Price Discovery in Cryptocurrencies - UBRI

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VECM and IS

In price discovery literature the model used to find the relationship between instruments of a chosen market is the Vector Error Correction Model (VECM). The main methodology used to find the level of information of a specific market is the Information Share (IS), which measures the relative impact of market shocks in each instruments.

The Autoregressive Model

The Autoregressive Model is a representation of a time-varying stochastic process.

An AR(1) model is defined by:

$$X_t = c + \varphi X_{t-1} + \varepsilon_t$$

An AR(p) model:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-1} + \varepsilon_t$$

where, $\varphi_1, ..., \varphi_p$ are the *parameters* of the model, c is constant, and ε_t is *white noise*.

The Vector Autoregressive Model

The Vector Autoregressive Model is a multivariate generalization of the univariate AR model.

An VAR(1) model is defined by:

$$y_{1,t} = c_1 + a_{1,1}y_{1,t-1} + a_{1,2}y_{2,t-1} + \varepsilon_{1,t}$$

$$y_{2,t} = c_2 + a_{2,1}y_{1,t-1} + a_{2,2}y_{2,t-1} + \varepsilon_{2,t}$$

The VAR(1) in the matrix form:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

The VAR(p) is defined by:

$$Y_t = c + A_1 y_{t-1} + A_2 Y_{t-2} + ... + A_p Yt - p + \varepsilon_t$$

$$Y_t = c + \sum_{i=1}^{p} A_i Y_{t-i} + \varepsilon_t$$

The Error Correction Model

The Error Correction Model is used when data used are cointegrated (with long-run stochastic trend).

If you have a non-stationary of process you can regress first-differences of y_t and x_t to solve the problem. If x_t and y_t are cointegrated, meand that there is a long-run equilibrium relationship between Y and X, meaning that you make a linear combination of them in such way:

$$Y^E = \alpha + \beta X^E$$

This means that there if there is some difference between the actual value of y_t and the equilibrium value of Y^E. This should be explained by some delayed reaction of y_t to changes in x_t. This relation can be written as:

$$y_t = c + \delta_1 x_t + \delta_2 x_{t-1} + \mu y_{t-1} + \varepsilon_t$$

The problem with the above expression is that it doesn't measure the dynamics of the economic relationship between y_t and x_t. Another problem is that, because neither y_t or x_t are stationary, there is the risk of spurious regression, since there will almost be a statistically significant value for δ_i.

To solve those problems we use an Error Correction Model:

$$y_t - y_{t-1} = c + \delta_1 x_t + \delta_2 x_{t-1} - (1-\mu) y_{t-1} + \varepsilon_t$$

$$\Delta y_t = c + \delta_1 x_t - (\delta_1 x_{t-1} + \delta_1 x_{t-1}) + \delta_2 x_{t-1} - (1-\mu) y_{t-1} + \varepsilon_t$$

$$\Delta y_t = c + \delta_1 \Delta x_t - \lambda (y_{t-1} - \alpha - \beta x_{t-1}) + \varepsilon_t$$

where $\lambda = 1 - \mu$ and $\beta = \frac{\delta_1 + \delta_2}{1-\mu}$.

If in the long term relationship between y_t and x_t they are cointegrated, then the term $y_{t-1} - \alpha - \beta x_{t-1}$ will be $I(0)$ and therefore will be stationary.

An economic interpretation of the ECM is that if $y_{t-1} > \alpha + \beta x_{t-1}$, then y_t is above the equilibrium value. This would mean that $\Delta y_t < 0$ and the error in the last period should be corrected to approximate to the equilibrium value.

