Additional Experiments

Methods		Dataset: Elec MCAR	Elec MAR	OT MCAR	OT MAR	Energy MCAR	Energy MAR
~~CP-MDA-Exact~~	Cov	0.885±.0	0.883±.0	~~0.665±.0~~	~~0.666±.0~~	0.952±.0	0.955±.0
	Width	0.602±.0	0.593±.0	21.30±.08	21.13±.08	300.56±5.42	302.61±5.42
	CovGap	0.016±.0	0.017±.0	0.228±.0	0.227±.0	0.050±.0	0.052±.0
	Winkler	0.703±.0	0.705±.0	37.22±.13	36.93±.12	464.78±8.14	468.62±8.02
~~CP-MDA-Reweighting~~	Cov	0.899±.0	0.902±.0	~~0.807±.0~~	~~0.807±.0~~	0.898±.0	0.898±.0
	Width	**0.584**±.0	0.603±.0	22.89±.13	22.68±.12	304.32±6.52	304.32±6.73
	CovGap	**0.002**±.0	**0.004**±.0	0.101±.0	0.101±.0	0.005±.0	0.005±.0
	Winkler	**0.692**±.0	0.723±.0	36.67±.14	36.42±.13	456.25±8.13	458.13±8.15
CP-MDA-Adaptive	Cov	0.895±.0	0.895±.0	0.894±.0	0.896±.0	0.896±.0	0.896±.0
	Width	0.605±.0	0.604±.0	28.44±.12	28.42±.12	296.43±5.52	296.62±5.63
	CovGap	0.007±.0	0.007±.0	0.011±.0	0.018±.0	0.008±.0	0.008±.0
	Winkler	0.732±.0	0.728±.0	34.12±.19	34.16±.18	441.09±8.13	441.85±8.16
MVSC	Cov	0.898±.0	0.893±.0	0.897±.0	0.909±.0	0.902±.0	0.902±.0
	Width	0.631±.0	0.628±.0	30.67±.18	34.34±.18	576.33±15.6	523.74±15.8
	CovGap	0.003±.0	0.010±.0	**0.002**±.0	0.231±.0	**0.005**±.0	0.005±.0
	Winkler	0.764±.0	0.772±.0	38.24±.58	40.13±.56	741.31±13.4	698.59±13.1
ECP-TSM	Cov	0.899±.0	0.900±.0	0.902±.0	0.890±.0	0.894±.0	0.896±.0
	Width	0.592±.0	**0.588**±.0	**25.71**±.04	**25.80**±.04	**292.53**±4.32	**293.13**±4.30
	CovGap	**0.002**±.0	**0.004**±.0	0.011±.0	**0.016**±.0	0.009±.0	**0.003**±.0
	Winkler	0.695±.0	**0.684**±.0	**27.48**±.23	**27.32**±.22	**406.91**±7.23	**408.33**±6.96

Table 1: Performance of the compared methods for the three benchmark datasets with miscoverage rate α =0.1, under two missing mechanisms. Results were obtained by 5 repetitions with different seeds. Bold numbers indicate the best result, while significant under-coverage is canceled out.

Method	1M Coverage	1M Width	6M Coverage	6M Width	1Y Coverage	1Y Width	5Y Coverage	5Y Width
CP-MDA-Adaptive	0.894	1.35	0.893	1.37	0.897	1.37	0.904	1.38
MVSC	0.902	4.12	0.898	4.42	0.903	4.56	0.907	4.72
ECP-TSM	0.895	**1.28**	0.902	**1.32**	0.899	**1.33**	0.898	**1.33**

Table 2: Comparison of coverage rates and prediction interval widths for large scale high frequency financial time series data under MCAR.

