

PURBANCHAL UNIVERSITY

Time-bound Home Exam 2020

B. E. (Civil/Computer/E&C/Electrical/B. Arch)/First Semester/Final

Time: 03:00 hrs. (+2 Hrs. for Submission)

Full Marks: 80 /Pass Marks: 32

BEG101SH: Engineering Mathematics-I (New Course)

Instructions:

Dear Students!

- This model of examination is for you as the end of your current semester. This examination allows you to write answer from your own place of residence. Follow the following instructions without fail.
- Do not write your name in the answer-sheet(s)
- All the answer-sheets should be sent to college through your approved email in which you have received your question paper.
- Do not write questions in the answer-sheet but mention clearly the question number.
- All the scan/photos of answer-sheets should be clearly visible. Any blur scan/photo will not be considered for evaluation. Responsibility lies with the students to make sure that scan/photos of the answer-sheet are of readable quality.
- Leave 1 inch margin on each side of the answer-sheet.
- Clearly mention your Roll no, subject, program, semester, page number at the right-top of each page as instructed by the Office of the Examination Management.
- Make sure that you send your answer-sheets within the given time. Any email received after the given time will not be acceptable.
- You are strictly advised to write with your own handwriting and that you are not using any unfair means to answer the questions.
- Do not consult during the examination period to any other person in answering the questions.
- Do not post any pictures of taking examination or your answer-sheets in any social-media. Found that may be taken action from University.

The figures in the margin indicate full marks.

Group A

Answer FIVE questions.

5×5=25

1. In a catenary $y = c \cosh \frac{x}{c}$, show that the length of the perpendicular from the foot of the ordinate on the tangent is of constant length and that the length of the normal at any point is y^2/c .
2. If $y = \sin^{-1} x$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.
3. Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x}}$.
4. Use guidelines, sketch the curve $y = \frac{2x^2}{x^2 - 1}$.
5. Find the radius of the curvature of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ at any point (x, y).
6. Prove that $e^x \sec x = 1 + x + x^2 + \frac{2}{3} x^3 + \dots$

Group B

Answer FIVE questions.

5×5=25

7. Evaluate (a) $\int_0^1 \frac{1-x^2}{1+x^2} dx$
(b) $\int \frac{x + \sin x}{1 + \cos x} dx$
8. Show that $\int_0^{\frac{\pi}{2}} \frac{x \tan x}{\sec x + \cos x} dx = \frac{\pi^2}{4}$

Contd. ...

(2)

9. Prove that $\int_0^{12} \cos^m x \sin mx dx = \frac{1}{2^{m+1}} \left(2 + \frac{2^2}{2} + \frac{2^3}{3} + \frac{2^4}{4} + \dots + \frac{2^m}{m} \right).$

10. Find the area of hypo cycloid $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1.$

11. Evaluate $\int_0^\infty \int_x^\infty e^{-y/y} dy dx$ by changing the order of integration.

12. Find the volume of the solid generated by the revolution of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ about } x\text{-axis}.$$

Group C

Answer THREE questions.

3×5=15

13. Transform the equation $2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0$ to parallel axes through (-2, 3).

14. If e and e' be the eccentricities of a hyperbola $x^2/a^2 - y^2/b^2 = 1$ and its conjugate prove that $1/e^2 + 1/e'^2 = 1.$

15. Find the vertices eccentricity, latus rectum and foci of the ellipse $8x^2 + 6y^2 - 16x + 12y + 13 = 0.$

16. Show that the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ if $ln = am^2.$

Group D

Answer THREE questions.

3×5=15

17. Find the volume of the parallelepiped whose co-terminal sides are given by the vectors $2\vec{i} + 3\vec{j} - 4\vec{k}, \vec{i} - \vec{j} + \vec{k}, \vec{i} + \vec{j} + \vec{k},$

18. Prove by vector method $\cos(A+B) = \cos A \cos B - \sin A \sin B.$

19. Find the Cartesian and cylindrical coordinates of the points whose spherical coordinates are $\left(\sqrt{3}, \frac{\pi}{2}, \frac{-\pi}{2}\right).$

20. Find the parametric and Cartesian equation of the line through (1,2,3) and parallel to the vector $3\vec{i} - \vec{j} + 2\vec{k},.$

