Computer: Exercise 12

Sunday, July 26, 2020 5:17 PM

2, 3, 17

Q 11 $\int \frac{x^2+2x-1}{2x^2+3x+1}\,dx$

Den $+ Ax + B$ / Den.

$= \int \frac{\frac{1}{2}(2x^2+3x+1) + 2x - 1 - \frac{3x}{2} - \frac{1}{2}}{2x^2+3x+1}\,dx$

$= \int \frac{1}{2}\,dx + \int \frac{(\frac{x}{2} - \frac{3}{2})\,dx}{2x^2+3x+1}$

$= \frac{x}{2} + \int \frac{\frac{1}{8}(4x+3) - \frac{3}{8} - \frac{3}{2}}{(2x^2+3x+1)}\,dx$

$= \frac{x}{2} + \frac{1}{8}\log(2x^2+3x+1) - \frac{15}{8}\int \frac{dx}{2x^2+3x+1}$

" $-\frac{15}{16}\int \frac{dx}{x^2+\frac{3}{2}x+\frac{1}{2}}$ $-\frac{9}{16}+\frac{1}{2}$

= " $-\frac{15}{16}\int \frac{dx}{(x+\frac{3}{4})^2-(\frac{1}{4})^2}$ $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$

2, 3, 17

$2\int \frac{1+x^2}{\sqrt{1-x^2}}\,dx$

$= \int \frac{2-(1-x^2)}{\sqrt{1-x^2}}\,dx = 2\int \frac{dx}{\sqrt{1-x^2}} - \int \sqrt{1-x^2}\,dx$

Q5 $\int \sqrt{\frac{a+x}{x}}\,dx$ <-- Q6 $\int \sqrt{\frac{a+x}{a-x}}\,dx$ 44. $\int \frac{1}{\sqrt{x}(1-x)}\,dx$

$\int \frac{(a+x)}{\sqrt{x(a+x)}}\,dx$

$= \int \frac{a+x}{\sqrt{x^2+ax}}\,dx$

$= \int \frac{a+x}{\sqrt{a^2-x^2}}\,dx$

$= a\int \frac{dx}{\sqrt{a^2-x^2}} + (-\frac{1}{2})\int \frac{-2x\,dx}{\sqrt{a^2-x^2}}\,dx$

$= a\sin^{-1}x/a - \frac{1}{2}\cdot 2\sqrt{a^2-x^2}$ $\int \frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)}$

Let, $x = y^2$

$dx = 2y\,dy$

$\because d(x^2+ax) = 2x + a$

$= \int \frac{\frac{1}{2}(2x+a) + a/2}{\sqrt{x^2+ax}}\,dx$

$= \frac{1}{2}\left(2\sqrt{x^2+ax}\right) + \frac{a}{2}\int \frac{dx}{\sqrt{x^2+2\cdot x\cdot\frac{a}{2}+(\frac{a}{2})^2-(\frac{a}{2})^2}}$

44)

$x = y^2$

$dx = 2y\,dy$

$\int \frac{dx}{\sqrt{x}(1+x)}$

$\int \frac{dx}{\sqrt{x}(1+x)}$

$dx = 2y\,dy$

$\int \frac{2y\,dy}{y(1-y^2)}$

$= 2\int \frac{dy}{1-y^2}$

Q 8 $\int \frac{\sin 2x\,dx}{a\sin^2 x + b\cos^2 x}$

$2\sin x\cos x\,dx$

Let $a\sin^2 x + b\cos^2 x = y$

=

(18) $\int \frac{\cos x\,dx}{\sin^2 x + 4\sin x + 5}$

Let $\sin x = y$

diff w.r.t x ....

$\cos x\,dx = dy$

$= \int \frac{dy}{y^2+4y+5}$

$= \frac{dy}{(y+2)^2+1}$

(i)

(25) $\int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$

Q $\int \frac{dx}{2+\sin^2 x}$

37 $\int \frac{\sin x\,dx}{\sqrt{a^2\cos^2 x + b^2\sin^2 x}}$

Let, $\cos x = y$

Q 25 $\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$= \int \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$

Let, $\tan x = y$

$\sec^2 x\,dx = dy$

$= \int \frac{dy}{a^2+b^2y^2}$

$= \frac{1}{b^2}\int \frac{dy}{(\frac{a}{b})^2 + y^2}$

$= \frac{1}{b^2}\cdot\frac{1}{\frac{a}{b}}\tan^{-1}\frac{y}{a/b} + c$

= y = Put ue

$\frac{d\tan x}{dx} = \sec^2 x$, $\frac{d\cot x}{dx} = -\operatorname{cosec}^2 x$

26, 28, 29

39

Q29 $\int \frac{dx}{5-4\sin x}$

den: $1 = \sin^2\frac{x}{2} + \cos^2 x/2$

$\sin x = 2\sin x/2\cos x/2$

$\cos x = \cos^2\frac{x}{2} - \sin^2 x/2$

$= \int \frac{dx}{5(\sin^2\frac{x}{2} + \cos^2\frac{x}{2}) - 4\,2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \int \frac{\sec^2 x/2\,dx}{5(\tan^2\frac{x}{2}+1) - 8\tan x/2}$

Let $\tan x/2 = y$

27. $\int \frac{\cos x\,dx}{5-3\cos x}$

$= \int \frac{-\frac{1}{3}(5-3\cos x) + 5/3}{5-3\cos x}\,dx$

$-\frac{1}{3}\int dx + \frac{5}{3}\int \frac{dx}{5-3\cos x}$

$\frac{A(\text{Den})+B}{\text{Den}}$

2015

18 a, b

19

P.U

(30) $\int \frac{\cos\theta\,d\theta}{}$

$-\frac{1}{3}\int dx + \frac{5}{3}\int \frac{dx}{5-3\cos x}$ (30)

Q No 29

A Den + B Den'

19

$$\int \frac{\cos\theta\, d\theta}{\cos\theta+\sin\theta}$$

$$= \int \frac{\frac{1}{2}(\cos\theta+\sin\theta)+\frac{1}{2}(-\sin\theta+\cos\theta)}{\cos\theta+\sin\theta}\, d\theta$$

$$= \frac{1}{2}\theta + \ln(\cos\theta+\sin\theta)\, d\theta$$

(38)

$$\int \frac{5\cos\theta+6}{2\cos\theta+\sin\theta+3}\, d\theta$$

38 (ii)

$$\int \frac{4\sin x+5\cos x}{3\sin x+4\cos x}\, dx$$

A =

B =

$$= \frac{A(3\sin x+4\cos x)+B(3\cos x-4\sin x)}{\text{Den.}}$$

Note: Sinx: $4 = 3A - 4B$

Cosx: $5 = 4A + 3B$

$A = \frac{32}{25}$

$B = -\frac{1}{25}$