Types of Random Variables (Models)

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1 Discrete Random Variables

Bernoulli(p)

Y has two possible outcomes "Success" or "Failure"

Y$\sim$ Bernoulli(p), p = P("Success"), in other words, Y = # success of 1 trial

• Bernoulli : Distribution
• p : parameter that characterizes distribution

$\mathbf{E}[Y] = p$
Var(Y) = p(1-p)

Binomial(n,p)

Y = # of successes in n independent Bernoulli(p) trials

Y$\sim$ Binomial(n,p), p = P("Success"), in other words, Y = # success of 1 trial

$\mathbf{E}[Y] = np$
Var(Y) = np(1-p)

$X_1, \dot{;} X_n \sim$ iid Bermoulli(p), then, Y = $\sum^n X_i \sim Bin(n,p)$

Poisson(λ)

Y is a random variable representing a count

Y$\sim$ Po(λ),
Y $\in$ {0,1,2}
$\mathbf{E}[Y] = Var$(Y) $= \lambda$

Note: the only discrete RV this is true for...

Geometric(p)

Y = # of trials (with Binary outcomes) needed before the 1^{st} success
Y $\sim$ Geometric(p)
p = probability of success

Y $\in$ { 1,2,3, ...}
$\mathbf{E}[Y] = \frac{1}{p}$
Var(Y) = $\frac{1-p}{p^2}$

Only one discrete distribution that is Memory-less!

Memoryless Property

A random variable Y is memoryless if P(Y>t|Y>s) = P(Y>t-s), t-s

- Discrete distribution: Geometric(p)
- Continuous distribution: Exponential(λ)

Negative Binomial(p, k)

Y = # of trials needed before k success (trials have only "successes" or "failures")
Y $\sim$ NegtiveBinomial(k)
p = probability of success

Y $\in$ { k,k+1, ...}
$\mathbf{E}[Y] = \frac{1}{p}$
Var(Y) = $\frac{1-p}{p^2}$

2 Continuous Random Variables

Uniform(a,b)

Y has equal chance of taking on any interval $[a, b]$
p(Y $\in [x, x + l]$) = $\frac{l}{b-a}$ Y$\sim$ Uniform(a,b) $\mathbf{E}[Y] = \frac{a+b}{2}$
Var(Y) = $\frac{(b-a)^2}{12}$

Normal(μ, σ^2)

Y$\sim$ $N(\mu, \sigma^2)$
if the density (histogram) of Y is bell-shaped

Exponential(λ)

Y = waiting time before an event occurs
Y $\sim$ Exp(λ)

λ is a mean waiting time AND the rate
Var(Y) = λ^2