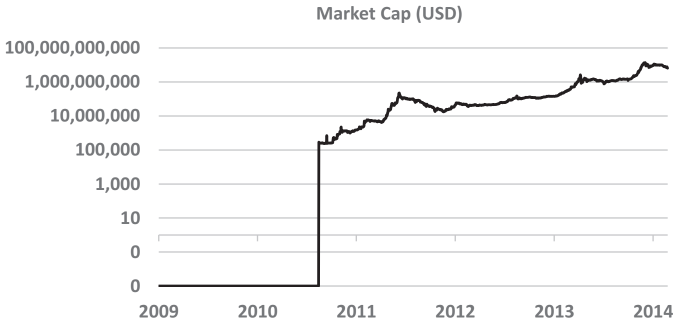


Fig. 1. Historic market capitalization in USD on logarithmic scale.

The less developed countries with lack of trust in the central bank or government should possibly gain the most by using Bitcoin. However, they do not adopt Bitcoin as fast as more developed countries, see Pekas (2013). One reason for this might be that technological and informational barrier such as lack of IT infrastructure, hinder adoption. However, the success of M-Pesa in Kenya, cf. Hughes and Lonie (2007), suggests that this difficulty can be overcomed and we might expect the first widespread adoption of Bitcoin in these countries.

Economic research investigating the phenomenon of Bitcoin has emerged recently with a slight lag to the introduction of Bitcoin itself. Halaburda and Gandal (2014) find that including Bitcoin into a diversified portfolio significantly increases risk-adjusted returns, due to both high average returns and low correlation with other assets. Yermack (2013) concludes that Bitcoin appears to behave more like a speculative investment than a currency, because the value of the currency is very high relatively to the transactions it facilitates. Bouoiyour and Selmi (2014) assess the lead-lag relationship between Bitcoin prices and transactions as well as the relationship between Bitcoin prices and investors' attractiveness and reach the same conclusion. This conclusion is also confirmed by Bouoiyour et al. (2014). Similarly, Ali et al. (2014) conclude that Bitcoin and other cryptocurrencies serve as money only to a limited extent and only for relatively few people, and therefore do not pose a material risk to monetary or financial stability in the United Kingdom. Briere et al. (2013) study the relationship between Bitcoin and other cryptocurrencies and find that during periods when Bitcoin appreciates against USD it also appreciates against other cryptocurrencies.

One of the features of the Bitcoin is that it is traded in many exchanges. Even though the Bitcoin itself is very different from fiat currencies, Bitcoin exchanges are essentially standard privately owned exchanges. However, the only study that briefly address this point is Briere et al. (2013), who note that Bitcoin price vary on different exchanges. The aim of our paper is to investigate this multi-exchange environment further and particularly to study the price discovery at Bitcoin exchanges. Knowing which exchange reacts most quickly to new information and therefore reflects value of Bitcoin most precisely is obviously important. Not surprisingly, this topic has received a lot of attention both among popular media and the Bitcoin community. However, to the best of our knowledge this is the first study of this topic.

The price discovery literature uses primarily two methodologies, the information share method by Hasbrouck (1995) and the permanent-transitory decomposition by Gonzalo and Granger (1995). In this paper we use the method of de Jong et al. (2001). The advantage of this method is that the information share calculated this way is uniquely defined, unlike information share of Hasbrouck (1995), but still takes into account the variance of innovations, unlike Gonzalo and Granger (1995).

The remainder of this paper is organized as follows. Section 2 briefly explains some historic events and characteristics of the Bitcoin exchange market. Section 3 describes the data used and the characteristics of the data. Section 4 describes the model, while Section 5 describes the implementation of this model to Bitcoin data. The results are presented in Section 6. Finally Section 7 draws a conclusion.

2. History

Bitcoin started to exist in January 2009. During its first year in existence it was traded solely privately, as illustrated in Fig. 1. In 2010, the first currency exchanges emerged, with Mt.Gox claiming the position of market leader. Throughout 2010, 2011 and 2012, Mt.Gox kept its position, holding a market share of more than 80%. During the same period most public Bitcoin trading was done in USD.

The Bitcoin has experienced four periods of major price increase within a short time span. As seen in Fig. 2, the price of Bitcoin (on Mt.Gox) reached parity with the US Dollar February 2011, after a surge from USD 0.1 during the latter months of 2010 and January 2011. During the spring of 2011 Bitcoin experienced another surge from $1 to $10 per Bitcoin. The price did not see another boom of this magnitude until the first quarter of 2013, where a similar 10-times surge from $10 to $100 happened. During October and November of 2013, Bitcoin jumped 10-times yet again, roughly from $100 to $1000. The price then hoovered around this milestone until February of 2014, when a series of events led to a dramatic price fall culminating in Mt.Gox declaring bankruptcy at the end of this month.

Fig. 2. Historic USD price at Mt.Gox, on logarithmic scale.

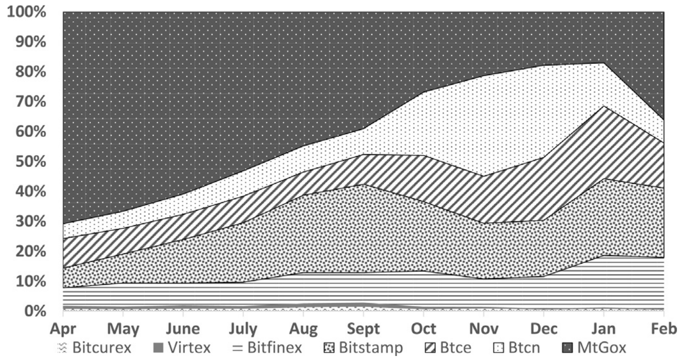
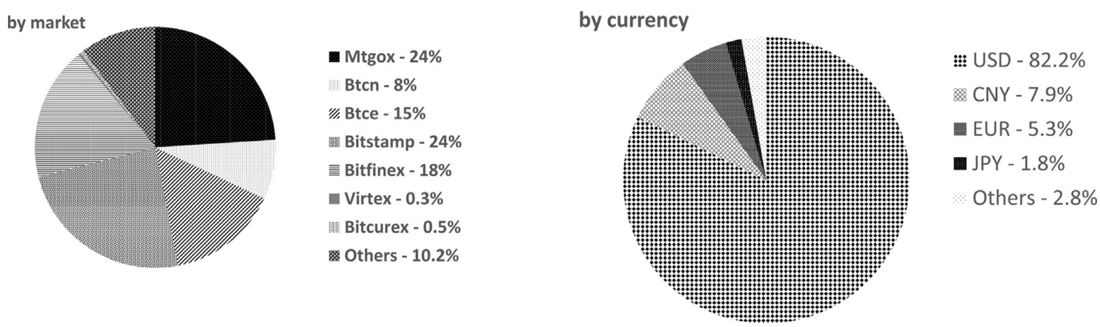


Fig. 3. Exchange market shares development April 2013–February 2014.

The surge in Bitcoin price in Q1 2013 brought the Bitcoin market capitalization past $1B and marked the start of a diversification of the exchange market, highlighted in Fig. 3. Throughout the year, a lot of smaller actors, specializing in different currencies and other cryptocoins have appeared. Mt.Gox lost a lot of market share on the BTC/USD market to Bitstamp and BTC-e during 2013, which is widely acknowledged as a result of issues related to withdrawal delays and troubles at Mt.Gox, see Gilson (2013). BTC China (Btcn) has emerged as another large actor as a result of increased Bitcoin interest in China.

The exchange volume distribution as of February is shown in Fig. 4. As seen, over 80% of Bitcoin trades are made in USD at the time of writing. However at times during the past year this figure has been significantly lower, notably during November and December of 2013 when roughly half of all Bitcoin trades were made in Chinese Yuan.

A notable characteristic is the development of the price differences between the exchanges. This is most apparent with Mt.Gox, where the price of Bitcoin has been as high as 7–14% above the price at Bitstamp during 2013, see Fig. 5. The withdrawal issues that arose during the summer of 2013, and continued to persist throughout 2013, are thought to be the main catalyst behind this. Mt.Gox traders were thus paying a premium for their Bitcoins. 2014 began with this difference



(a) Market Distribution (b) Currency Distribution

Fig. 4. Exchange volume distribution. Source: bitcoincharts.com.

