Data Flow Analysis

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Data flow analysis

- Derives information about the **dynamic** behavior of a program by only examining the **static** code
- Intraprocedural analysis
- Flow-sensitive: sensitive to the control flow in a function

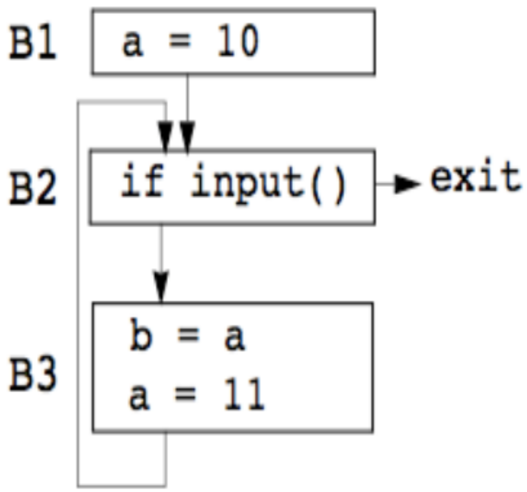
```
    a := 0
L1: b := a + 1

    c := c + b
    a := b * 2
    if a < 9 goto L1
    return c
```

- **Examples**
  – Live variable analysis
  – Constant propagation
  – Common subexpression elimination
  – Dead code detection

- How many registers do we need?
- Easy bound: # of used variables (3)
- Need better answer

Data flow analysis



- Statically: finite program
- Dynamically: can have infinitely many paths
- Data flow analysis abstraction
  - For each point in the program, combines information of all instances of the same program point

Example 1: Liveness Analysis

Liveness Analysis

Definition

- A variable is **live** at a particular point in the program if its value at that point will be used in the future (**dead**, otherwise).
  - To compute liveness at a given point, we need to look into the future

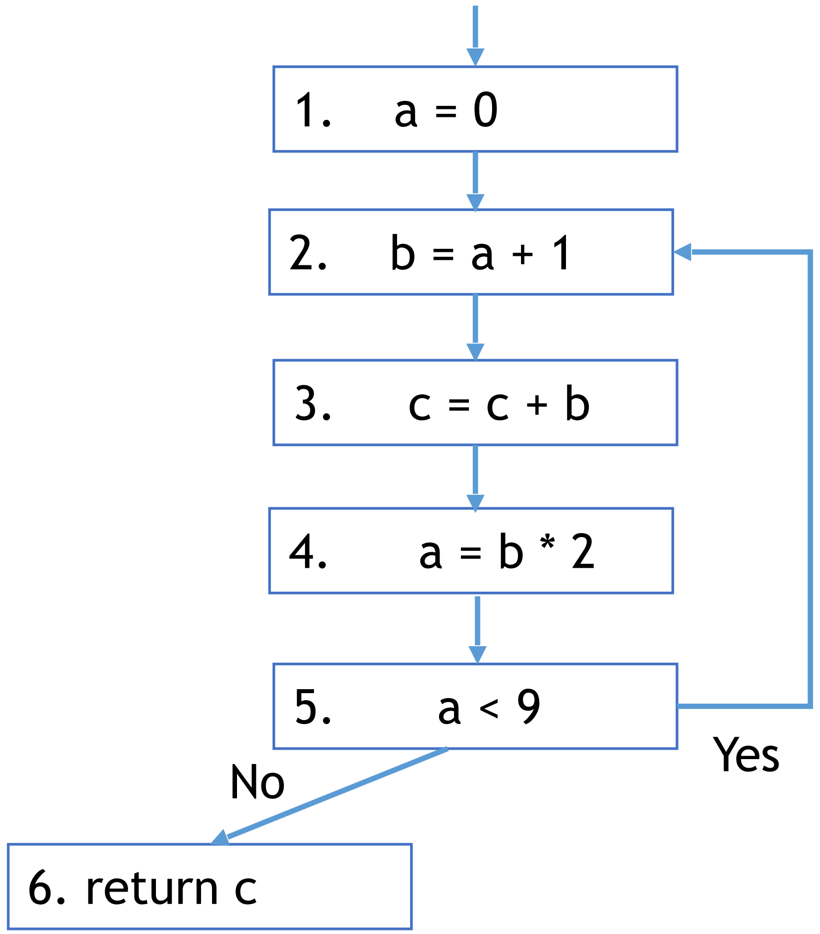
Motivation: Register Allocation

- A program contains an unbounded number of variables
- Must execute on a machine with a bounded number of registers
- Two variables can use the same register if they are never in use at the same time (*i.e*, never simultaneously live).
  - Register allocation uses liveness information

Control Flow Graph

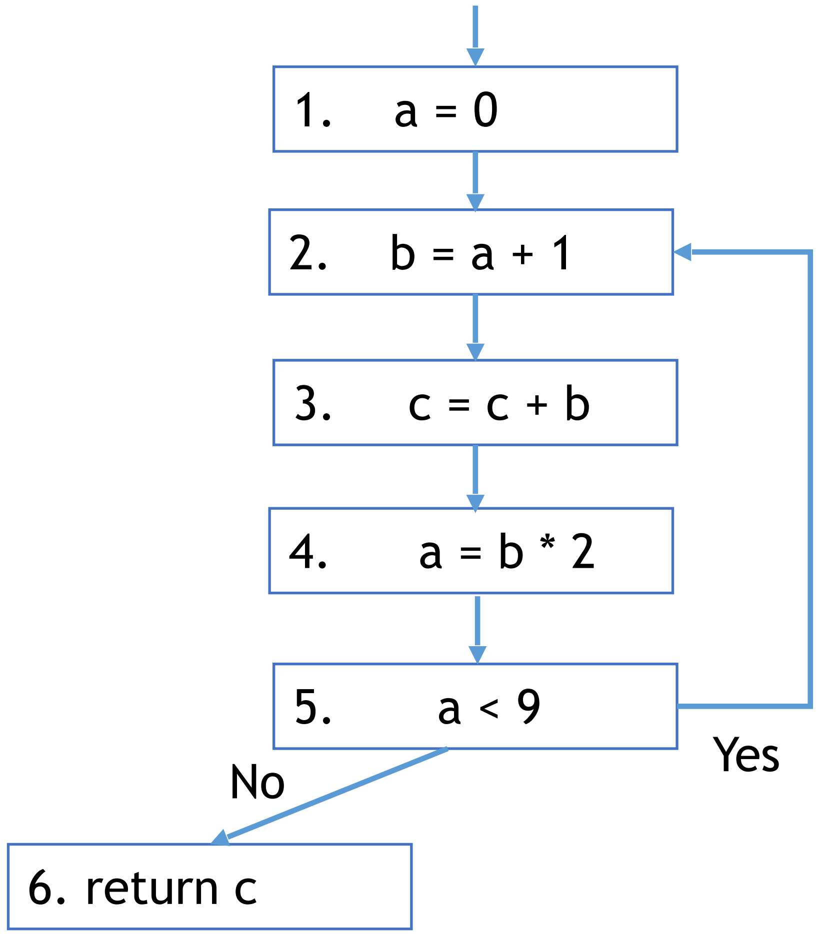
- Let’s consider CFG where nodes contain program statement instead of basic block.
-

1. a := 0
2. L1: b := a + 1
3. c:= c + b
4. a := b * 2
5. if a < 9 goto L1
6. return c



