Acoustics

- **Sound:**
- Sound waves are longitudinal mechanical waves which propagate in a medium due to compression and rarefaction of the medium.
- The pressure is highest at the point of compression and is lowest at the point of rarefaction.
- In case of propagating sound wave the pressure at a point varies periodically.
- The mechanical waves of frequency less than 20Hz are called infrasonic waves.
- The sound waves of frequency above 20KHz are called ultrasonic waves.
- The sound waves of frequency range 20Hz to 20KHz are audible sound waves.
- Normal human ear can hear the sound of frequency of 20Hz to 20 KHz.
- **Speed of Sound:**
- Consider the sound wave is travelling in a gaseous (or fluid) medium from left to right along x direction through a tube with velocity v as shown in figure.

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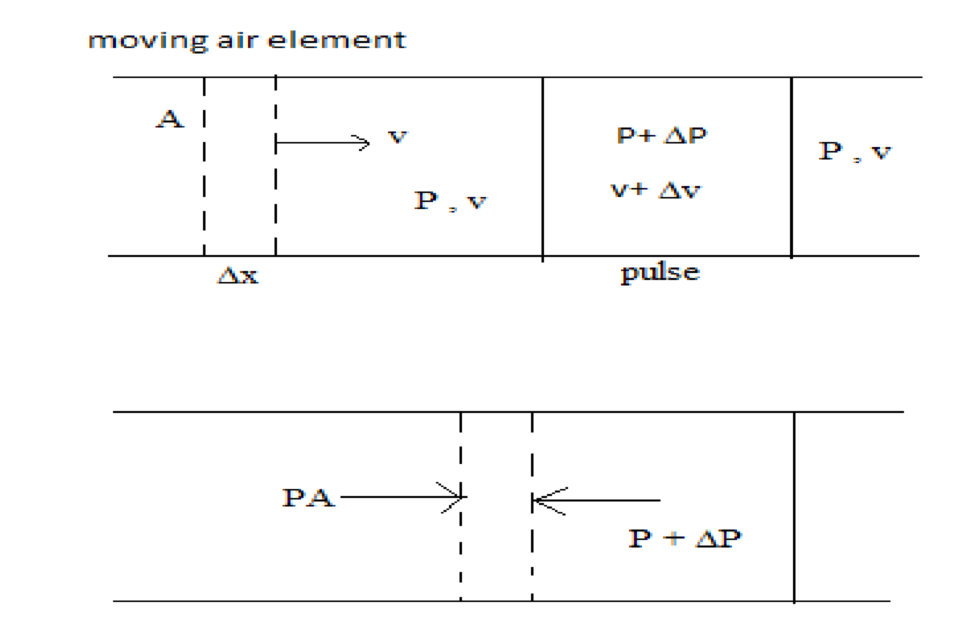

- The pressure of undisturbed air be P and inside the pulse be P + ΔP where ΔP is positive due to compression.
- Let a slice of air of thickness Δx and face area A moving towards the pulse with speed v.
- When the fluid element enters into the pulse, its leading face encounters a region of higher pressure which slows it to speed v + Δv. Where Δv is negative.
- This slowing completes when the rear face reaches the pulse which requires time interval,
- $\Delta t = \dfrac{\Delta x}{v}$ ……………………(1)
- During this interval force on trailing face of element is PA towards right and force on the leading face is (P + ΔP) A towards left. So the net force on the element in Δt is

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- $F = PA - (P + \Delta P)A = -(\Delta P)A$ ……….....(2)
- The average acceleration of the element during Δt is
- $a = \frac{\Delta v}{\Delta t}$ ……………....(3)
- The volume of the element, $V = A(\Delta x)$, then mass of the element is
- $\Delta m = \rho A(\Delta x) = \rho A\, v(\Delta t)$ …………..... (4)
- where ρ is density of medium.
- Now from Newton's second law of motion
- $F = ma$
- From equation (2), (3) and (4) we have
- $-(\Delta P)A = \rho A v(\Delta t)\left(\frac{\Delta v}{\Delta t}\right)$

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- $\rho v^2 = -\frac{\Delta P}{\frac{\Delta V}{V}}$ ……………......(5)
- The volume of element, V = Av Δt outside the pulse is compressed by ΔV = A(Δv)Δt inside the pulse. Thus,
- $\frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V} = \frac{A(\Delta v)\Delta t}{Av\Delta t} = \frac{\Delta v}{v}$
- Thus, equation (5) becomes
- $\rho v^2 = -\frac{\Delta P}{\frac{\Delta V}{V}} = \frac{\text{Stress}}{\text{Strain}} = B$ (Bulk modulus)
- or, $v = \sqrt{\frac{B}{\rho}}$ ………......(6)
- This is speed of sound in fluid medium of density ρ.