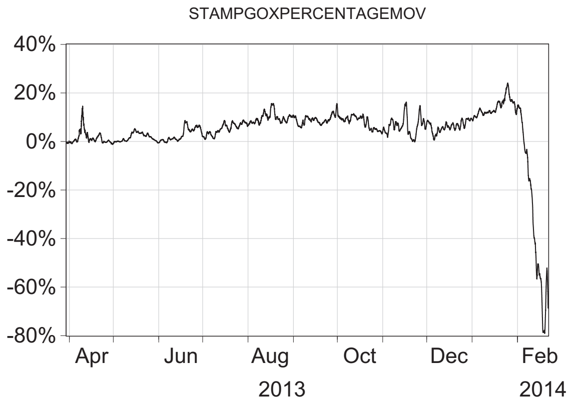


Fig. 5. Daily spread (in percentage) between MtGox and Bitstamp.

increasing to over 20%, before Mt.Gox prices fell to below those at Bitstamp in February 2014, only weeks before declaring bankruptcy.

Notice from Fig. 5 that the spread spiked in April and November 2013 and late January and February 2014, the same months that we saw the highest volatility. In 2013 these high volatility periods caused large, however temporary spreads (20:30 GMT time April 10th 2013 Mt.Gox prices were 143% above the ones at Bitstamp).

On May 14th the US government seized funds that Mt.Gox held at Dwolla (an online payment processor) and Wells Fargo, after accusations of operating an illegal money service business, see Buterin (2013). This made Mt.Gox traders convert their at-risk dollars into easy-to-withdraw Bitcoins and the spread spiked. The spread went back to approximately zero shortly after, but on June 20th Mt.Gox suspended USD withdrawals. Traders could transfer their dollars in, but they could not get them out again. This caused to spread to go back up, and it remained high until February 2014. Therefore Mt.Gox no longer could be said to be a straightforward measure of a Bitcoin's value, but a measure of a Bitcoin plus the desperation of Mt.Gox traders. Despite this, Mt.Gox held their position as one of the largest Bitcoin exchanges up until the end of February 2014.

February 7th 2014 Mt.Gox halted all Bitcoin withdrawals. The price at Mt.Gox then first fell below the price at Bitstamp before falling below the rest of the market during the next couple of days. On February 10th 2014 Mt.Gox went on to blame flaws in the Bitcoin protocol for their withdrawal issues, namely the "transaction malleability" (see Bradbury (2014) for a good non-technical article about the transaction malleability), however leading Bitcoin experts and Bitcoin core developers hit back with strong criticism of Mt.Gox, see Rizzo (2014a). The price at Mt.Gox kept falling drastically after this, until stopping all trading at February 25th. Then, in a further twist, Mt.Gox bottomed out with the news that it appears to have lost over seven hundred thousand Bitcoins, most of which were customer funds, and rumours of insolvency surfaced Rizzo (2014b). It took three days before Mt.Gox filed for bankruptcy and at the time of writing it is unclear if or when it will ever reopen.

There are variety of variables that may effect the price of Bitcoin on the exchanges and lead to the price differences observed. Market size, exchange volume, price of entry and currency of trade are such variables. As mentioned Mt.Gox have had prices significantly higher than Bitstamp, however Btc-e on the other hand have traditionally had prices somewhat lower than Bitstamp. BTC China has also experienced similar price differences. When the media reported of an all time high of $265 in April, the price at BTC China was CNY 1944 equivalent to approximately $320. The price increase in late October and November saw similar differences between the USD exchanges and BTC China. At the same time BTC China has had days with over 50% market share, and went from being a minor regional player to the world's largest exchange in terms of volume in November and December 2013. Note that trades on BTC China are denominated in Chinese Yuan. It follows that only traders with Yuan can transact trades on that exchange, and it would be of limited interest for foreign traders to sell their Bitcoin for Yuan given that one can only withdraw funds from BTC China to a Chinese bank. After some major Chinese companies announced it would accept Bitcoins as payment, cf. Spaven (2013b) and Chang (2013), it triggered increased demand from China, creating a seller's market on BTC China and resulting in Bitcoins beginning to trade at a premium there.

After this booming period from mid November until mid December of 2013, regulatory issues began influencing BTC China. On December 16th 2013 the Peoples Bank of China (PBOC) met with top third party payment companies where the PBOC directed these companies not to do business with Bitcoin exchanges in China Spaven (2013a). Consequently BTC China shut off Yuan deposits two days later. It wasn't until January 30th 2014 BTC China again accepted Yuan deposits, however its market share have yet to reach its previous highs.

Given such factors there is a reason to believe arbitrage opportunities are common in the Bitcoin market. However even though this sounds lucrative in theory, doing it in practice is harder. Partially to blame is that while Bitcoins and other digital currencies are aiming to disrupt traditional fiat money, they are still closely tied to the existing financial infrastructure. It's easy to send Bitcoins around the world, but if you want to buy them from someone on the other side of the globe, you need to send them fiat. As a result, traditional arbitrage does not work. If it did, observed spreads would not exist, or at least be

Table 1
Descriptive stats.

	Mean	Median	Max	Min	St. Dev	Skew	Kurtosis	ρ_1	ρ_2
Bitcurex	5.80E−05	0	0.51	−0.30	0.011	1.65	176.9	−0.14	−0.09
Bitfinex	2.51E−05	0	0.47	−0.41	0.010	−0.14	238.4	−0.20	−0.06
Bitstamp	1.87E−05	0	0.61	−0.37	0.008	3.80	543.8	−0.07	−0.07
Btce	1.98E−05	0	0.36	−0.39	0.008	−0.60	295.6	−0.11	−0.08
Btcn	2.29E−05	0	0.44	−0.19	0.007	2.67	275.6	0.00	−0.02
Mtgox	4.01E−06	0	0.38	−0.49	0.010	−0.72	207.0	−0.16	−0.03
Virtex	5.10E−05	0	0.21	−0.33	0.017	−0.29	24.5	−0.14	−0.09

much smaller. Of course one would expect some traders are willing to make the effort, but such opportunities are risky due to transaction time, transaction fees, trading fees, fiat currency exchange fees, withdrawal time and not least the volatile nature of the Bitcoin price itself.

The development in the Bitcoin exchange market forms the basis of the hypotheses investigated in this paper. We examine whether Mt.Gox is as dominant in information share as in market share from April and throughout the summer. Further we investigate whether the fall in market share for Mt.Gox lead to decreased information share and what impact the issues that arose in February 2014 had on the price discovery process. Another subject we study is how much impact the increasing exchange activity in China has had. It has been widely reported in popular media Rooney (2013) that Chinese traders drove the rapid price increase in November, and we aim to answer whether it is as such. Our final hypothesis is that traders at large exchanges have an information advantage over traders at small exchanges.

3. Data

Transparency is one of the key features of Bitcoin. All code is open source and all transactions ever made are publicly available. This manner of transparency is kept intact on 3rd party applications, in the form of open APIs providing real time data. High frequency data is easily available for free from public sites. The data for this paper is primarily downloaded from www.bitcoincharts.com, one dataset for each exchange. This data contains raw tick data, that is timestamp (in unixtime), price and volume for each exchange.

We include seven exchanges in our study. Five of them are obvious choices – Bitfinex, Bitstamp, BTC-e (Btce), BTC China (Btcn) and Mt.Gox (Mtgox) have had by far the biggest traded volume lately and are therefore natural to include. We also wanted to look at a couple of smaller exchanges to check if there are any differences in behaviour between these and the others. Thus we also included Bitcurex and Canadian Virtual Exchange (Virtex) in our dataset, which are much smaller than the five major but still among the top 10 biggest exchanges. Together these exchanges make up around 90% of Bitcoins traded publicly, and for our purposes we make the assumption that they cover the whole Bitcoin exchange market. Note that 4 of the exchanges are denominated in USD, whereas Btcn is in CNY, Bitcurex is in PLN and Virtex is in CAD.

Regarding the sampling interval, it is important to use short enough intervals to correctly capture the high frequency behaviour of the data, but at the same time long enough that there are enough observations and to avoid noise, see Goodhart and O'Hara (1997) for a discussion. Andersen (2000) argues that 5 min intervals are the best trade off between these, and that is also the length we have chosen. In order to get our tick data to 5 min intervals, we use the last trade price in each interval as the price for this interval, and calculate return according to this. If no trades occur in a given 5 min interval, we treat it as a missing observation.

There are data available for all the exchanges for as long as they have existed, but it was not until early 2013, and especially around the rising interest in April that the high frequency data became interesting to use. Before this there would be too many missing observations. In light of this, the period we are studying is April 1st 2013–February 25th 2014. Our period ends with the bankruptcy of Mtgox, i.e. the last day in our study is also the last day that trade was executed at Mtgox. It is also worth to note that before this period, Mtgox was the by far biggest exchange, meaning that a model of price discovery probably would not be as interesting to investigate. We ran our model on the full period, and subsequently for each month to get the development of the exchanges over time. We also investigate what happens after a shock in the market, in our case what happens in the days after the owner of the infamous marketplace Silk Road was arrested. See Albanesius (2013) for a good article about Silk Road.