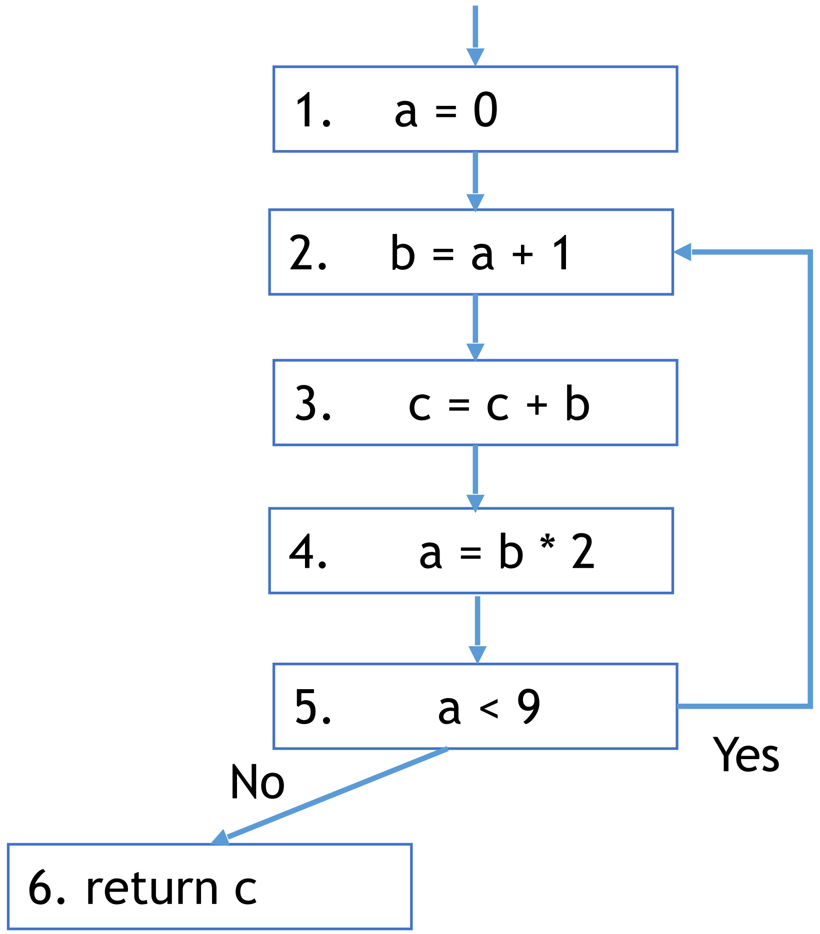
Liveness by Example

- Live range of b
  - Variable b is read in line 4, so b is live on 3->4 edge
  - b is also read in line 3, so b is live on (2->3) edge
  - Line 2 assigns b, so value of b on edges (1->2) and (5->2) are not needed. So b is **dead** along those edges.
- b’s live range is (2->3->4)



Liveness by Example

- Live range of a
  - (1->2) and (4->5->2)
  - a is dead on (2->3->4)

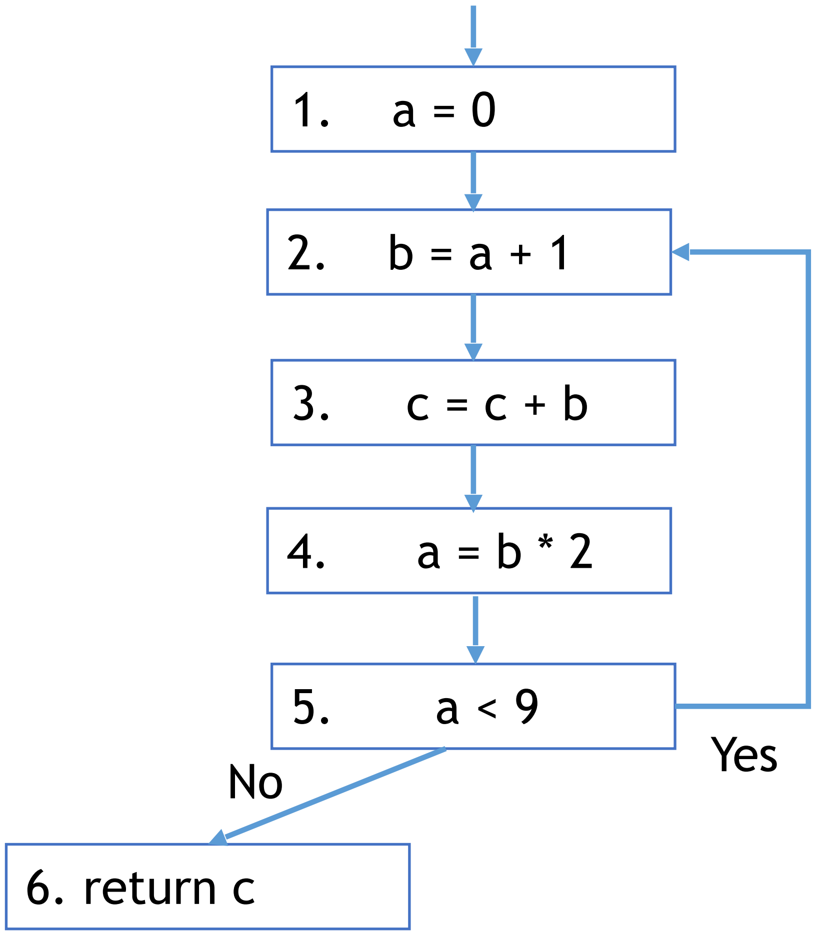


Terminology

- Flow graph terms
  - A CFG node has **out-edges** that lead to **successor** nodes and **in-edges** that come from **predecessor** nodes
  - pred[n] is the set of all predecessors of node n
  - succ[n] is the set of all successors of node n

Examples

– Out-edges of node 5: (5→6) and (5→2)
- succ[5] = {2,6}
- pred[5] = {4}
- pred[2] = {1,5}



Uses and Defs

Def (or definition)

- An **assignment** of a value to a variable
- def[v] = set of CFG nodes that define variable v
- def[n] = set of variables that are defined at node n

```
a = 0
```

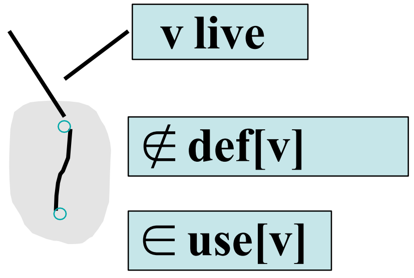
Use

- A **read** of a variable's value
- use[v] = set of CFG nodes that use variable v
- use[n] = set of variables that are used at node n

```
a < 9
```

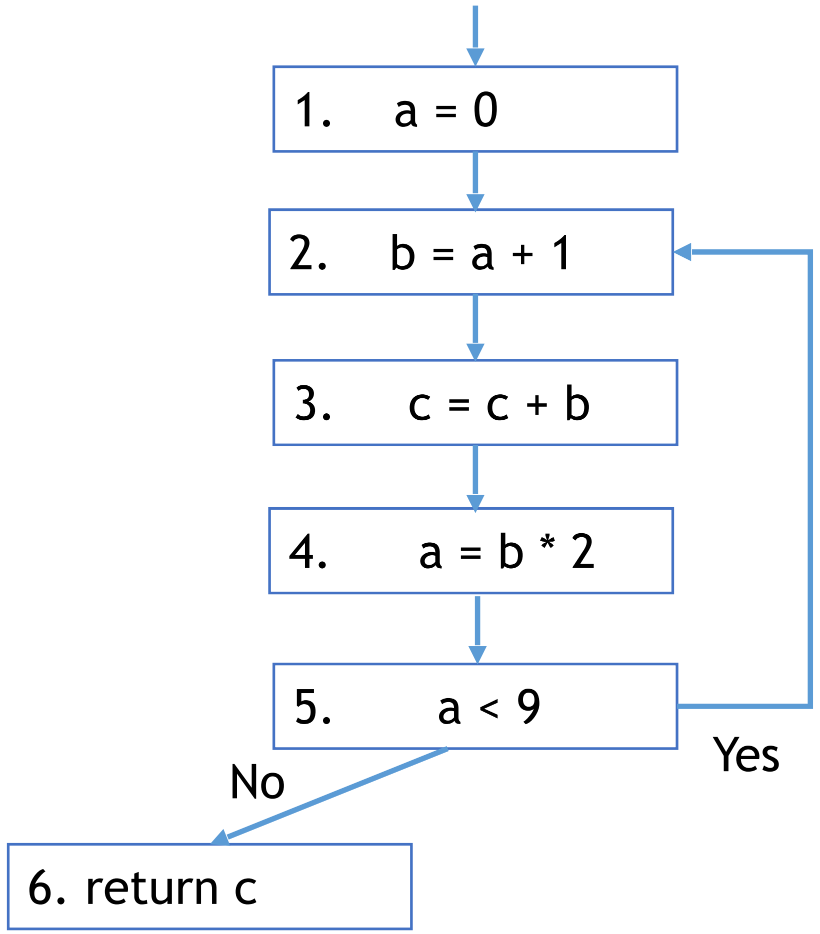
More precise definition of liveness

- A variable v is live on a CFG edge if
  (1) ∃ a directed path from that edge to a use of v (node in use[v]), and
  (2) that path does not go through any def of v (no nodes in def[v])