The ECM(p) is defined by:

$$\Delta y + t = \delta_1 + \sum_{i=0}^{p} \delta_i \Delta x_{t-i} + \sum_{j=1}^{k} \mu_j \Delta y_{t-j} - \lambda (y_t - 1 - \alpha - \beta x_{t-1}) + \varepsilon_t$$

The Vector Error Correction Model

The ECM approach by Engle-Granger is restricted to univariate processes. Also, it requires tests to check if the time series variables are $I(0)$ or $I(1)$. Using the Johansen's procedure all variables are treated endogenously and there is no need for testing cointegration. The model adds the error correction features from the ECM to a VAR model.

This methodlogy follows as: 1. Estimate an unrestricted VAR involving potentially non-stationary variables 2. Test for cointegration using Johansen test 3. Form and analyse the VECM

For the unit root test for the VAR(p) model we have:

$$Y_t = c + \Phi D_t + \sum_{i=1}^{p} A_i Y_{t-1} + \varepsilon_t$$

where D_t is a deterministic (trend) component.

Applying the error correction models there are two possible specifications:

1. The longrun VECM:

$$\Delta Y_t = c + \Phi D_t + AY_{t-p} + \Gamma_{p-1}\Delta Y_{t-p+1} + ... + \Gamma_1 \Delta Y_{t-1} + \varepsilon_t$$

$$\Delta Y_t = c + \Phi D_t + AY_{t-p} + \sum_{i=1}^{p-1} (\Gamma_i + \Delta Y_{t-i}) + \varepsilon_t$$

where

$$\Gamma_i = A_1 + .. + A_i - I; i = 1, ..., p-1$$

2. The shortrun (or transitory) VECM:

$$\Delta Y_t = c + \Phi D_t - \Gamma_{p-1}\Delta Y_{t-p+1} - ... - \Gamma_1 \Delta D_{t-1} + AY_{t-1} + \varepsilon_t$$

$$\Delta Y_t = c + \Phi D_t + AY_{t-1} - \sum_{i=1}^{p-1}(\Gamma_i + \Delta Y_{t-i}) + \varepsilon_t$$

where

$$\Gamma_i = A_{i+1} + .. + A_p - I; i = 1, ..., p-1$$

in both terms $A = A_1 + ...A_p - I$

Information Share Measure

Let $Y_{i,t} = (y_{1,t}, y_{2,t})'$ be a vector of logged prices of the same instrument in 2 distinct markets.

$$Y_{i,t} = (y_{1,t}, y_{2,t}) \sim I(1)$$

Since the prices represent the same instrument, then the two prices are cointegrated with a linear relationship expressed as

$$y_{1,t} = \beta y_{2,t} + \varepsilon_t$$

where $\varepsilon_t \sim I(0)$ and β is the cointegrating vector, $\beta = (1, -\beta)$‘.

The VECM representation for p lags is given as

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_p \Delta y_{t-p} + \varepsilon_t$$

where α is the coeficient of the error correction term, ε is a $2x1$ vector of the residuals with $\varepsilon_t \sim N(0, \Omega)$, since the price changes are logged the covariance is assumed to be stationary and the vector moving average (or the Wold) representation is given as:

$$\Delta p_t = \Psi(L)\varepsilon_t$$

using the Beverdige-Nelson decomposition where $\Psi(L) = \sum_{s=0}^{\infty} \Psi_S L^S$, the comomon trends of the price levels can be represented as

$$y_t = y_0 + \Psi(1)\sum_{s=0}^{t} \varepsilon_s + \Psi^*(L)\varepsilon_t$$

where, $\Psi(1) = \sum_{s=0}^{\infty} \Psi_p$. In the above, the matrix $\Psi(1)$ contains the cumulative impact of innovation ε_t on all future prices, measuring the long-run impact of ε_t on prices.