If we take a look at some of the characteristics of the dataset, summarized in Table 1, we notice that there are some outliers/extreme trades in the dataset. The most extreme return values for instance, positive and negative, often occur after each other. One reason why this might happen is someone placing a order quite much below or above the last price, and then the price will revert back within the next interval. There are several of these trades, but because the volume of these trades is so small, they will not interfere with of our model. They will however have impact on the descriptive statistics. The standard deviation in the smaller exchanges is larger than in the big, as can be expected. We also note that the skewness is quite different among the exchanges, Bitstamp has a positive skewness of 3.80, whereas Mtgox has a negative skewness of −0.72. The reason for this might be that Mtgox was quite dominating in April, when a lot of the unusual activity occurred. The number of extreme negative returns would therefore be expected to be higher for an exchange which has experienced the sudden burst of a bubble, than those which in this aftermath grew bigger and bigger.

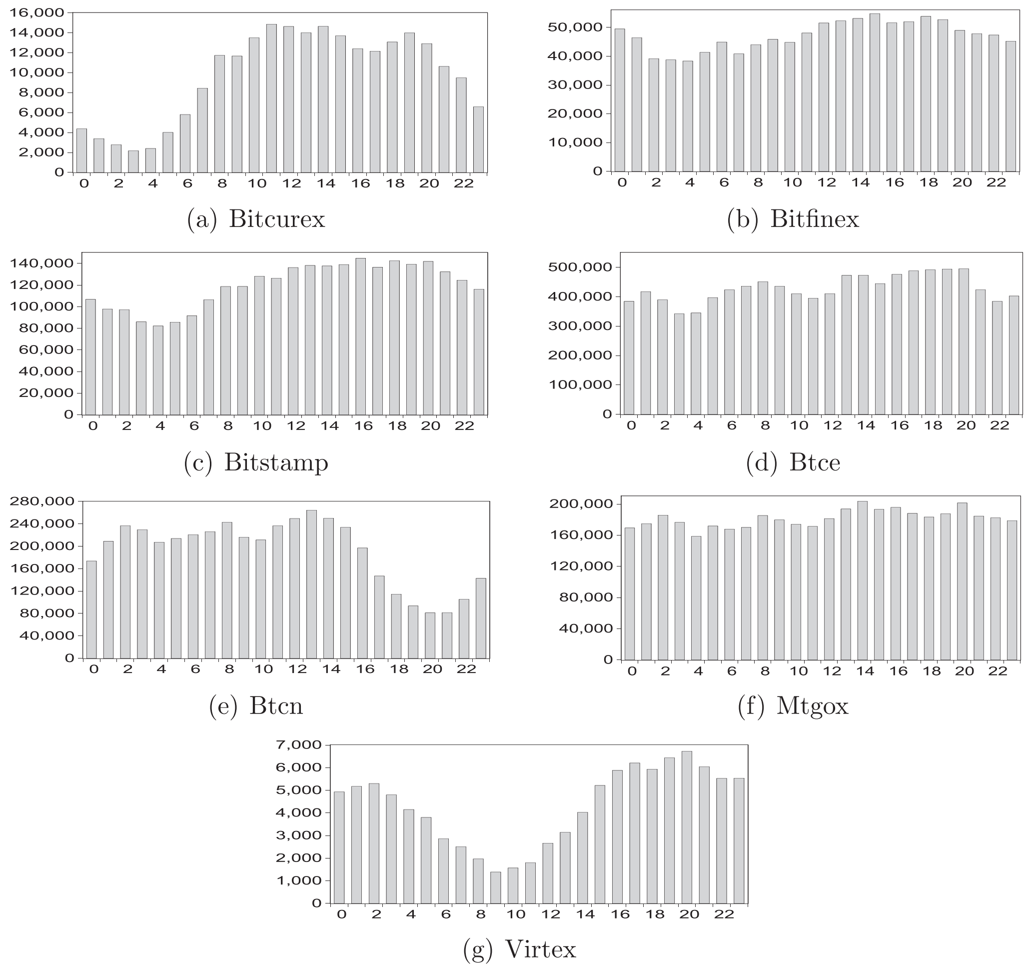


Fig. 6. Trade distribution over 24 h.

Since we are investigating one asset traded on multiple exchanges, it seems safe to assume that they are cointegrated. When testing the time series of each exchange against the others, we find that all of them are indeed cointegrated, except Mtgox against all the others. The reason for this is probably as previously mentioned that Mtgox has had lots of trouble during the sample period, leaving the Bitcoins traded here either going for a premium or at a discount price, see Fig. 5.

Bitcoin exchanges are open 24/7, and it might be interesting to look at the distribution for when the trades on the different exchanges occur. These result are presented in Fig. 6. These charts indicate that some of the exchanges are more global than others, with trading occurring with no special pattern regarding at which hourly interval it happens. This can be clearly seen at Mt.Gox, Btce and to some degree Bitfinex – they appear to have no pattern at all, trades are executed at all times of the day. At the other exchanges however there seem to be a notable dip in trading activity whenever the homeland of exchange is at nighttime. Bitcurex (Poland), Btcn (China) and Virtex (Canada) are all showing these signs. Bitstamp (Slovenia) also has indications, but the dip in trades is much smaller and the overall picture looks like something in between global and local. The reason why Bitcurex, Btcn and Virtex seem to mostly be used by local traders might be that they are all denominated in their local currency, complicating the trading process for foreign users.

Fig. 7 summarizes the distribution of volume at the individual exchanges. Notice that low volumes (<0.1 BTC) is in general by far the most common trade volume. However the different exchanges have some significant differences which may give some valuable insight. Notice for instance that large trades (>5 BTC) accounts for a lot of the activity on Virtex. Also note from Table 2 that integer sized trades are much less common on Btce than the others, whereas Btcn and Virtex have over 15% of trades that are integer sized.

One Bitcoin can be divided down to 8 decimal places where the smallest unit is denominated one satoshi. Some Bitcoin related sites, e.g. Bitcoinity (2015), have lately even changed the standard unit to one mBTC (0.001 BTC), and it has been subject of a lot of discussion in the Bitcoin community whether this should be the new standard, see Razick (2013). The reasoning behind this is related to the rapid price increase seen the last year. It has been argued that such a high price may lead to barriers for new traders to enter the market. It has even been discussed whether the general public even know that