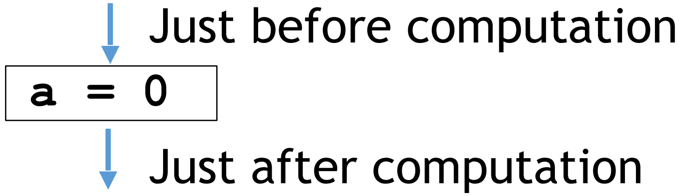


The Flow of Liveness

- Data-flow
  - Liveness of variables is a property that flows through the edges of the CFG
- Direction of Flow
  - Liveness flows backwards through the CFG, because the behavior at future nodes determines liveness at a given node

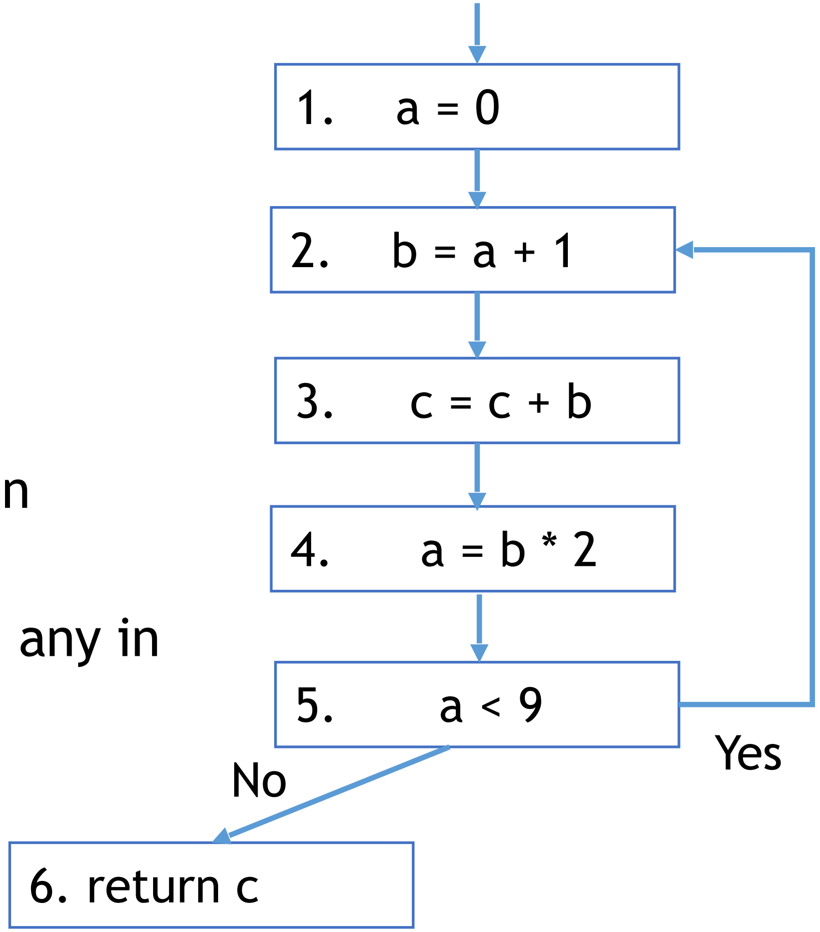


Liveness at Nodes



Two More Definitions

- A variable is **live-out** at a node if it is live on any out edges
- A variable is **live-in** at a node if it is live on any in edges



Computing Liveness

- Generate liveness: If a variable is in use[n], it is live-in at node n
- Push liveness across edges:
  - If a variable is live-in at a node n
  - then it is live-out at all nodes in pred[n]
- Push liveness across nodes:
  - If a variable is live-out at node n and not in def[n]
  - then the variable is also live-in at n
- Data flow Equation:

$$\mathbf{in[n] = use[n]} \cup \mathbf{(out[n] - def[n])}$$

$$\mathbf{out[n]} = \bigcup_{s \in \mathrm{succ}[n]} \mathbf{in[s]}$$

Solving Dataflow Equation

for each node n in CFG

in[n] = ∅; out[n] = ∅

Initialize solutions

repeat

for each node n in CFG

in'[n] = in[n]

out'[n] = out[n]

Save current results

in[n] = use[n] ∪ (out[n] – def[n])

