LEXICAL ANALYSIS

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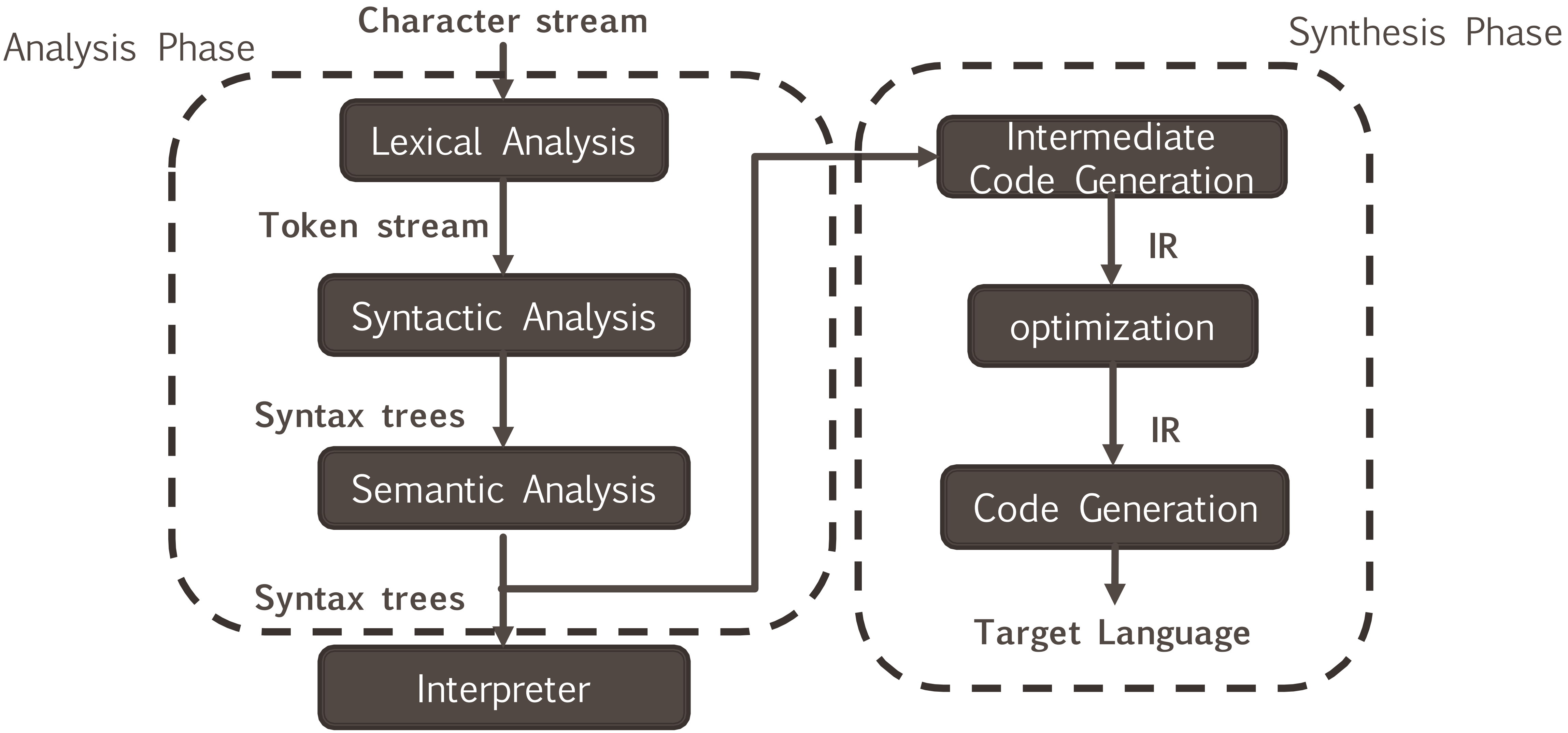
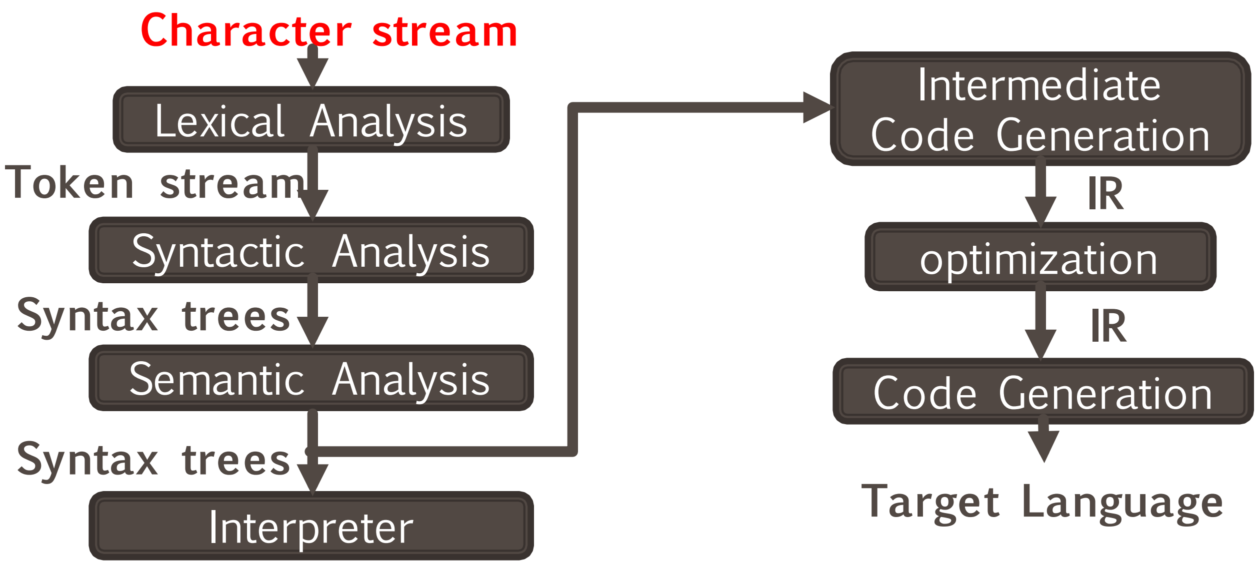
Structure of a Typical Compiler



Input to Compiler

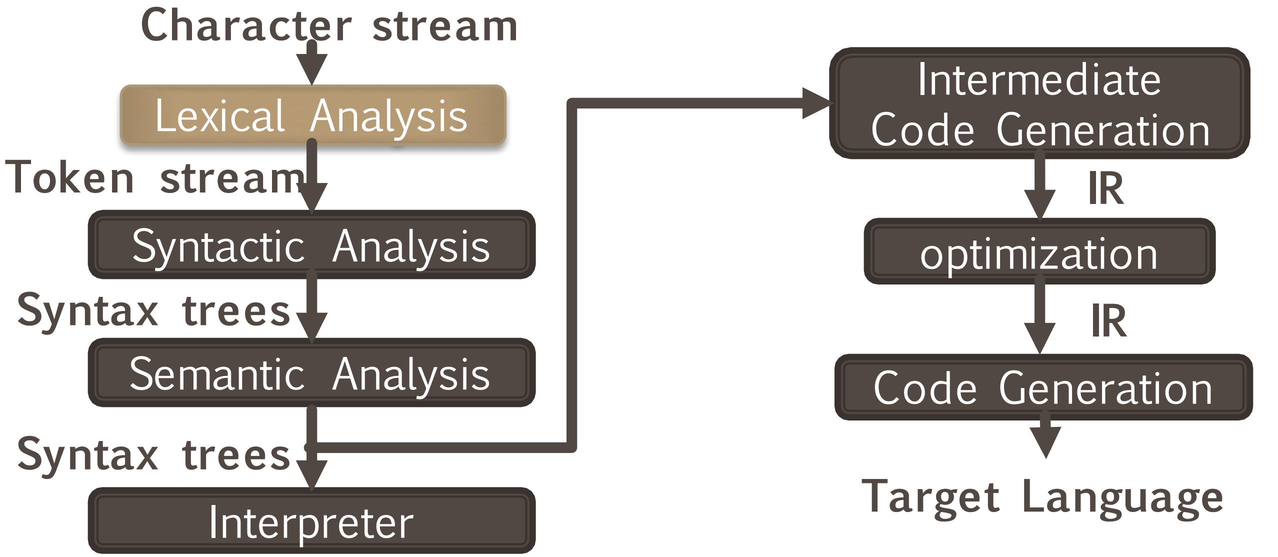



```
if(i == j)
  z = 0;
else
  z = 1;
```

i f (i = = j)\n\t z = 0 ; \n\t else z = 1 ;