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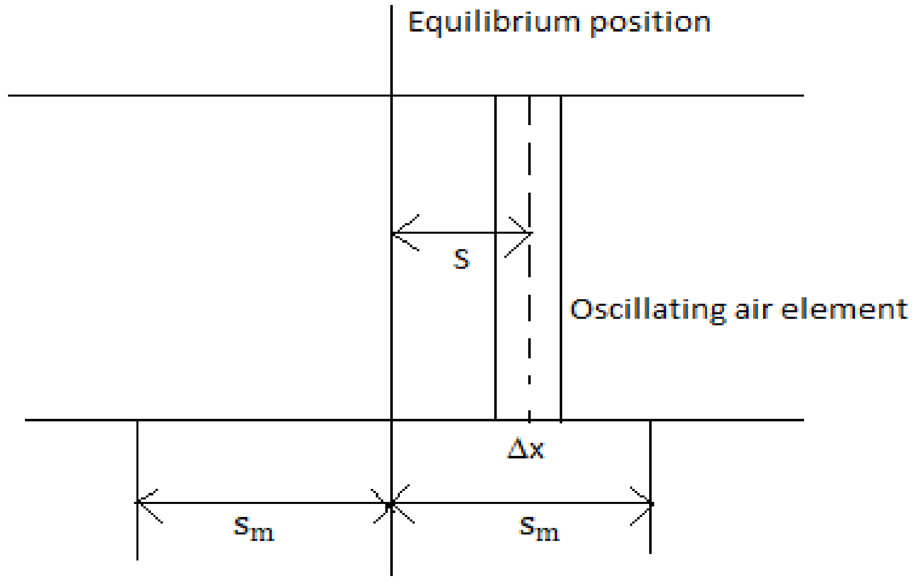
- **Travelling sound waves (pressure amplitude):**
- Consider sound wave is travelling along positive x-direction through a long tube filled with fluid as shown in figure.
- An oscillating element of fluid of cross-sectional area A and thickness Δx with its centre displaced from its mean position by S.
- The equation for longitudinal displacement is
- $S = S_m \cos(kx - \omega t)$ ………………..(1)
- where S_m is displacement amplitude.
- Again bulk modulus, $B = -\dfrac{\Delta P}{\frac{\Delta V}{V}}$



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- $\Delta P = -B\,\dfrac{\Delta V}{V}$..............(2)
- The volume of the element is $V = A(\Delta x)$.
- The change in volume of element, $\Delta V = A(\Delta S)$
- Where, ΔS is the difference in displacement of two faces of the element.
- So from equation (2) we have $\Delta P = -B\,\dfrac{A(\Delta S)}{A(\Delta x)} = -B\,\dfrac{\Delta S}{\Delta x}$
- In differential limit, $\Delta P = -B\,\dfrac{\partial S}{\partial x}$
- Now using equation (1) we have
- $\Delta P = -B\,\dfrac{\partial}{\partial x}[S_m \cos(kx - \omega t)]$
- $\Delta P = BkS_m \sin(kx - \omega t)$

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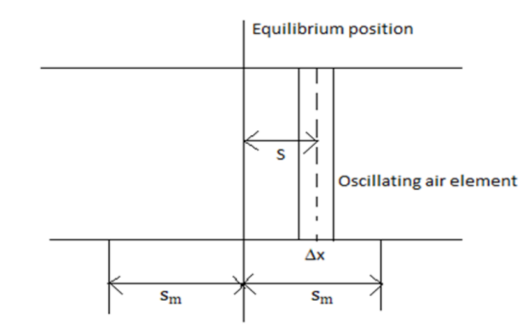
- Since, $B = \rho v^2$ then
- $\Delta P = \rho v^2 k S_m \sin(kx - \omega t)$
- $\Delta P = \Delta P_m \sin(kx - \omega t)$ ……........(3)
- Where $\Delta P_m = \rho v^2 k S_m$ is pressure amplitude of sound wave.
- This equation (3) shows that the sound wave may be considered as pressure wave.
- **Intensity of Sound Wave**
- Consider a thin slice of fluid of thickness dx and area of cross–section A oscillating back and forth in a long air filled tube in which sound is propagating with velocity v.
- The longitudinal displacement of the element is given by
- $S = S_m \cos(kx - \omega t)$ ………………......(1)
- The kinetic energy of the slice of element is
- $dk = \frac{1}{2} dm\, v_s^2$, where dm = mass of the element, v_s= velocity of thin slice of air.