$$\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$

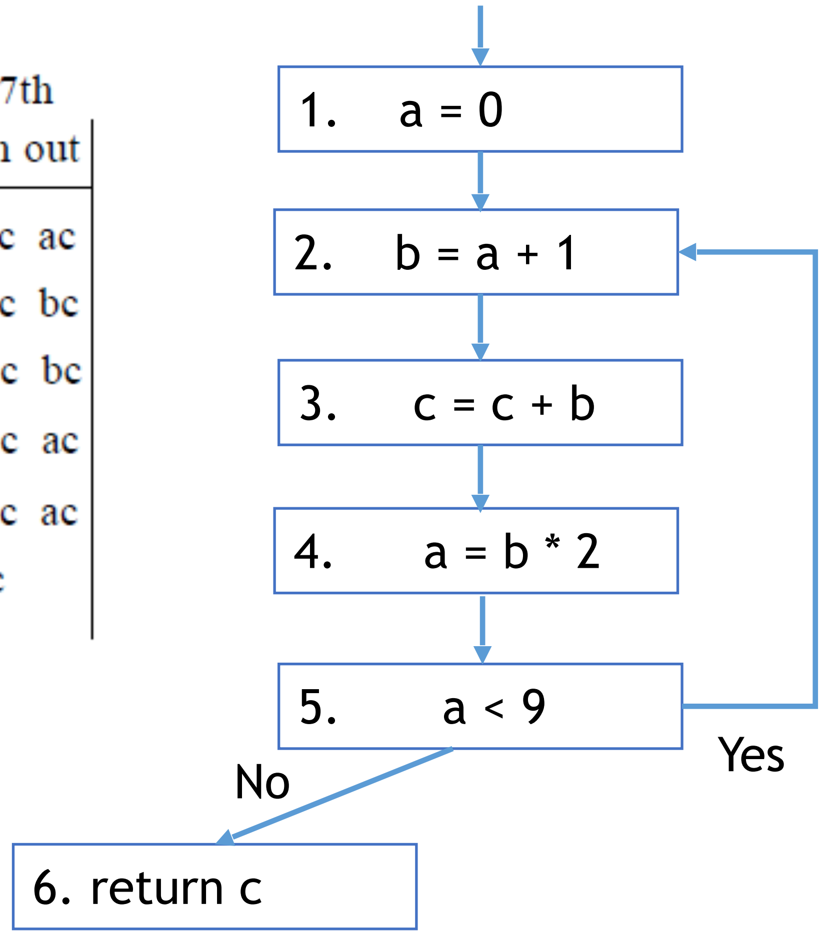
Solve data-flow equation

until in'[n]=in[n] and out'[n]=out[n] for all n

Test for convergence

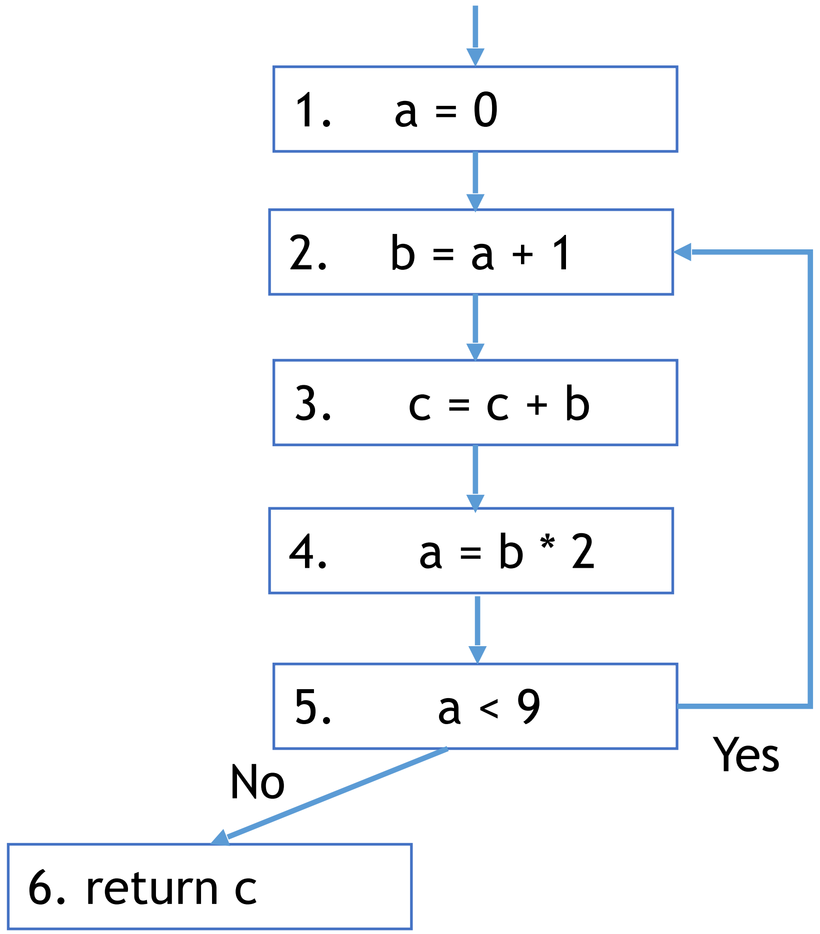
Computing Liveness Example

			1st		2nd		3rd		4th		5th		6th		7th	
node #	use	def	in	out	in	out	in	out	in	out	in	out	in	out	in	out
1		a				a		a		ac	c	ac	c	ac	c	ac
2	a	b	a		a	bc	ac	bc	ac	bc	ac	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	b	bc	b	bc	bc	bc	bc
4	b	a	b		b	a	b	a	b	ac	bc	ac	bc	ac	bc	ac
5	a		a	a	a	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac
6	c		c		c		c		c		c		c		c	



Iterating Backwards: Converges Faster

			1st		2nd		3rd	
node #	use	def	out	in	out	in	out	in
6	c			c		c		c
5	a		c	ac	ac	ac	ac	ac
4	b	a	ac	bc	ac	bc	ac	bc
3	bc	c	bc	bc	bc	bc	bc	bc
2	a	b	bc	ac	bc	ac	bc	ac
1		a	ac	c	ac	c	ac	c



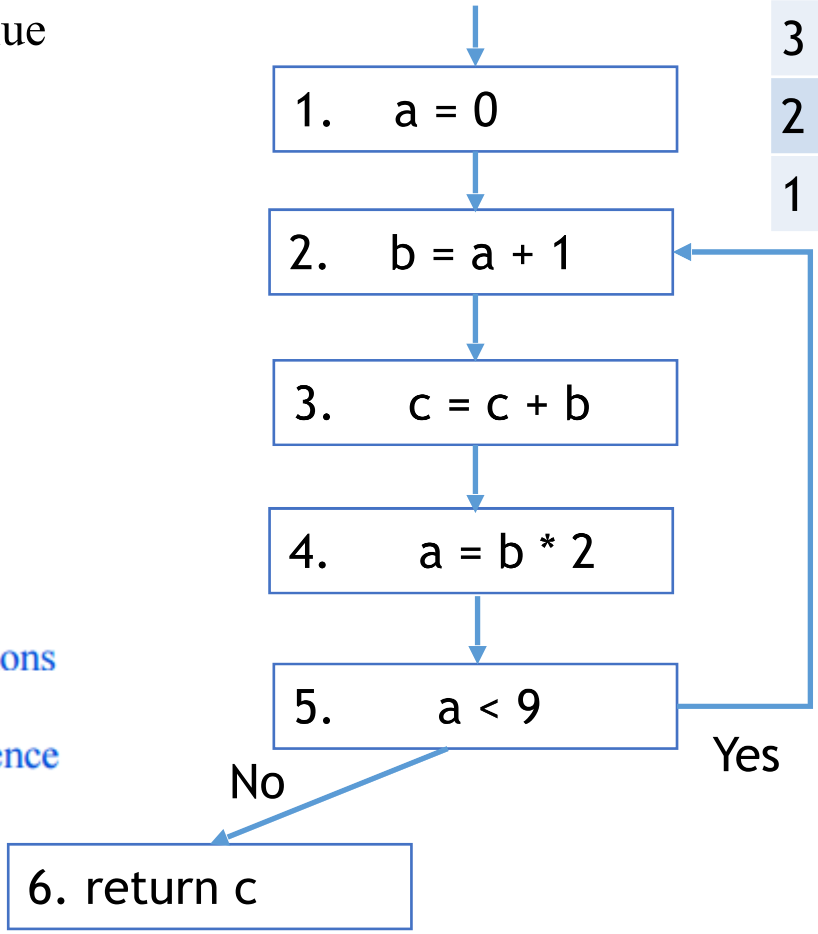
Liveness Example: Round1

Node	use	def
6	c	
5	a	
4	b	a
3	bc	c
2	a	b
1		a

A variable is **live** at a particular point in the program if its value at that point will be used in the future (**dead**, otherwise).

Algorithm

for each node n in CFG
  in[n] = ∅; out[n] = ∅ — Initialize solutions
repeat
  for each node n in CFG **in reverse topsort order**
    in'[n] = in[n]
    out'[n] = out[n] — Save current results
    out[n] = $\bigcup_{s \in succ[n]}$ **in[s]**
    in[n] = use[n] ∪ (out[n] – def[n]) — Solve data-flow equations
until in'[n]=in[n] and out'[n]=out[n] for all n — Test for convergence



Liveness Example: Round1

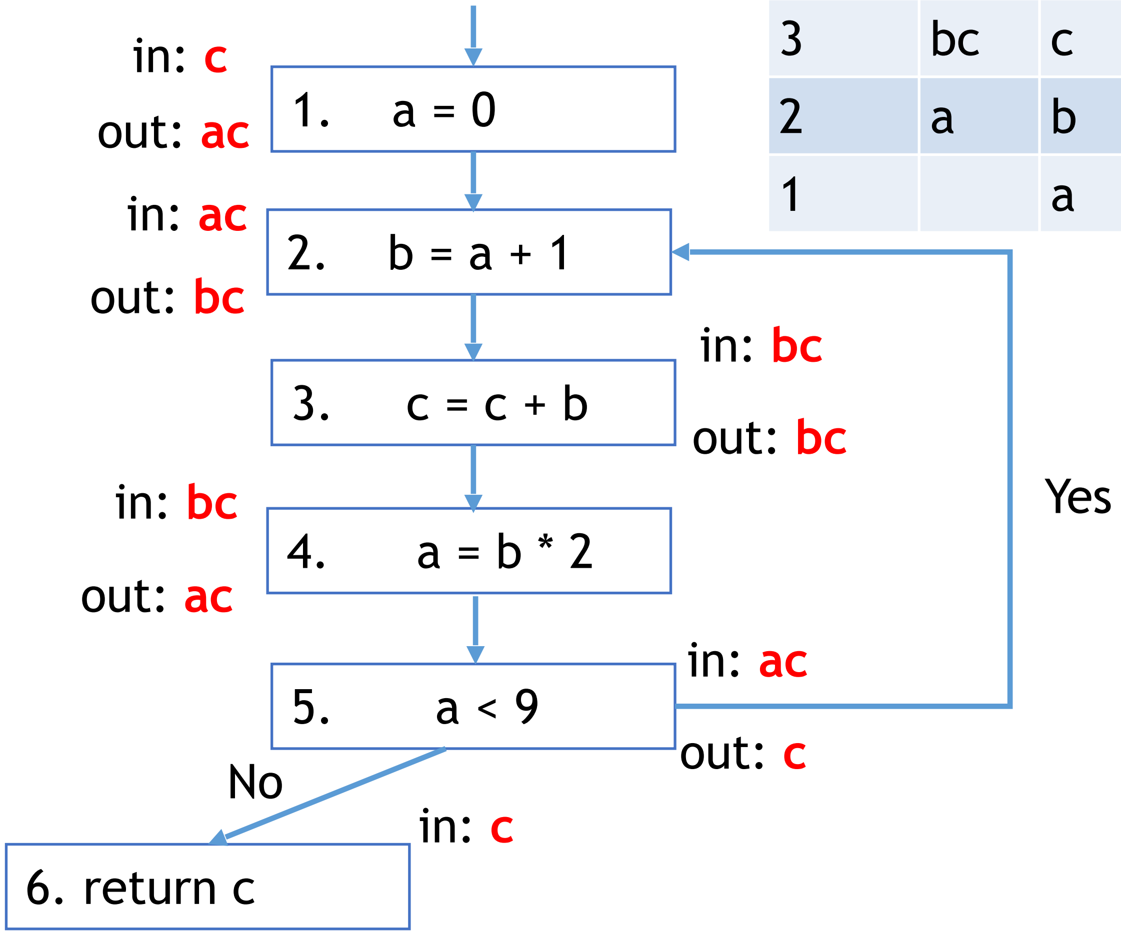
Node	use	def
6	c	
5	a	
4	b	a
3	bc	c
2	a	b
1		a