Lexical Analysis

Break character stream into tokens ("words")




```
if(i == j)
  z = 0;
else
  z = 1;
```

i f (i = = j)\n\t z = 0 ; \n\t else z = 1 ;

1. Identify the substrings

'if' '(' 'i' '==' 'j' ')' '\n' '\t' 'z' '=' '0' ';' '\n' '\t' 'else' 'z' '=' '1' ';'

2. Identify the token classes

keyword<if> LPAR identifier<i> op<==> identifier<j> RPAR whitespaces identifier<z> op<=> number<0> <;> whitespaces keyword<else> identifier<z> op<=> number<1> ';'

Token Class

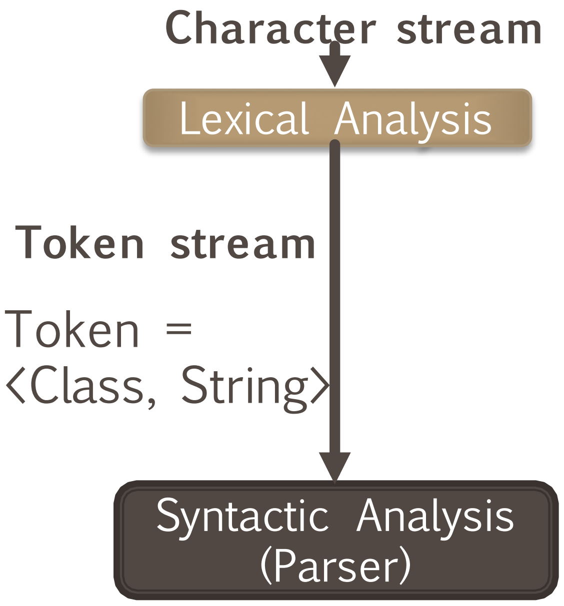
- In Programming Language
  - keywords, identifiers, LPAR, RPAR, const, etc.
- In English?
  - Noun, verb, ...

Token Class

- Each class corresponds to a set of strings
- Identifier
  - Strings are letters or digits, starting with a letter
  - Eg:
- Numbers:
  - A non-empty strings of digits
  - Eg:
- Keywords
  - A fixed set of reserved words
  - Eg:
- Whitespace
  - A non-empty sequence of blanks, newlines, and tabs

Lexical Analysis (Example)

- Classify program substrings according to roles (token class)
- Communicate tokens to parser



Z = 1

- ⟨Id, “Z”⟩
- ⟨Op, “=”⟩
- ⟨Numbers, “1”⟩

“Z”, “=”, “1” are called lexemes (an instance of the corr. token class)

Lexical Analysis: Examples

Here is a photo of my house;
<p>
see More Picture if you liked that one.</p>

⟨text, "Here is a photo of"⟩
⟨nodestart, b⟩
⟨text, "my house"⟩
⟨nodeend, b⟩
⟨nodestart, p⟩
⟨selfendnode, img⟩
⟨selfendnode, br⟩
⟨text, "see"⟩
⟨nodestart, a⟩
⟨text, "More Picture"⟩
⟨nodeend, a⟩
⟨text, "if you liked that one."⟩
⟨nodeend, p⟩

Lexical Analysis: Examples

- Usually, given the **pattern** describing the lexemes of a token, it is relatively simple to recognize matching lexemes when they occur on the input.
- However, in some languages, it is not immediately apparent when we have seen an instance of a lexeme corresponding to a token.

```
FORTRAN RULE: White Space is insignificant: VA R1 == VAR1

DO 5 I = 1.25

DO 5 I = 1,25
```

- Lexical analysis may require to “look ahead” to resolve ambiguity.
  - Look ahead complicates the design of lexical analysis
  - Minimize the amount of look ahead

Lookahead

- Lexical analysis tries to partition the input string into the logically units of the language. This is implemented by reading left to right. “scanning”, recognizing one token at a time.
- “Lookahead” is required to decide where one token ends and the next token begins.

```
if(i == j)
  z = 0;
else
  z = 1;
```

==/=?

Keyword/Identifier?

Lexical Analysis: Examples

- C++ template Syntax:
  - Foo<Bar>
- C++ stream Syntax:
  - cin >> var

- Ambiguity
  - Foo<Bar<Bar1>>
  - cin >> var

Summary So Far

- The goal of Lexical Analysis
  - Partition the input string to lexeme
  - Identify the token class of each lexeme

- Left-to-right scan => look ahead may require

REGULAR LANGUAGES

- Lexical structure of a programming language is a set of token classes.
- Each token class consists of some set of strings.
- How to map which set of strings belongs to which token class?
  - Use regular languages
- Use Regular Expressions to define Regular Languages.

Regular Expressions

- Single character
  - ‘c’ = {“c”}
- Epsilon
  - ε = {“”}
- Union
  - A + B = {a | a ϵ A} ∪ {b | b ϵ B}
- Concatenation
  - AB = {ab | a ϵ A ^ b ϵ B}
- Iteration (Kleene closure)
  - A* = $\cup_{i\geq 0} A^i$ = A.....A (i times)
  - $A^0 = \varepsilon$ (empty string)

Regular Expressions

- **Def:** The regular expressions over Σ are the smallest set of expressions including

$$
\begin{aligned}
R = {} & \varepsilon \\
& \mid \text{‘c’},\ \text{‘c’} \in \Sigma \\
& \mid R + R \\
& \mid RR \\
& \mid R^*
\end{aligned}
$$

Regular Expression Example

- $\Sigma = \{0,1\}$
  - 1^*
  - $(0+1)1$
  - 0^*+1^*
  - $(0+1)^*$

- There can be many ways to write an expression

Formal Languages

- **Def**: Let Σ be a set of character (alphabet}. A language over Σ is a set of strings of characters drawn from Σ.
  - Regular languages is a formal language
- Alphabet = English character, Language = English Language
  - Is it formal language?
- Alphabet = ASCII, Language = C Language

Formal Language

‘c’ = {“c”}

ε = {“”}

A + B = {a | a ϵ A} ∪ {b | b ϵ B}

AB = {ab | a ϵ A ^ b ϵ B}

$$\underbrace{A^*}_{\text{expression}} = \underbrace{\bigcup_{i\geq 0} A^i}_{\text{Set}}$$

Formal Language

$$L(\text{'c'}) = \{\text{"c"}\}$$

$$L(\varepsilon) = \{\text{""}\}$$

$$L(A + B) = \{a \mid a \in L(A)\} \cup \{b \mid b \in L(B)\}$$

$$L(AB) = \{ab \mid a \in L(A) \wedge b \in L(B)\}$$

$$\underbrace{L(A^*)}_{\text{expression}} = \underbrace{\bigcup_{i \geq 0} L(A^i)}_{\text{Set}}$$

L: Exp -> Set of strings

- **Meaning function L maps syntax to semantics**
- **Mapping is many to one**
- **Never one to many**

Lexical Specifications

- Keywords: “if” or “else” or “then” or “for” ….
  - Regular expression = ‘i’ ‘f’ + ‘e’ ‘l’ ‘s’ ‘e’
    = ‘if’ + ‘else’ + ‘then’

- Numbers: a non-empty string of digits
  - digit = ‘0’+’1’+’2’+’3’+’4’+’5’+’6’+’7’+’8’+’9’
  - digit*
  - How to enforce *non-empty string*?
    - digit digit* = digit$^+$

Lexical Specifications

- Identifier: strings of letters or digits, starting with a letter
  - letter = ‘a’ + ‘b’ + ‘c’ + …. + ‘z’ + ‘A’ + ‘B’ + …. + ‘Z’
    = [a-zA-Z]
  - letter (letter + digit)*

- Whitespace: a non-empty sequence of blanks, newline, and tabs
  - $(\text{‘ ’} + \text{‘\textbackslash n’} + \text{‘\textbackslash t’})^+$

PASCAL Lexical Specification

- digit = '0'+'1'+'2'+'3'+'4'+'5'+'6'+'7'+'8'+'9'
- digits = digit$^+$
- opt_fraction = ('.' digits) + ε = ('.' digits)?
- opt_exponent = ('E' ('+' + '-' + ε) digits) + ε

  = ('E' ('+' + '-')? digits)?