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- Now, $dm = \rho A dx$ and $v_s = \frac{\partial s}{\partial t} = \frac{\partial}{\partial t}[S_m \cos(kx - \omega t)] = \omega S_m \sin(kx - \omega t)$

 Where, ρ is the density of air. Then,

- $dk = \frac{1}{2}\rho A dx\, \omega^2 S_m^2 \sin^2(kx - \omega t)$
- The rate at which this kinetic energy is transported along with the wave is
- $\frac{dk}{dt} = \frac{1}{2}\rho A \frac{dx}{dt}\, \omega^2 S_m^2 \sin^2(kx - \omega t)$
- $\frac{dk}{dt} = \frac{1}{2}\rho A v \omega^2 S_m^2 \sin^2(kx - \omega t)$, where v is the velocity of sound.
- The average rate at which this kinetic energy is transported along with the wave is
- $\left(\frac{dk}{dt}\right)_{avg} = \frac{1}{2}\rho A v \omega^2 S_m^2 [\sin^2(kx - \omega t)]_{avg}$
- $\left(\frac{dk}{dt}\right)_{avg} = \frac{1}{4}\rho A v \omega^2 S_m^2$ …………(2)
- The average rate at which kinetic energy is transported is equal to the average rate at which

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- the potential energy is transported along with the wave.
- The intensity of the sound wave which is the average rate at which both kinds of energies are transported per unit area is given by
- $I = \frac{2\left(\frac{dk}{dt}\right)_{avg}}{A} = \frac{1}{2}\rho v\omega^2 S_m^2$ ……………(3)
- Where S_m is the displacement amplitude of the sound wave.
- **Intensity level:**
- The intensity of threshold of hearing is 10^{-12} watt/m^2.
- The loudness of sound is directly proportional to the logarithm of intensity.
- If I and I_0 are intensities of sound having corresponding loudness L and L_0 then $(L - L_0)$ is called intensity level or sound level.
- Thus, sound level = $L - L_0 = K(\log_{10} I - \log_{10} I_0) = K \log_{10}\left(\frac{I}{I_0}\right)$

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- Where K is constant depends on unit, I_0 is standard reference intensity taken as 10^{-12} w/m^2.
- If K is taken equal to 1, the sound level is expressed in bel and its $\frac{1}{10}$ is called decibel(dB).
- So, in terms of decibel sound level can be written as
- Sound level = $10 \log_{10}\left(\frac{I}{I_0}\right)$ (in dB) When $I = I_0$
- Sound level = $10\log_{10}\left(\frac{I_0}{I_0}\right) = 0$
- In terms of decibel our standard reference level corresponds to zero decibel.

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- **Beats:**
- The phenomenon of periodic variations in the intensity of sound due to superposition of two waves of slightly different frequencies are called beats.
- One maximum and one succeeding minimum (or vice-versa) constitute one beat.
- The time interval in which one beat occurs is called beat period and the number of beats per second is called beat frequency.
- If f_1 and f_2 are frequencies of superposing sound waves then the beat frequency is found to be $(f_1 – f_2)$. Thus beat frequency $= f_1 – f_2$
- **Mathematical analysis:**
- Consider two superposing sound waves of frequency f_1 and f_2 as
- $y_1 = a \sin\omega_1 t = a \sin(2\pi f_1 t)$ and $y_2 = a \sin\omega_2 t = a \sin(2\pi f_2 t)$
- From superposition principle,
- $y = y_1 + y_2 = a \sin(2\pi f_1 t) + a \sin(2\pi f_2 t)$

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- $y = 2a \sin\left[\frac{2\pi(f_1+f_2)t}{2}\right] \cos\left[\frac{2\pi(f_1-f_2)t}{2}\right]$
- $y = 2a \cos\left[\frac{2\pi(f_1-f_2)t}{2}\right] \sin\left[\frac{2\pi(f_1+f_2)t}{2}\right]$
- $y = A \sin\left[\frac{2\pi(f_1+f_2)t}{2}\right]$
- where $A = 2a\cos\left[\frac{2\pi(f_1-f_2)t}{2}\right]$ is resultant amplitude.
- (i) A is maximum when $\cos\left[\frac{2\pi(f_1-f_2)t}{2}\right] = 1 = \cos n\pi$
- $\frac{2\pi(f_1-f_2)t}{2} = n\pi$; $n = 0, 1, 2, 3$ ....
- $t = \frac{n}{f_1-f_2} = 0, \frac{1}{f_1-f_2}, \frac{2}{f_1-f_2}, \frac{3}{f_1-f_2}, \ldots\ldots\ldots\ldots\ldots$
- At these instants intensity of sound will be maximum. The interval between any two consecutive maxima is $\frac{1}{f_1-f_2}$, where $f_1 - f_2$ is beat frequency or frequency of maxima.