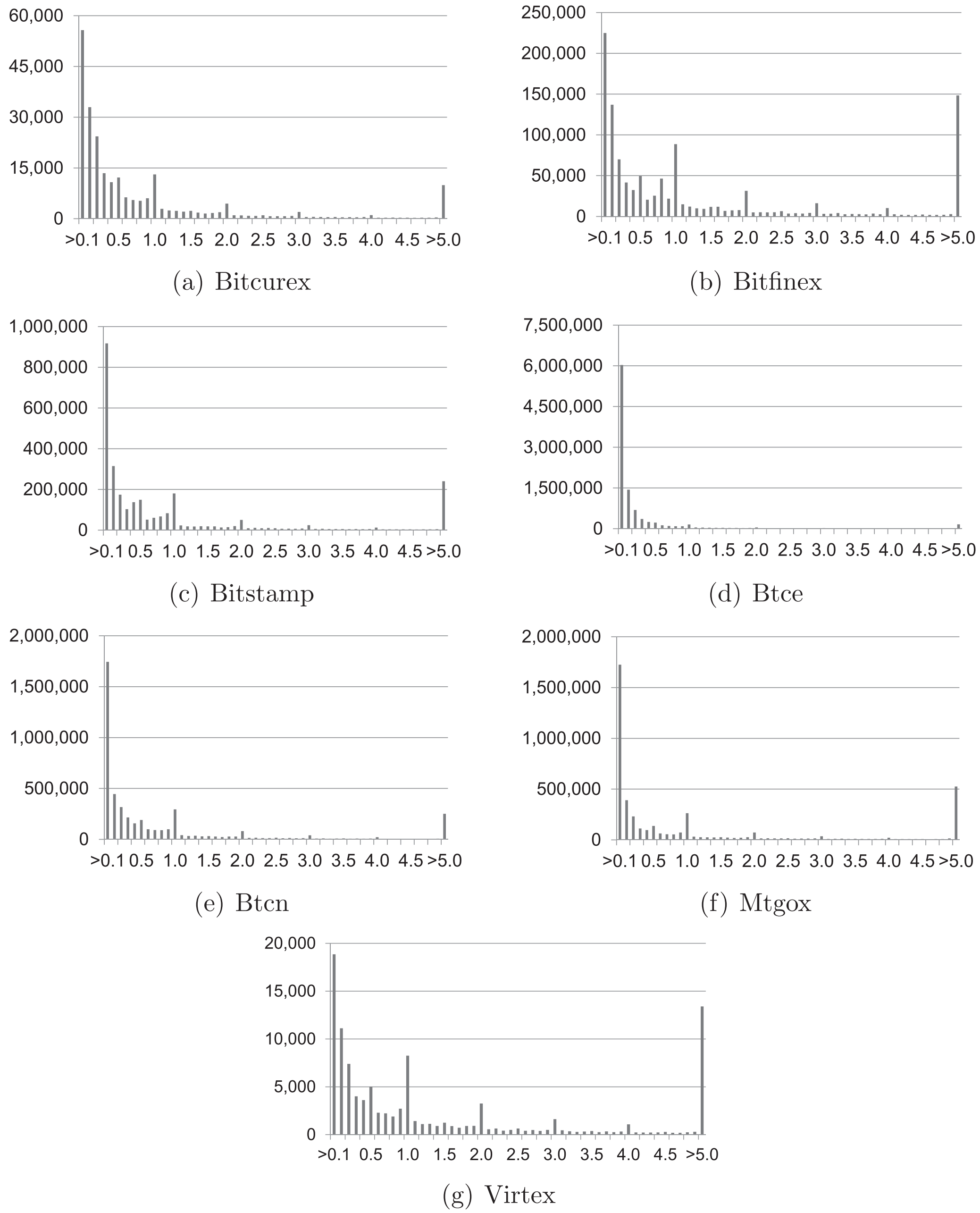


Fig. 7. Distribution of volume.

Table 2
Percentage of trades that have integer volume.

	Percentage of trades that have integer volume (%)
Bitcurex	7.8
Bitfinex	16.2
Bitstamp	9.6
Btce	1.6
Btcn	10.2
Mtgox	11.0
Virtex	17.5

Bitcoins can be traded in fractions, see Inedible (2013). One therefore may think that integer sized trades are the result of new participants entering the market with the goal of "buying 1 (or 2, 3, etc.) Bitcoin(s)". These new participants would be less informed than experienced traders that may have been around since the early days. Noticing that these integer sized trades are more likely to appear at certain exchanges indicating where these new traders are trading.

4. Methodology

In this section we outline an unobserved components price discovery model as proposed by de Jong et al. (2001). The model is a multivariate time series model that aims for an assessment of the contribution of various exchanges to the information generated by the whole market.

The main assumption is that the prices are composed of two components, one common underlying random walk component and an idiosyncratic component specific for each exchange. Two names are used for the random walk component throughout the paper, we will refer to it either as efficient price or the fundamental news component. Two reasons emerge to be beneficial for this approach. The common random walk component makes the price vectors of the individual exchanges cointegrated by construction. Combining that component with a idiosyncratic component makes it suited to the negative first order serial correlation in the price changes, see Zhou (1996).

The main idea in the model is that the price at a given time interval for each exchange can be derived from one common efficient price. However the efficient price is unobserved and the price at every exchange equals the efficient price in addition to an idiosyncratic component. The idea of introducing the idiosyncratic component and separate it from the efficient price was proposed by Hasbrouck (1995). The idiosyncratic component can reflect specific conditions at an exchange, strategic behaviour of the traders at that exchange, noise or shocks.

In our model we look at n individual exchanges as well as m corresponding markets, with $m=n$. A market for an exchange is defined as all the other exchanges. As outlined in Section 3 we make the assumption that the seven exchanges covers the whole Bitcoin market. To fix some notation, we let P be a vector of prices and U the vector of idiosyncratic components. Since we need to consider both exchanges and markets in the model both P and U will actually be two vectors, one for exchanges and one for markets. We therefore denote P^e as the vector of exchange prices and P^m as the vector of market prices. Correspondingly we have U^e and U^m. We let the ith element of P^e and U^e refer to exchange i and the jth element of P^m and U^m refer to market j.

We let P^* denote the efficient price. Letting $p^e=\ln P^e$, $u^e=\ln U^e$ and $p^*=\ln P^*$, we have that the logarithm of the n-vector of exchange prices equals

$$p_t^e = p_t^* + u_t^e \tag{1}$$

and similary the logarithm of the m-vector of market prices equals

$$p_t^m = p_t^* + u_t^m \tag{2}$$

The main assumption in the model is that p^* is a random walk. It follows from (1) and (2) that the model therefore has a specific unobserved components structure, namely random walk plus noise, see Harvey (1989) for a discussion of such models. It also follows from the random walk assumption that the efficient price is unpredictable, which is a standard assumption in the price discovery literature, again see Hasbrouck (1995).

As described previously, the Bitcoin price often differ substantial from one exchange to another. However the spread is assumed to be stationary. Therefore the term u takes into account the exchange or market specific spread as well as all other temporary deviations from the efficient price. We expect u to revert to its normal spread in the long term, however the temporary deviations can be both substantial and lasting on intraday data.

We notice that the price of all the exchanges and markets share the same random walk component, p^*. Therefore the price series are cointegrated by construction. Since we expect the price of each exchange and market to revert to the same efficient price plus its specific spread in the long run, this is an economically intuitive assumption. This is also necessary for the model as the price series are indeed cointegrated as described earlier.

Below follows some definitions of the model. Let the change of the efficient price over the interval $(t-1, t)$ be denoted by

$$r_t = p_t^* - p_{t-1}^* \tag{3}$$

A retained assumption in the model is that the unconditional serial covariances of r_t and u_t^e and u_t^m are stable in the given sampling interval. Given that assumption we make the following definitions:

$$E[r_t^2] = \sigma^2 \tag{4a}$$

$$E[r_t u_{it}^e] = \psi_i \tag{4b}$$

$$E[r_t u_{jt}^m] = \psi_j \tag{4c}$$

$$E[r_t u_{i,t+l}^e] = \gamma_{li}, \quad l \geq 0 \tag{4d}$$

$$E[r_t u_{j,t+l}^m] = \gamma_{lj}, \quad l \geq 0 \tag{4e}$$

$$E[r_t u^e_{i,t-k}] = 0, \quad k > 0 \tag{4f}$$

$$E[r_t u^m_{j,t-k}] = 0, \quad k > 0 \tag{4g}$$

$$E[u^e_{it}] = \Omega^e \tag{4h}$$

$$E[u^e_{it} u^m_{jt}] = \Omega, \quad i = j \tag{4i}$$

$$E[u^e_{i,t-k}] = 0, \quad k \neq 0 \tag{4j}$$

$$E[u^e_{it} u^m_{j,t-k}] = 0, \quad k \neq 0 \tag{4k}$$

ψ, γ_l and Ω are $(n \times 1)$ matrices. It follows from these definitions that r_t is serially uncorrelated, which it should be since it is the return of a random walk. As de Jong et al. (2001) we denote r_t as the fundamental news component. Further our definitions state that the fundamental news and the idiosyncratic component may be correlated at leads of r_t to future u_t, but is otherwise uncorrelated. Introducing serial correlation in in r_t and cross correlation between unobserved components at lead and lags would lead to an underidentified model, according to Harvey (1989), who discusses the structure of unobserved components models in detail. Therefore there is only concurrent correlation between the fundamental news and the idiosyncratic component. The idiosyncratic components are serially uncorrelated. The intuition behind this is that the idiosyncratic component reflects the noise present in intradaily data.

The only observable parameter so far is p. We therefore need to find an expression for p so that the definitions (4a)–(4k) can be of use. We let

$$y_{it} = p_{it} - p_{i,t-1} = p^*_t + u_{it} - p^*_{t-1} - u_{it-1} = r_t + u_{it} - u_{it-1} \tag{5}$$

In the model we therefore consider a vector of prices for exchanges and a vector for market prices.

$$Y^e_t = \iota r_t + u^e_t - u^e_{t-1} \tag{6a}$$

$$Y^m_t = \iota r_t + u^m_t - u^m_{t-1} \tag{6b}$$

where ι is a vector of ones with n = m elements. Given the definitions (4a)-(4k) the serial covariances of Y_t are

$$E[Y_t Y'_t] = \sigma^2 \iota\iota' + \iota\psi' + \psi\iota' + 2\Omega \tag{7a}$$

$$E[Y_t Y'_{t-1}] = -\psi\iota' - \Omega + \gamma\iota' \tag{7b}$$

$$E[Y_t Y'_{t-2}] = -\gamma\iota' \tag{7c}$$

Further we are only interested in the serial covariance between an exchange and its corresponding market, or in other words the covariance between an element in vector Y^e and the corresponding element in vector Y^m. From (7) we can write this out as

$$E[y_{jt} y_{it}] = \sigma^2 + 2\omega_{ij} + \psi_j + \psi_i \tag{8a}$$

$$E[y_{jt} y_{i,t-1}] = -\omega_{ij} - \psi_j + \gamma_j \tag{8b}$$

$$E[y_{jt} y_{i,t-2}] = -\gamma_j \tag{8c}$$

We also consider the first order autocorrelation for exchanges. It is given by

$$\rho_{1,ii} = \frac{-(\omega^e_i + \psi_i - \gamma_i)}{\sigma^2 + 2(\omega^e_i + \psi_i)} \tag{9}$$

One parameter of special interest so far is ψ_i. As stated in (4b) and (4c) it is the covariance between the fundamental news component and the idiosyncratic component. This parameter determines what the market learns, and how it reacts given a price update from the individual exchanges. The higher ψ_i, the stronger the signal to the market by a price change from an exchange. Stronger signal in this sense means that the price change is informative.