λ_{reg}	0.1		1		10		100	
c	Coverage	Width	Coverage	Width	Coverage	Width	Coverage	Width
100	0.898	25.68	0.902	25.71	0.897	25.76	0.902	25.83
1000	0.902	25.78	0.897	25.63	0.904	25.94	0.902	25.82

Table 3: Comparison of coverage rates and prediction interval widths for Oil Temperature Data under MCAR with different hyperparameters

			0	1	2	3	4	5
BRITS	CP-MDA-Adaptive	Cov	0.905±.0	0.892±.0	0.896±.0	0.904±.0	0.894±.0	0.898±.0
		Width	27.63±.04	27.64±.04	27.34±.04	27.96±.04	27.93±.04	26.82±.04
	MVSC	Cov	0.892±.0	0.893±.0	0.904±.0	0.902±.0	0.896±.0	0.895±.0
		Width	32.32±.23	31.78±.23	32.23±.24	31.95±.24	32.35±.24	31.84±.24
	ECP-TSF	Cov	0.896±.0	0.884±.0	0.896±.0	0.902±.0	0.902±.0	0.898±.0
		Width	**25.12**±.04	**25.76**±.04	**25.43**±.04	**26.05**±.04	**25.59**±.04	**25.23**±.04
GRU-D	CP-MDA-Adaptive	Cov	0.905±.0	0.912±.0	0.898±.0	0.904±.0	0.903±.0	0.903±.0
		Width	30.12±.04	29.84±.04	29.14±.04	29.85±.04	29.54±.04	29.23±.04
	MVSC	Cov	0.904±.0	0.896±.0	0.898±.0	0.896±.0	0.903±.0	0.892±.0
		Width	33.281±.23	33.86±.23	33.12±.24	33.95±.23	32.94±.24	33.32±.23
	ECP-TSF	Cov	0.895±.0	0.900±.0	0.894±.0	0.908±.0	0.898±.0	0.894±.0
		Width	**26.92**±.04	**26.04**±.04	**25.96**±.04	**26.13**±.04	**26.27**±.04	**24.63**±.04

Table 4: Group conditional coverage of the compared CP algorithms ($\alpha = 0.1$). Each column represents a group categorized by the number of missing values.

Method	5 steps		10 steps		25 steps		50 steps	
	Coverage	Width	Coverage	Width	Coverage	Width	Coverage	Width
CP-MDA-Adaptive	0.891	28.63	0.901	28.52	0.896	28.32	0.903	29.03
MVSC	0.894	33.15	0.898	33.46	0.902	34.67	0.897	33.75
ECP-TSM	0.891	**25.90**	0.892	**25.96**	0.892	**25.81**	0.889	**25.58**

Table 5: Comparison of coverage rates and prediction interval widths for multi-step forecasting tasks on the Oil Temperature dataset under the MCAR missingness mechanism. Bold values indicate the best (smallest) prediction interval width.

	MCAR			MAR			MNAR		
Methods	Cov	Width	CovGap	Cov	Width	CovGap	Cov	Width	CovGap
Ours	89.66	25.40	0.032	89.73	25.80	0.016	90.23	26.14	0.018
w.o. Group Features	89.23	25.38	0.083	89.62	26.12	0.036	89.45	26.02	0.057

Table 6: Ablation studies under different missing data mechanisms for Oil Temperature Data.

	MCAR			MAR			MNAR		
Methods	Cov	Width	CovGap	Cov	Width	CovGap	Cov	Width	CovGap
Ours	89.40	292.53	0.009	89.60	293.13	0.003	89.58	293.06	0.004
w.o. Group Features	89.56	292.86	0.005	89.42	293.05	0.006	89.47	293.15	0.006

Table 7: Ablation studies under different missing data mechanisms for Energy Data.