Algorithm

for each node n in CFG
$\quad in[n] = \varnothing;\ \ out[n] = \varnothing$ } Initialize solutions

repeat
$\quad$ **for each** node n in CFG **in reverse topsort order**
$\quad\quad in'[n] = in[n]$
$\quad\quad out'[n] = out[n]$ } Save current results
$\quad\quad \mathbf{out[n]} = \bigcup_{s \in succ[n]} \mathbf{in[s]}$
$\quad\quad \mathbf{in[n] = use[n] \cup (out[n] - def[n])}$ } Solve data-flow equations

until in'[n]=in[n] and out'[n]=out[n] for all n } Test for convergence

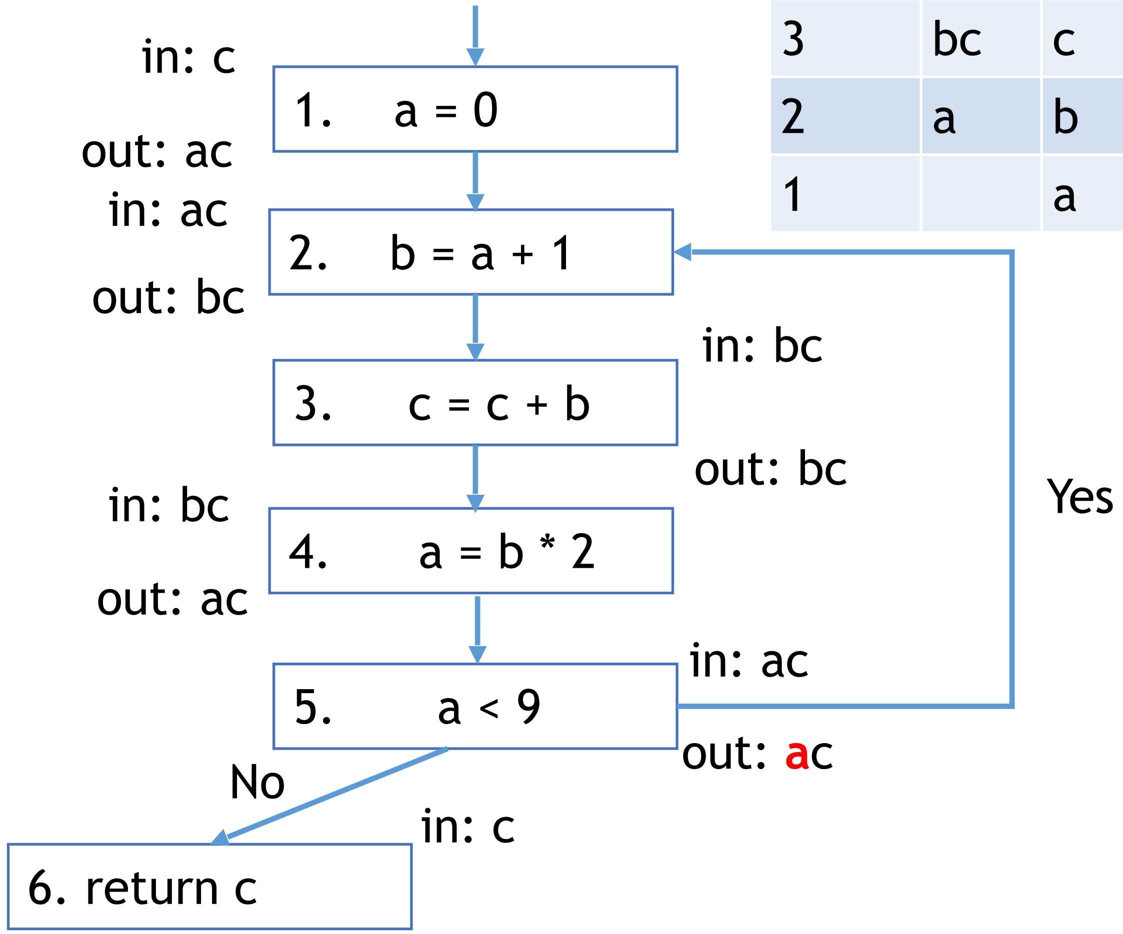


Liveness Example: Round1

Node	use	def
6	c	
5	a	
4	b	a
3	bc	c
2	a	b
1		a

Algorithm

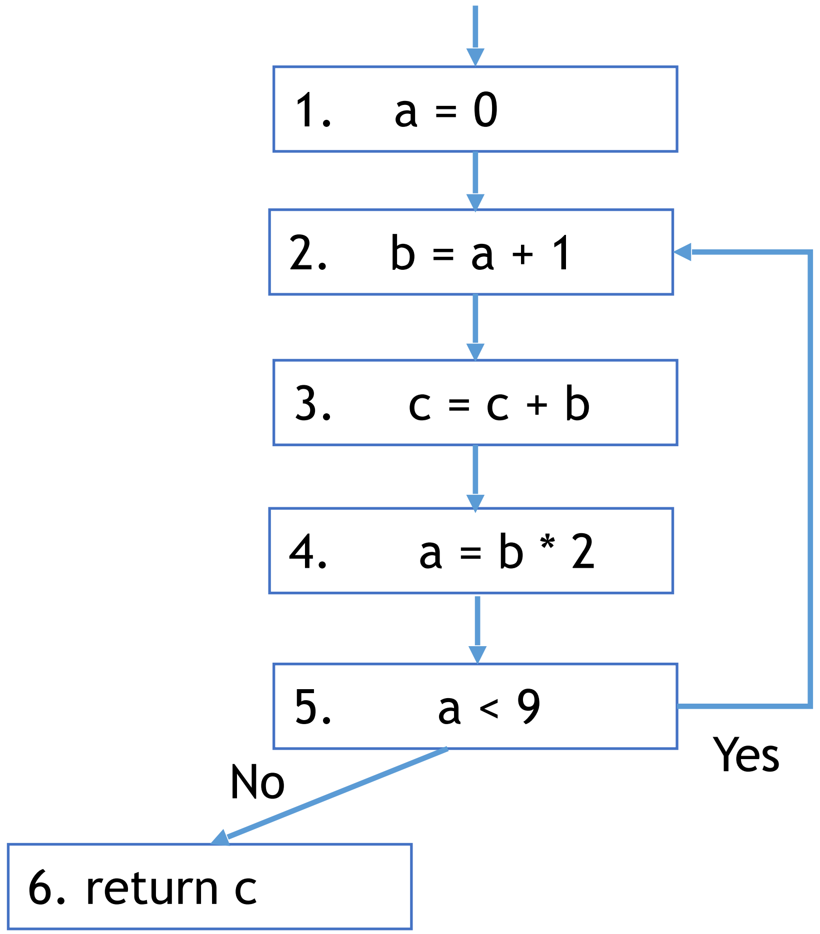
for each node n in CFG
    in[n] = ∅; out[n] = ∅   } Initialize solutions
repeat
    for each node n in CFG **in reverse topsort order**
        in'[n] = in[n]
        out'[n] = out[n]   } Save current results
        out[n] = $\bigcup_{s \in succ[n]}$ **in[s]**
        in[n] = use[n] ∪ (out[n] – def[n])   } Solve data-flow equations
until in'[n]=in[n] and out'[n]=out[n] for all n   } Test for convergence



