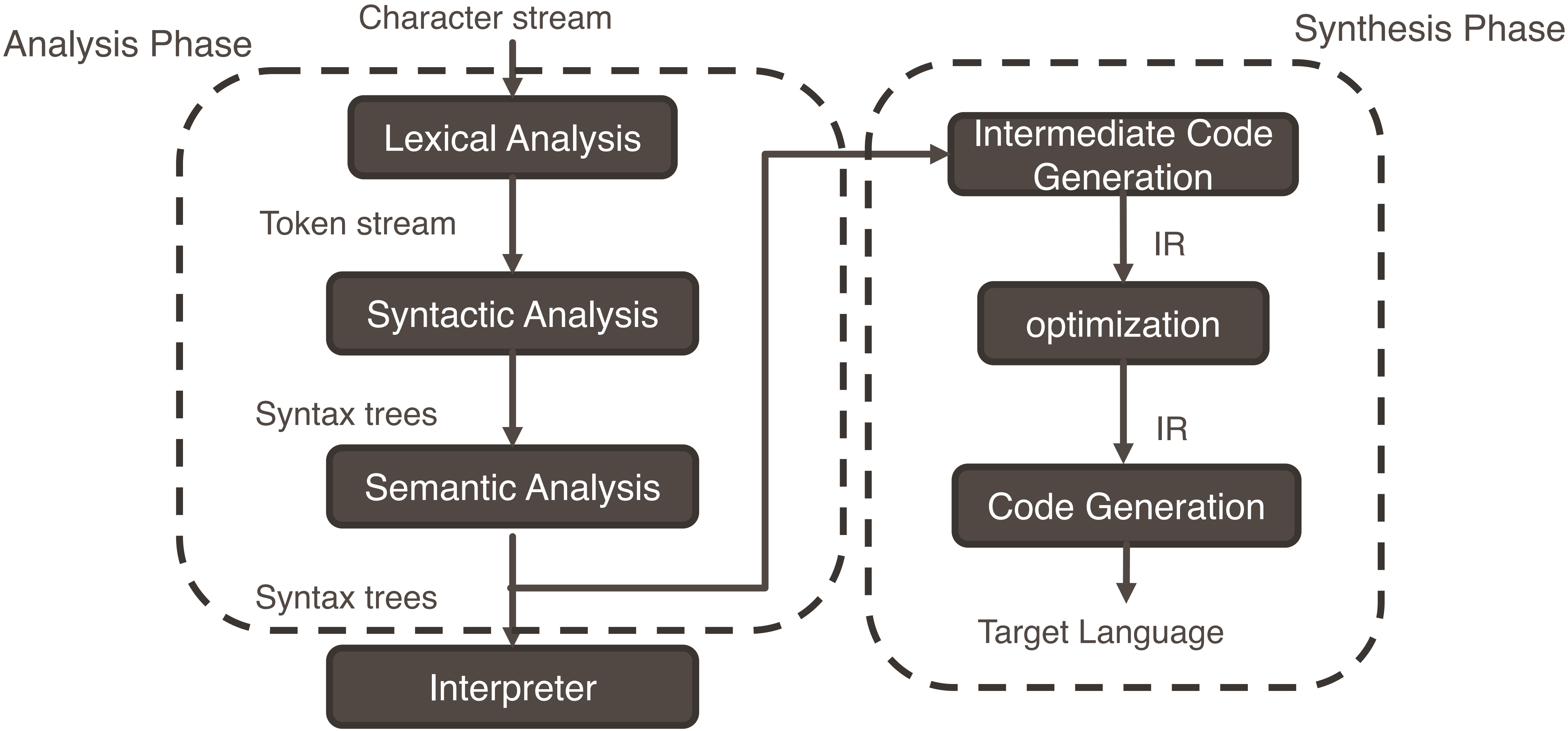
LEXICAL ANALYSIS

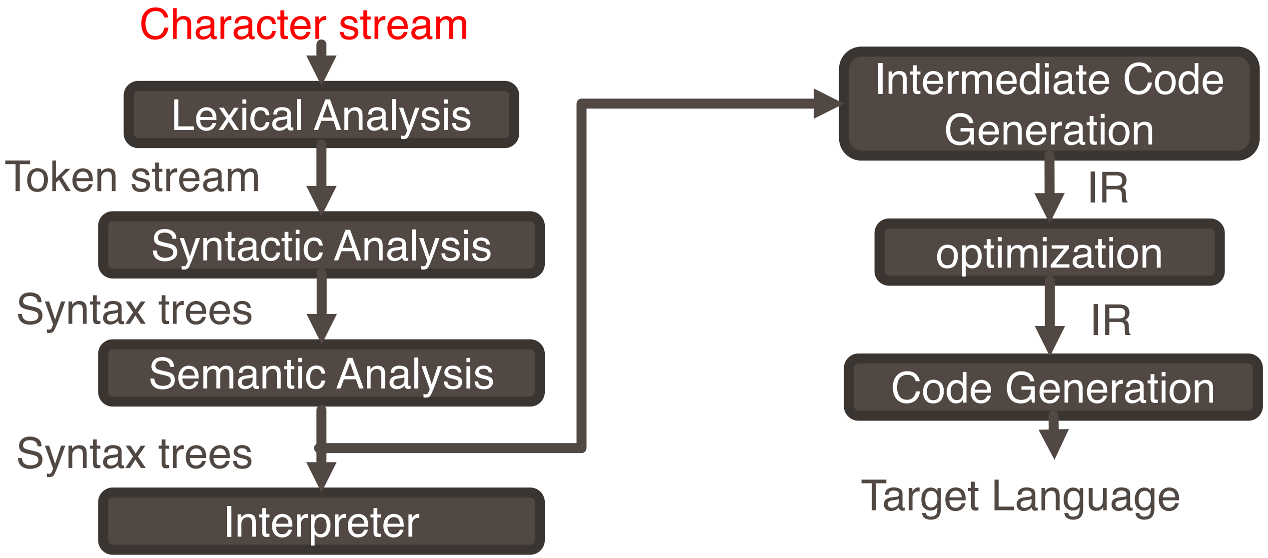
Baishakhi Ray

These slides are motivated from Prof. Alex Aiken: Compilers (Stanford)

Structure of a Typical Compiler



Input to Compiler

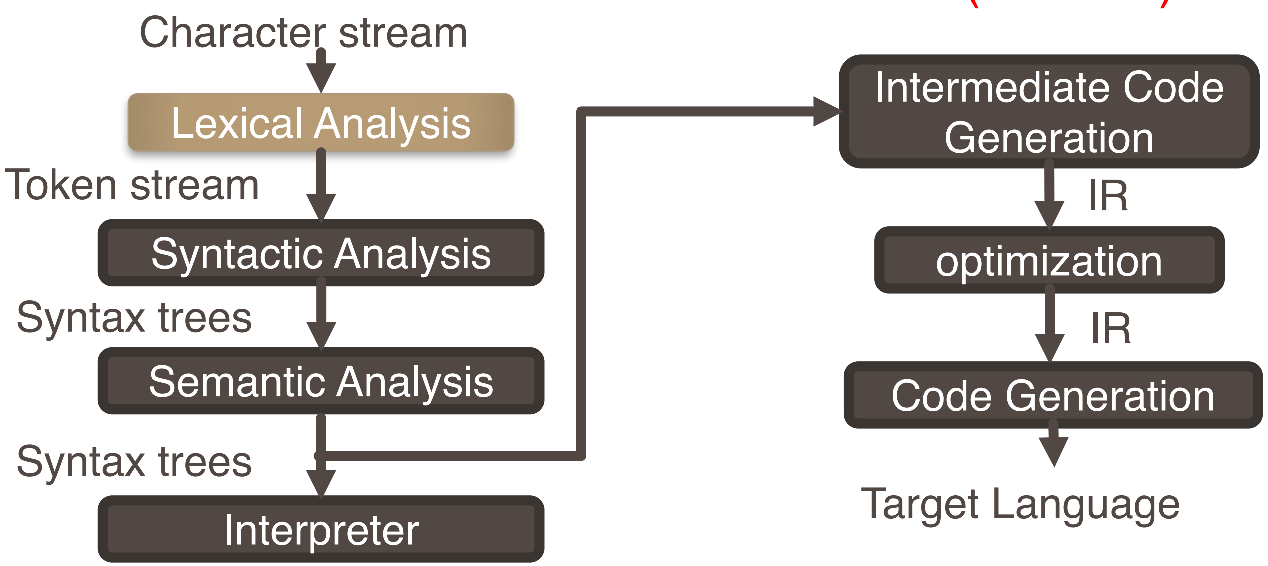



```
if(i == j)
 z = 0;
else
 z = 1;
```

if (i = = j)\n\t z = 0 ; \n else\n\t z = 1 ;

Lexical Analysis

Break character stream into tokens ("words")




```
if(i == j)
 z = 0;
else
 z = 1;
```

if (i = = j)\n\t z = 0 ; \n else\n\t z = 1 ;

'if' '(' 'i' '==' 'j' ')' '\n' '\t' 'z' '=' '0' ';' '\n' 'else' '\n' '\t' 'z' '=' '1' ';'

keyword<if> LPAR identifier<i> op<==> identifier<j> RPAR
whitespaces identifier<z> op<=> number<0> <;> whitespaces
keyword<else> identifier<z> op<=> number<1> ';'

1. Identify the substrings

2. Identify the token classes

Token Class

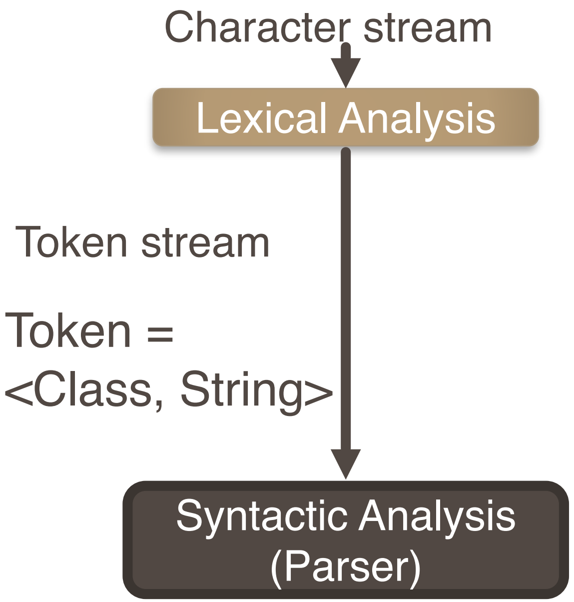
- In English?
  - Noun, verb, adjectives, …
- In Programming Language
  - keywords, identifiers, LPAR, RPAR, const, etc.