- num = digits opt_fraction opt_exponent

Common Regular Expression

- At least one $A^+ \equiv AA^*$
- Union: $A \mid B \equiv A + B$
- Option: $A? \equiv A + \varepsilon$
- Range: 'a' + … + 'z' = [a-z]
- Excluded range: complement of [a-z] ≡ [^a-z]

Lexical Specification of a language

1. Write a regex for the lexemes of each token class
   - Number = digit$^+$
   - Keywords = 'if' + 'else' + ..
   - Identifiers = letter (letter + digit)*
   - LPAR = '('

Lexical Specification of a language

2. Construct **R**, matching all lexemes for all tokens

   R = Number + Keywords + Identifiers + ...

   $= R_1 + R_2 + R_3 + \ldots$

3. Let input be $x_1 \ldots x_n$.

   For $1 \leq i \leq n$, check $x_1 \ldots x_i \in L(R)$

4. If successful, then we know that

   $x_1 \ldots x_i \in L(R_j)$ for some j

5. Remove $x_1 \ldots x_i$ from input and go to step 3.

Lexical Specification of a language

- How much input is used?
  - $x_1 \ldots x_i \in L(R)$
  - $x_1 \ldots x_j \in L(R)$, $i \neq j$
  - Which one do we want? (e.g., == or =)
  - Maximal munch: always choose the longer one

- Which token is used if more than one matches?
  - $x_1 \ldots x_i \in L(R)$ where $R = R_1 + R_2 + .. + R_n$
  - $x_1 \ldots x_i \in L(R_m)$
  - $x_1 \ldots x_i \in L(R_n)$, $m \neq n$
  - Eg: Keywords = 'if', Identifier = letter (letter + digit)*, if matches both
  - Keyword has higher priority
  - Rule of Thumb: Choose the one listed first

Lexical Specification of a language

- What if no rule matches?
  - $x_1 \ldots x_i \notin L(R)$ … compiler typically tries to avoid this scenario
  - Error = [all strings not in the lexical spec]
  - Put it in last in priority

Summary so far

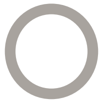
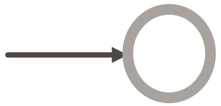
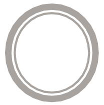
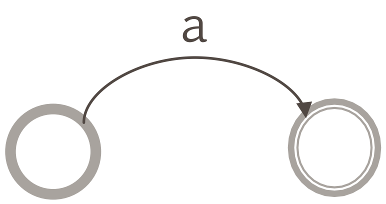
- Regular Expressions are concise notations for the string patterns

- Use in lexical analysis with some extensions
  - To resolve ambiguities
  - To handle errors

- Implementation?
  - We will study next

Finite Automata

- Regular Expression = specification
- Finite Automata = implementation

- A finite automaton consists of
  - An input Alphabet: Σ
  - A finite set of states: S 
  - A start state: n 
  - A set of accepting states: $F \subseteq S$ 
  - A set of transitions state: state1 $\xrightarrow{input}$ state2 

Transition

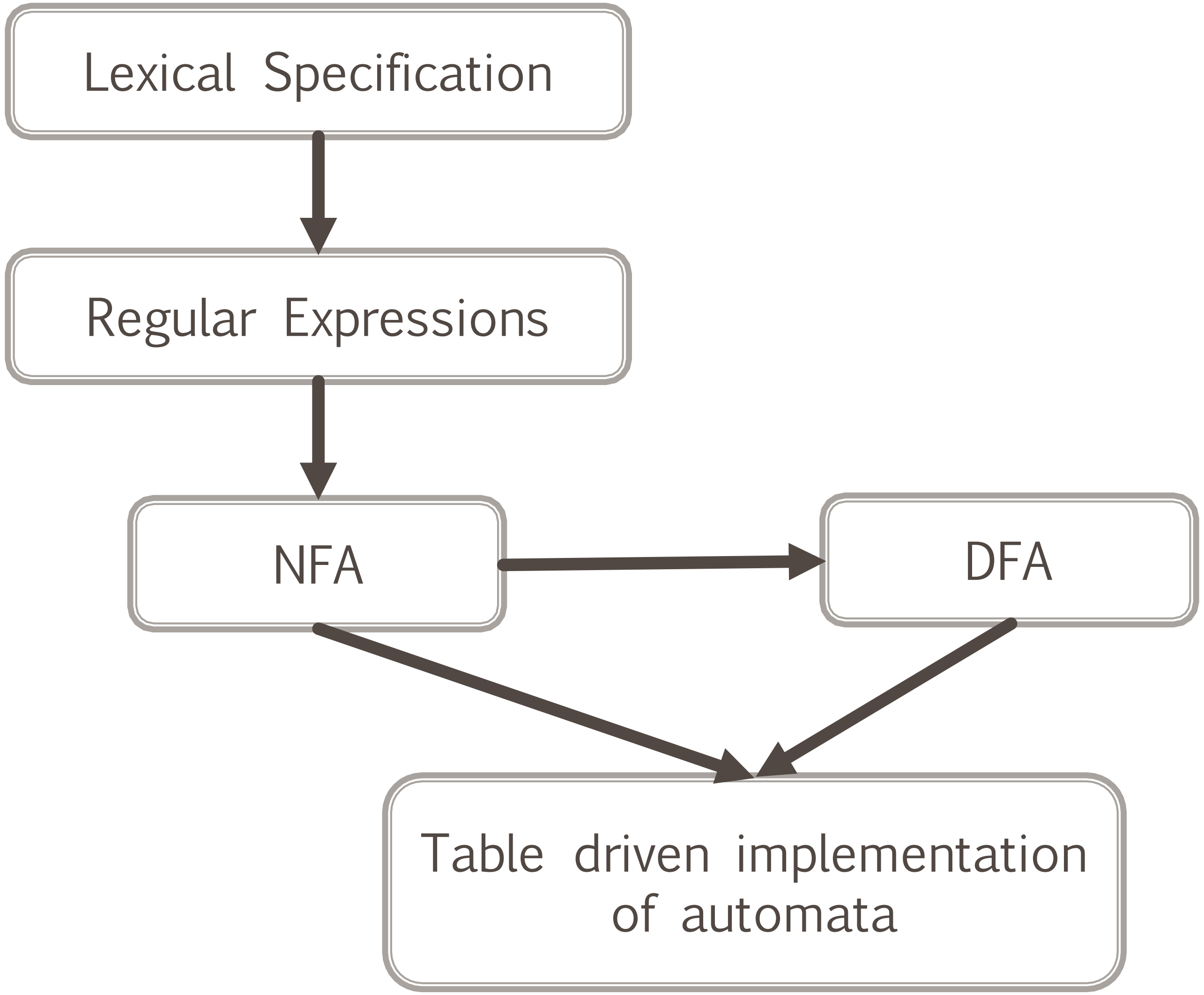
- s1 $\xrightarrow{a}$ s2 (state s1 on input a goes to state s2)
- If end of the input and in final state, the input is accepted
- Otherwise reject

- Language of FA = set of strings accepted by that FA

Example Automata

- a finite automaton that accepts only "1"