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- (ii) A is minimum when $\cos\left[\frac{2\pi(f_1-f_2)t}{2}\right] = 0 = \cos(2n+1)\frac{\pi}{2}$
- $\frac{2\pi(f_1-f_2)t}{2} = (2n+1)\frac{\pi}{2}$; n = 0, 1, 2, 3 ....
- $t = \frac{(2n+1)}{2(f_1-f_2)} = \frac{1}{2(f_1-f_2)}, \frac{3}{2(f_1-f_2)}, \frac{5}{2(f_1-f_2)}, \ldots\ldots\ldots\ldots\ldots$
- At these instants intensity of sound will be minimum. The interval between any two consecutive minima is $\frac{1}{f_1-f_2}$, where $f_1 - f_2$ is beat frequency or frequency of minima.
- Thus number of maxima per second is equal to number of minima per second which is equal to beat frequency $f_1 - f_2$.

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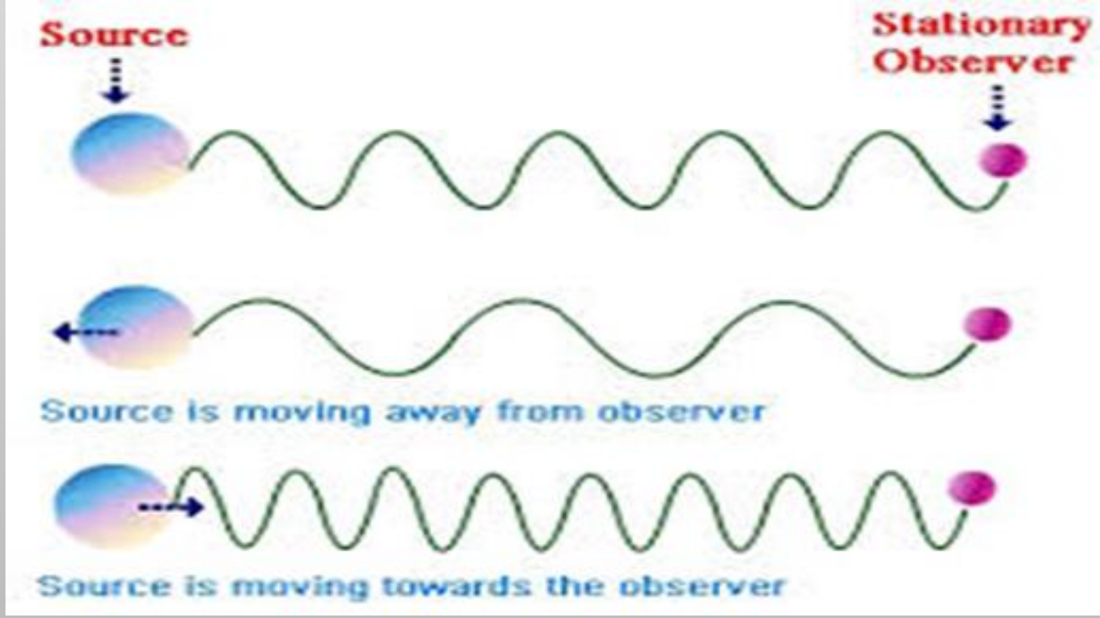
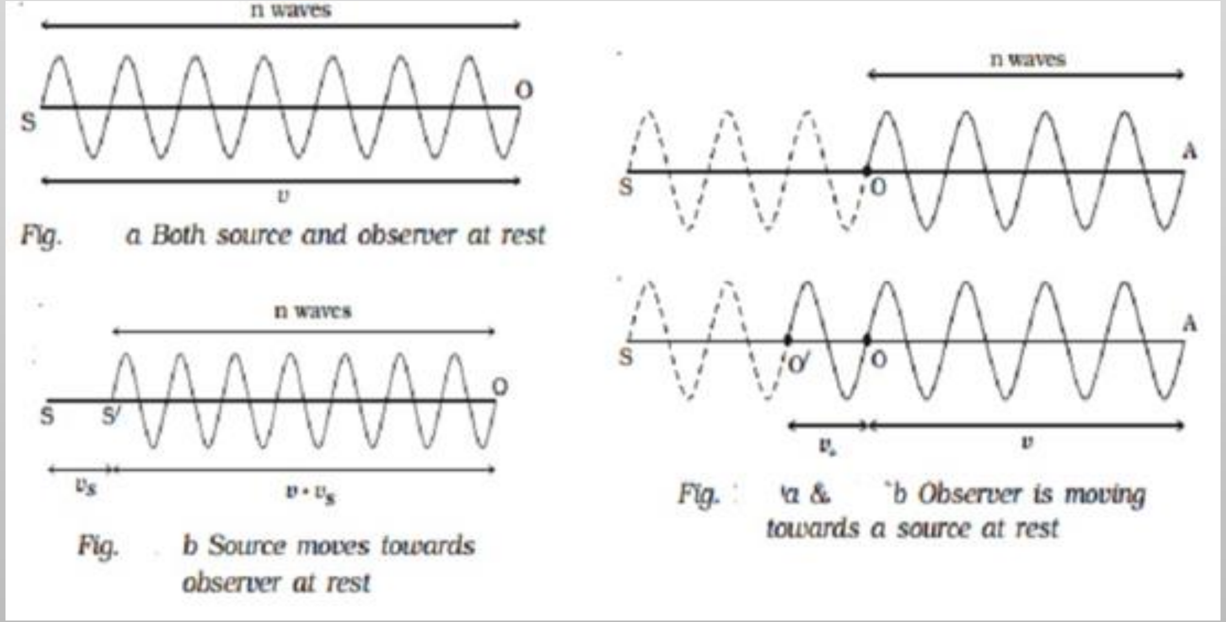
- Velocity of Sound in air (Newton's formula): $v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.01\times10^5}{1.293}} = 280$ m/s
- Laplace Correction: $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4\times1.01\times10^5}{1.293}} = 332$ m/s
- Factors affecting velocity of sound in air:
- Temperature: $v = \sqrt{T}$
- Pressure: v is independent of pressure at constant temperature
- Density: $v = \frac{1}{\sqrt{\rho}}$
- Humidity or moisture: v increases with humidity
- Wind direction: Velocity of sound will increase if the wind is in the direction of sound and decrease if the wind is against its direction.