Token Class

- Each class corresponds to a set of strings
- Identifier
  - Strings are letters or digits, starting with a letter
  - Eg:
- Numbers:
  - A non-empty strings of digits
  - Eg:
- Keywords
  - A fixed set of reserved words
  - Eg:
- Whitespace
  - A non-empty sequence of blanks, newlines, and tabs

Lexical Analysis (Example)

- Classify program substrings according to roles (token class)
- Communicate tokens to parser



Z = 1

- <Id, “Z”>
- <Op, “=”>
- <Numbers, “1”>

“Z”, “=”, “1” are called lexemes (an instance of the corr. token class)

Lexical Analysis: HTML Examples

```
Here is a photo of <b> my house </b>
<p><img src="house.gif"/><br/>
see <a href="morePix.html">More Picture</a> if you liked that
one.</p>
```

<text, "Here is a photo of">
<nodestart, b>
<text, "my house">
<nodeend, b>
<nodestart, p>
<selfendnode, img>
<selfendnode, br>
<text, "see">
<nodestart, a>
<text, "More Picture">
<nodeend, a>
<text, "if you liked that one.">
<nodeend, p>

Exercise

```
x = p;
while ( x < 100 ) { x++ ; }
```

Exercise

```
if(i == j)
 z = 0;
else
 z = 1;
```

==/=?

Keyword/Identifier?

Lookahead

- Lexical analysis tries to partition the input string into the logical units of the language. This is implemented by reading left to right. “scanning”, recognizing one token at a time.
- “Lookahead” is required to decide where one token ends and the next token begins.

```
if(i == j)
 z = 0;
else
 z = 1;
```

==/=?

Keyword/Identifier?

Lookahead: Examples

- Usually, given the pattern describing the lexemes of a token, it is relatively simple to recognize matching lexemes when they occur on the input.
- However, in some languages, it is not immediately apparent when we have seen an instance of a lexeme corresponding to a token.

```
FORTRAN RULE: White Space is insignificant: VA R1 == VAR1

DO 5 I = 1,25

DO 5 I = 1.25
```

- Lexical analysis may require to “look ahead” to resolve ambiguity.
  - Look ahead complicates the design of lexical analysis
  - Minimize the amount of look ahead

Lexical Analysis: Examples

- C++ template Syntax:
  - Foo<Bar>
- C++ stream Syntax:
  - cin >> var
- Ambiguity
  - Foo<Bar<Barq>>
  - cin >> var

Summary So Far

- The goal of Lexical Analysis
  - Partition the input string to lexeme
  - Identify the token class of each lexeme

- Left-to-right scan => look ahead may require
  - In reality, lookahead is always needed
  - Our goal is to minimize thee amount of lookahead

REGULAR LANGUAGES

- Lexical structure of a programming language is a set of token classes.
- Each token class consists of some set of strings.
- How to map which set of strings belongs to which token class?
  - Use regular languages
- Use Regular Expressions to define Regular Languages.

Regular Expressions

- Single character
  - ‘c’ = {“c”}
- Epsilon
  - ε = {“”}
- Union
  - A + B = {a I a ϵ A} ∪ {b I b ϵ B}
- Concatenation
  - AB = {ab I a ϵ A ^ b ϵ B}
- Iteration (Kleene closure)
  - A* = $\bigcup_{i>=0} A^i$ = A…..A (i times)
  - Ap = ε (empty string)

Regular Expressions

- Def: The regular expressions over Σ are the smallest set of expressions including

  R = ε

  | ‘c’, ‘c’ ϵ Σ

  | R + R

  | RR

  | R*

Regular Expression Example

- Σ = {p,q}
  - q*
  - (p+q)q
  - p*+q*
  - (p+q)*

- There can be many ways to write an expression

Exercise

Choose the regular languages that are equivalent to the given regular language: $(p + q)^*q(p + q)^*$

A. $(pq + qq)^*(p + q)^*$

B. $(p + q)^*(qp + qq + q)(p + q)^*$

C. $(q + p)^*q(q + p)^*$

D. $(p + q)^*(p + q)(p + q)^*$

Formal Languages

- Def: Let Σ be a set of character (alphabet). A language over Σ is a set of strings of characters drawn from Σ.
  - Regular languages is a formal language
- Alphabet = English character, Language = English Language
  - Is it formal language?
- Alphabet = ASCII, Language = C Language

Formal Language

$\text{‘c’} = \{\text{“c”}\}$

$\varepsilon = \{\text{“”}\}$

$A + B = \{a \mid a \epsilon A\} \cup \{b \mid b \epsilon B\}$

$AB = \{ab \mid a \epsilon A \wedge b \epsilon B\}$

$$\underbrace{A^*}_{\text{expression}} = \underbrace{\bigcup_{i>=0} A^i}_{\text{Set}}$$

Formal Language

$L(\text{'c'}) = \{\text{"c"}\}$

$L(\varepsilon) = \{\text{""}\}$

$L(A + B) = \{a \mid a \in L(A)\} \cup \{b \mid b \in L(B)\}$

$L(AB) = \{ab \mid a \in L(A) \wedge b \in L(B)\}$

$$\underbrace{L(A^*)}_{\text{expression}} = \underbrace{\bigcup_{i>=0} L(A^i)}_{\text{Set}}$$

L: Expressions -> Set of strings

- Meaning function L maps syntax to semantics
- Mapping is many to one
- Never one to many

Lexical Specifications

- Keywords: “if” or “else” or “then” or “for” ….
  - Regular expression = ‘i’ ‘f’ + ‘e’ ‘l’ ‘s’ ‘e’
    = ‘if’ + ‘else’ + ‘then’

- Numbers: a non-empty string of digits
  - digit = ‘1’+’0’+’2’+’3’+’4’+’5’+’6’+’7’+’8’+’9’
  - digit*
  - How to enforce non-empty string?
    - digit digit* = digit^{+}

Lexical Specifications

- Identifier: strings of letters or digits, starting with a letter
  - letter = ‘a’ + ‘b’ + ‘c’ + …. + ‘z’ + ‘A’ + ‘B’ + …. + ‘Z’
    = [a-zA-Z]
  - letter (letter + digit)*
- Whitespace: a non-empty sequence of blanks, newline, and tabs
  - $(\text{‘ ’} + \text{‘\textbackslash n’} + \text{‘\textbackslash t’})^+$

PASCAL Lexical Specification

- digit = ‘0’+‘1’+‘2’+‘3’+‘4’+‘5’+‘6’+‘7’+‘8’+‘9’
- digits = digit^{+}
- opt_fraction = (‘.’ digits) + ε = (‘.’ digits)?
- opt_exponent = (‘E’ (‘+’ + ‘-’ + ε) digits) + ε

  = (‘E’ (‘+’ + ‘-’)? digits)?
- num = digits opt_fraction opt_exponent

Common Regular Expression

- At least one $A^{+} \equiv AA^{*}$
- Union: $A \mid B \equiv A + B$
- Option: $A? \equiv A + \varepsilon$
- Range: 'a' + … + 'z' = [a-z]
- Excluded range: complement of [a-z] ≡ [^a-z]

Summary of Regular Languages

- Regular Expressions specify regular languages

- Five constructs
  - Two base expression
    - Empty and 1-character string

  - Three compound expressions
    - Union, Concatenation, Iteration

Lexical Specification of a language

1. Write a regex for the lexemes of each token class
   - Number = digit$^+$
   - Keywords = ‘if’ + ‘else’ + ..
   - Identifiers = letter (letter + digit)*
   - LPAR = ‘(‘

Lexical Specification of a language

2. Construct R, matching all lexemes for all tokens

   R = Number + Keywords + Identifiers + …

   $= R_1 + R_2 + R_3 + \ldots$

3. Let input be $x_q \ldots x_n$.

   For $1 \leq i \leq n$, check $x_1 \ldots x_i \in L(R)$

4. If successful, then we know that

   $x_1 \ldots x_i \in L(R_j)$ for some j

5. Remove $x_1 \ldots x_i$ from input and go to step 3.

Lexical Specification of a language

- How much input is used?
  - $x_1 \ldots x_i \in L(R)$
  - $x_1 \ldots x_j \in L(R)$, $i \neq j$
  - Which one do we want? (e.g., == or =)
  - Maximal munch: always choose the longer one

- Which token is used if more than one matches?
  - $x_1 \ldots x_i \in L(R)$ where $R = R_1 + R_2 + .. + R_n$
  - $x_1 \ldots x_i \in L(R_m)$
  - $x_1 \ldots x_i \in L(R_n)$, $m \neq n$
  - Eg: Keywords = 'if', Identifier = letter (letter + digit)*, if matches both
  - Keyword has higher priority
  - Rule of Thumb: Choose the one listed first

Lexical Specification of a language

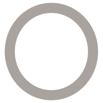
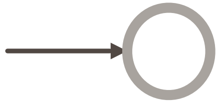
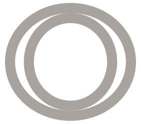
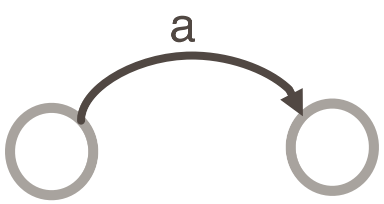
- What if no rule matches?
  - $x_1 \ldots x_i \notin L(R)$ … compiler typically tries to avoid this scenario
  - Error = [all strings not in the lexical spec]
  - Put it in last in priority

Summary so far

- Regular Expressions are concise notations for the string patterns
- Use in lexical analysis with some extensions
  - To resolve ambiguities
  - To handle errors
- Implementation?
  - We will study next

Finite Automata

- Regular Expression = specification
- Finite Automata = implementation

- A finite automaton consists of
  - An input Alphabet: Σ
  - A finite set of states: S 
  - A start state: n 
  - A set of accepting states: $F \subseteq S$ 
  - A set of transitions state: state1 $\xrightarrow{input}$ state2 