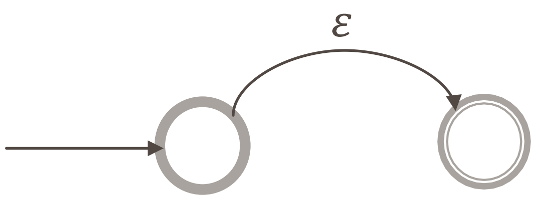
- A finite automaton that accepting any number of "1" followed by "0"


Lexical Specification
Regular Expressions
NFA
DFA
Table driven implementation of automata

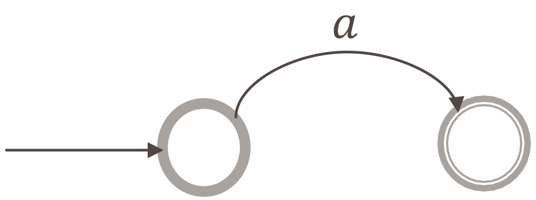
Regular Expression to NFA

- For each kind of regex, define a NFA
  - Notation: NFA for regex M

- For ε

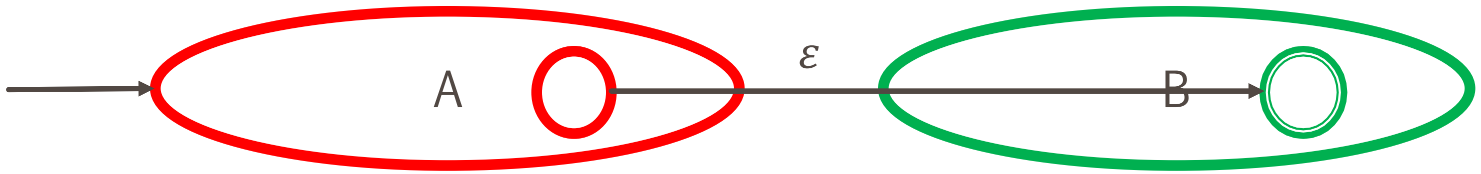


- For input a

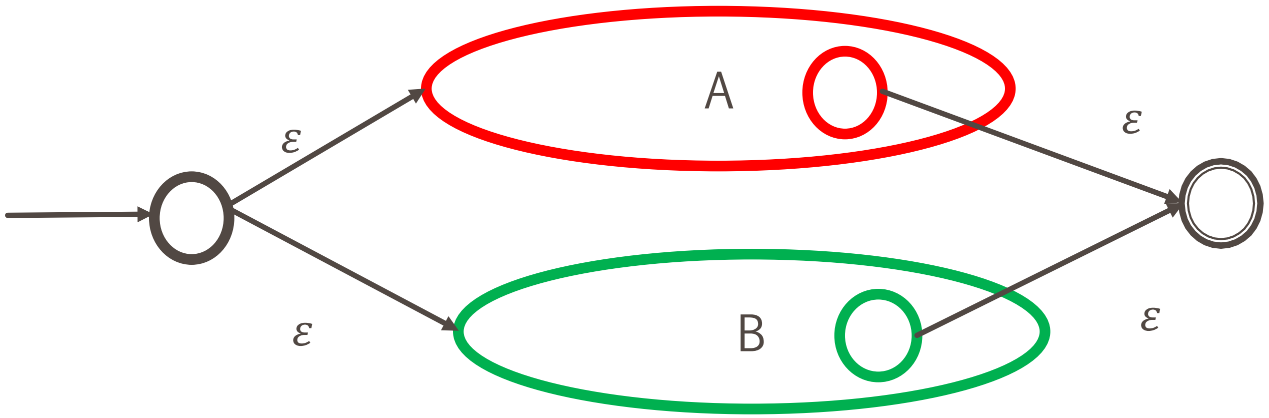