To illustrate this point lets consider the covariance between the fundamental news component and a price change at an exchange

$$Cov(y_{it}, r_t) = \sigma^2 + \psi_i \tag{10}$$

This covariance is derived from (4) and (5). Given observed price changes, inference on the fundamental news component is largely driven by this covariance. By considering the average covariance between the price of an arbitrarily selected exchange and the fundamental news a natural restriction for identifying the parameters can be derived. This average covariance is given by

$$\sum_{i=1}^{n} \pi_i Cov(y_{it}, r_t) = \pi'(\sigma^2 \iota + \psi) = \sigma^2 + \pi'\psi \tag{11}$$

Table 3
Model results.

	π	ψ	IS	IS/AS ratio
Bitcurex	0.009	−2.2E−05	0.006	0.71
Bitfinex	0.073	−4.3E−05	0.031	0.42
Bitstamp	0.146	−1.5E−05	0.118	0.81
Btce	0.287	9.34E−06	0.322	1.12
Btcn	0.177	−1.1E−05	0.152	0.86
Mtgox	0.302	1.61E−05	0.366	1.21
Virtex	0.006	−2.4E−05	0.004	0.68

Table 4
Model parameters.

σ^2	7.55E−05						
	π	ψ_i	Market ψ_j	ω_i^e	ω_{ij}	IS	IS/AS ratio
Bitcurex	0.009	−2.2E−05	2.78E−05	3.42E−05	−2.95E−05	0.006	0.71
Bitfinex	0.073	−4.3E−05	3.55E−05	6.95E−05	−3.35E−05	0.031	0.42
Bitstamp	0.146	−1.5E−05	3.17E−05	1.97E−05	−3.48E−05	0.118	0.81
Btce	0.287	9.34E−06	4.48E−05	3.21E−05	−5.18E−05	0.322	1.12
Btcn	0.177	−1.1E−05	4.25E−05	9.67E−06	−4.55E−05	0.152	0.86
Mtgox	0.302	1.61E−05	5.73E−05	3.29E−06	−6.35E−05	0.366	1.21
Virtex	0.006	−2.4E−05	2.87E−05	2.59E−05	−2.59E−05	0.004	0.68

where π is a vector with n elements of weights adding to one. The selection of π_i (i=1,. . .,n) is described in Section 5. To derive the restriction the importance of σ^2 is investigated. One would expect that the information generated in each interval by each price update should equal σ^2 because it is the variance of r_t namely the fundamental news component. Therefore σ^2 can be interpreted as the total information generated in the market. From that interpretation it follows that imposing the restriction $\pi'\psi = 0$ is natural. This restriction is also sufficient to identify the parameters. It further leads to a definition of the information share if we let π_i represent the activity share of the individual exchanges. Activity share in its simplest form would be the fraction of trades that happened on one exchange, however it can be defined in more than one way. But for now let π_i be the fraction of trades that happened on exchange i, or in other words the probability that a trade happened on exchange i. Multiplying this probability with the covariance between the fundamental news component and the price change of exchange i, Eq. (10), gives a natural measure of how much information is generated by the price update of exchange i. Dividing this with σ^2, the total information generated in the market, the information share of exchange i is obtained.

$$IS_i = \frac{(\sigma^2 + \psi_i)\pi_i}{\sigma^2} = \pi_i\left(1 + \frac{\psi_i}{\sigma^2}\right) \tag{12}$$

As described by de Jong et al. (2001) this definition of information share has some appealing properties. First of all the information shares add to 1, because we imposed the restriction $\pi'\psi = 0$. That makes it easy to interpret the number directly. It also makes it trivial to add or remove exchanges from the model. Further if all other conditions are the same, an exchange with high activity share will have high information share. Whether information share is larger or smaller than the activity share depends solely on ψ. Setting $\psi_i = 0$ gives that $IS_i = \pi_i$. So it follows that an exchange with a contemporaneous covariance between its idiosyncratic component and the fundamental news greater than zero ($\psi_i > 0$) has a higher information share than activity share. The higher the value of ψ_i the higher the information share relative to its activity share. Another appealing property is that it is easy to derive joint information shares, the joint information share of two exchanges is equal to the sum of their individual information shares.

5. Implementation

This section describes how is the model applied to the data. The application of the model proceeds in two steps. First the serial covariances and the autocorrelations have to be estimated. This is a straightforward procedure. From last section notice that only the autocorrelations and the covariances with the market are needed. So we need to estimate $\rho_{1,ii}$ and $E[y_{jt}, y_{i-k}]$, where $k = 0, 1, 2$. Using Eqs. (8) and (9) these estimates allow for identification of the unknown parameters using a non-linear programming model.

From the previous section remember that σ^2 is defined as the variance of r_t further interpreted as the total information generated in the whole market. This is of importance, because given the assumption that the seven exchanges in our dataset represent the whole Bitcoin market, σ^2 can be observed directly as the variance of the aggregated return of the seven exchanges. Similarly γ defined in (4d) and (4e) can be observed directly as the covariance between a market and its corresponding exchange lagged two intervals. This implies that only ω_i^e, ω_{ij}, ψ_i and ψ_j are unknown and needs to be identified.

The choice of objective function in the model is not obvious, however as we will see it is not of great importance. The choice only affects the estimates slightly given the constraint precision of the solver. The model will not give unique solutions, but