Conservative Approximation

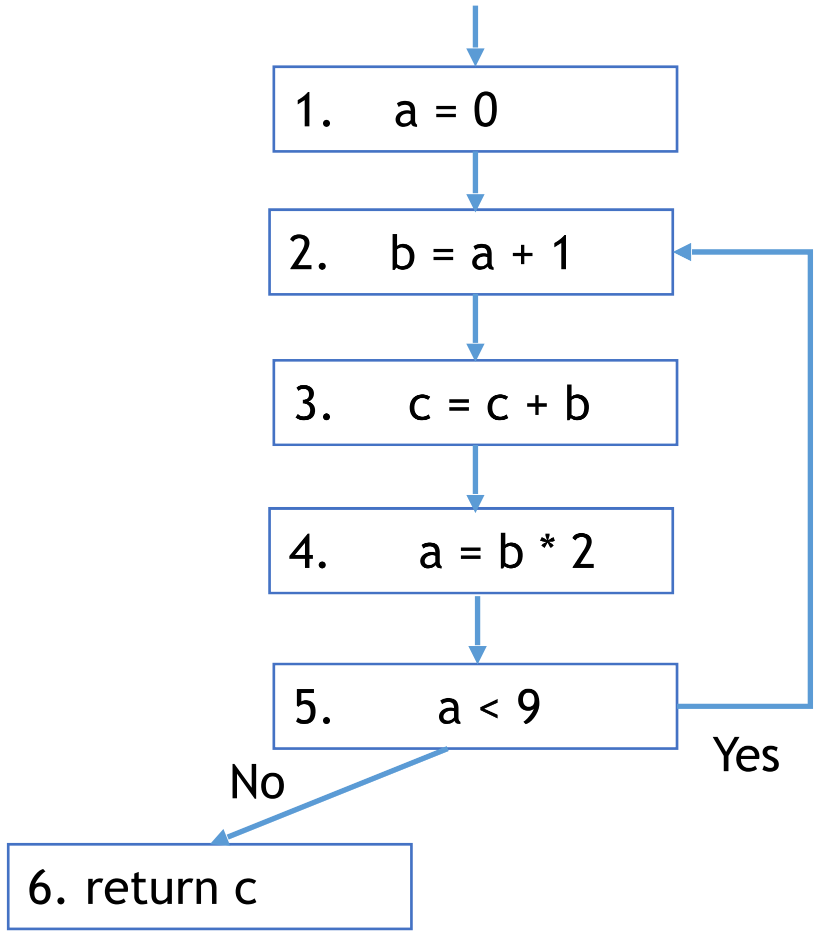
			X		Y		Z	
node #	use	def	in	out	in	out	in	out
1		a	c	ac	cd	acd	c	ac
2	a	b	ac	bc	acd	bcd	ac	b
3	bc	c	bc	bc	bcd	bcd	b	b
4	b	a	bc	ac	bcd	acd	b	ac
5	a		ac	ac	acd	acd	ac	ac
6	c		c		c		c	

Solution X:
- From the previous slide



Conservative Approximation

node #	use	def	X in	X out	Y in	Y out	Z in	Z out
1		a	c	ac	cd	acd	c	ac
2	a	b	ac	bc	acd	bcd	ac	b
3	bc	c	bc	bc	bcd	bcd	b	b
4	b	a	bc	ac	bcd	acd	b	ac
5	a		ac	ac	acd	acd	ac	ac
6	c		c		c		c	

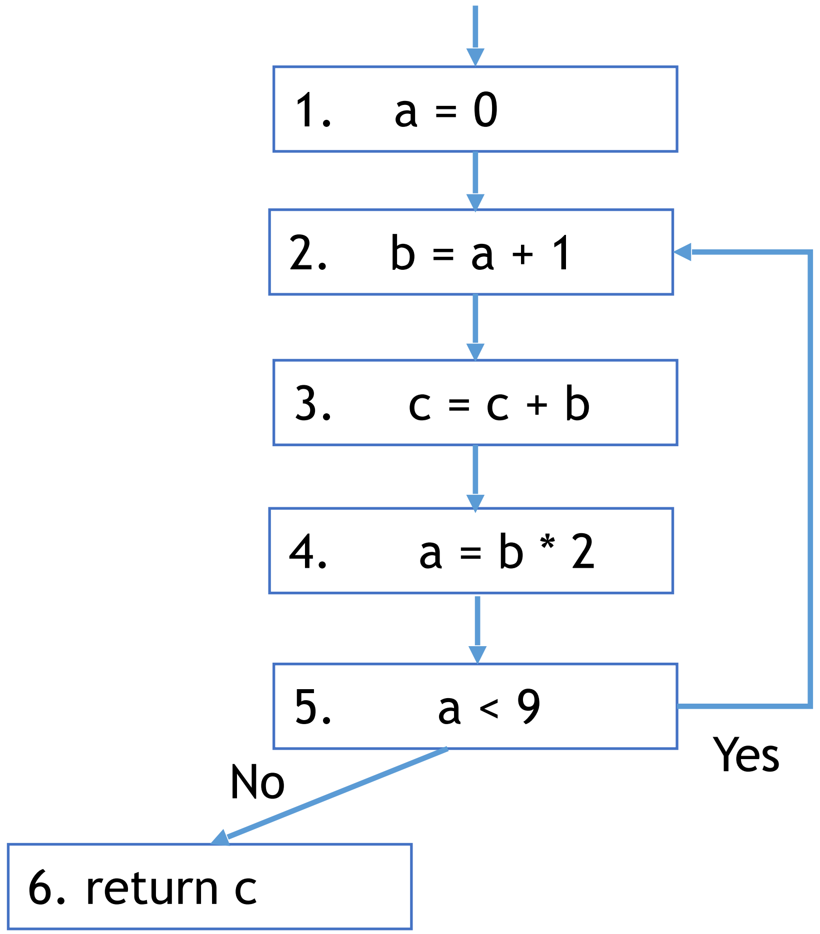


Solution Y:
Carries variable d uselessly
– Does Y lead to a correct program?

Imprecise conservative solutions ⇒ sub-optimal but correct programs

Conservative Approximation

			X		Y		Z	
node #	use	def	in	out	in	out	in	out
1		a	c	ac	cd	acd	c	ac
2	a	b	ac	bc	acd	bcd	ac	b
3	bc	c	bc	bc	bcd	bcd	b	b
4	b	a	bc	ac	bcd	acd	b	ac
5	a		ac	ac	acd	acd	ac	ac
6	c		c		c		c	

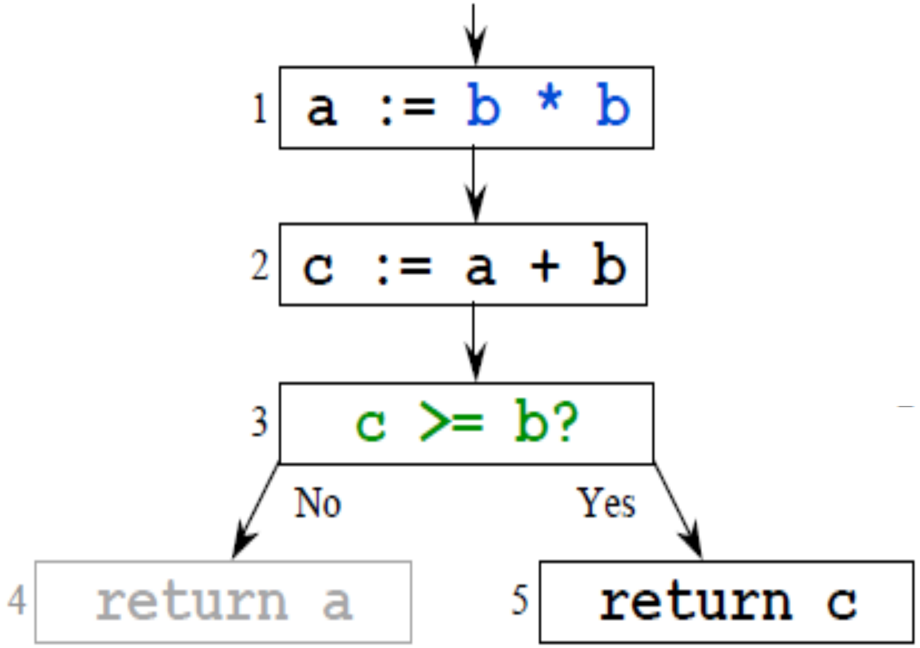


Solution Z:
Does not identify c as live in all cases
– Does Z lead to a correct program?