Regular Expression to NFA

- For AB

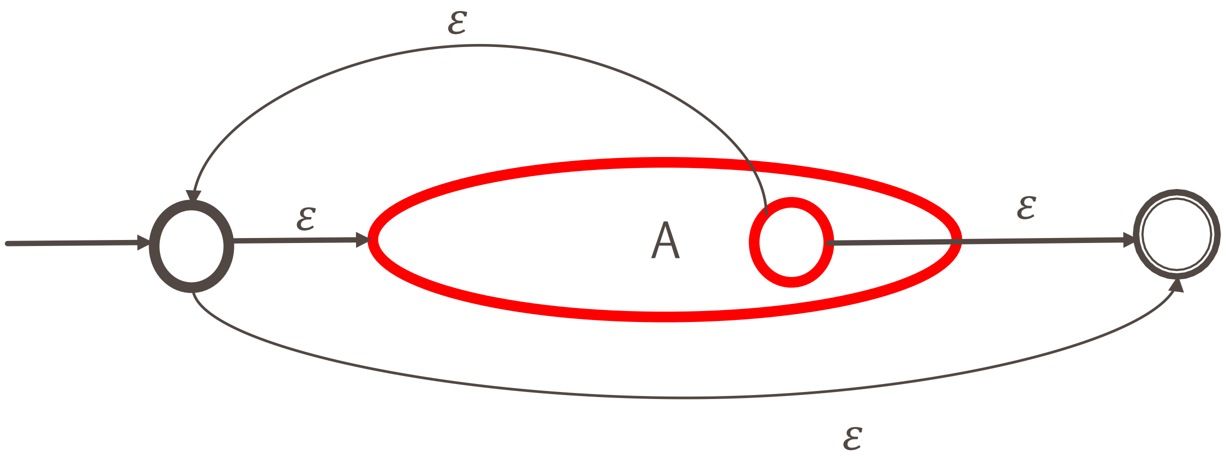


- For A + B



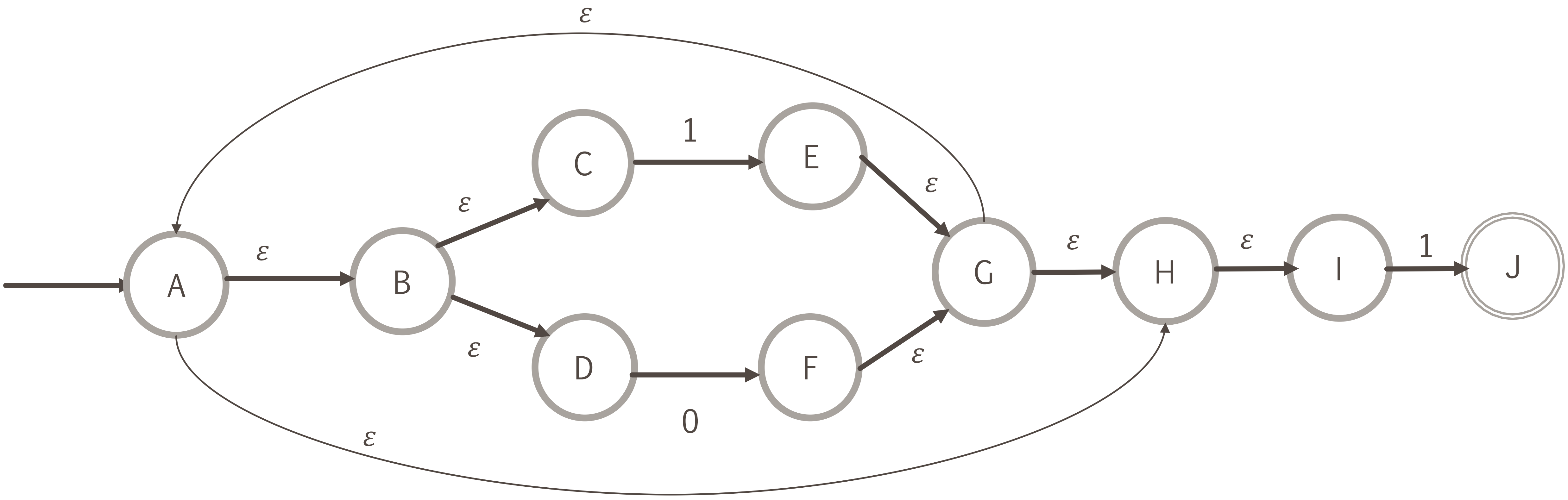
Regular Expression to NFA

- For A*

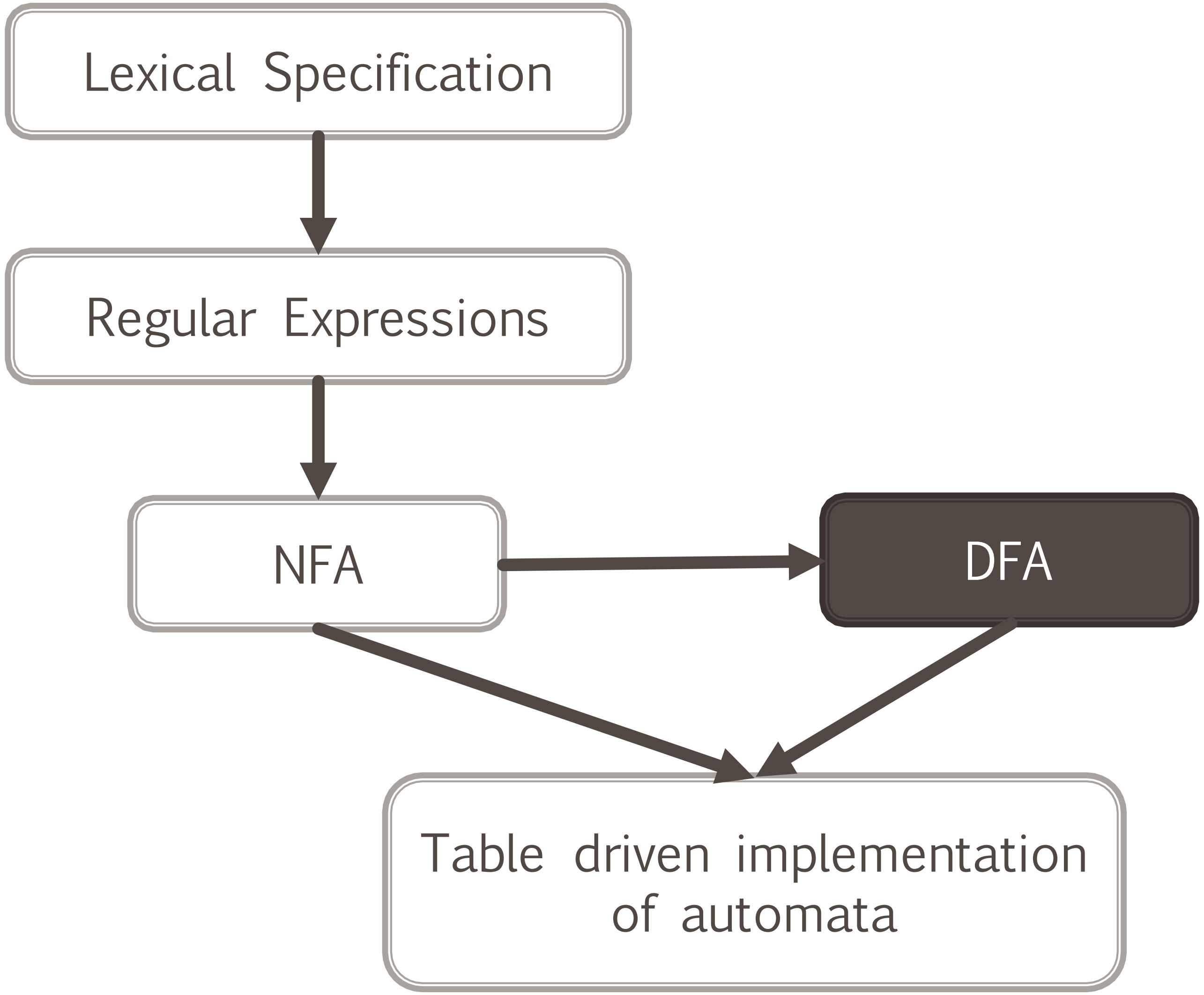


Example

- (1+0)*1

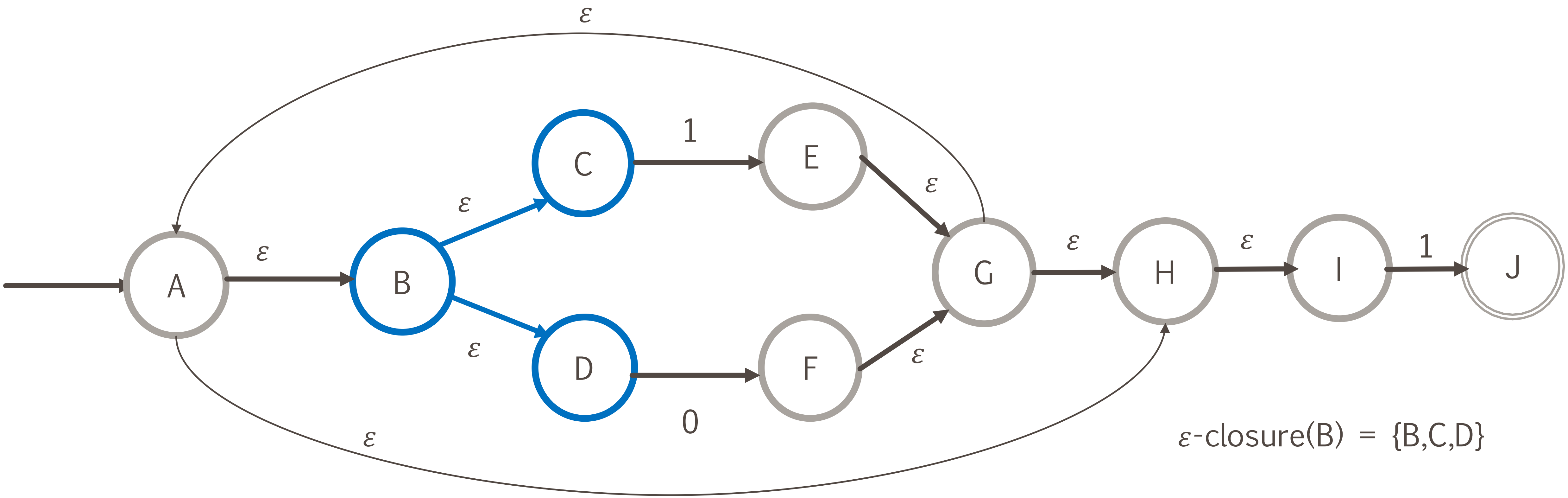


NFA to DFA


Lexical Specification
Regular Expressions
NFA
DFA
Table driven implementation of automata

ε-closure

- ε-closure of a state is all the state I can reach following ε move.



