Project 2. Automated Car Parking

1. Continuous-time OCP

$$p_x = \vartheta \cos\theta$$
$$p_y = \vartheta \sin\theta$$
$$\dot{\theta} = \frac{\vartheta}{L} \tan\theta$$

optimal control problem

$$\underset{u(t),\, x(t)}{\text{minimise}} \quad J(x(t), u(t))$$

$$\text{subject to} \quad \dot{x}(t) = f(x(t), u(t)), \quad t \in [0, T]$$
$$h(x(t), u(t)) \le 0, \quad t \in [0, T]$$
$$r(x(0), x(T)) = 0$$

with

$$f(x(t), u(t)) = \begin{bmatrix} u_1(t) \cdot \cos x_3(t) \\ u_2(t) \cdot \sin x_3(t) \\ u_1(t) \cdot u_2(t) \end{bmatrix}$$

$$J(x(t), u(t)) = \int_0^T u_1^2(t) + u_2^2(t) \, dt$$

$$h(x(t), u(t)) = \begin{bmatrix} -(x_1(t) - \sigma \cos x_3(t) + 3.5)^2 - (x_2(t) - \sigma \sin x_3(t))^2 + 4 \\ -(x_1(t) - \sigma \cos x_3(t) - 3.5)^2 - (x_2(t) - \sigma \sin x_3(t))^2 + 4 \\ -(x_1(t) + \sigma \cos(x_3(t)) + 3.5)^2 - (x_2(t) + \sigma \sin(x_3(t))^2) + 4 \\ -(x_1(t) + \sigma \cos(x_3(t) - 3.5)^2 - (x_2(t) + \sigma \sin x_3(t))^2 + 4 \\ -(x_1(t) - 3.5)^2 - (x_2(t))^2 + 4 \\ -(x_1(t) + 3.5)^2 - (x_2(t))^2 + 4 \\ u_1(t) - 0.5 \\ -u_1(t) - 0.5 \\ u_2(t) - 0.33 \\ -u_2(t) - 0.33 \end{bmatrix}$$

$$r(x(0), x(T)) = \begin{bmatrix} x_1(0) \\ x_2(0) - 2 \\ x_3(0) - 0.01 \\ x_1(T) \\ \vdots \end{bmatrix}$$

2. Discrete time OCP

using sequential approach

$$X_{k+1} = X_k + \Delta t \begin{bmatrix} u_1(k) \cdot \cos x_3(k) \\ u_1(k) \cdot \sin x_3(k) \\ u_1(k) \cdot u_2(k) \end{bmatrix}$$

$$\underset{X(0),\,u}{\text{minimise}} \quad \sum_{k=0}^{N-1} u_1^2(k) + u_2^2(k)$$

subject to
$$\begin{bmatrix} x_1(0) \\ x_2(0) - 2 \\ x_3(0) - 0.01 \\ x_1(T) - \\ x_2(T) \\ x_3(T) \end{bmatrix}$$

$$h(u) = \begin{bmatrix} -(x_1(k) - \sqrt{\ } \cos x_3(k) + 3.5)^2 - (x_2(k) - \sqrt{\ } \sin x_3(k))^2 + 4 \\ -(x_1(k) - \sqrt{\ } \cos x_3(k) - 3.5)^2 - (x_2(k) - \sqrt{\ } \sin x_3(k))^2 + 4 \\ -(x_1(k) + \sqrt{\ } \cos x_3(k) + 3.5)^2 - (x_2(k) + \sqrt{\ } \sin x_3(k))^2 + 4 \\ -(x_1(k) + \sqrt{\ } \cos x_3(k) - 3.5)^2 - (x_2(k) + \sqrt{\ } \sin x_3(k))^2 + 4 \\ -(x_1(k) - 3.5)^2 - (x_2(k))^2 + 4 \\ -(x_1(k) + 3.5)^2 - (x_2(k))^2 + 4 \\ u_1(k) - 0.5 \\ -u_1(k) - 0.5 \\ u_2(k) - 0.33 \\ -u_2(k) - 0.33 \end{bmatrix}$$