Transition

- s1 $\stackrel{a}{\to}$ s2 (state s1 on input a goes to state s2)
- If end of the input and in final state, the input is accepted
- Otherwise reject

- Language of FA = set of strings accepted by that FA

Example Automata

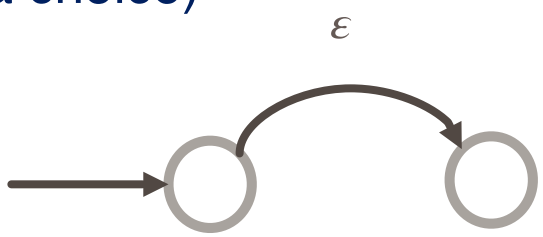
- a finite automaton that accepts only “1”

Example Automata

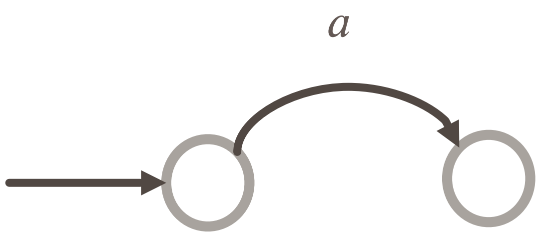
- A finite automaton that accepting any number of “1” followed by “0”

Regular Expression to NFA

- For ε (it's a choice)



- For input a



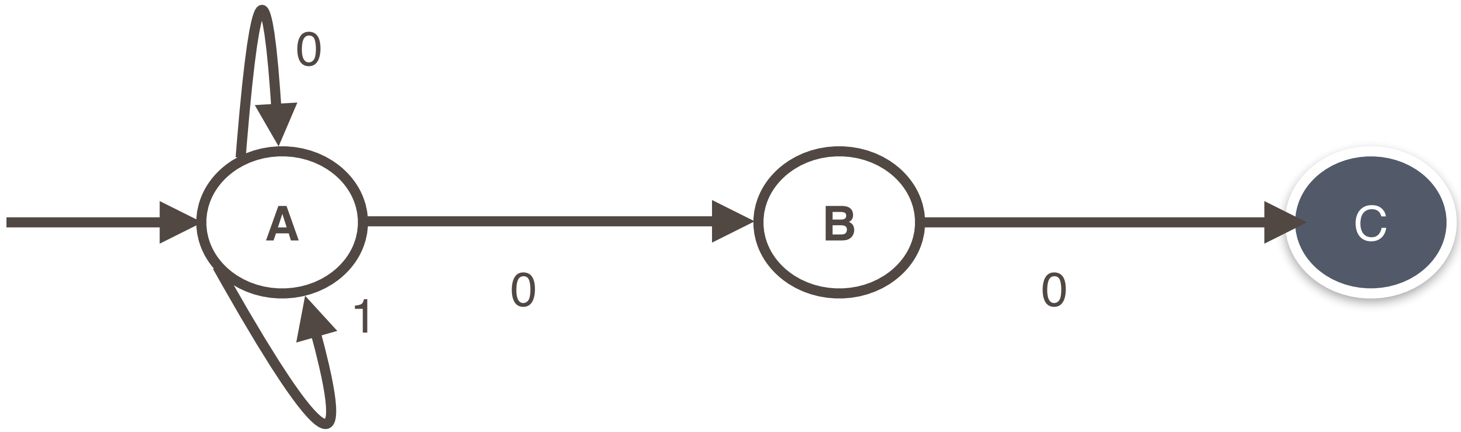
Finite Automata

- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
  - Takes only one path through the state graph

- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
  - Can choose which path to take
    - An NFA accepts if some of these paths lead to accepting state at the end of input.

Finite Automata

- An NFA can get into multiple states



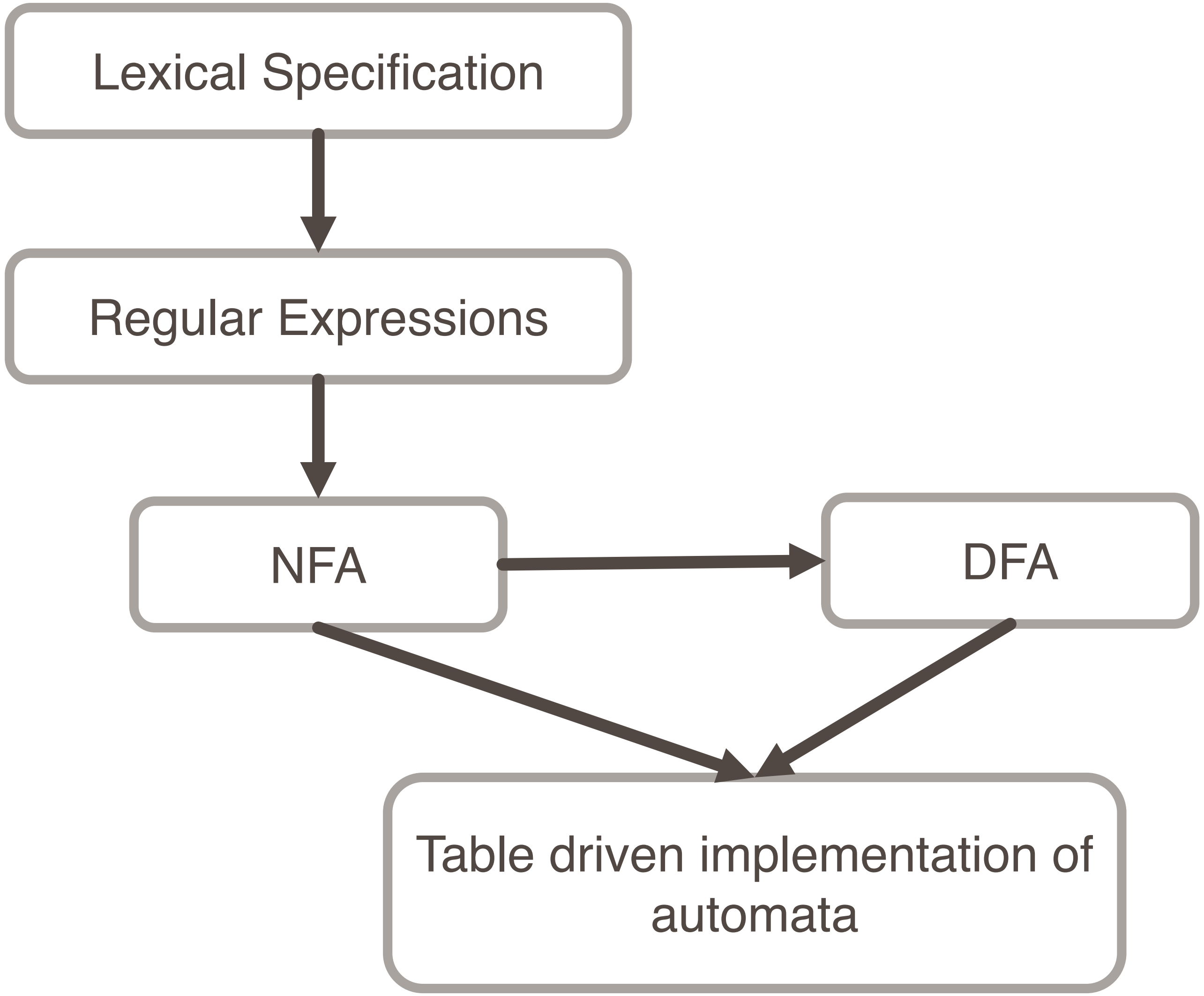
- Input: 1   0   0
- Output: {A}.   {A,B}   {A,B,C}