ε-closure(B) = {B,C,D}

ε-closure

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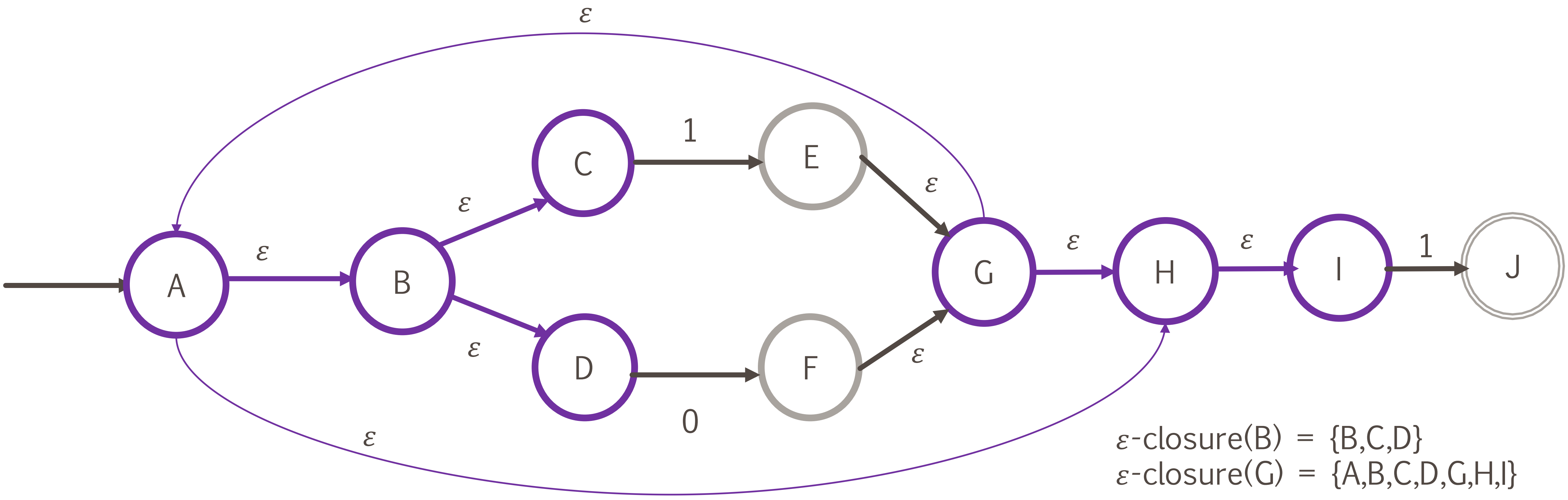


ε-closure(B) = {B,C,D}
ε-closure(G) = {A,B,C,D,G,H,I}

NFA

- An NFA can be in many states at any time

- How many different states?
  - If NFA has N states, it reaches some subset of those states, say S
  - $|S| \le N$
  - There are $2^N - 1$ possible subsets (finite number)

NFA to DFA

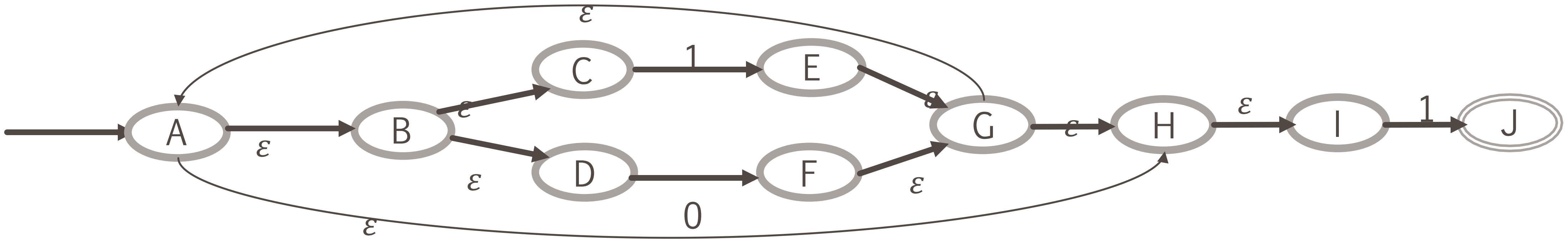
NFA

- States S
- Start s
- Final state F
- Transition state
  - a(X) = {y | x ∈ X ∧ x $\overset{a}{\rightarrow} y$)
- $\varepsilon - closure$

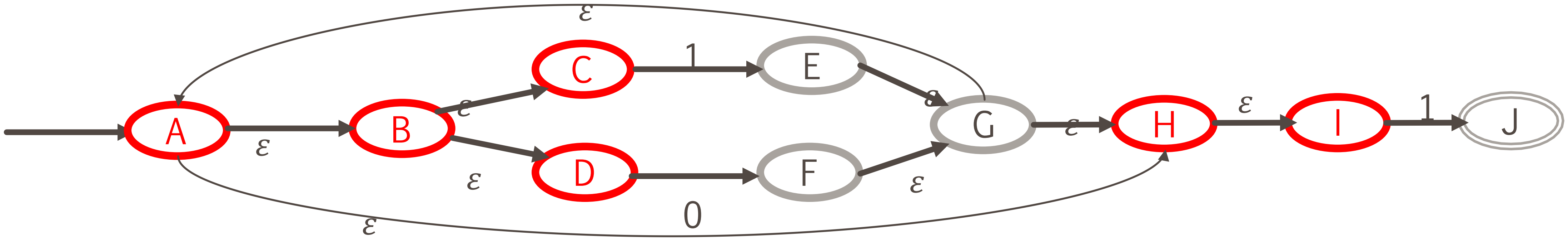
DFA

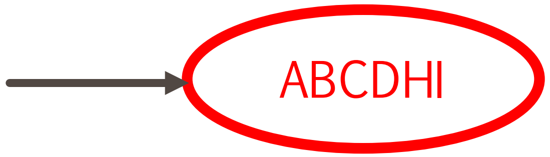
- States will be all possible subset of S except empty set
- Start state = $\varepsilon - closure$(s)
- Final state $\{\mathrm{X} \mid X \cap F = \emptyset\}$
- X $\overset{a}{\rightarrow} Y$ if
  - Y = $\varepsilon - closure\ (\mathrm{a}(\mathrm{X}))$