- **Doppler's Effect:**
- In 1842, C. Doppler investigated that pitch of sound appears to change due to relative motion between source of sound and observer.
- This apparent change in pitch (frequency) of sound due to relative motion between observer and the source of sound is called Doppler's effect.
- In general, the apparent frequency of sound observed by observer is increased when the distance between observer and the source of sound decreases and vice-versa.





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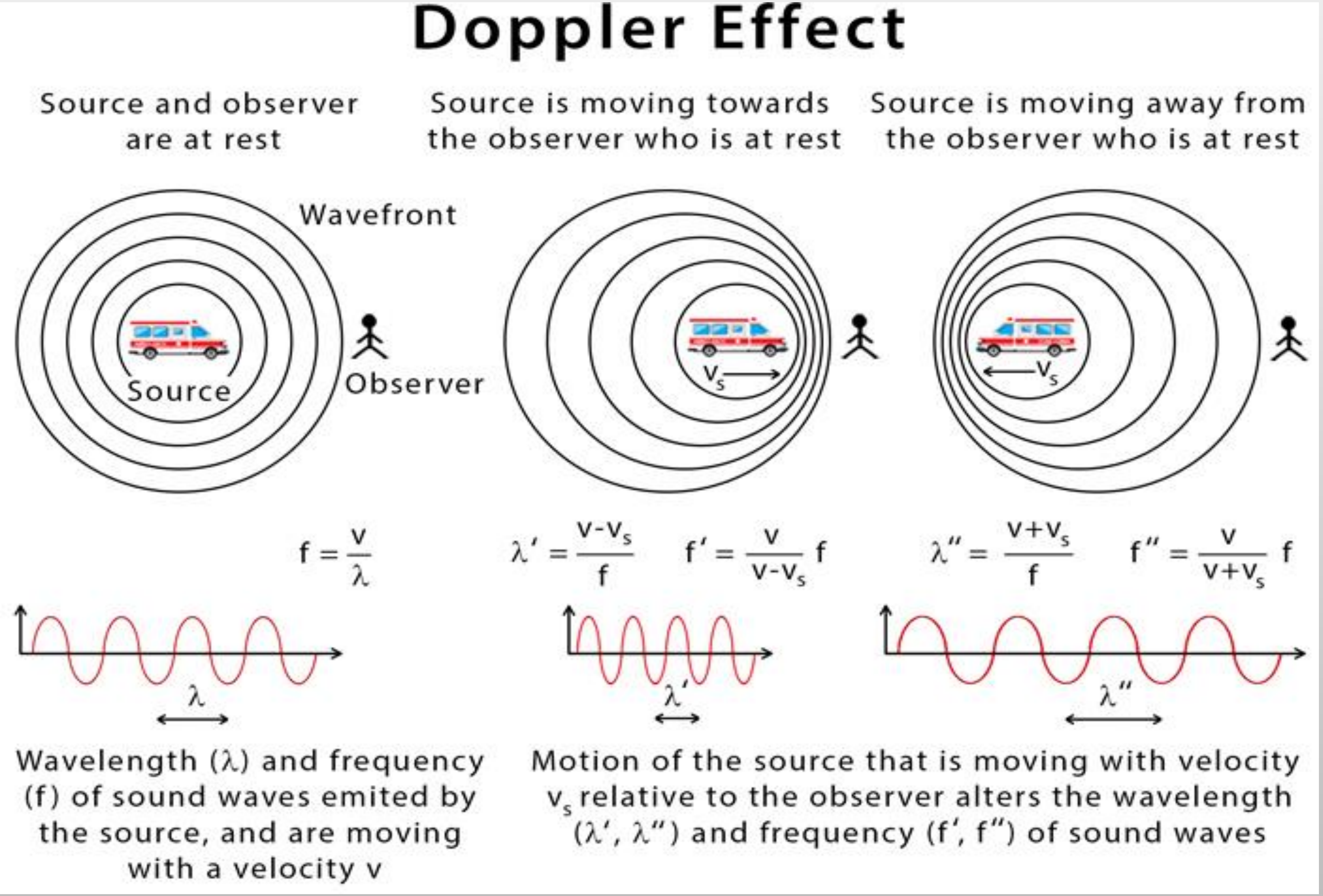

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- **Source in motion, observer at rest:**
- Let v = velocity of sound, f = true frequency of sound, v_s = velocity of source when the source is at rest, actual wavelength of sound is $\lambda = \frac{v}{f}$
- When the source moves towards observer, then all f waves produced per second will now be confined in a distance $v - v_s$. The apparent wavelength $\lambda' = \frac{v - v_s}{f}$
- So the apparent pitch of the sound heard by the observer is given by
- $f' = \frac{v}{\lambda'} = \frac{v}{\frac{v - v_s}{f}} = \frac{v}{v - v_s} f$
- Since $f' > f$, pitch of sound heard by the observer increases.
- If the source moves away from the observer apparent wavelength will be $\lambda' = \frac{v + v_s}{f}$
- So the apparent frequency will be $f' = \frac{v}{v + v_s} f$
- Thus the apparent frequency decreases

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- **Observer in motion, source at rest:**
- Let v = velocity of sound, f = true frequency of source and v_o = velocity of observer.
- The actual wavelength of sound is $\lambda = \frac{v}{f}$
- When the observer moves with velocity v_o towards the source, the observer receives more waves in one second.