Let $\psi = (\psi_1, \psi_2)$ be a common row vector of $\Psi(1)$, and $\psi\varepsilon_t$ the incremental change in price that permanently added into the instrument price due to new information. Hasbrouck 1995 proposes the use of the structure variance to dervie the measure of price discovery.

$$var(\psi\varepsilon_t) = \psi\Omega\psi$$

If Ω is diagonal, if would mean that prices innovations are not correlated, then we would have that the the jth market's information share is defined as $IS_j = \frac{\psi_j^2\Omega_{jj}}{\psi\Omega\psi'}$. However, if Ω is not diagonal then the measure has the problem of attributing the covariance terms to each market. To overcome this problem Hasbrouck 1995 suggested the use of the Cholesky decomposition of Ω and measure IS using the orthogonalized innovations.

Let F be a lower triangular matrix such that $FF' = \Omega$. The IS for the jth market is then

$$IS = \frac{([\psi' F]_i)^2}{\Psi\Omega\Psi'}$$

Annotated Bibliography

"Does Bitcoin dominate the price discovery of the Cryptocurrencies market? A time-varying information share analysis", Chang and Shi

In this article, the authors examine the dynamic information shares of the four top cryptocurrencies, being them Bitcoin (BTC), Ethereum (ETH), Ripple (XRP) and Litecoin (LTC). The new contribution to literature is the analysis of different crypto (instead of only Bitcoin) in the dynamics of price discovery. There are two main methodologies for examining price discovery, the Information Share (IS) and the Total Spillover Index (TSI). The TSI measures the relative variations on one instrument due to the shock of another instrument. In contrast, the IS measures the relative efficiency of each instrument as a whole after incorporating common shocks in the market. Running a modified version of IS the authors found that although BTC is the main contributor to price discovery, that role has been decreasing. The second largest contribution to price discovery comes from XRP, followed by LTC at most of the time and then ETH. This result is different from the findings of Agosto and Cafferata 2020, for example, which suggest a central role of ETH in co-explosivity of cryptocurrencies during bubble periods. One suggestion of further work is the investigation of the nature of the less sensitivity to market shocks of ETH, which differs from other cryptos.

"Price discovery in Bitcoin: The impact of unregulated markets", Alexander and Heck

The paper by Alexander and Heck analyzes the minute-level and multi-dimension information flows within Bitcoin (BTC) spot and it's derivatives.Their work extends the work of Alexander, Choi, et al. 2020 confirming their results with respect to spot exchanges, but also shows that there information flows are lost by restricting analysis only to the BitMEX perpetual derivatives. The types of derivatives annalyzed are the *perpetual swaps*, the BTC futures traded on unregulated market and the CME BTC futures. Their findings indicate that unregulated markets led the price discovery of BTC whilst derivatives of the CME and the US-based spot exchanges reacted to these price movements. They also found a faster speed of adjustment and information absorption on the unregulated spot and derivatives than on CME. Using a generalization information share metric proposed by Hasbrouck 1995, they found that different from traditional derivative markets, bitcoin spot leads information in comparision to CME futures, the delay to incorporate new information can reach even 4 minutes, this is consistent with the dominance of unregulated derivative products over the regulated futures.

"Price discovery of cryptocurrencies: Bitcoin and beyond", Brauneis and Mestel

Although the title says about price discovery, this article actually measures the liquidity and market efficeinty of the cryptocurrencies. It tests Fama's weak form efficiency by using the random walk hypothesis. THe data used is from 73 cryptocurrencies from 08/31/2015 to 11/30/2017. After performing many statistical test to investigate the properties of the timeeries they perform a non-parametric test for market efficency proposed by Godfrey 2017.The metric Measure Of Efficeincy (MOE), which ranges from 0 (fully inneficient prices) to one (fully efficent).The MOE measures how well the passive strategy performs relative to active (speculative) trading. Bitcoin prices passes most of the statistical test of price randomness, whilst show that for earlier timespan, the prices were inefficient. The other 73 cryptocurrencies passes different tests for randomness, but not all. The pattern revelaled was that ineffficency of cryptocurrencies rises as they become less capitalize. Their findings suggests that markets with more liquidity and volume have more efficency, and that Bitcoin tends to become more efficient over time.

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