NFA to DFA

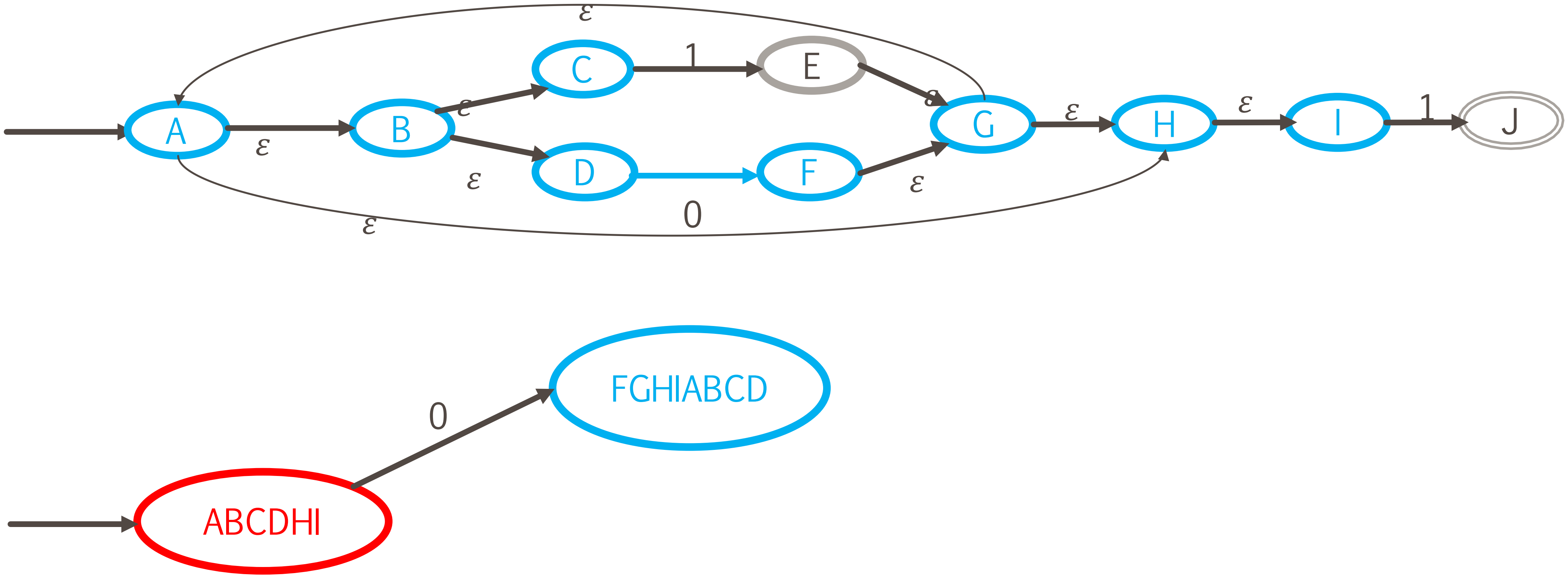


NFA to DFA

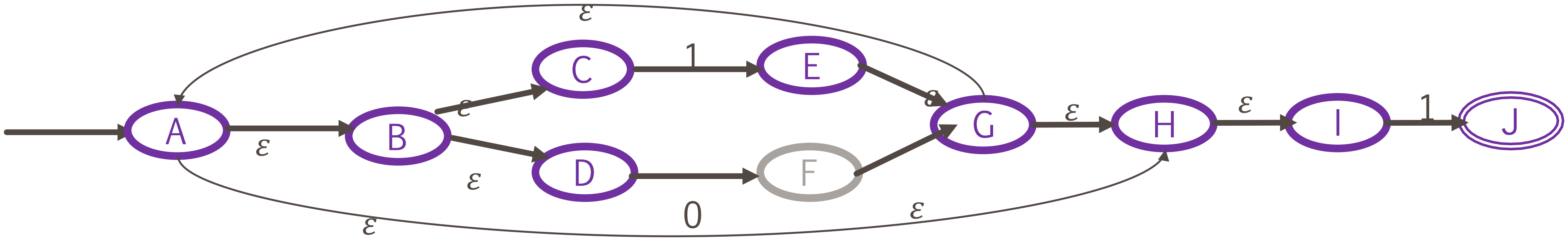


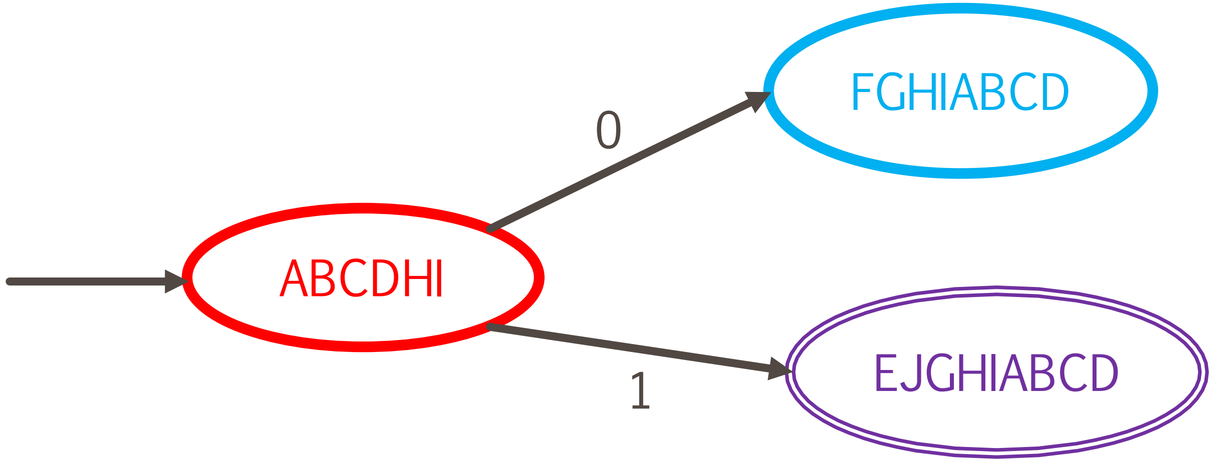


NFA to DFA

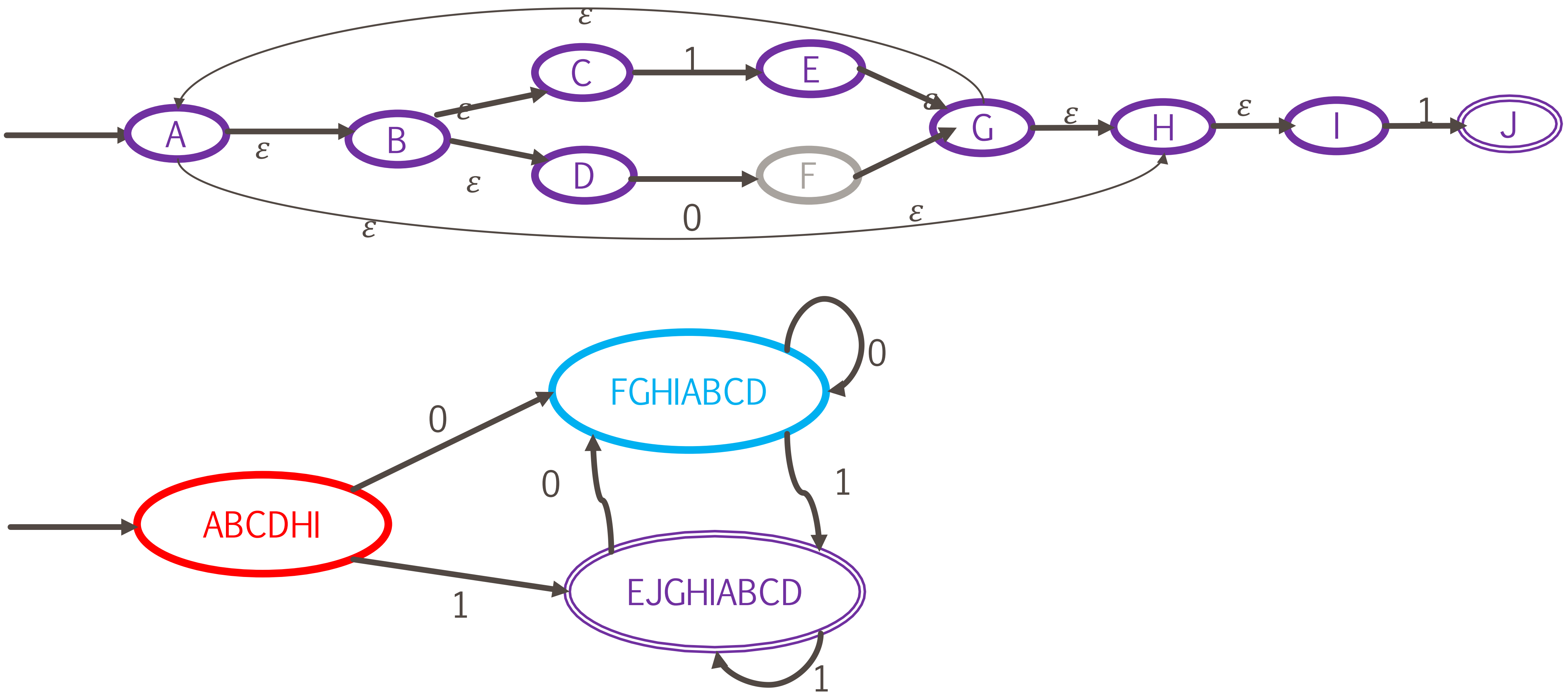


NFA to DFA



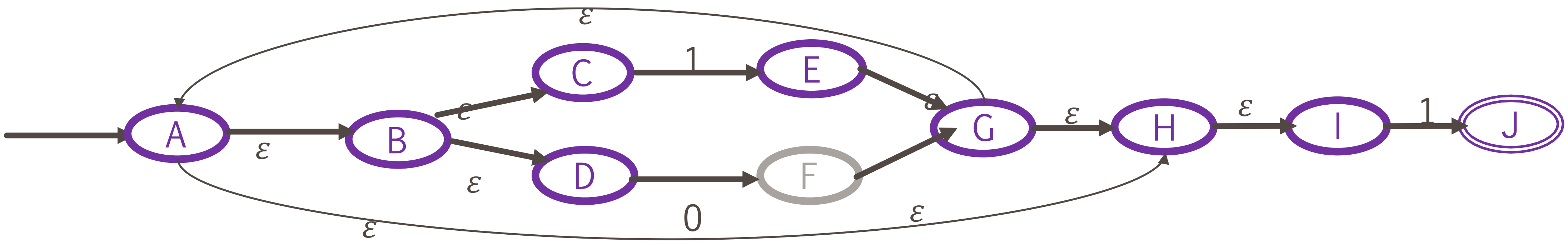


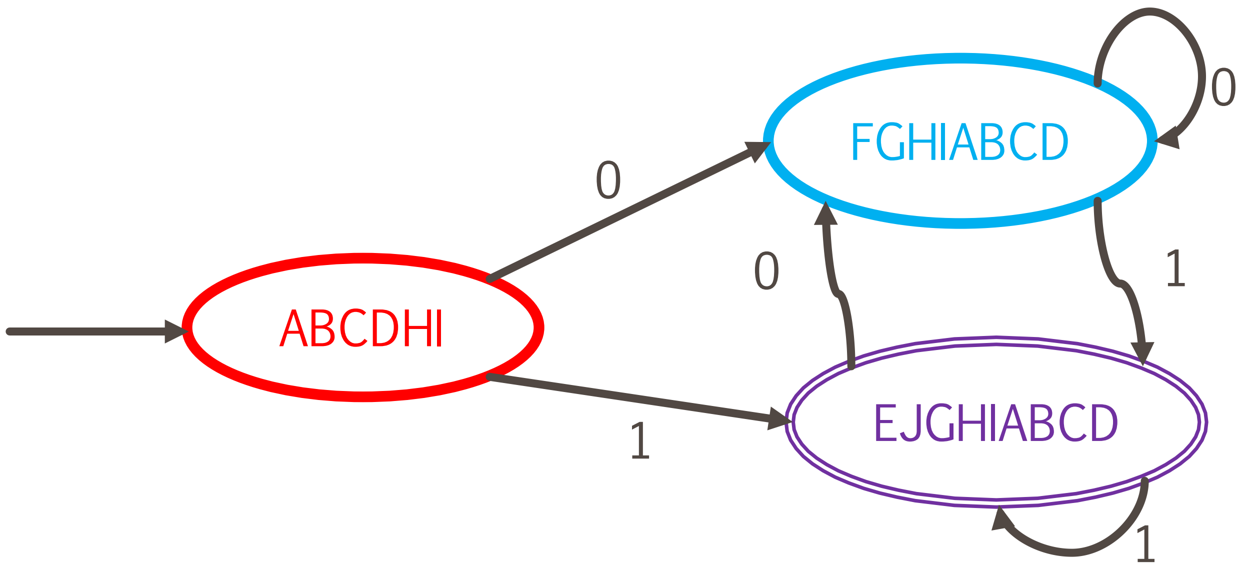
NFA to DFA



ε
A
B
C
D
E
F
G
H
I
J
1
0
ε
ε
ε
ε
ε
ε
ε
ε
1
FGHIABCD
0
ABCDHI
0
0
1
EJGHIABCD
1
1

NFA to DFA





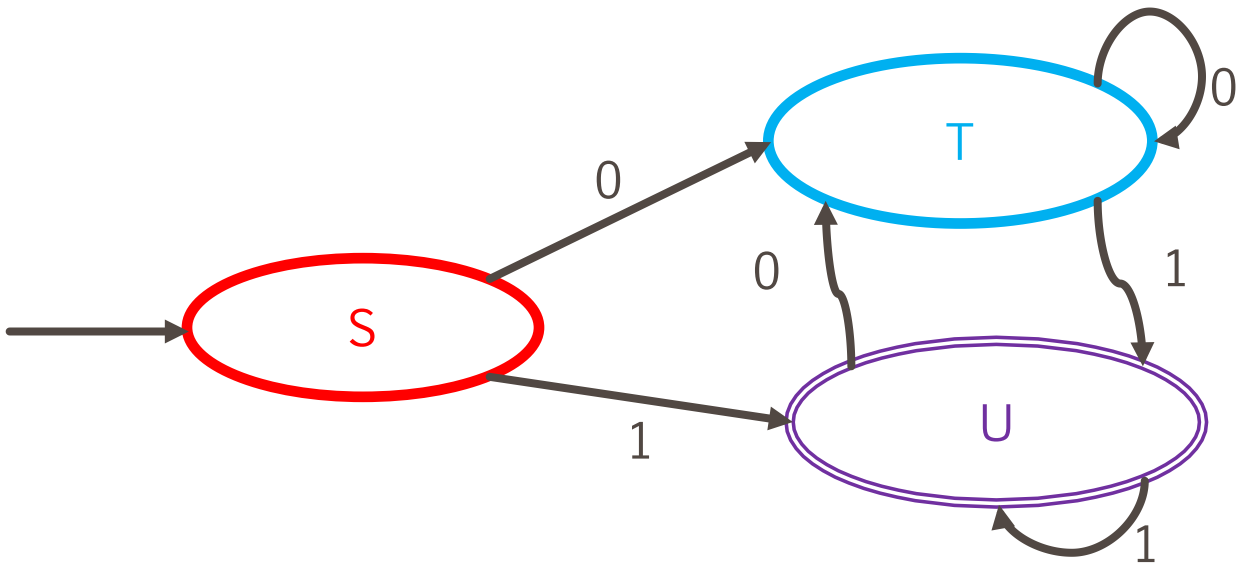
Implementing DFA

- A DFA can be implemented by a 2D table T
  - One dimension is states
  - Another dimension is input symbol
  - For every transition $s_i \rightarrow^a s_k$: define T[i,a] = k