- The additional waves confined in a distance v_0 is given by $\frac{v_o}{\lambda} = \frac{v_o}{\frac{v}{f}} = \frac{v_o}{v} f$
- Apparent frequency $f' = f + \frac{v_o}{v} f = \left(\frac{v+v_o}{v}\right) f$
- Thus the apparent frequency increases
- If the observer moves away from the stationary source apparent frequency will be
- $f' = f - \frac{v_o}{v} f = \left(\frac{v-v_o}{v}\right) f$
- The apparent frequency will be decreased.

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- **Source and observer both in motion.**
- When the source is at rest and observer is in motion the apparent frequency is given by
- $f' = \left(\frac{v \pm v_o}{v}\right) f$
- When the observer is at rest and source is in motion the apparent frequency is given by
- be $f' = \frac{v}{v \mp v_s} f$
- When both source and observer are in motion
- $f' = \frac{v \pm v_o}{v \mp v_s} f$
- Where upper sign (+ numerator, – denominator) corresponds to source and observer moving towards each other and the lower sign in the direction away from each other.

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- **Ultra-Sound:**
- The human ear can hear the sound wave of frequency between 20Hz to 20KHz. This range is audible range.
- The sound wave having frequency below 20Hz is called infrasonic wave and the sound wave having frequency above 20 KHz is called ultrasonic wave.
- **Production of ultrasonic waves:**
- With the help of mechanically oscillating body Production of ultrasonic waves is very difficult.
- This is because maintaining high frequency oscillation only by mechanical method is impossible.
- There are two methods for producing ultrasonic waves which use electrical, mechanical and magnetic properties of vibrating body.