Methods		Dataset					
		Elec		OT		Energy	
		MCAR	MAR	MCAR	MAR	MCAR	MAR
CP-MDA-Adaptive	Cov	0.895±.0	0.895±.0	0.894±.0	0.896±.0	0.896±.0	0.896±.0
	Width	0.605±.0	0.604±.0	28.44±.12	28.42±.12	296.43±5.52	296.62±5.63
	CovGap	0.007±.0	0.007±.0	0.011±.0	0.018±.0	0.008±.0	0.008±.0
MVSC	Cov	0.898±.0	0.893±.0	0.897±.0	0.909±.0	0.902±.0	0.902±.0
	Width	0.631±.0	0.628±.0	30.67±.18	34.34±.18	576.33±15.6	523.74±15.8
	CovGap	0.003±.0	0.010±.0	**0.002**±.0	0.231±.0	**0.005**±.0	0.005±.0
CP-MDA-SPCI	Cov	0.896±.0	0.897±.0	0.897±.0	0.903±.0	0.892±.0	0.904±.0
	Width	0.597±.0	0.598±.0	27.62±.14	27.64±.14	298.15±4.03	298.73±4.11
	CovGap	0.003±.0	0.008±.0	0.006±.0	**0.006**±.0	0.012±.0	0.008±.0
ECP-TSM	Cov	0.899±.0	0.900±.0	0.902±.0	0.890±.0	0.894±.0	0.896±.0
	Width	0.592±.0	**0.588**±.0	**25.71**±.04	**25.80**±.04	**292.53**±4.32	**293.13**±4.30
	CovGap	**0.002**±.0	**0.004**±.0	0.011±.0	**0.016**±.0	0.009±.0	**0.003**±.0

Table 8: Performance of the compared methods for the three benchmark datasets with miscoverage rate α =0.1, under two missing mechanisms. Results were obtained by 5 repetitions with different seeds. Bold numbers indicate the best result, while significant under-coverage is canceled out.

	MCAR			MAR			MNAR		
Methods	Cov	Width	CovGap	Cov	Width	CovGap	Cov	Width	CovGap
Ours	89.66	25.40	0.032	89.73	25.80	0.016	90.23	26.14	0.018
w.o. coverage loss	86.82	25.20	0.056	87.05	25.12	0.053	86.68	24.45	0.063
w.o. inefficiency loss	92.72	26.62	0.013	92.35	26.82	0.025	92.13	26.46	0.024

Table 9: Ablation studies under different missing data mechanisms for Oil Temperature Data.

Methods		Dataset					
		Elec		OT		Energy	
		MCAR	MAR	MCAR	MAR	MCAR	MAR
CP-MDA-Adaptive	Cov	$0.903\pm_{.0}$	$0.894\pm_{.0}$	$0.903\pm_{.0}$	$0.892\pm_{.0}$	$0.899\pm_{.0}$	$0.904\pm_{.0}$
	Width	$0.603\pm_{.0}$	$0.604\pm_{.0}$	$27.85\pm_{.12}$	$28.23\pm_{.12}$	$298.33\pm_{4.83}$	$298.73\pm_{4.88}$
	CovGap	$0.007\pm_{.0}$	$0.007\pm_{.0}$	$0.012\pm_{.0}$	$0.016\pm_{.0}$	$0.006\pm_{.0}$	$0.007\pm_{.0}$
MVSC	Cov	$0.903\pm_{.0}$	$0.902\pm_{.0}$	$0.896\pm_{.0}$	$0.895\pm_{.0}$	$0.898\pm_{.0}$	$0.904\pm_{.0}$
	Width	$0.633\pm_{.0}$	$0.629\pm_{.0}$	$32.23\pm_{.18}$	$32.35\pm_{.18}$	$565.37\pm_{14.7}$	$545.67\pm_{14.7}$
	CovGap	$0.003\pm_{.0}$	$0.08\pm_{.0}$	$\mathbf{0.005}\pm_{.0}$	$0.212\pm_{.0}$	$\mathbf{0.007}\pm_{.0}$	$0.009\pm_{.0}$
ECP-TSM	Cov	$0.902\pm_{.0}$	$0.898\pm_{.0}$	$0.902\pm_{.0}$	$0.897\pm_{.0}$	$0.903\pm_{.0}$	$0.898\pm_{.0}$
	Width	$\mathbf{0.598}\pm_{.0}$	$\mathbf{0.592}\pm_{.0}$	$\mathbf{26.42}\pm_{.04}$	$\mathbf{26.35}\pm_{.04}$	$\mathbf{293.58}\pm_{4.25}$	$\mathbf{293.46}\pm_{4.22}$
	CovGap	$\mathbf{0.002}\pm_{.0}$	$\mathbf{0.006}\pm_{.0}$	$0.008\pm_{.0}$	$\mathbf{0.012}\pm_{.0}$	$0.009\pm_{.0}$	$\mathbf{0.006}\pm_{.0}$

