The figures in the margin indicate full marks.

Group A

Answer FIVE questions.   5×5=25

1. In a catenary $y = c\cosh\dfrac{x}{c}$, show that the length of the perpendicular from the foot of the ordinate on the tangent is of constant length and that the length of the normal at any point is y^2/c .

2. *If* $y = \sin^{-1} x$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.

3. Evaluate: $\lim_{x \to 0} \left(\dfrac{tanx}{x}\right)^{\frac{1}{x}}$.

4. Use guidelines, sketch the curve $y = \dfrac{2x^2}{x^2-1}$.

5. Find the radius of the curvature of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ at any point (x, y).

6. Prove that $e^x Secx = 1 + x + x^2 + \dfrac{2}{3}x^3 + \ldots\ldots$

Group B

Answer FIVE questions.   5×5=25

7. Evaluate (a) $\displaystyle\int_0^1 \dfrac{1-x^2}{1+x^2} dx$

    (b) $\displaystyle\int \dfrac{x + \sin x}{1 + \cos x} dx$

8. Show that $\displaystyle\int_0^{\wedge} \dfrac{xTanx}{Secx + \cos x} dx = \dfrac{\wedge^2}{4}$

9. Prove that $\int_0^{\wedge 12} \cos^m x \sin mx\, dx = \frac{1}{2^{m+1}}(2 + \frac{2^2}{2} + \frac{2^3}{3} + \frac{2^4}{4} + ... + \frac{2^m}{m})$.

10. Find the area of hypo clycloid $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

11. Evaluate $\int_0^\infty \int_x^\infty e^{-y}/y\, dy\, dx$ by changing the order of integration.

12. Find the volume of the solid generated by the revolution of the ellipse

    $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about $x - axis$.

Group C

Answer THREE questions.   3×5=15

13. Transform the equation $2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0$ to parallel axes through (-2, 3).

14. If e and e' be the eccentricities of a hyperbola $x^2/a^2 - y^2/b^2 = 1$ and its conjugate prove that

    $1/e^2 + 1/e'^2 = 1$.

15. Find the vertices eccentricity, latus rectum and foci of the ellipse $8x^2 + 6y^2 - 16x + 12y + 13 = 0$.

16. Show that the line lx + my + n = 0 touches the parabola y² = 4ax if In = am².

Group D

Answer THREE questions.   3×5=15

17. Find the volume of the parallerlopiped whose co-terminal sides are given by the vectors
    $2\vec{i} + 3\vec{j} - 4\vec{k}, \ \vec{i} - \vec{j} + \vec{k}, \ \vec{i} + \vec{j} + \vec{k}$,

18. Prove by vector method Cos (A+B) = Cos A Cos B - Sin A Sin B.

19. Find the Cartesian and cylindrical coordinates of the points whose spherical coordinates are

    $\left(\sqrt{3}, \frac{\pi}{2}, \frac{-\pi}{2}\right)$.

20. Find the parametric and Cartesian equation of the line through (1,2,3) and parallel to the
    vector $3\vec{i} - \vec{j} + 2\vec{k}$,.

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