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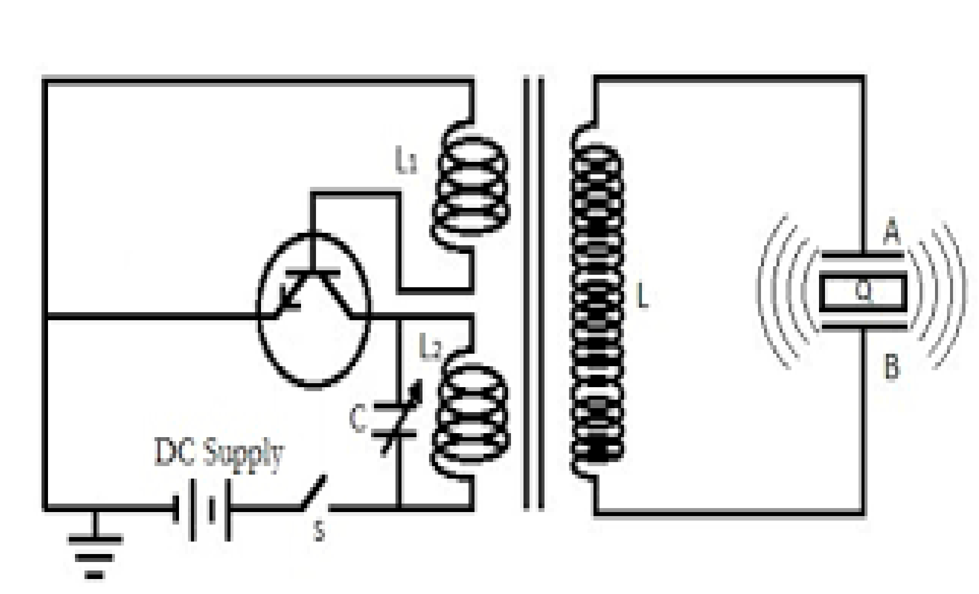
- **Piezoelectric or electrostriction method:**



- The phenomenon of electrical changes on certain crystals like tourmaline, quartz etc. when heated or cooled is called piezoelectricity.
- When such crystals are stretched or compressed along certain axis, electric potential difference is produced along a perpendicular axis.
- Alternately when an alternating potential difference is applied along electric axis, crystal vibrates along mechanical axis. If the frequency of electric oscillation coincide with natural frequency of crystal vibration amplitude will be large. This phenomenon is used for the production of ultrasonic waves. The experimental arrangement is shown in figure.
- A variable capacitor is adjusted such that natural frequency of crystal becomes equal to frequency of oscillating circuit.

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- At this condition amplitude of vibration will be large. The vibration is longitudinal in nature.
- If Y and ρ are young's modulus of elasticity and density of crystal respectively then the frequency of vibration is
- $f = \frac{k}{2l}\sqrt{\frac{Y}{\rho}}$ where, k = 1,2,3…(mode of vibration)
- The velocity of longitudinal wave in crystal is given by $v = \sqrt{\frac{Y}{\rho}}$
- In case of quartz the velocity $v = 5.5\times 10^3$m/s. Thus for a crystal of length 0.05m, the frequency for the first mode of vibration will be
- $f = \frac{1\times5.5\times10^3}{2\times0.05} = 5.5\times 10^4$ HZ
- The other modes of frequency are integral multiples of 5.5×10^4 Hz

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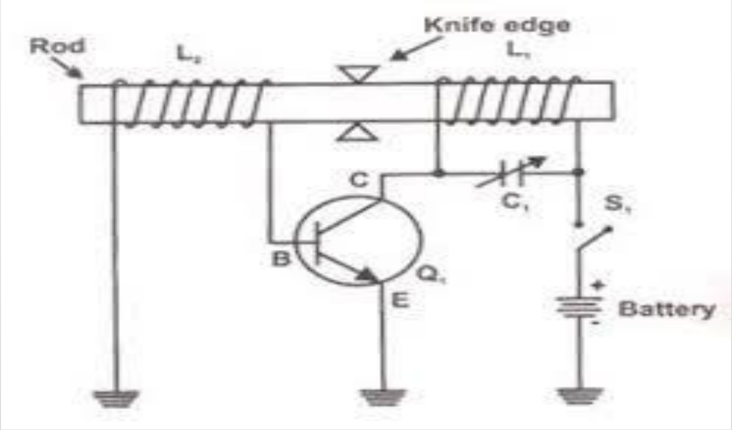
- **Magnetostriction Method:**
- When a rod of ferromagnetic substance like iron, nickel etc. is placed in a magnetic field parallel to its length, contraction or extension occurs.