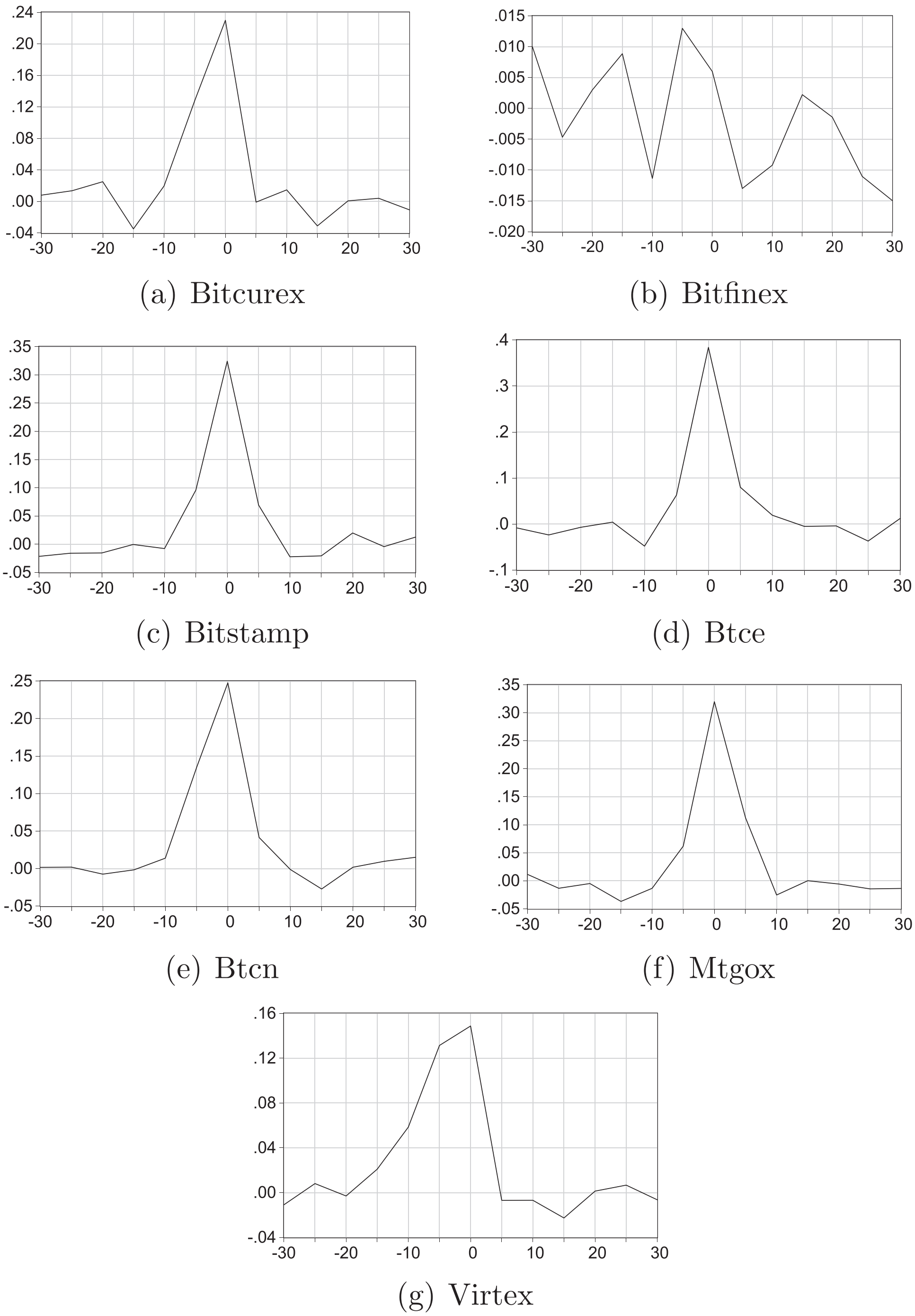


Fig. 8. Exchange correlation with market for the whole period.

this is not considered a problem because the solution space is consistent. Therefore the model is not considerably sensitive of neither the choice of objective function nor the choice of start values. Most solvers also allow problems to be solved given no objective function at all, for instance we could have $\pi'\psi = 0$ as a restriction and just skip the objective function, but for clarity it is stated in the model description below.

$$Z = \sum_{i=1}^{n} |\pi_i \psi_i| = 0,$$

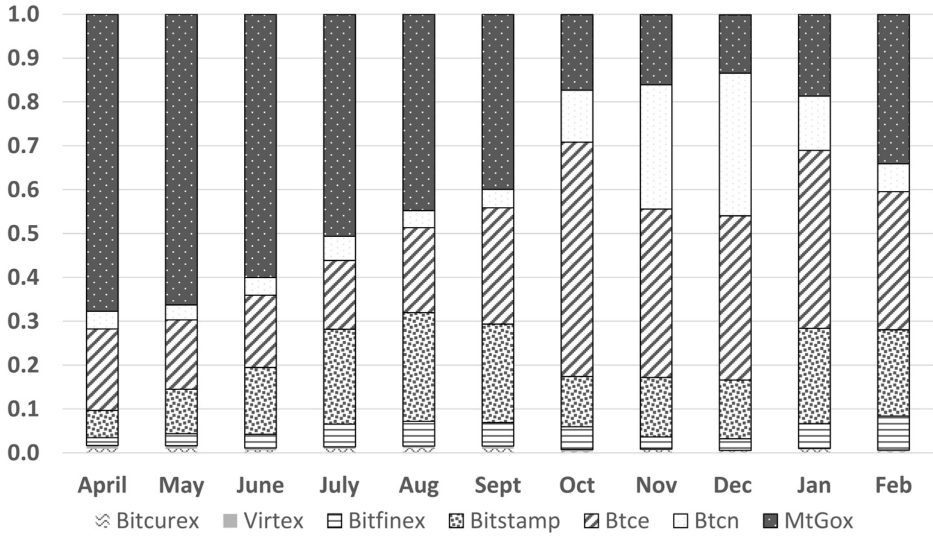


Fig. 9. Information share development.

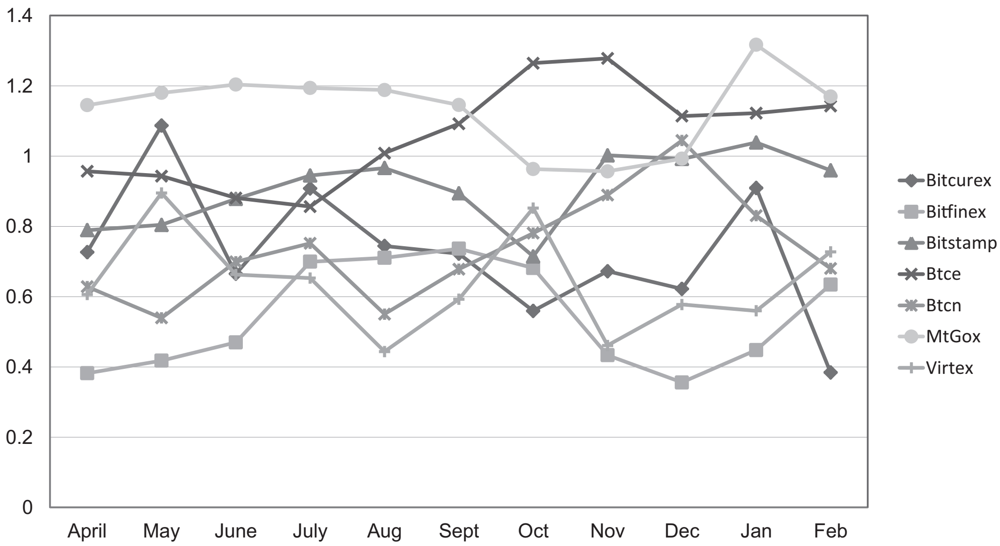


Fig. 10. Information share/activity share ratio development.

given

$$
\begin{aligned}
\rho_{1,ii} &= \frac{-(\omega_i^e + \psi_i - \gamma_i)}{\sigma^2 + 2(\omega_i^e + \psi_i)} && (i = 1, \ldots, n) \\
E[y_{jt} y_{i,t-1}] &= -\omega_{ij} - \psi_j && (i = j = 1, \ldots, n) \\
E[y_{jt} y_{i,t-2}] &= -\gamma_j && (i = j = 1, \ldots, n) \\
E[y_{jt} y_{i,t-2}] &= -\gamma_j && (i = j = 1, \ldots, n) \\
E[y_{it} y_{j,t-2}] &= -\gamma_i && (i = j = 1, \ldots, n) \\
\omega_i^e &\geq 0 && (i = 1, \ldots, n)
\end{aligned}
$$

Notice that the information shares are not found directly from the model above. From (12) the only unknown parameter needed to be found is ψ. A possible alternative objective function would be to let the information shares summarize to 1 given the fitted ψ from the restrictions. However this leads to minimal differences in the estimated parameters and the results are consistent independent of that choice. The restrictions comes directly from the definitions in (8) and (9). We also

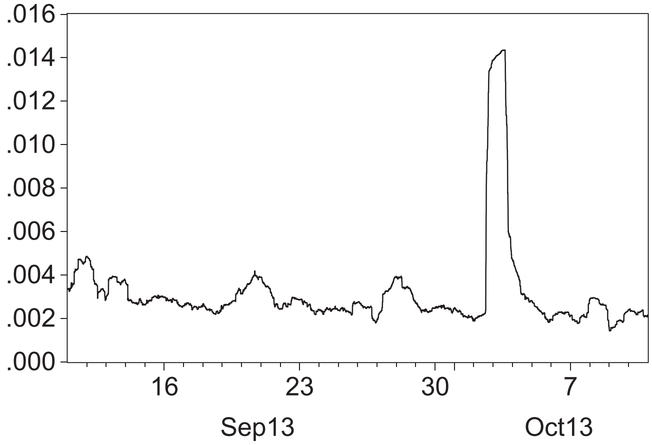


Fig. 11. Variance of the fundamental news component in September and October.

Table 5
Silk Road model results.

	π	ψ	IS	IS/AS ratio
Bitcurex	0.009	−2.04E−05	0.007	0.770
Bitfinex	0.109	−3.69E−05	0.064	0.584
Bitstamp	0.215	−3.17E−05	0.138	0.643
Btce	0.240	6.2E−05	0.408	1.699
Btcn	0.064	−2.5E−05	0.046	0.719
Mtgox	0.352	−6.4E−06	0.327	0.928
Virtex	0.009	−3.5E−06	0.009	0.960

impose the restrictions that ω_i^e is greater than zero. This is intuitive because ω_i^e is just the variance of the exchange specific idiosyncratic component, and variances cannot be negative.

Remember from Section 4 that π_i is a measure of exchange specific activity relative to the total market. Thus π_i is the link between the trade process and price updates, and should ideally capture all relevant features. The literature suggests several measurements to capture all the relevant features. Easley and O'Hara (1987) argues that trade size is what provides information. Jones et al. (1994) on the other hand strongly suggests that it is the frequency of transactions that contains all the information relevant for the pricing process. Chan and Fong (1999) argues that it is premature to conclude that the size of trades has no information feature beyond the one contained in the number of trades, and further implies that order imbalance (the difference between buy and sell orders) may contain relevant information. The confusion is further amplified by Barclay and Warner (1993) and their discussion of optimal sized trades placed by informed traders. They argue that informed traders prefer to break up trades for camouflage purposes resulting in medium sized orders. Several other measurements have been proposed, see for instance Kempf and Korn (1999) and Dufour and Engle (2000).