Non-conservative solutions ⇒ incorrect programs

Need for approximation

- Static vs. Dynamic Liveness: **b*b** is always non-negative, so **c** >= **b** is always true and **a**'s value will never be used after node

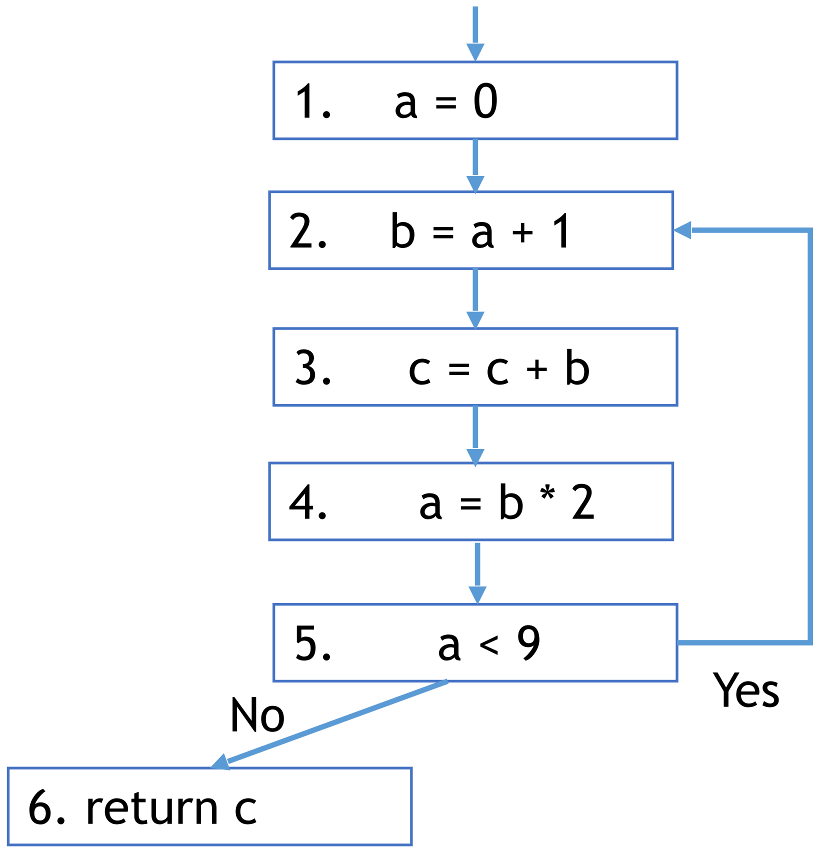


No compiler can statically identify all infeasible paths

Liveness Analysis Example Summary

- Live range of a
  - (1->2) and (4->5->2)
- Live range of b
  - (2->3->4)
- Live range of c
  - Entry->1->2->3->4->5->2, 5->6

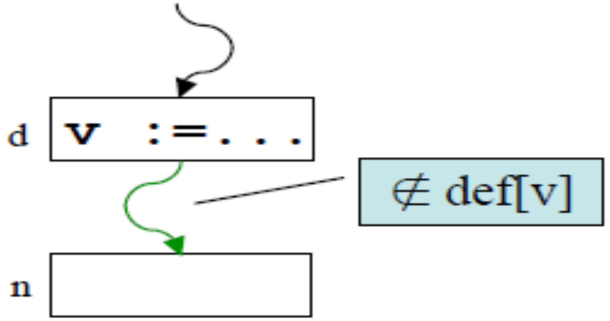
You need **2** registers Why?



Example 2: Reaching Definition

Definition

– A definition (statement) d of a variable **v** **reaches** node n if there is a path from d to n such that **v** is not redefined along that path

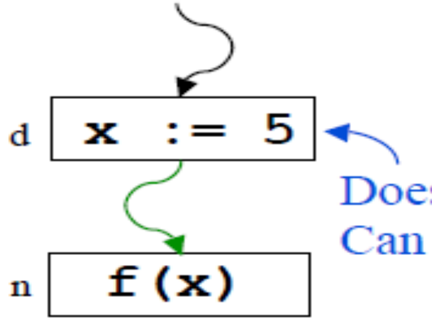


Uses of reaching definitions

– Build use/def chains

– Constant propagation

– Loop invariant code motion



Does this def of **x** reach n?
Can we replace n with **f(5)**?

```
1   a = . . .;
2   b = . . .;
3   for (. . .) {
4         x = a + b;
5         . . .
6   }
```

Reaching definitions of **a** and **b**

To determine whether it's legal to move statement 4 out of the loop, we need to ensure that there are no reaching definitions of **a** or **b** inside the loop

Computing Reaching Definition

- Assumption: At most one definition per node

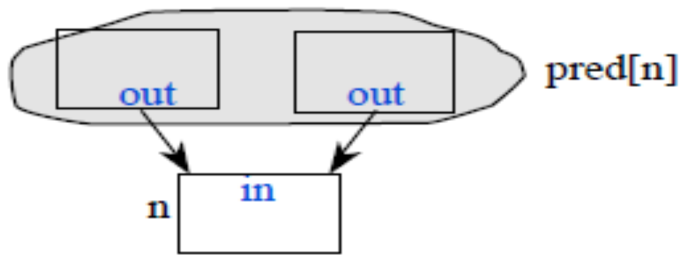
- **Gen[n]:** Definitions that are generated by node n (at most one)
- **Kill[n]:** Definitions that are killed by node n

statement	gen's	kills
x:=y	{y}	{x}
x:=p(y,z)	{y,z}	{x}
x:=*(y+i)	{y,i}	{x}
*(v+i):=x	{x}	{}
x := f(y_1,...,y_n)	{f,y_1,...,y_n}	{x}

Data-flow equations for Reaching Definition

The in set

– A definition reaches the beginning of a node if it reaches the end of **any** of the predecessors of that node

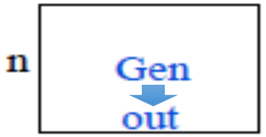


The out set

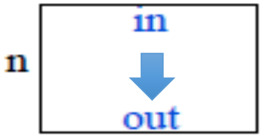
– A definition reaches the end of a node if (1) the node itself **generates** the definition **or** if (2) the definition reaches the beginning of the node and the node does **not kill** it

$$in[n] = \bigcup_{p \in pred[n]} out[p]$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$



(1)



(2)

Recall Liveness Analysis

- Data-flow Equation for liveness

$$in[n] = \mathbf{use}[n] \cup (out[n] - \mathbf{def}[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

- **Liveness equations in terms of Gen and Kill**

$$\left.\begin{aligned} in[n] &= \mathbf{gen}[n] \cup (out[n] - \mathbf{kill}[n]) \\ out[n] &= \bigcup_{s \in succ[n]} in[s] \end{aligned}\right\}$$

A use of a variable generates liveness
A def of a variable kills liveness

Gen: New information that's added at a node
Kill: Old information that's removed at a node

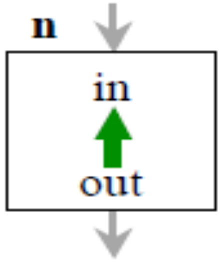
Can define almost any data-flow analysis in terms of Gen and Kill

Direction of Flow

Backward data-flow analysis

– Information at a node is based on what happens later in the flow graph
  i.e., in[] is defined in terms of out[]

$$in[n] = gen[n] \cup (out[n] - kill[n])$$
$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

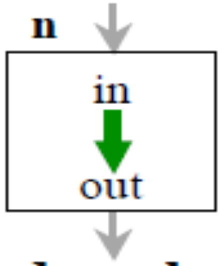


liveness

Forward data-flow analysis

– Information at a node is based on what happens earlier in the flow graph
  i.e., out[] is defined in terms of in[]

$$in[n] = \bigcup_{p \in pred[n]} out[p]$$
$$out[n] = gen[n] \cup (in[n] - kill[n])$$



reaching definitions

Some problems need both forward and backward analysis

– *e.g.*, Partial redundancy elimination (uncommon)