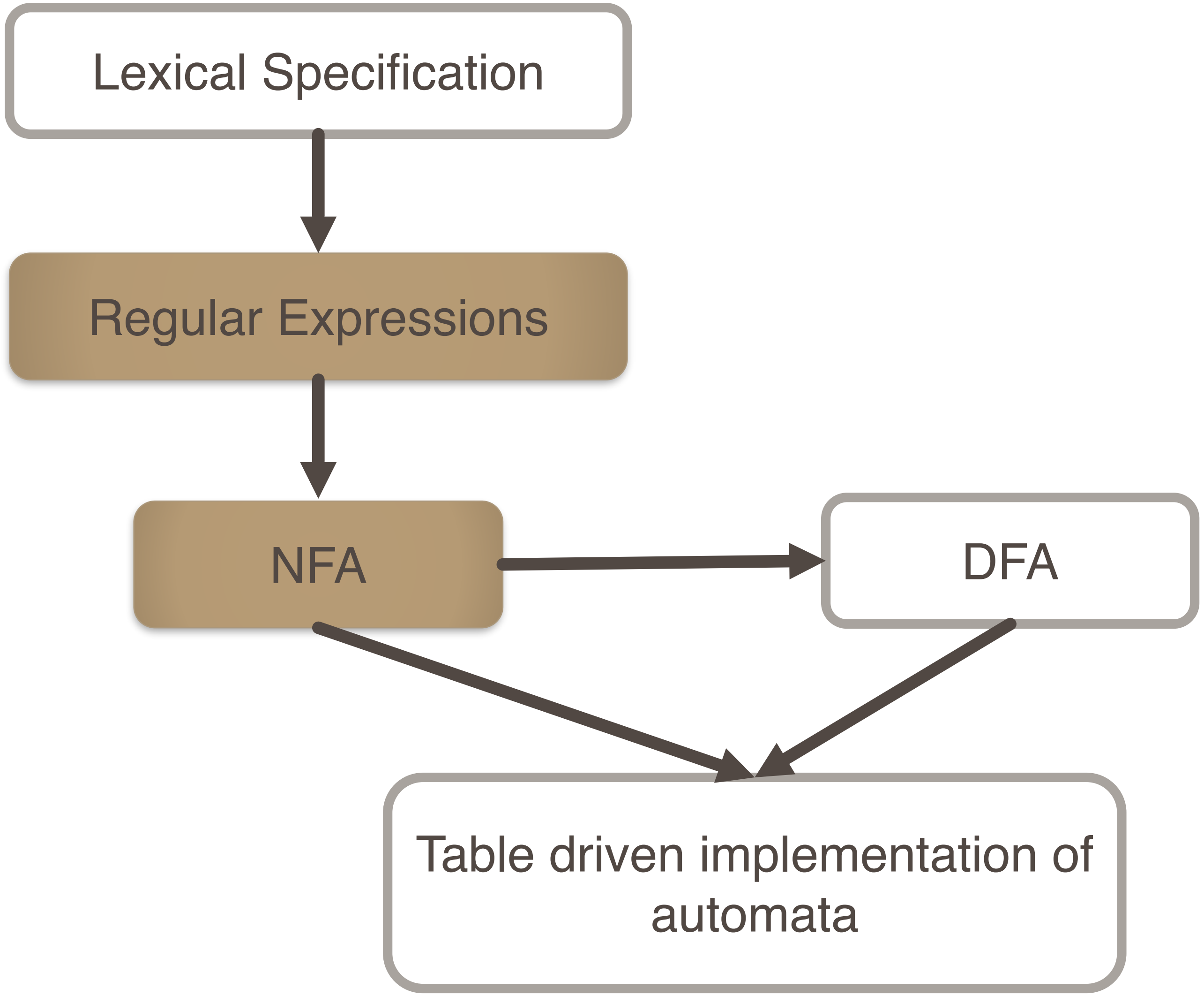
NFA vs. DFA

- NFAs and DFAs recognize the same set of regular languages

- DFAs are faster to execute
  - No choices to consider

- NFAs are, in general, small


Lexical Specification
Regular Expressions
NFA
DFA
Table driven implementation of automata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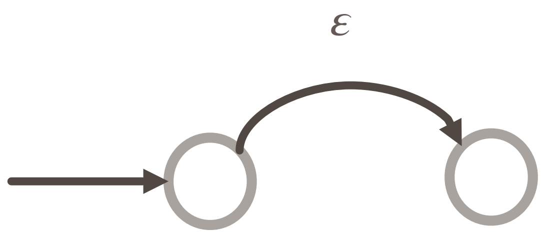
Finite Automata

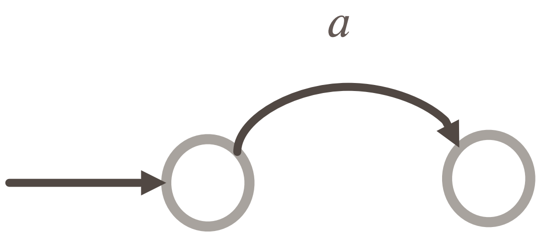
- For each kind of regex, define an equivalent NFA
  - Notation: NFA for regex M

Regular Expression to NFA

- For ε

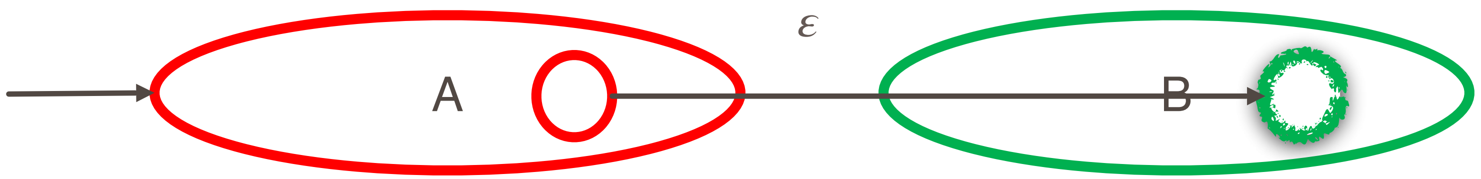


- For input a

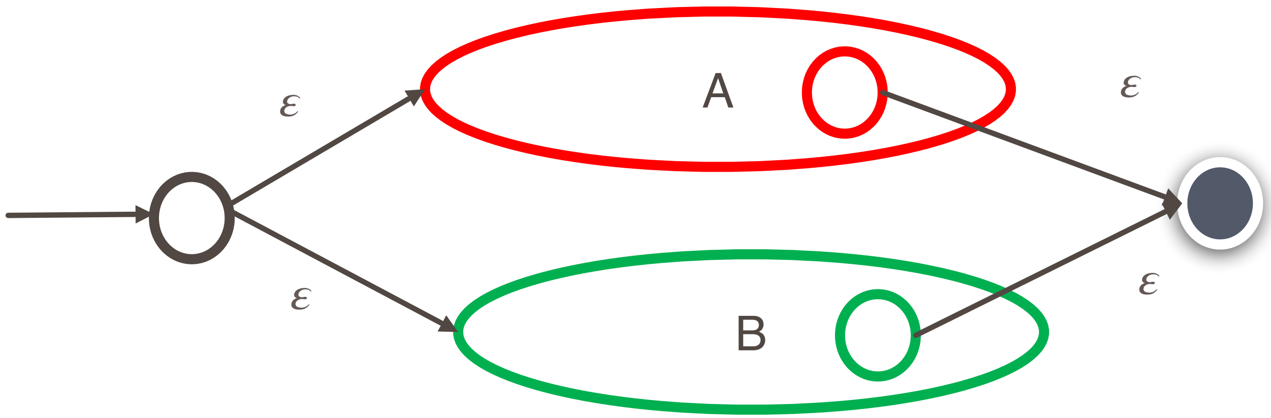


Regular Expression to NFA

- For AB

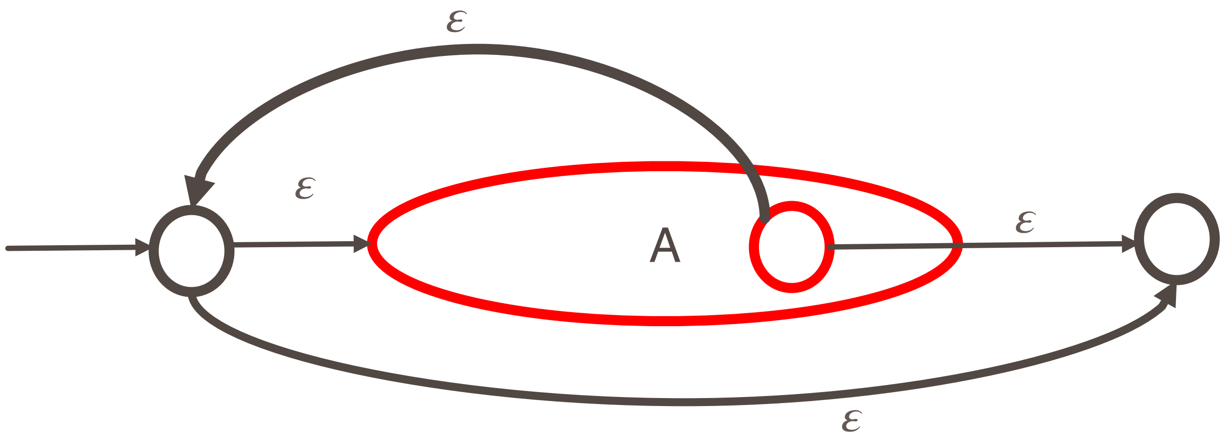


- For A + B



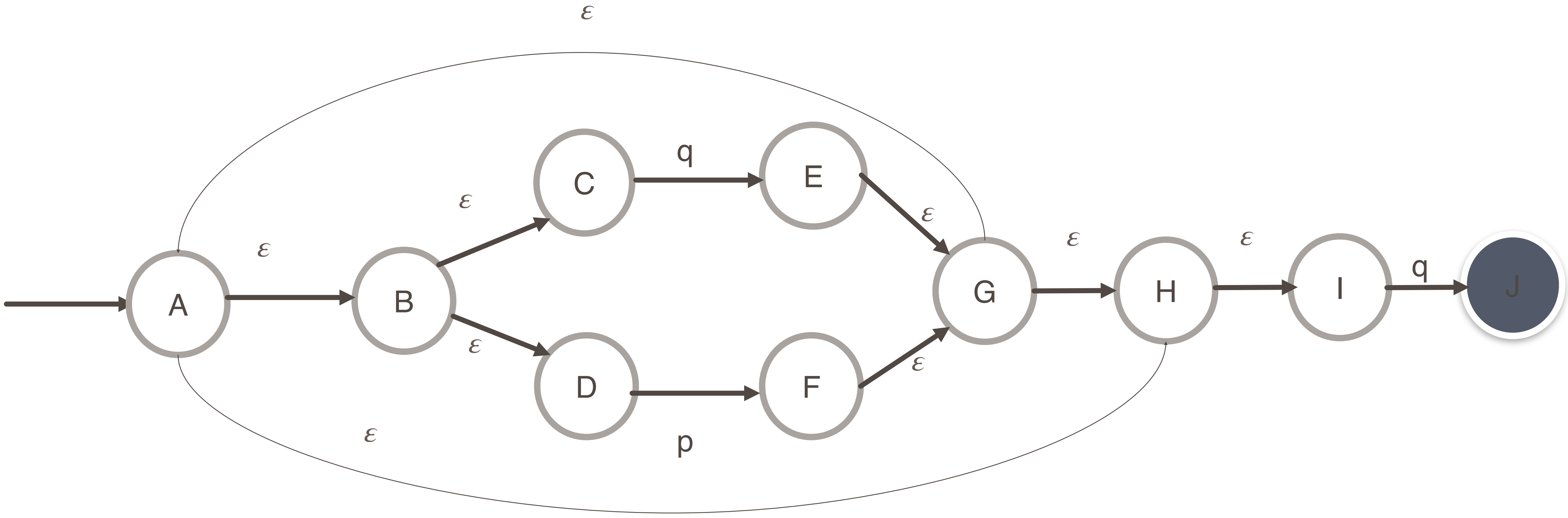
Regular Expression to NFA

- For A*



Example

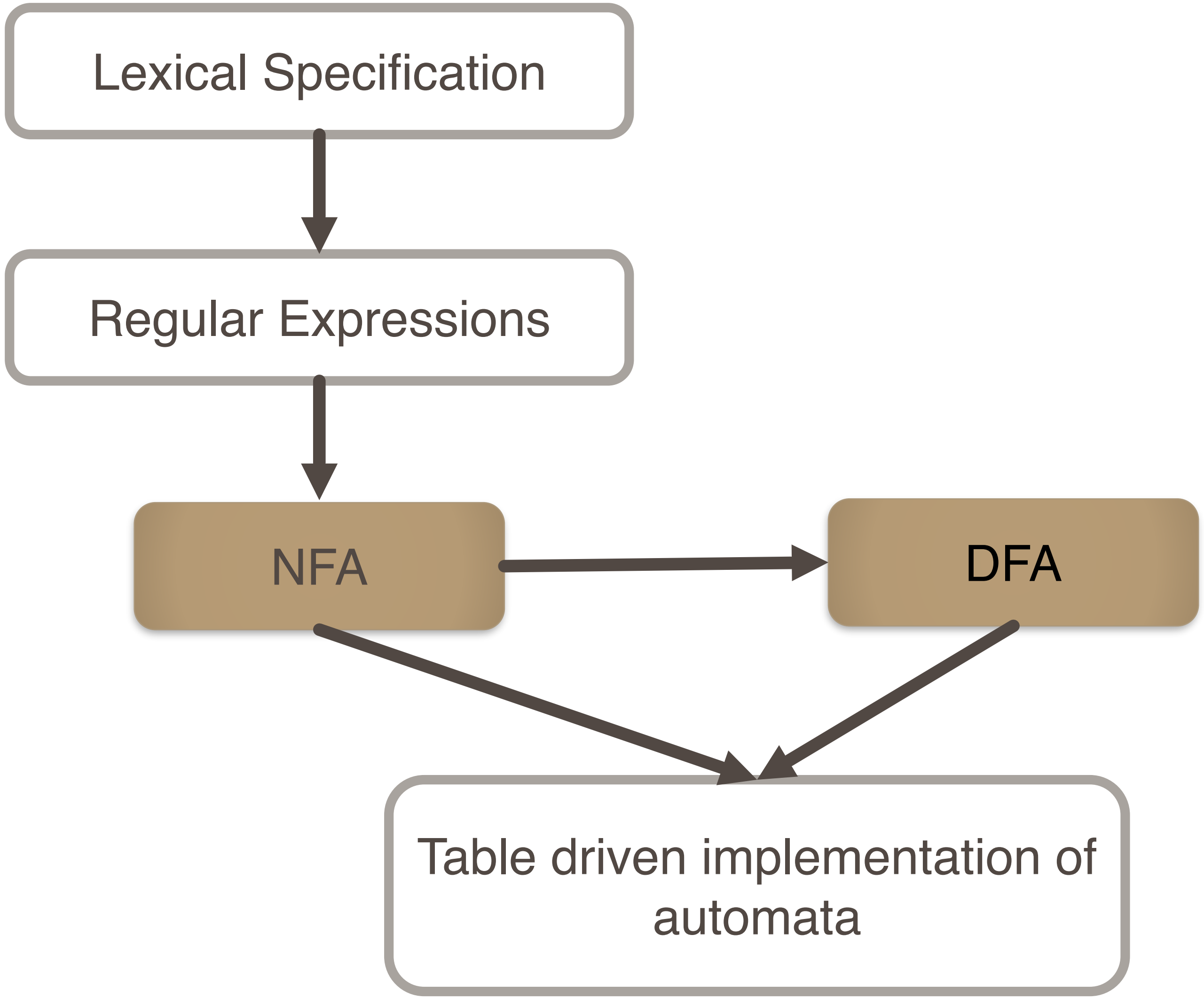
- (q+p)*q



Example

Choose the NFA that accepts the regular expression: 1* + 0.

NFA to DFA



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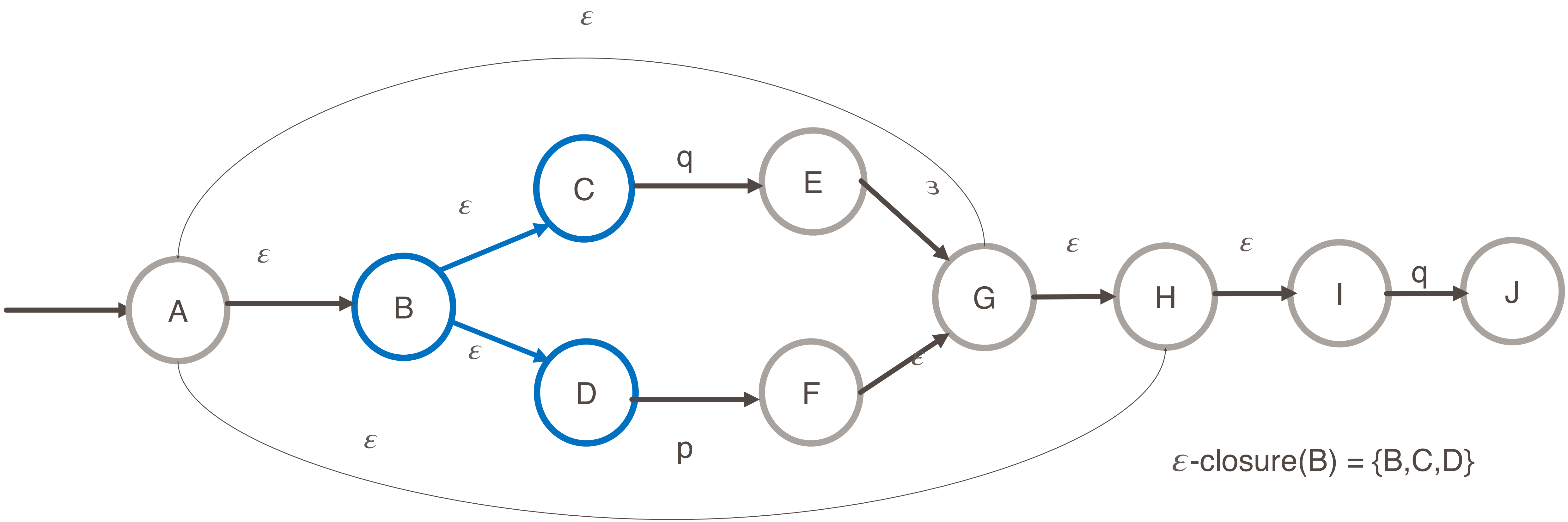
ε-closure

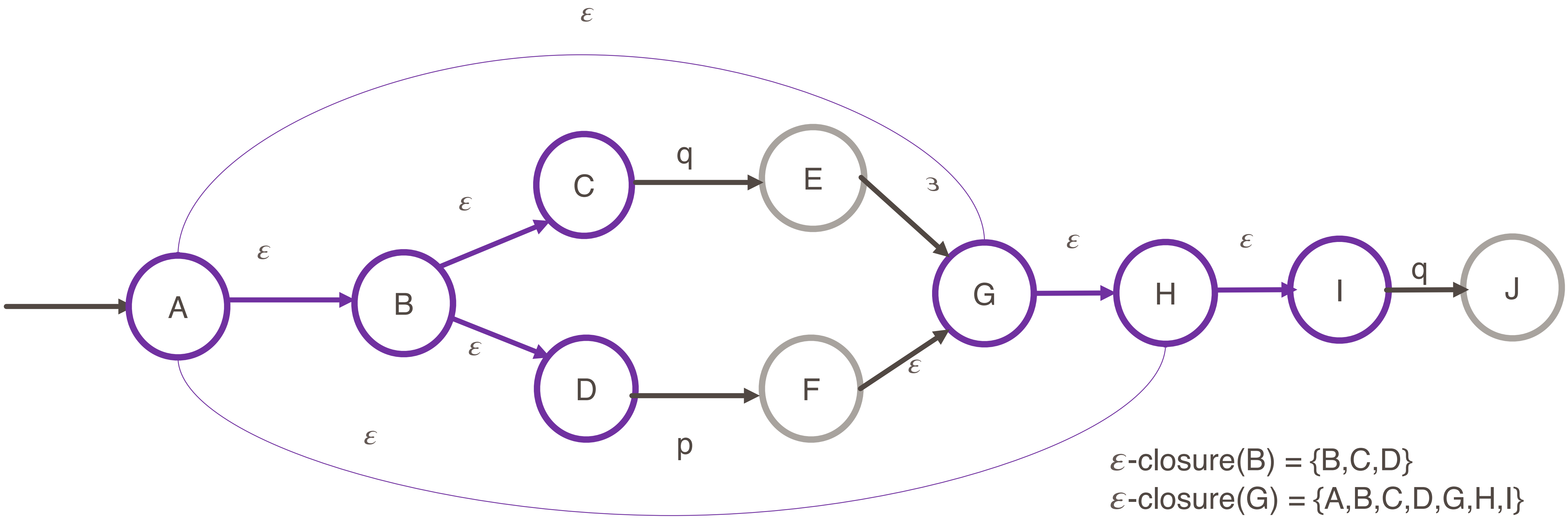
- ε-closure of a state is all the state I can reach following ε move .



ε-closure(B) = {B,C,D}

ε-closure

- ε-closure of a state is all the state I can reach following ε move .



ε-closure(B) = {B,C,D}
ε-closure(G) = {A,B,C,D,G,H,I}

NFA

- An NFA can be in many states at any time

- How many different states?
  - If NFA has N states, it reaches some subset of those states, say S
  - |S| $\leq \quad N$
  - There are $2^N – 1$ possible subsets (finite number)

NFA to DFA

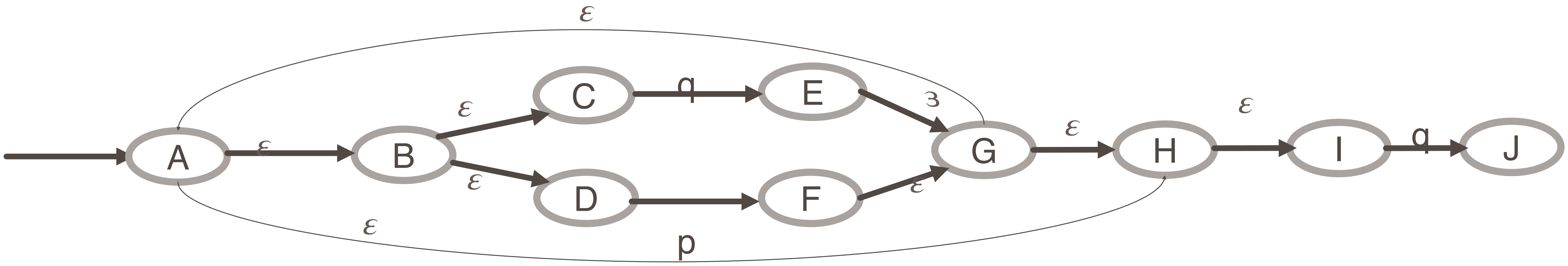
NFA

- States S
- Start s
- Final state F
- Transition state
  - a(X) = {y | x ∈ X $\bigwedge$ x $\xrightarrow{a}$ y)
- $\varepsilon - closure$

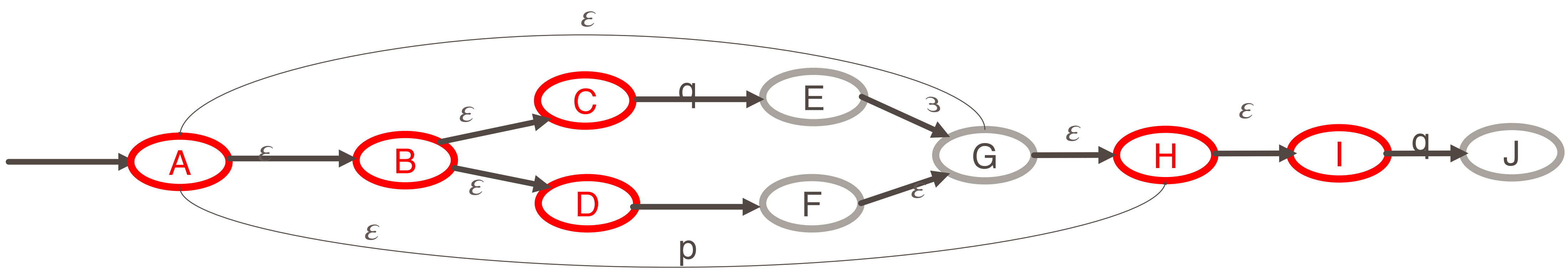
DFA

- States will be all possible subset of S except empty set
- Start state = $\varepsilon - closure$(s)
- Final state $\{\mathrm{X} \mid X \cap F = \varnothing\}$
- X $\xrightarrow{a}$ Y if
  - Y = $\varepsilon - closure(a(X))$

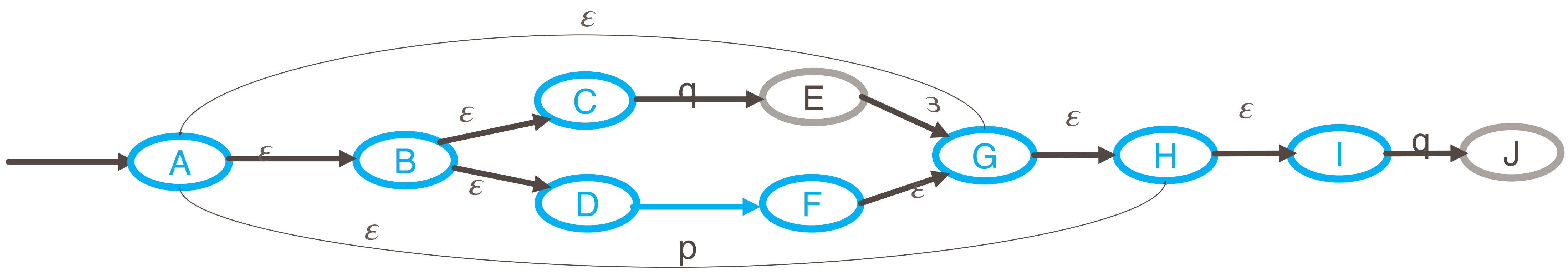
NFA to DFA

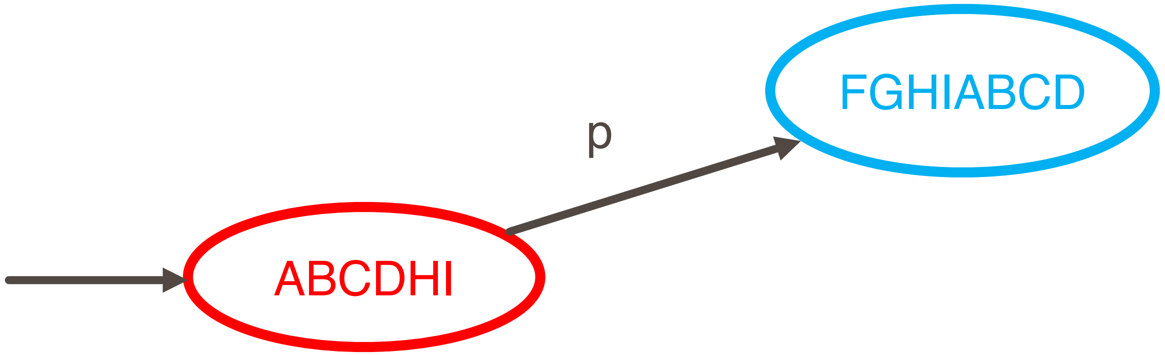


NFA to DFA

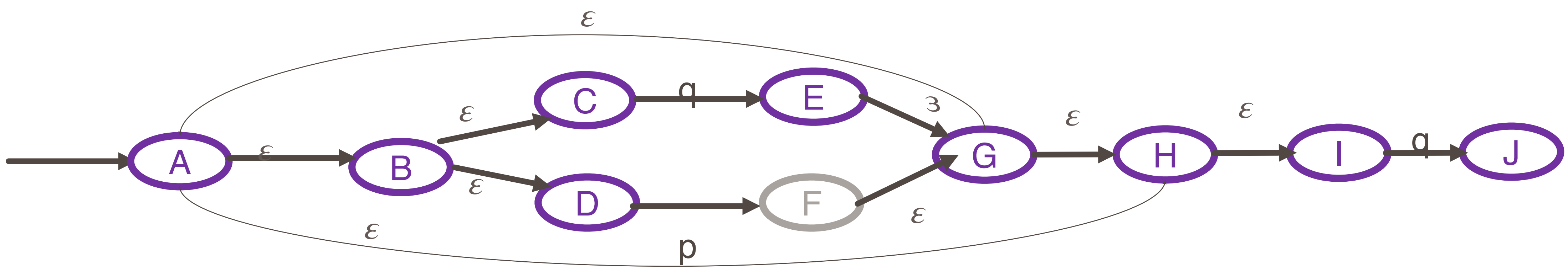


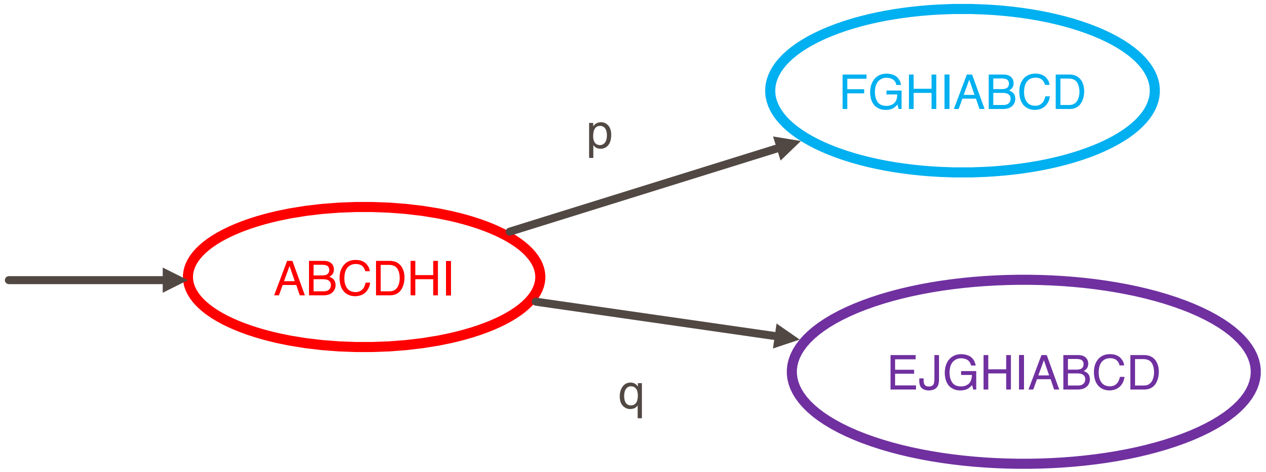
NFA to DFA



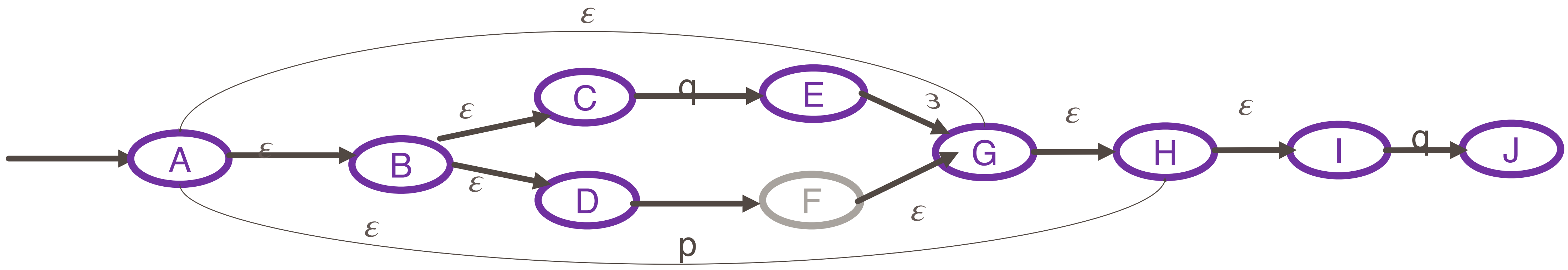


NFA to DFA

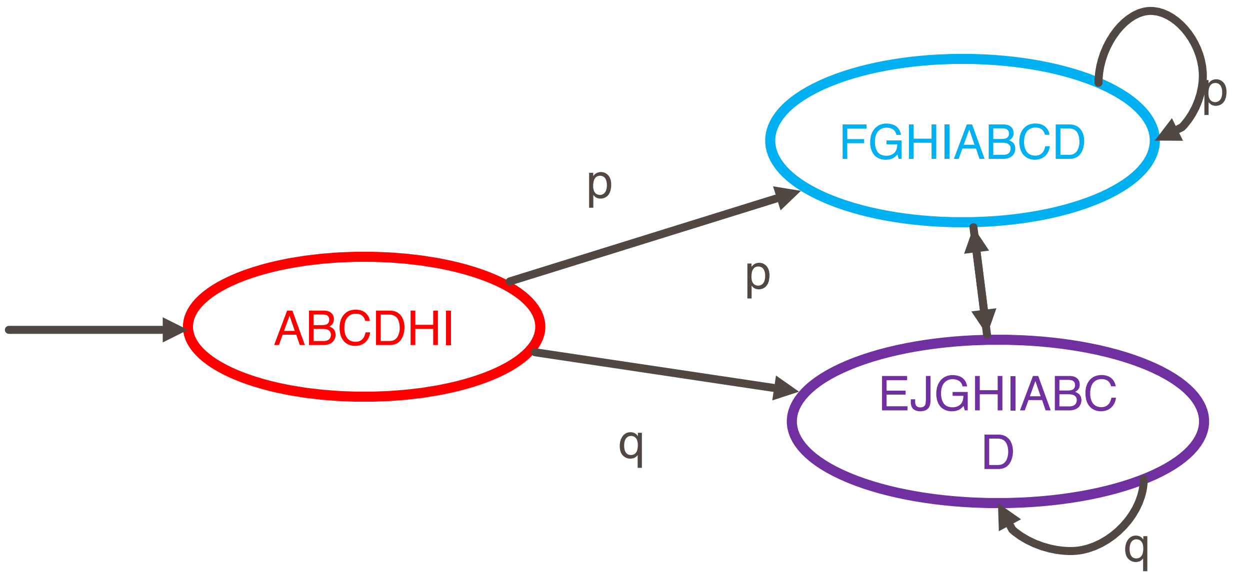




NFA to DFA

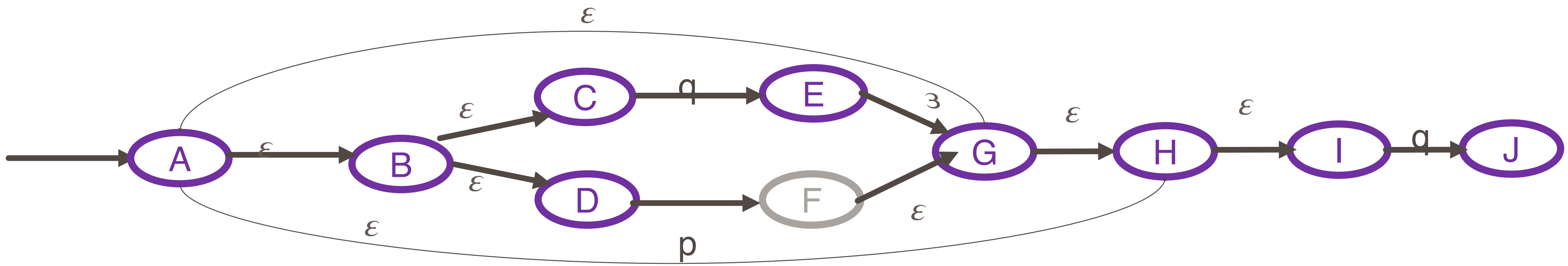


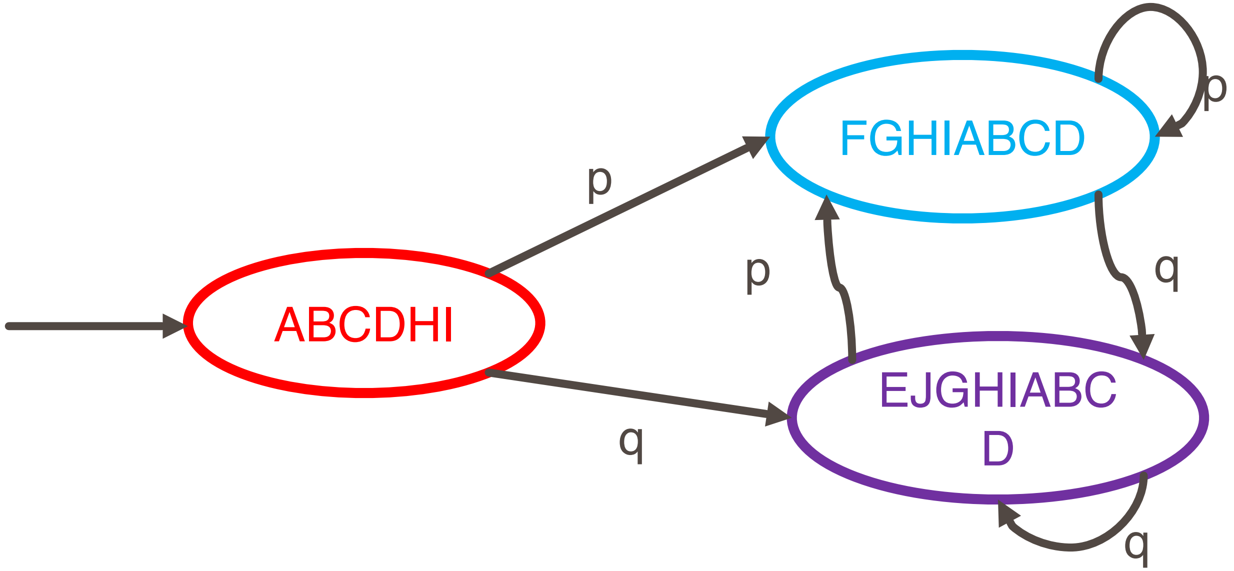
ε
A
ε
B
ε
C
q
E
ε
ε
D
F
ε
G
ε
H
ε
I
q
J
ε
p