These conflicting findings suggest that for our purpose neither trade volume nor trade frequency alone is satisfactory as measurements of trading activity at a specific exchange relative to all trading activity in the market. Market participants instead learn and update their beliefs from more complex interactions between trade features and some of these features may not be directly observable. Ideally historical order book data should be included, but was not found to be available. Therefore to capture most information from the trade process with the data obtained, a linear combination of volume and frequency are used in the model. However it is worth noting that the choice of π only affects the magnitude of the information share, and not the relation between information- and activity share (whether ψ_i is positive or negative). Thus if identification of the ψ vector is of most importance, the value of π may be chosen arbitrary, for instance equal value for every exchange. In our opinion such tests are a fine way to measure the consistency of the model.

6. Results

In order to decide whether an exchange is a leader or a follower, the following way of thinking can be used: If the exchange correlates more with lagged market returns than with concurrent market returns, then the exchange is a follower. If the opposite is true, exchange is leading the market. However, in this setting even an exchange with a symmetrical correlation is thought to be an informative exchange. This is due to the fact that if you are mostly moving together with the market, some of the price discovery is bound to happen here. If the correlation is skewed to the right, then there are indications of lag relationship and with left skewness there are indications of leader relationship.

Fig. 8 graphs this for the whole period. Although all exchanges except Bitfinex have their highest correlation with the concurrent market, there are differences among exchanges. Virtex for instance is lagging behind, having almost the same

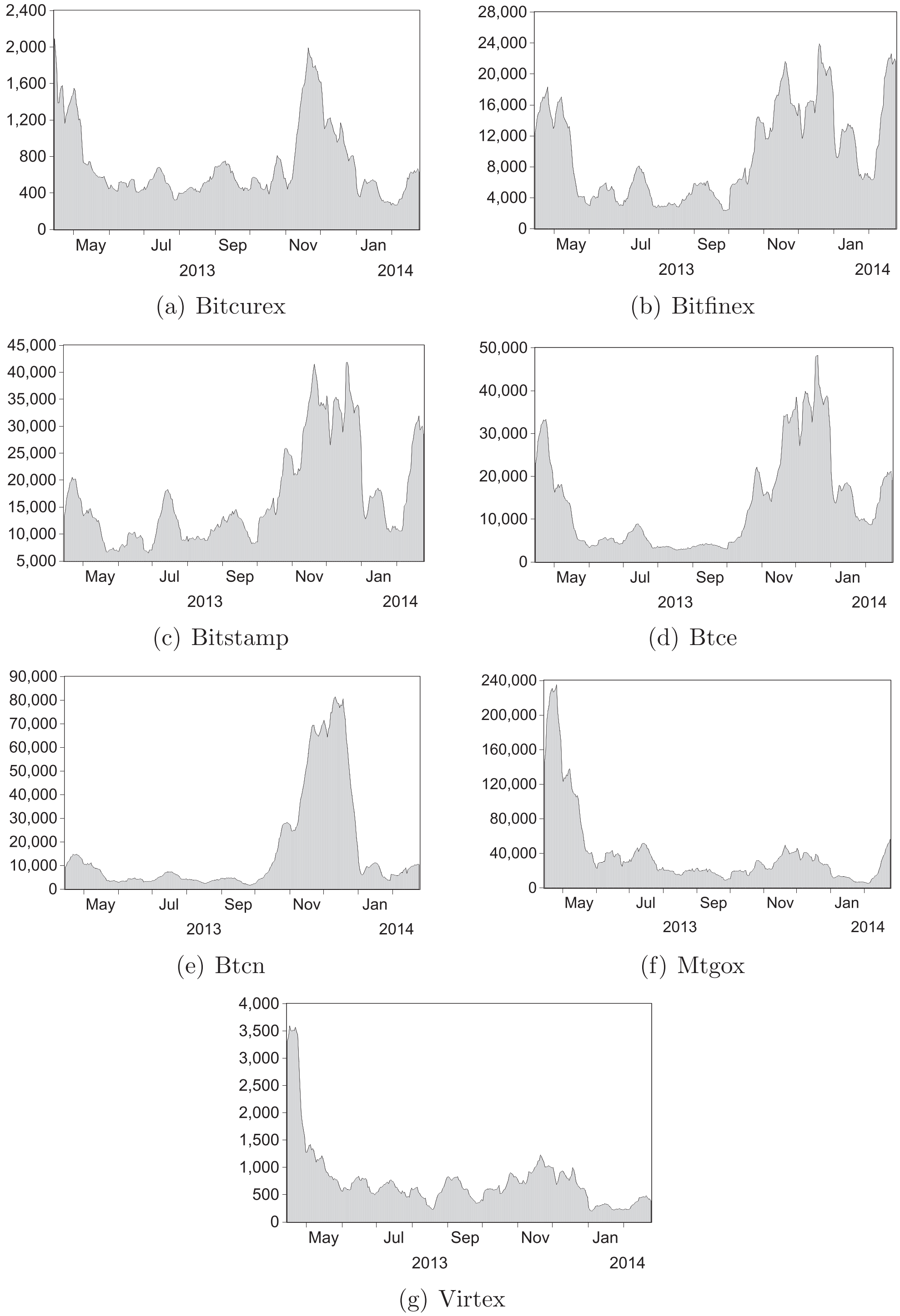


Fig. 12. Daily volume 14 days moving average.

correlation with past as with concurrent market movements. We can see that Btcn lags behind as well. Btce and Mtgox on the other hand have higher correlation with future market moves than with past market moves. Bitstamp seem to be neither a follower nor a leader. Regarding Bitfinex, it does not indicate any relationship – it is not correlated with the market at all.

The empirical results from our model are summarized in Table 3, while all the output values from the model can be found in Table 4. The variance of the fundamental news component, σ^2, is estimated at 7.55E−05. This is quite much larger than the estimates of the noise variance for each of the exchanges, ω_i^e, reflecting the fact the Bitcoin price in itself has been really volatile in this sample period.