Implementing DFA

Table A

	0	1
S	T	U
T	T	U
U	T	U




```
i = 0;
state = 0;
while(input[i])
   state = A[state,input[i]]
```

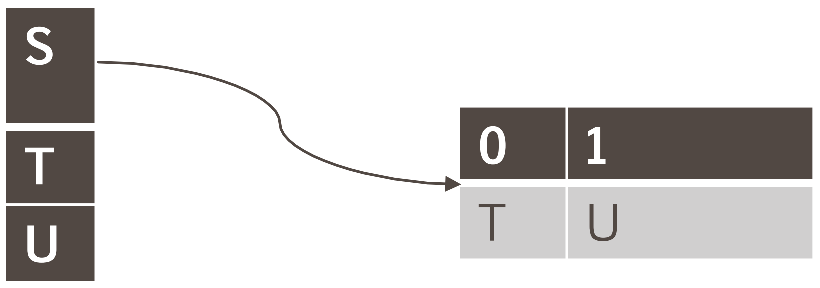
Implementing DFA

Table A

	0	1
S	T	U
T	T	U
U	T	U

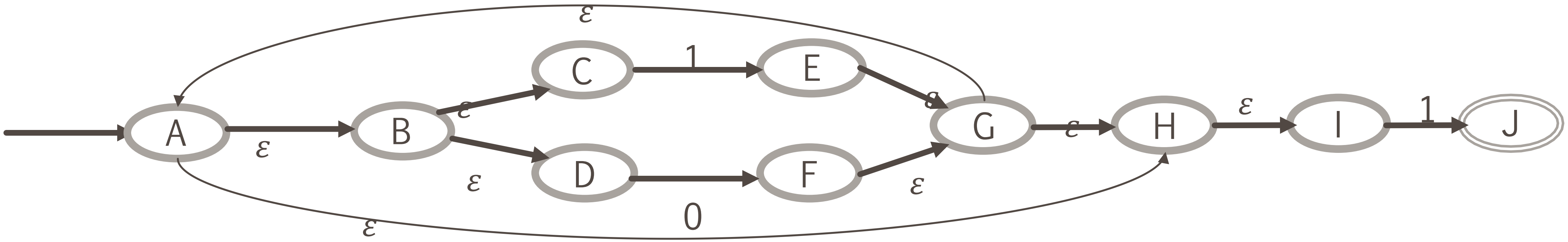
A lot of duplicate entries

Table B



Compact but need an extra indirection
- Inner loop will be slower

Implementing DFA



	0	1	ε
A			{B,H}
B			{C,D}
C		{E}	
…			

Deal with set of states rather than single state-→ inner loop is complicated

Deterministic Finite Automata: Example

```
{
   type token = ELSE | ELSEIF
}

rule token =
  parse "else"{ ELSE }
       | "elseif"{ ELSEIF }
```