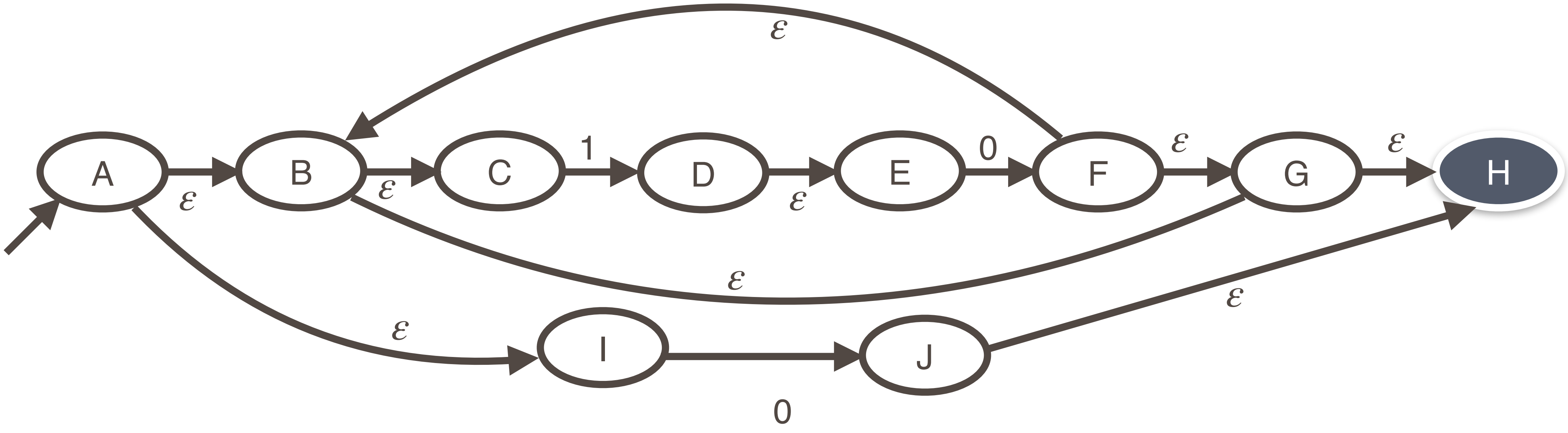
FGHIABCD
p
p
p
ABCDHI
EJGHIABC
D
q
q

NFA to DFA

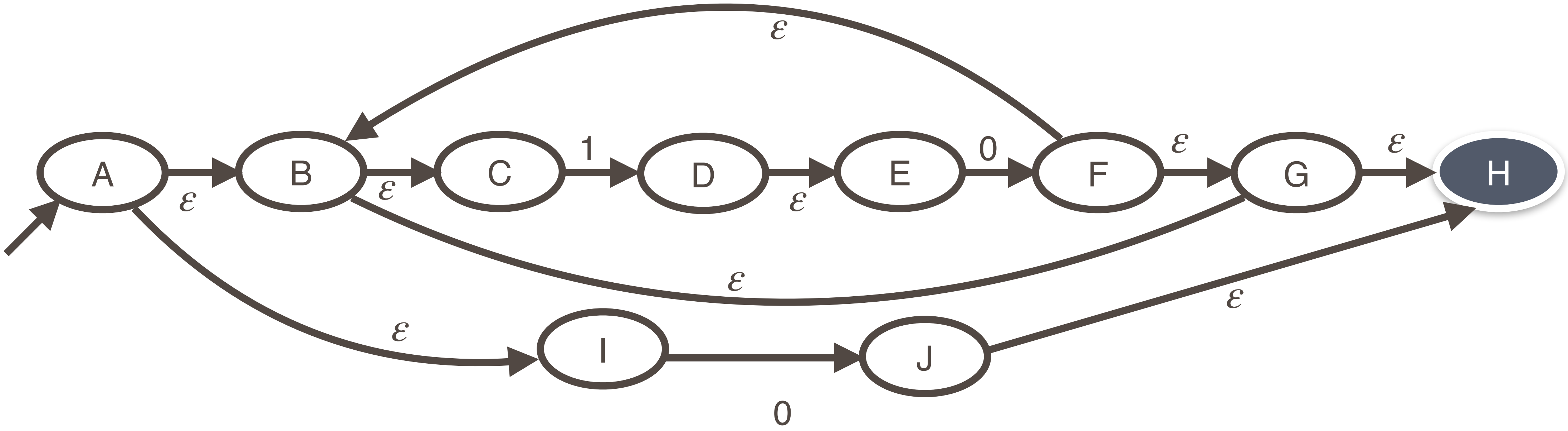


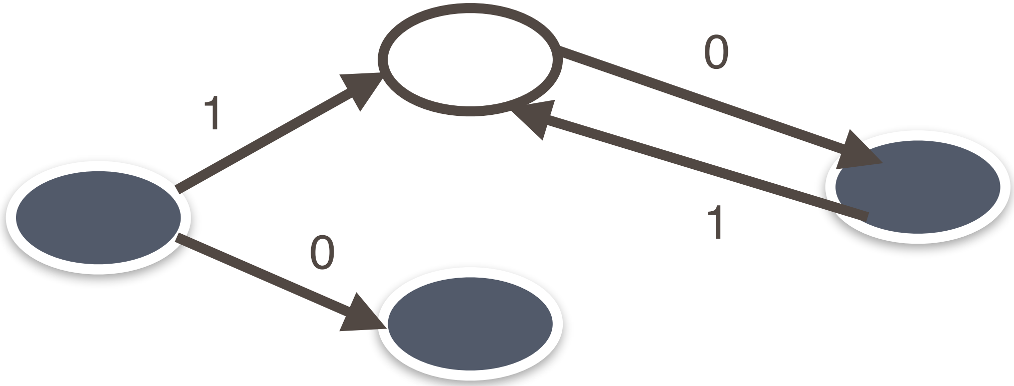


Example: NFA to DFA



Example: NFA to DFA





NFA to DFA

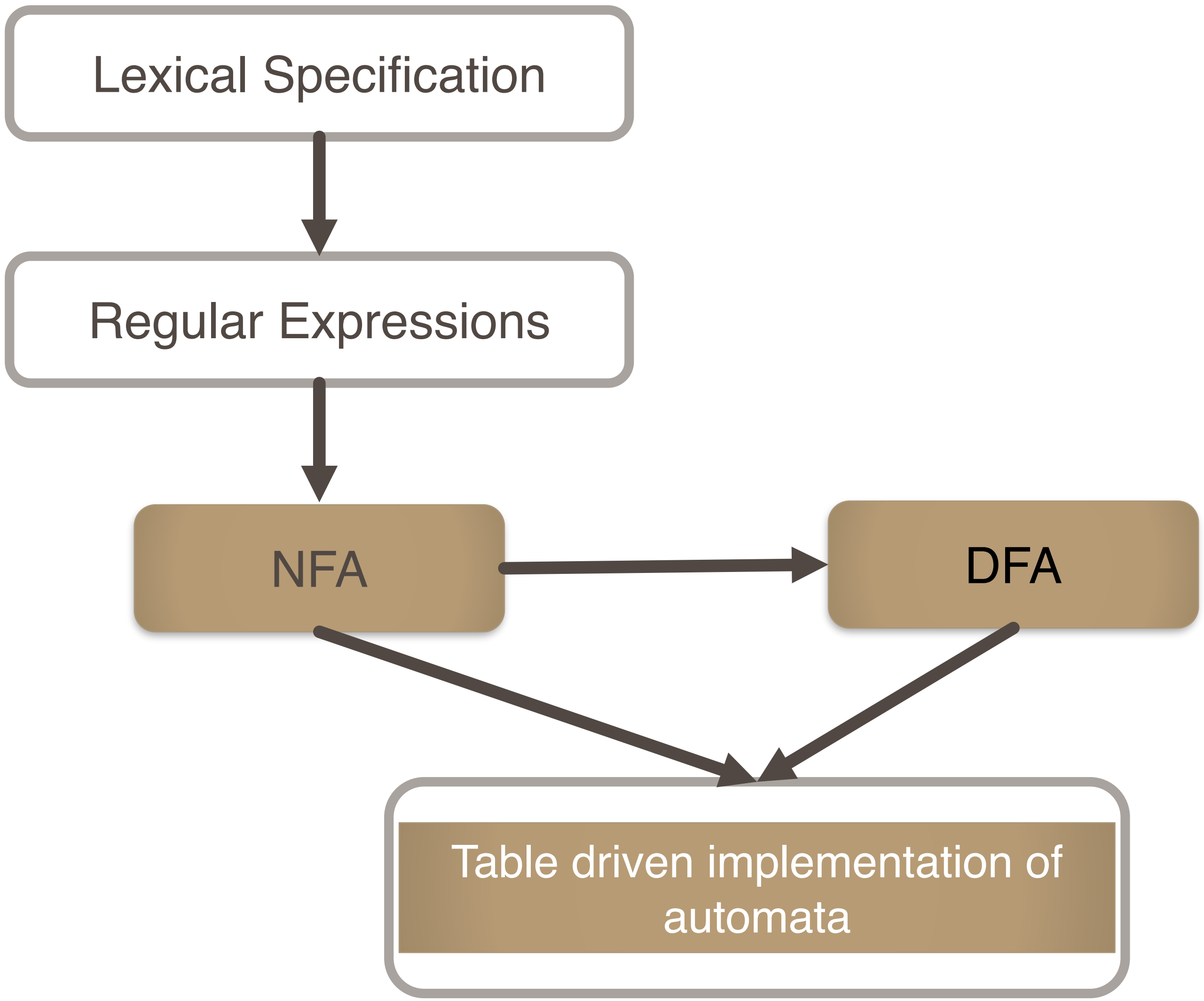


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Table driven implementation of automata

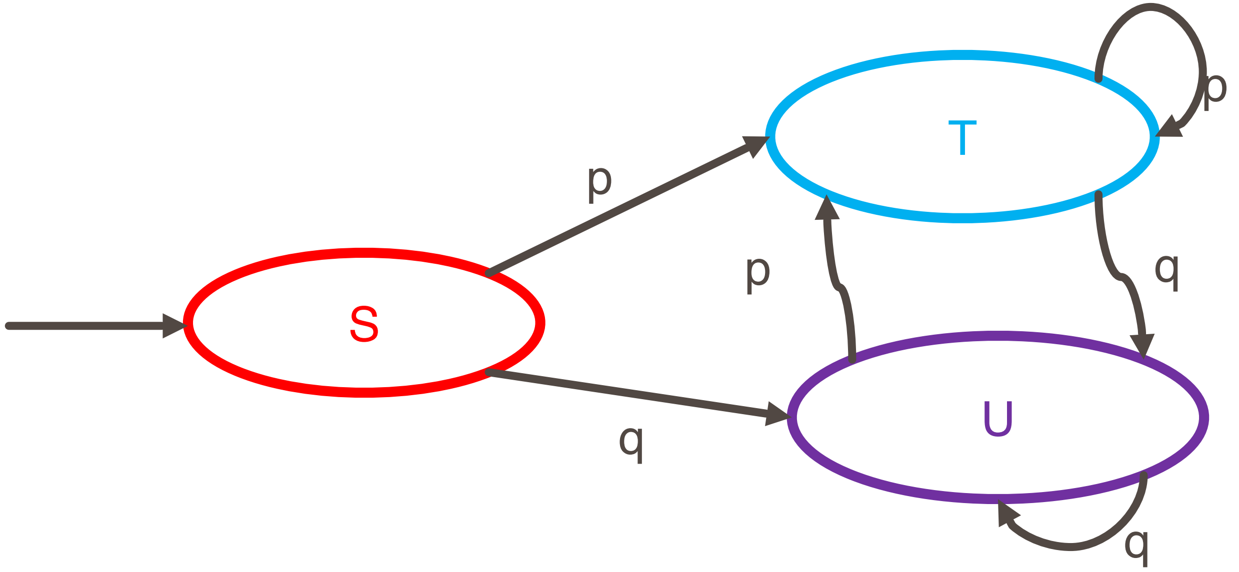
Implementing DFA

- A DFA can be implemented by a 2D table T
  - One dimension is states
  - Another dimension is input symbol
  - For every transition $s_i \to^a s_k$: define T[i,a] = k

Implementing DFA

Table A

	p	q
S	T	U
T	T	U
U	T	U




```
i = p;
state = 0;
while(input[i]) {
        state = A[state,input[i]];
        i++;
    }
```

Implementing DFA

Table A

	p	q
S	T	U
T	T	U
U	T	U

A lot of duplicate entries

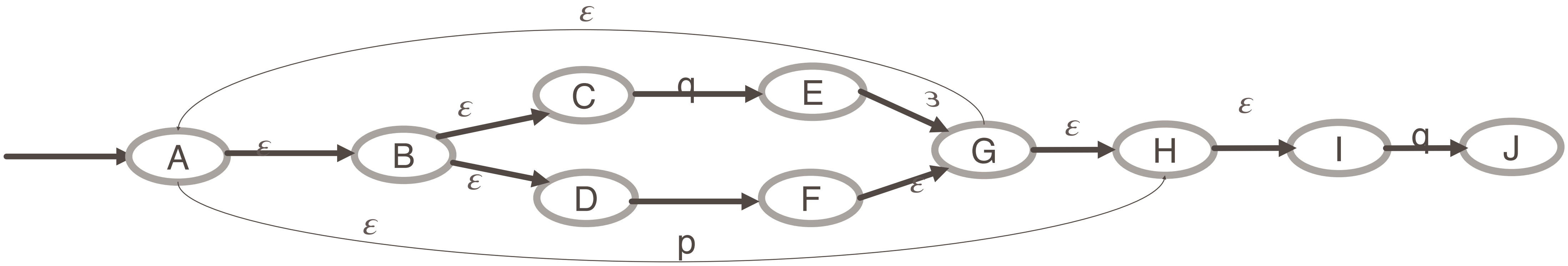
Table B

S
T
U

p	q
T	U

Compact but need an extra indirection
- Inner loop will be slower

Implementing DFA



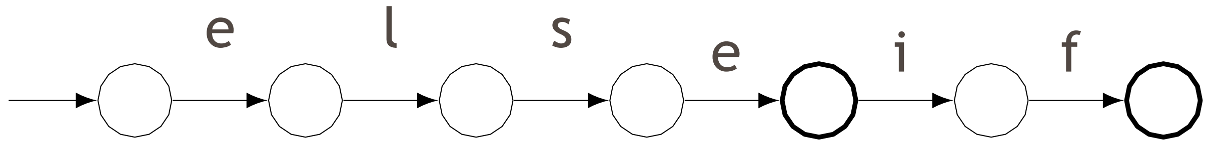
	p	q	
A			{B,H}
B			{C,D}
C		{E}	
…			

Deal with set of states rather than single state-→ inner loop is complicated

Deterministic Finite Automata: Example

```
{
   type token = ELSE | ELSEIF
}


rule token =
  parse "else"{ ELSE }
       | "elseif"{ ELSEIF }
```



Deterministic Finite Automata

```
{ type token = IF | ID of string | NUM of string }

rule token =
  parse "if"{ IF }
      | ['a'-'z'] ['a'-'z' '0'-'9'] as lit { ID(lit) }
      | ['0'-'9']+                   * as num { NUM(num) }
```