$$k \in N_{[0, 1, \ldots, N-1]}$$

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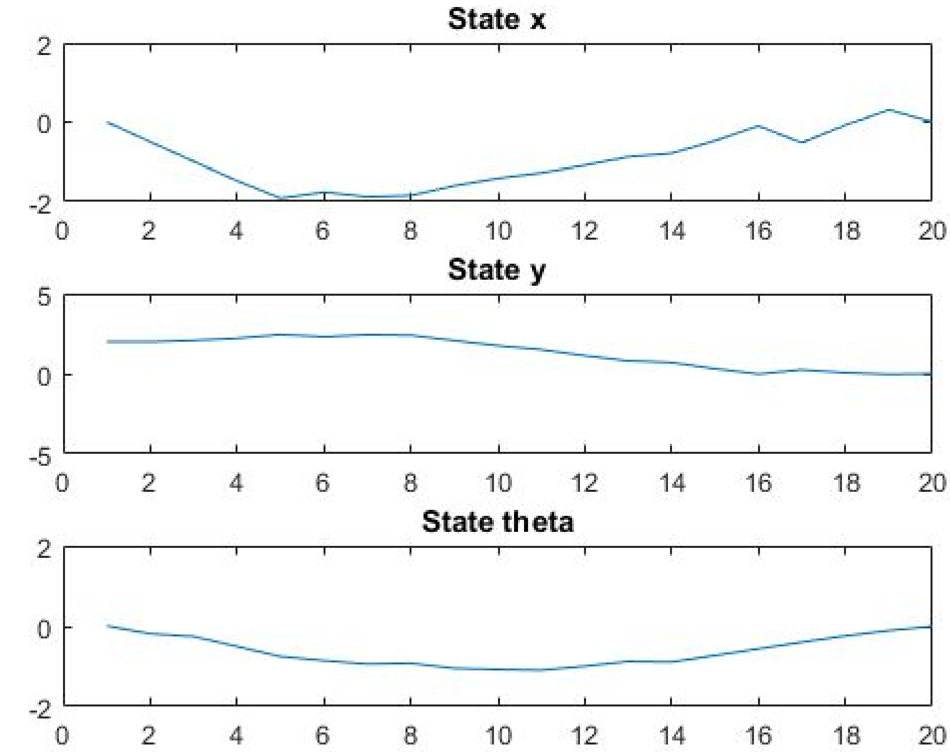
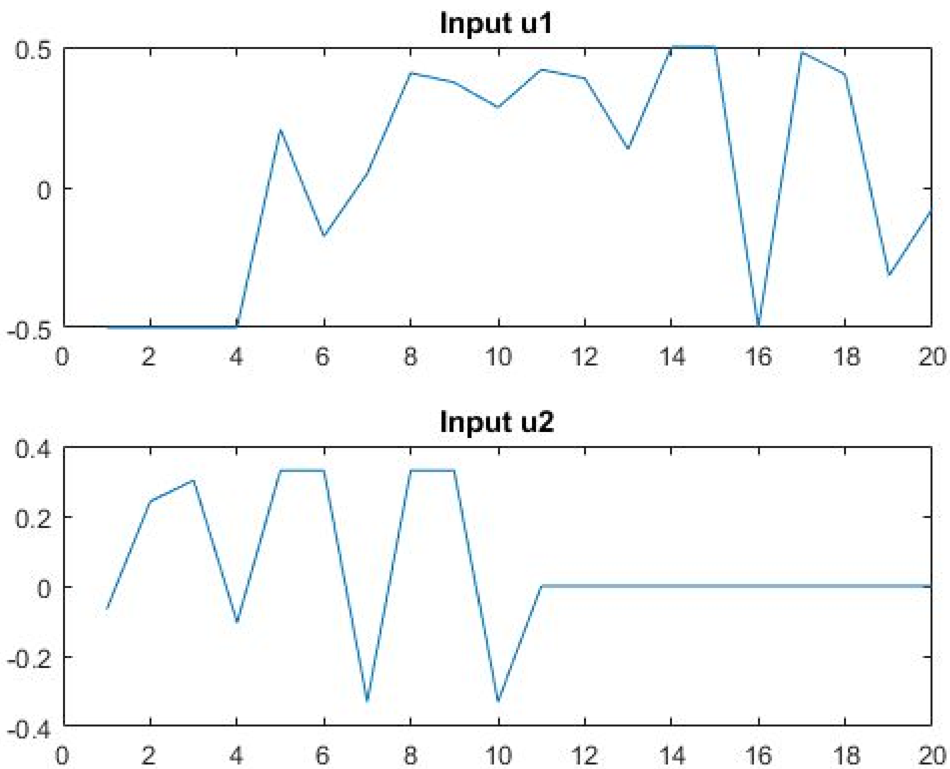
Task 3.
Below the plot of the state and inputs are given.





Below is given the continuous and discrete time OCP formulation: