Bell Inequality Test

CSC 791 – Quantum Networking

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***Abstract*—Bell's theorem refers to a group of closely related theorems which determine that Quantum Mechanics is incompatible with local hidden-variable theories. Various experiments have, over the years, broken inequality limit to prove this. In this project we attempt to simulate the quantum network communication in order to verify Bell's Theorem by breaking Bell Inequality. To do this, we have designed a Discrete Event Simulation where entangled photons are generated at the Entanglement Source (ES) and distributed to two receivers, Alice and Bob. Noise is added tothe system in the form of extraneous photons generated by an Abstract Light Source (AS). Each simulation is carried out for 30 seconds. The receivers measure in varying basis per simulation run and report each detection to the Coincidence Monitor (CM). This experiment was performed with 4 different states of entangled EPR pairs, ranging from maximally entangled to non-maximallyentangled, with and without mixtures. We used python and qiskit for our simulation.**

***Index Terms*—quantum, bell inequality, entangled photons**

I. Introduction

Quantum entanglement has puzzled scientists for decades and raised a classical debate to explain the correlation between the entangled photons over long distances. This strange correlation was referred to as "spooky action at a distance" by Einstein. It all started when in 1935, Einstein et al raised a question, "Can Quantum-Mechanical description of physical reality be considered complete?" [1]. In this paper authors argued that photons are carrying a property or local hidden variables that determine this correlation. This argument was based on the principle of locality which states that the object is influenced directly only by its immediate surrounding.

This theory was challenged by Bell and changed the nature of debate. Bell represented Einstein's view of local realism into the algebraic predictions famously known as Bell's inequality. He experimentally proved that EPR gedanken experiment contradicts the bell inequality hence proving the non-locality [2]. Over the years researchers have studied quantum entanglement with improving entanglement source generators and polarizers over increasing distance and had repeatedly confirmed the contradiction of bell inequality [3]–[8]. In this project, we attempt to test the bell inequality for different set of entangled photons using simulation.

II. Problem Statement

Our objective in this paper is to break the Bell Inequality and prove that quantum communication does not depend on local variables. To prove the Bell Inequality, multiple experiments have been proposed over the past 50 years. One such experiment is the CHSH inequality. In this experiment, a source creates pairs of entangled photos and channels these pairs to two participants, Alice and Bob. Depending on the observables set for Alice and Bob, corresponding measurements will be recorded at their respective detectors

For this experiment, we have designed a simulated quantum network which generates entangled photons and distributes them to two receivers, Alice and Bob. Each receiver is equipped to detect and measure the photons in an arbitrary basis based on some pre-defined probabilities. We rotate the basis of measurement till we detect some basis where the Bell Inequality is violated. Our simulated quantum network contains 1 *Entanglement source (ES)*. which generates the entangled photons from Poisson statistics with an average rate of $15,000$ entanglements per second. An *Abstract Source (AS)* also generated non-entangled photons to simulate noise. We experimented with 4 different entanglement pairs:

- **Maximally entangled EPR pair**:
  $|\Phi^{+}\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$
- **Maximally entangled EPR pair with mixture**:
  $\rho = 0.7\,|\Phi^{+}\rangle\,\langle\Phi^{+}| + 0.15\,|HV\rangle\,\langle HV| + 0.15\,|VH\rangle\,\langle VH|$
- **Non-maximally entangled EPR pair**:
  $|\phi\rangle = \frac{1}{\sqrt{5}}\,|HH\rangle + \frac{2}{\sqrt{5}}\,|VV\rangle$

- **Non-maximally entangled EPR pair with mixture**: $\rho' = 0.7 |\phi\rangle \langle\phi| + 0.15 |HV\rangle \langle HV| + 0.15 |VH\rangle \langle VH|$

Here H and V refer to Horizontal and Vertical Polarization respectively.

III. Methodology

In this section, we describe in detail the components of our simulated quantum network. We have considered 4 major components, viz., Entanglement Source, Abstract Source, Receiver, Co-Incidence Monitor. Figure 1 shows a complete example of our experimental setup.

Entanglement Source
This is the generator for all entanglements. Entanglement source generates approximately $15,000$ entanglements per second and distributes them to the receptors. The source runs for a total of 30 seconds and follows a Poisson distribution.

Abstract Source
Abstract source refers to the ambient light that create noise during the experiment as a result of not being entangled. This photons hit the receivers and generate a detection, even though they do not form any part of an entangled pair. As a result the fidelity of the system is reduced.

Receiver
Receivers are the actual receptors which detect photons sent to them by the entanglement source. In this experiment, we use two receivers - Alice and Bob. Each receiver is equipped with a polarizer which is set to measure photons in a particular basis. As a result, they do not detect every photon that hits them. Additionally, receivers do not have 100% efficiency when detecting photons.

Receivers are also hindered by the dead time, which is referred to as the time that must elapse before another detection event can take place. Receivers detect both entangled photons, and ambient light. This data is sent to the co-incidence monitor for analysis.

Coincidence Monitor
The Co-incidence monitor is the analysis center for the experiment. The data for photon clicks is sent to the co-incidence monitor to analyse how many entangled pairs have been detected. For our experiment we predefined the coincidence window to be $4ns$. Thus, if both Alice and Bob detect photons within this time, this counts as coincidence click.

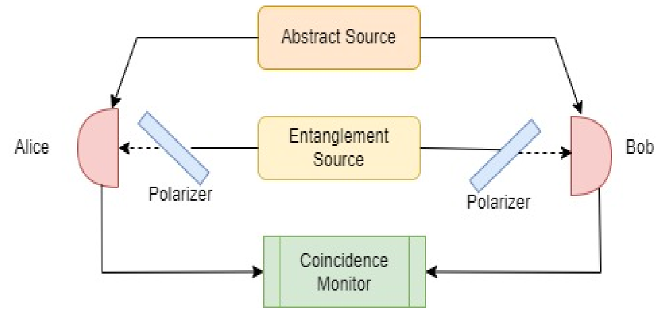

Fig. 1. Diagram of a generic Bell test. A source emits pairs of entangled photons. Each photon after passing a one-channel polarizer, which its direction can be set by experimenter, is detected by a detector. Coincidence counter monitors coincidences due to a predefined interval.

IV. Simulation

A. Design

We have designed a discrete event simulation in python to execute our simulation. Discrete event simulation is a type of memoryless process where each state is affected only by the previous state. In our simulation, we have developed a Simulation Engine which keeps track of every event that needs to be executed. As one event is processed, it generates further events which are inserted into the engine based on time. Figure 2 shows an example of a discrete event simulation.

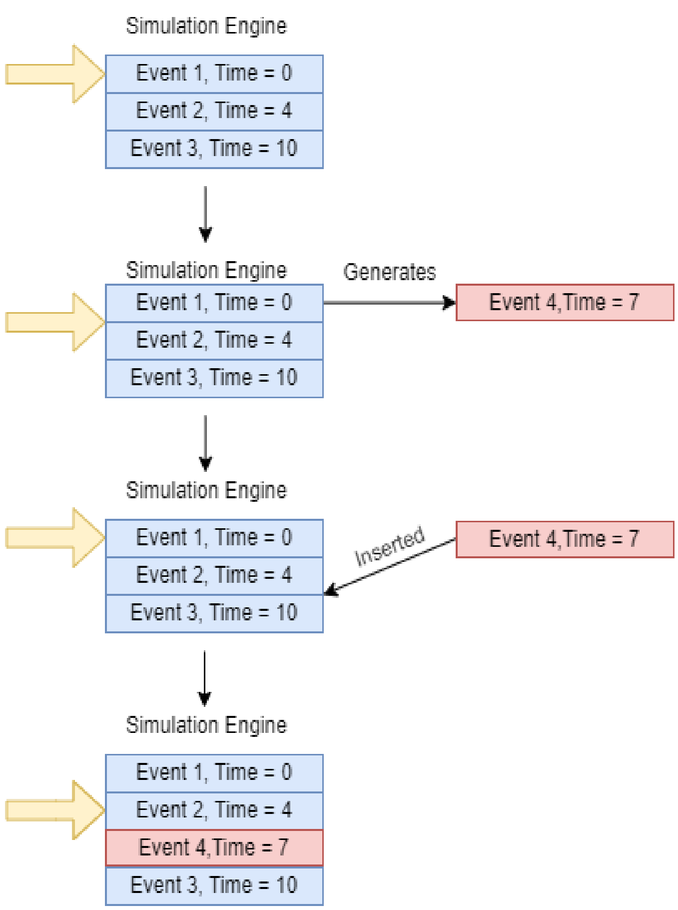

Fig. 2. Discrete Event Simulation

Initially, the Engine class executes the Entanglement Source class to generate entanglements, encapsulated in Send Entanglement Events based on Poisson distribution. The Abstract Source class is then used to generate noise as Send Dark

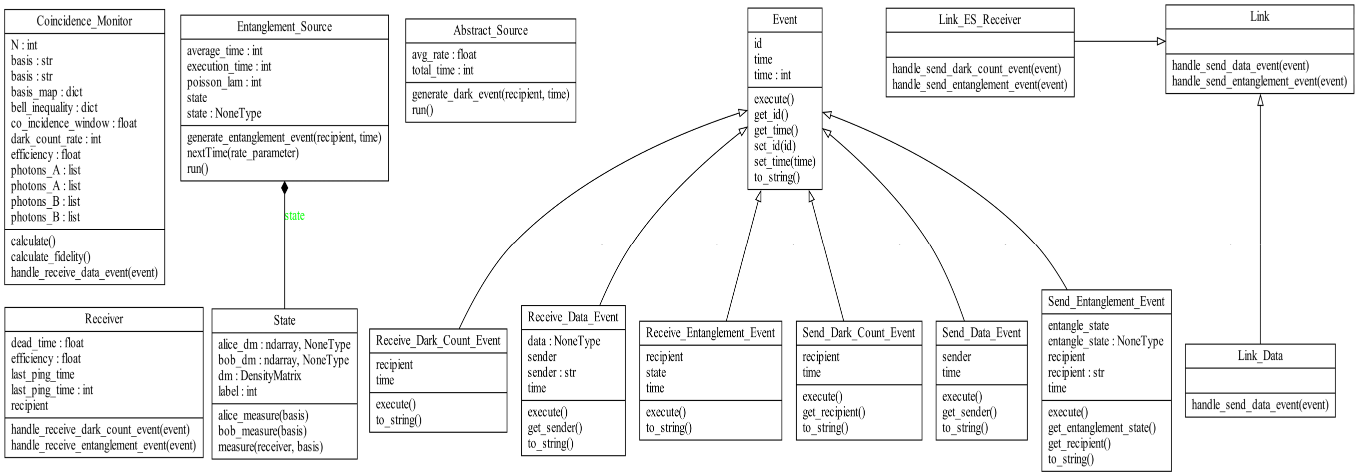

Fig. 3. Class structures created for the Bell Test in our simulation program

Count Events. Each of these events, when processed, generate Receive Entanglement and Receive Dark Count Events, which are handled by the Receiver. The Receiver accepts each photon probabilistically, based on efficiency and dead time. Furthermore, if the event is an entanglement event, the Receiver detects it only if the entanglement passes through the polarizer which was set before the simulation was started.

The Link class is used to simulate the channel between Source and Receivers, while Link Data is used for the channel between Receiver and Coincidence Monitor. Send Data events are used to send the photons detected at the receiver to the Coincidence Monitor, which in turn calculates Fidelity and Bell Inequality to send back to the engine. The entire class diagram has been provided in Figure 3.

Additionally, we have used used Python's Qiskit library to perform various measurements on the state and density matrices transformation and encapsulate the quantum elements used in this project. Qiskit is a hardware-agnostic software development kit that gives users the ability to build, compile, run, and analyze quantum circuits and quantum programs. It encodes useful quantum measurement routines such as partial traces and and state measurements. We used Python's Qiskit library to perform various measurements on the state and density matrices transformation.

B. Results

The simulation is executed for $30s$ per state per basis in order to calculate the number of coincidence clicks. The measurement basis considered are HH, HV, VH and VV. For each state fidelity is calculated as

$$Fidelity = \frac{CC_{HH} + CC_{VV}}{CC_{HH} + CC_{HV} + CC_{VH} + CC_{VV}}$$

where CC = Coincidence Clicks. Figure 4 shows how Fidelity changes when for different states. Here dead time was set as $4\mu s$ while efficiency was 10%. As expected states without mixture achieve a higher fidelity.

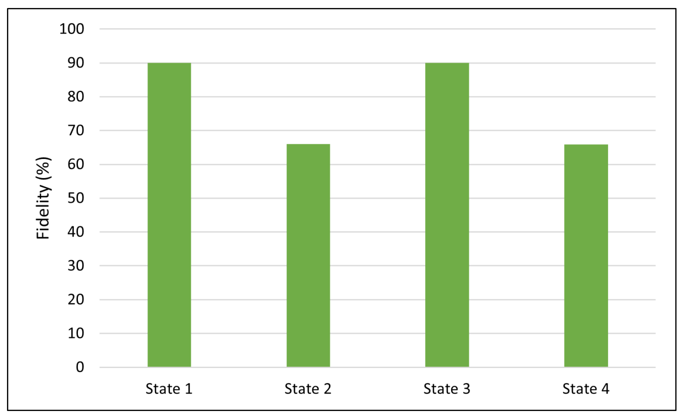

Fig. 4. Fidelity achieved for each state

Varying Efficiency

Next we decided to vary both dead time and efficiency in order to discern the effects on Fidelity. Owing to a lack of time, we could only test a limited number of cases, and were limited to State 1, i.e., maximally entangled EPR pair. Figure 5 shows how coincidence clicks vary when efficiency is changed. Figure 6 shows how fidelity changes with change in efficiency. In this case dead time was fixed at $1\mu s$.

While coincidence clicks changed noticeable with change in efficiency, change in fidelity was significantly lower. This is as expected, since, when we change the efficiency we are also altering the probability at which dark count is detected. Thus there is no net change in the ratio of actual entanglements to dark count detected at each receiver.

Varying Dead Time

Curiously, varying dead time did not alter the coincidence clicks of the receiver, as can be seen in 7. From this it can be

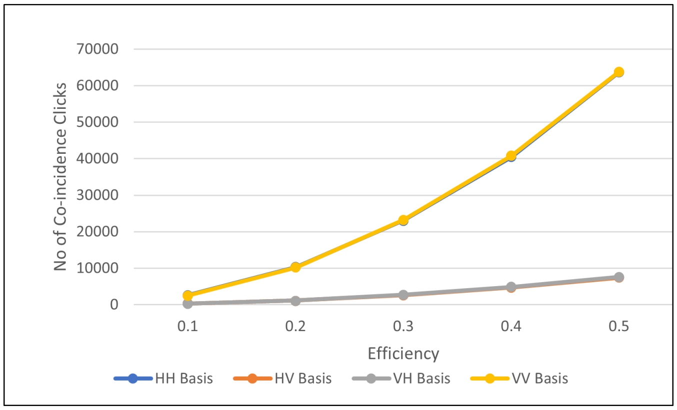

Fig. 5. Efficiency vs Coincidence Clicks

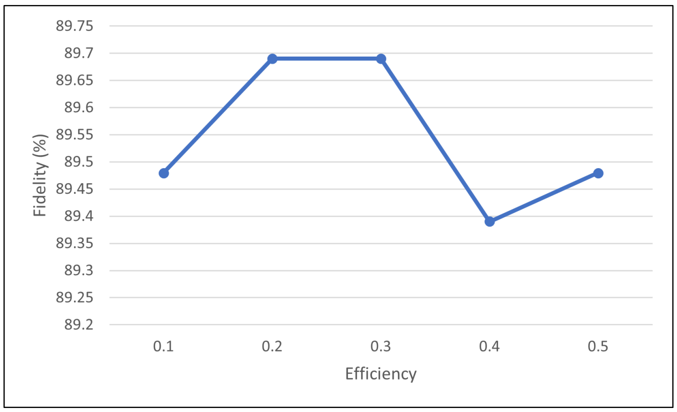

Fig. 6. Efficiency vs Coincidence Clicks

inferred that the coincidence window and not the dead time is a more significant contributor to coincidence clicks. Change in fidelity (Figure 8 was similar to that observed when varying efficiency. This is even more obvious since coincidence clicks themselves did not change.

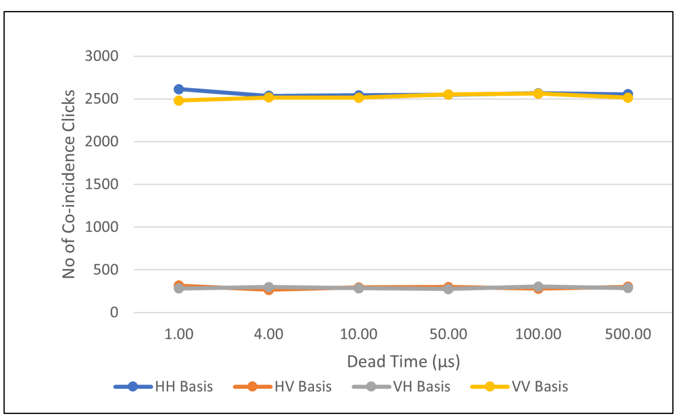

Fig. 7. Dead Time vs Coincidence Clicks

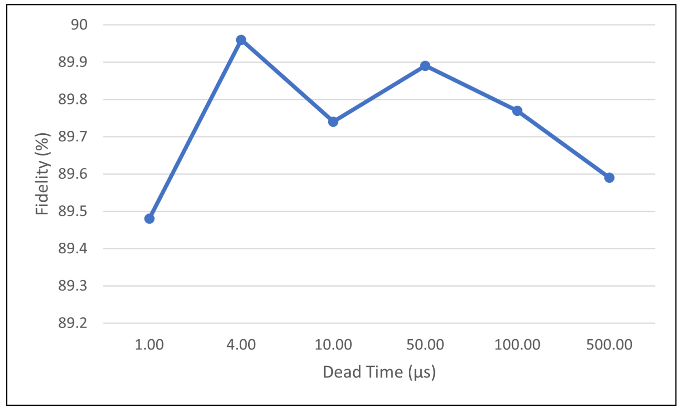

Fig. 8. Dead Time vs Fidelity

C. Bell Inequality

To prove that quantum elements do not depend on local variables, our simulation also aimed to violate Bell Inequality using the CHSH game. While initially we wished to simulate the design created by Aspect et al. [9], it was soon clear that our setup was not quite alike. Hence, we turned to a blog post by Qiskit [10] which used a similar methodology as us.

We divided the $360°$ space into 15 equal angles. For each angle, we rotated Alice's bases by the angle specified with respect to Bob. Bob would always measure in Z or X basis. And then we counted the number of coincidence click for each pair of bases. The CHSH value for the angle would thus be:

$$|\langle CHSH\rangle| = |\langle AB\rangle - \langle Ab\rangle + \langle aB\rangle + \langle ab\rangle| \leq 2$$

where A and a were the bases at Alice, while B and b were Bob's bases. Each basis was simulated for $10s$ and the resulting curve can be seen at Figure 9.

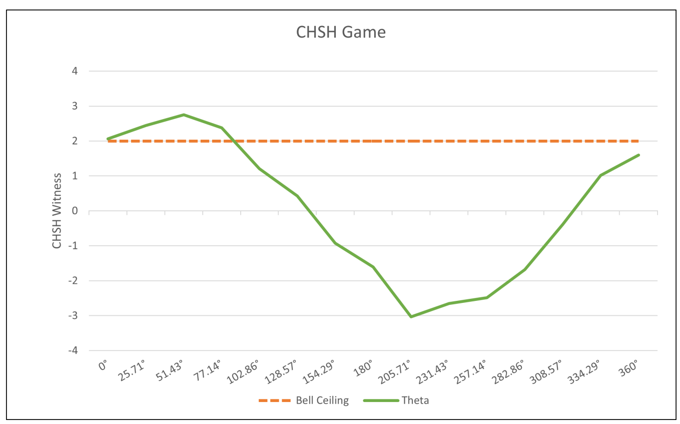

Fig. 9. Plotting CHSH Values

To verify our results, we further generated CHSH values on an actual IBM Quantum Computer. We ran executed simulations with Ideal, Noisy and (Noise) Mitigated qubits. The curves for all three have been compared in Figure 10

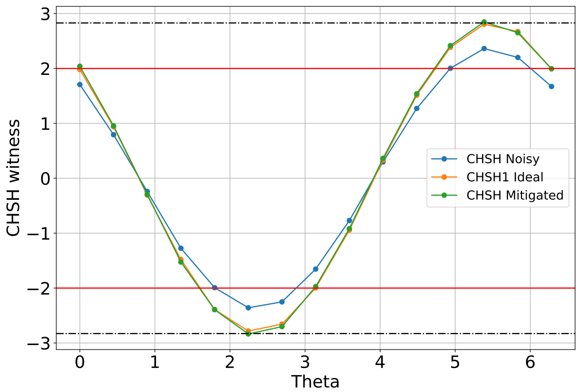

Fig. 10. Plotting CHSH Values on IBM Quantum Computer

As is clearly evident the curve obtained in our simulation is similar to the one generated on a quantum computer.

V. Conclusion

We have designed a discrete event simulation which simulates an actual quantum network in order to generate and distribute entanglements to two receivers. We have also simulated noise in the form of ambient life and, using the observable detection of photons, calculated coincidence clicks and fidelity of the system.

Our results show that we have achieved a high fidelity where states were entangled without mixtures. Introducing mixtures decreased the fidelity, as expected. We have also presented a study on how changing the physical constraints of the receivers can change the number of clicks detected, as well as, the fidelity. We have also proved that quantum elements do not have local hidden variables by breaking the Bell Inequality

While the work was not novel, it has granted us a deeper understanding of how quantum networks function and how entangled pairs can affect each other when measured at a detector. Simulating this experiment has shown us how exactly bases and angles need to be rotated when measuring qubits with the goal of breaking Bell Inequality. By plotting the curve ourselves we can clearly see how the angles affect CHSH values.

VI. Future work

There are significant avenues of exploration in our simulation. For starters, we have only shown affect of variation of dead time and co-incidence clicks at the receiver for one particular quantum state. We can continue testing similar variation for all four states mentioned in this paper.

An obvious expansion of our work is, of course, increasing the number of users receiving entanglement at the same time, as was shown in Wengerowsky et al. [11]. We can also vary other parameters to check potential ramifications. For now both receivers have the same parameters. We can try changing individual parameters and see how this changes results. We can also try increasing or decreasing the coincidence window to see if this is significantly affects the data we obtain.

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