- This phenomenon is known as magnetostriction.
- When the rod is placed inside a coil carrying alternating current, some change in length occurs on the rod for each half cycle of alternating current.
- If the frequency of alternating current is same as natural frequency of rod, amplitude of vibration of rod becomes large.
- The rod sends out ultrasonic waves from its ends when the applied frequency is of order of ultrasound.
- The frequency of oscillator can be adjusted with the help of variable capacitor.
- The experimental arrangement is shown in figure.

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- The natural frequency is given by
- $f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$
- Where, *l* = length of the rod, Y = young's modulus of rod and ρ = density of rod
- By adjusting the length of the rod and capacitance of the capacitor for resonance, high frequency oscillations of different frequencies can be obtained.
- A nickel rod, 10 cm long can produce frequency of about 25 kHz.
- **Application of ultrasonic waves:**
- **Detection of submarines iceberg, aircraft and other objects in sea:**
- Sharp ultrasonic wave is sent in various directions into the sea.
- The reflected beam from different objects can be detected with the help of quartz receiver.
- The interval between generation of ultrasonic waves and their return after reflection gives the idea of distance of the body.

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- Thus using ultrasonic waves presence of a body at a certain depth into the sea can be detected.
- **Depth of sea:**
- Determination of depth of sea using ultrasonic wave can be done by recording time interval between sending wave and reflected wave from the bottom of sea.
- If v is velocity of wave in water and t is time interval, then depth of sea can be estimated by $\frac{vt}{2}$.
- **Directional signaling:**
- Ultrasonic wave has smaller wavelength. Ultrasonic waves can be concentrated into a sharp beam. Hence, ultrasonic waves can be used for signaling in a particular direction.
- **Detection of flaws in metals:**
- The flaw in metal produces a change in medium due to which ultrasonic waves reflect.
- When ultrasonic waves pass through a metal having hole or crack inside it, an appreciable reflection occurs.

Acoustics contd.

- The reflection also takes place at back surface of the metal.
- The reflected waves are picked up by receiver and are suitably amplified.
- The pulses are applied to one set of plates of cathode ray oscilloscope.
- The transmitted and reflected signal from flaw and back surface of metal produce a peak each.
- If there are no flaws, reflected pulses can be observed at regular interval.
- But if there are flaws extra pulses will be observed.
- **Cleaning and clearing:**
- Ultrasonic waves can be used for cleaning utensils, washing clothes, removing dusts and soot from chimney.
- **Soldering and metal cutting:**
- Due to high frequency of ultrasonic waves, these waves can be used for drilling and cutting metals. These waves can be used for soldering aluminium which cannot be soldered by normal method. Ultrasonic welding can be done at room temperature.

Acoustics contd.

- **Chemical use:**
- A colloid solution of two immiscible liquid like oil and water can be formed by simultaneously subjecting to ultrasonic waves.
- Polishes, paints, food products, pharmaceutical preparations etc. can be prepared by using ultrasonic mixing.
- The constituents of alloys having widely different densities can be kept mixed uniformly by a beam of ultrasonic waves.
- Thus, it is easy to get alloy of uniform composition.
- **Medical use:**
- The body parts affected due to neurologic pains on being exposed to ultrasonic waves get great relief from pain.
- Abnormal growth in brain, certain tumors can be detected by ultrasonic waves.
- Dentists use ultrasonic waves for proper extraction of broken teeth.
- Nowadays, ultrasonic waves are also used in performing bloodless brain surgery.

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- **Destruction of lower life:**
- The animals like rat, frog, fish, bacteria etc. can be killed or injured by using high intensity ultrasonic waves.
- **Properties of ultrasound**
- They are highly energetic.
- Their speed of propagation depends upon frequency.
- They show negligible diffraction due to their small wavelength. So they can be transmitted over long distance without any appreciable loss of energy.
- Intense ultrasonic radiation has a disruptive effect in liquids by causing bubbles to be formed.
- When ultrasonic wave travels in a liquid bath, stationary wave pattern is formed due to reflection of wave from other end. The density of liquid varies from layer to layer along the direction of propagation. The plane diffraction grating is formed which can diffract light.