Table 10: Performance of the compared methods for the three benchmark datasets with miscoverage rate α =0.1, under two missing mechanisms with 20% missingness. Results were obtained by 5 repetitions with different seeds. Bold numbers indicate the best result, while significant under-coverage is canceled out.

Methods		Dataset					
		Elec		OT		Energy	
		MCAR	MAR	MCAR	MAR	MCAR	MAR
CP-MDA-Adaptive	Cov	$0.892\pm_{.0}$	$0.901\pm_{.0}$	$0.897\pm_{.0}$	$0.909\pm_{.0}$	$0.904\pm_{.0}$	$0.906\pm_{.0}$
	Width	$0.612\pm_{.01}$	$0.618\pm_{.01}$	$28.50\pm_{.32}$	$28.59\pm_{.31}$	$303.03\pm_{4.03}$	$307.49\pm_{4.07}$
	CovGap	$0.015\pm_{.0}$	$0.012\pm_{.0}$	$0.014\pm_{.0}$	$\mathbf{0.005}\pm_{.0}$	$0.012\pm_{.0}$	$0.012\pm_{.0}$
MVSC	Cov	$0.905\pm_{.0}$	$0.902\pm_{.0}$	$0.900\pm_{.0}$	$0.897\pm_{.0}$	$0.908\pm_{.0}$	$0.910\pm_{.0}$
	Width	$0.623\pm_{.00}$	$0.627\pm_{.0}$	$32.78\pm_{.31}$	$32.26\pm_{.29}$	$509.69\pm_{4.10}$	$506.80\pm_{4.07}$
	CovGap	$0.010\pm_{.0}$	$0.008\pm_{.0}$	$0.011\pm_{.0}$	$0.009\pm_{.0}$	$\mathbf{0.007}\pm_{.0}$	$0.008\pm_{.0}$
ECP-TSM	Cov	$0.899\pm_{.0}$	$0.903\pm_{.0}$	$0.905\pm_{.0}$	$0.899\pm_{.0}$	$0.900\pm_{.0}$	$0.901\pm_{.0}$
	Width	$\mathbf{0.605}\pm_{.01}$	$\mathbf{0.603}\pm_{.01}$	$\mathbf{26.80}\pm_{.30}$	$\mathbf{26.92}\pm_{.30}$	$\mathbf{297.68}\pm_{3.98}$	$\mathbf{299.23}\pm_{3.99}$
	CovGap	$\mathbf{0.008}\pm_{.0}$	$\mathbf{0.007}\pm_{.0}$	$\mathbf{0.006}\pm_{.0}$	$\mathbf{0.005}\pm_{.0}$	$0.009\pm_{.0}$	$\mathbf{0.007}\pm_{.0}$

Table 11: Performance of the compared methods for the three benchmark datasets with miscoverage rate $\alpha = 0.1$, under two missing mechanisms. Results were obtained by 5 repetitions with different seeds. Bold numbers indicate the best result, while significant under-coverage is canceled out.

Method	0% Missing		20% Missing		40% Missing		60% Missing	
	Coverage	Width	Coverage	Width	Coverage	Width	Coverage	Width
CP-MDA-Adaptive	0.897	28.50	0.903	27.85	0.894	28.44	0.904	28.41
MVSC	0.887	32.26	0.896	32.23	0.897	30.67	0.907	31.80
ECP-TSM	0.905	26.80	0.902	26.42	0.902	25.71	0.898	26.40

Table 12: Comparison of coverage rates and prediction interval widths for different missing rates across three methods for Oil Temperature Data under MCAR.