Data-Flow Equation for reaching definition

Symmetry between reaching definitions and liveness

- Swap in[] and out[] and swap the directions of the arcs

Reaching Definitions

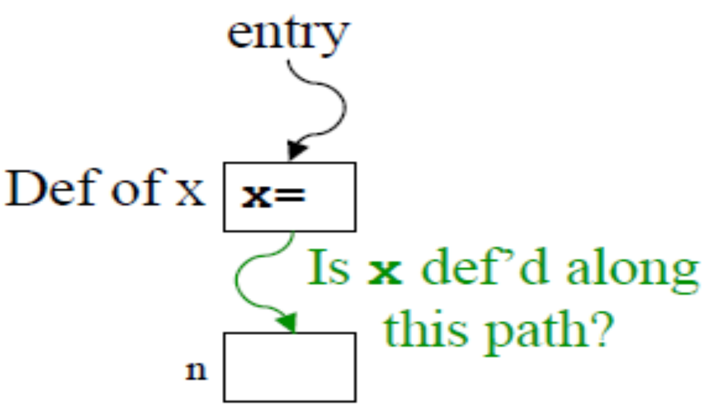
$$in[n] = \bigcup_{p \in pred[n]} out[s]$$

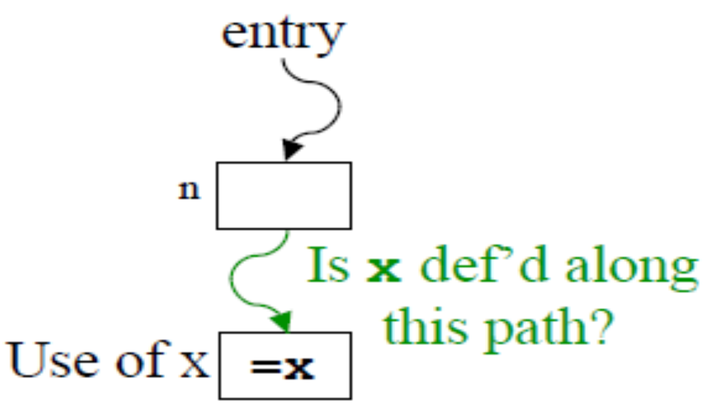
$$out[n] = gen[n] \cup (in[n] - kill[n])$$

Live Variables

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

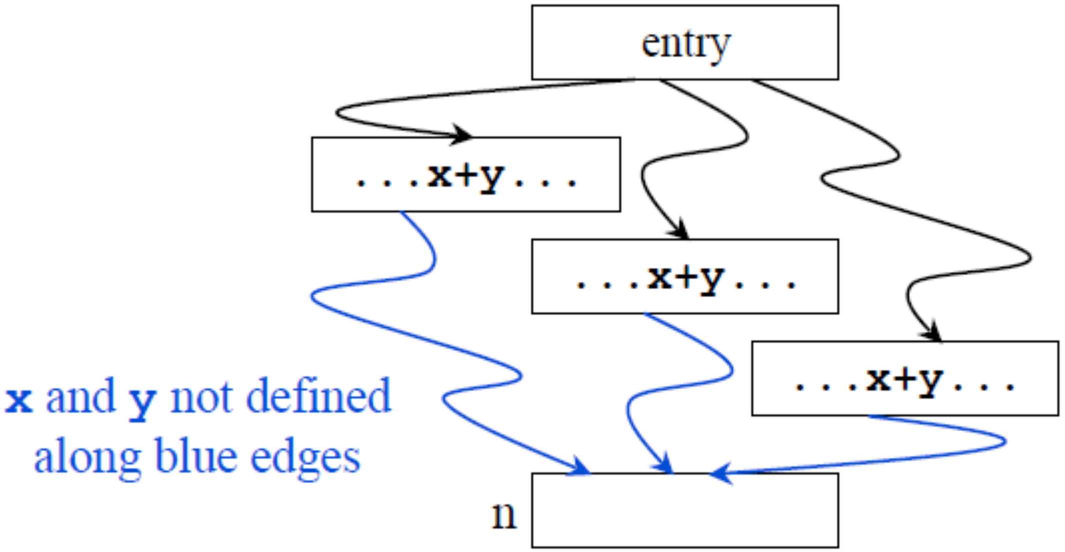
$$in[n] = gen[n] \cup (out[n] - kill[n])$$





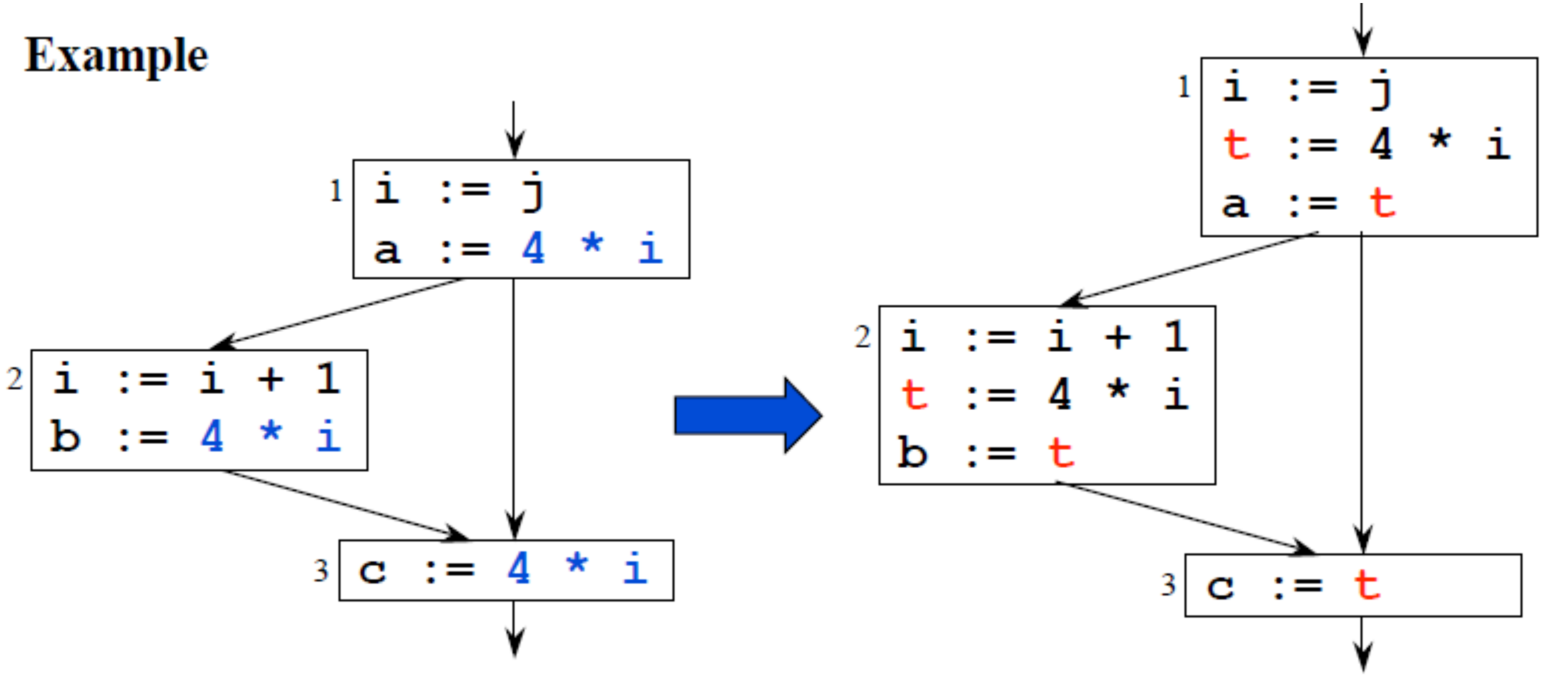
Available Expression

- An expression, **x+y**, is **available** at node n if every path from the entry node to n evaluates **x+y**, and there are no definitions of **x** or **y** after the last evaluation.



Available Expression for CSE

- Common Subexpression eliminated
  - If an expression is available at a point where it is evaluated, it need not be recomputed



Must vs. May analysis

- **May information:** Identifies possibilities
- **Must information:** Implies a guarantee

	May	Must
Forward	Reaching Definition	Available Expression
Backward	Live Variables	Very Busy Expression