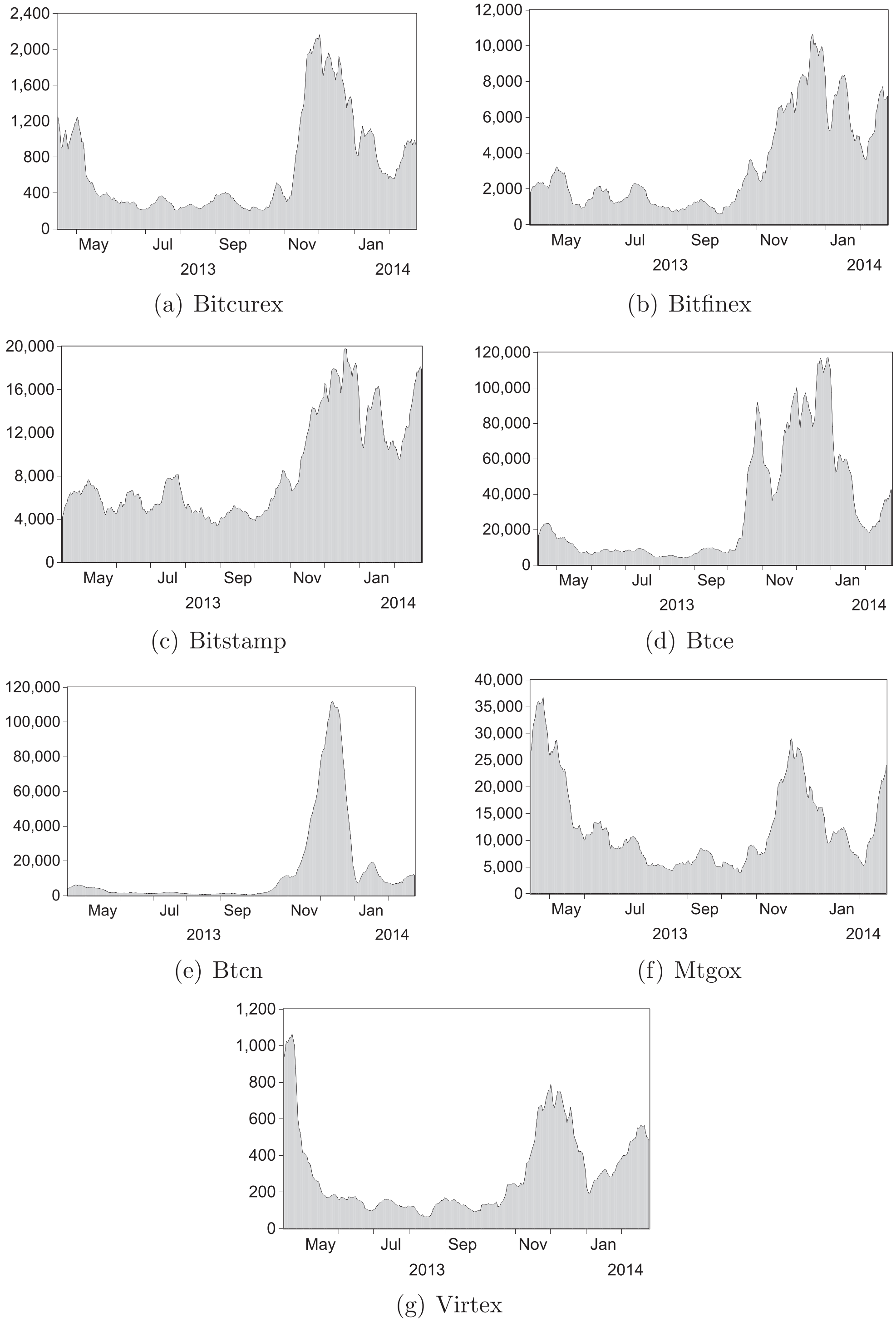
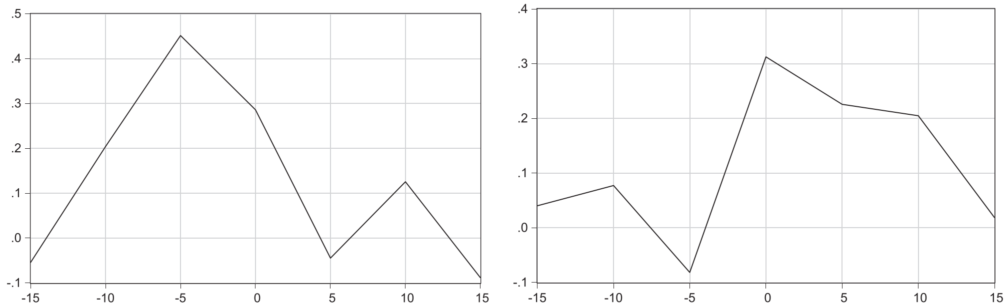


Fig. 13. Daily number of trades 14 days moving average.

We now take a look at the covariance between the fundamental price change and the idiosyncratic shocks, ψ_i, which forms the basis for the information share. As we can see from Table 3, the two exchanges with positive ψ for the entire period are Btce and Mtgox. All the other exchanges have negative ψ, indicating that the information that comes from these exchanges are less informative than the information coming from Btce and Mtgox. This is also reflected in the information share, Btce and Mtgox both have higher information share than activity share. However, even if the other exchanges have negative ψ and lower information share than activity share, they still provide information to the market, only less informative. Bitstamp for instance



(a) Bitstamp (b) Btce

Fig. 14. Exchange correlation with market for the Silk Road period.

has got an information share of 0.118, meaning that 11.8% of all the information in the market comes from this exchange. Mtgox and Btce have the by far biggest information share, indicating that most information is generated/incorporated at these two exchanges.

In order to investigate the development of information share of the exchanges over time, we estimate the model for each month. The information share distribution for each month is illustrated in Fig. 9. The reason why information share of Btcn sharply increased and decreased is the following. In October Chinese companies, and Baidu in particular Spaven (2013b), started accepting Bitcoin as payment. This lead to Btcn being the largest Bitcoin exchange measure in volume in November and December 2013, and the information share to go from 0.040 in April to 0.325 in December. However, after the Chinese government banned payment companies from clearing Bitcoin (see Section 1), the information share fell to 0.124 in January.

Mtgox dominated the market at the beginning, having an information share 0.667. It was gradually losing its dominant position to other exchanges. The reason why information rose sharply in January and February is the fact that during this time the main source of Bitcoin uncertainty was Mtgox itself. Mtgox indeed went bankrupt shortly after.

The ratio between information share and activity share (IS/AS ratio) is another variable that is interesting to take a look at. If this ratio is larger than one the exchange is providing the market with more information than the share of activity on this exchange indicates. These results are presented in Fig. 10. Mtgox and Btce are the only exchanges which consistently have an IS/AS ratio bigger than 1. The main reason for the IS/AS ratio volatility is probably caused by the fact that the Bitcoin economy is still early in the adoption phase.

6.1. Silk road

We also investigate what happens during and after a strong shock in the market. October 2nd 2013 the owner of the Silk Road marketplace was arrested by American authorities and the site was shut down Konrad (2013). Silk Road was believed to be a vital part of the Bitcoin ecosystem and the news caused the price to fall rapidly, before recovering over a two-day period. Events such as this, which produce large headlines in mass media, are particularly interesting to investigate, as one would expect all traders to get the information.

The news was announced around 5 pm GMT time, which is early afternoon for American, late afternoon for European, and night time for Asian traders. Remember from Section 3 that Bitcoin exchanges are open 24/7 and the frequency of trades are consistent throughout the day for the large exchanges trading the BTC/USD pair, see Fig. 6. One could therefore expect that the traders at the most active exchanges during the hours a market shock happens would have an advantage over traders at less active exchanges during those hours. Such information differentials could be the result of information advantages held by some traders, because these traders may be closer (geographically and time-wise) to both the rumours prior to the announcement and the announcement itself. Given such a situation traders at Btcn would have a information disadvantage given the time and location of this announcement. See Melvin and Covrig (1998) and de Jong et al. (2001) for more detailed discussions of the price discovery process on days with suspected strong information differentials.

Fig. 11 shows that during this event there is a very high volatility in the fundamental news component. Even for Bitcoin such high values are unusual, and for our dataset this period is only challenged by mid-April 2013, mid- to late-November 2013 and February 2014. The estimated parameters for this period is reported in Table 5.

Notice that Btce is the only exchange with positive ψ in this period, and has a significant higher information share than activity share. Bitstamp on the other hand has a significant lower information share than activity share. This implies a leader–follower relationship for the two during this short period. As seen in Fig. 9 this relationship is further confirmed by the information share for September and October. The market correlation graphs for the two, displayed in Fig. 14, again confirms these findings. One can of course only speculate over the reasons for this. These results give indications that either informed traders switched their preferred exchange in this period, or that traders at Btce suddenly became more informed. Notice also from Figs. 12 and 13 that Btce had a rapid increase in trading activity in October. Btce is also renowned in the Bitcoin community for having a good API for traders to place trading bots. Such bots can react extremely quickly. It is also worth noting that Btce and Mtgox's joint information share accounts for 70% of the information generated in the market during this event.

As discussed, these empirical results suggests that Btce was contributing a lot more to the price discovery process than its activity share should indicate in this period. The rest of the exchanges on the other hand has negative IS/AS ratio. This is as expected for the regional exchanges, but it is a little surprising for the large and global exchanges with consistent 24 h trading. So other explanations must come into play here, for instance those mentioned above.

7. Conclusion

In this paper we have investigated the role of different exchanges in the price discovery process of Bitcoin. The information share of exchange measures the fraction of the price discovery which happens at this particular exchange. Since it is natural that a higher fraction of the price discovery happens at exchanges with higher trading volume, we study also the ratio between the information share and the activity share (the IS/AS ratio). Since Bitcoin is still early in the adoption phase and there is rapid development in the Bitcoin economy, we investigate also how the information shares of exchanges change over time. Moreover, we study price discovery during a large shock to the market.

For the whole sample period Mtgox and Btce are the prominent price leaders. The rest of the exchanges are less informative, but they still provide information to the market. Mtgox dominated the price discovery process in the early part of the sample, but later we find a downward trend in the relative amount of information provided to the market. However Mtgox again influenced the Bitcoin exchange market early in 2014, with information share higher than activity share. We can conclude that Mtgox contributed with significant information share right until its bitter end. As hypothesized, the increasing Chinese interest in Bitcoin lead to a spike in Btcn's information share. In these months Btcn's IS/AS ratio exceed 1 for the first time, indicating that it then was more informative than the rest of the period. When it comes to differences between the larger and smaller exchanges, the smaller exchanges provide the market with less information and they usually follow the market with a lag.

Btce has a unique position in the price discovery of Bitcoin. It has maintained a position as one of the most informative exchanges over the whole sample period. Moreover, during a big shock to the market, the shutdown of the Silk Road, trades at Btce were much more informative than at other exchanges.

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