Excess - 3

ASCII

EBCDIC

$$\prod_{k} \sum_{i} x^{ik}$$

Evaluate

$$\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix}$$

$$= \frac{1}{a^2b^2c^2} \begin{vmatrix} (ab+ac)^2 & c^2b^2 & b^2c^2 \\ c^2a^2 & (bc+ba)^2 & a^2c^2 \\ b^2a^2 & a^2b^2 & (ac+bc)^2 \end{vmatrix}$$

$$= \frac{1}{a^2b^2c^2} \begin{vmatrix} (ab+ac)^2 - b^2c^2 & 0 & b^2c^2 \\ 0 & (bc+ba)^2 - a^2c^2 & a^2c^2 \\ b^2a^2 - (ac+bc)^2 & a^2b^2 - (ac+bc)^2 & (ac+bc)^2 \end{vmatrix}$$

$\cancel{R_3' \to R_3 - R_1}$

$C_1' \to C_1 - C_3$

$C_2' \to C_2 - C_3$

$$= \frac{1}{a^2b^2} \begin{vmatrix} (ab+bc+ca)(ab+ac-bc) & 0 & b^2 \\ 0 & (bc+ba+ac)(bc+ba-ac) & a^2 \\ (ba+ac+bc)(ba-ac-bc) & (ab+a+bc)(ab-ac-bc) & (a+b)^2 \end{vmatrix}$$

$$= \frac{1}{a^2b^2}(ab+bc+ca)^2 \begin{vmatrix} ab+ac-bc & 0 & b^2 \\ 0 & bc+ba-ac & a^2 \\ ba-ac-bc & ab-ac-bc & (a+b)^2 \end{vmatrix}$$

$$= \frac{2(ab+bc+ca)^2}{a^2b^2} \begin{vmatrix} ab+ac-bc & 0 & b^2 \\ 0 & bc+ab-ac & a^2 \\ -2ab & -2bc & 2ab \end{vmatrix}$$

$$= \frac{2(ab+bc+ca)^2}{a^2b^2} \begin{vmatrix} ab+ac-bc & 0 & b^2 \\ 0 & bc+ab-ac & a^2 \\ -ab & -bc & ab \end{vmatrix}$$

$$= \frac{2(ab+bc+ca)^2}{a^2b^2} \begin{vmatrix} a/b\,(ab+ac-bc) & 0 & ab \\ 0 & b/a\,(bc+ab-ac) & ab \\ -ab & -bc & ab \end{vmatrix}$$

$$= \frac{2(ab+bc+ca)^2}{a^2b^2} \begin{vmatrix} \frac{a}{b}(ab+ac) & bc & 0 \\ ab & \frac{b}{a}(bc+ab) & 0 \\ -ab & -bc & ab \end{vmatrix} \quad R_1' \to R_1 + \quad R_2' \to R_2 +$$

$$= \frac{2(ab+bc+ca)^2}{a^2b^2}(ab)\left\{(ab+ac)(bc+ab) - ab^2c\right\}$$

$$= \frac{2(ab+bc+ca)^2}{ab}\left(a\cancel{b^2}c + a^2b^2 + abc^2 + a^2bc - \cancel{ab^2c}\right)$$

$$= \frac{2(ab+bc+ca)^2}{\cancel{ab}} \cdot (ab+bc+ca)$$

$$\begin{vmatrix} a^2 & \dots & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}, \quad s = \frac{a+b+c}{2}$$

If we put $s=0$, $\Delta = 0$ $\begin{vmatrix} a^2 & a^2 & a^2 \\ b^2 & b^2 & b^2 \\ c^2 & c^2 & c^2 \end{vmatrix}$ s^2 is factor.

If we put $s = a$.

$a+b+c = 2a$

$b+c = a$

$a-b = c$

$a-c = b$

$$\Delta = \begin{vmatrix} a^2 & 0 & 0 \\ c^2 & b^2 & c^2 \\ b^2 & b^2 & c^2 \end{vmatrix} = 0$$

If we put $s = b$.

$a+b+c = 2b$

$\Rightarrow a+c = b$

$\Rightarrow$

$$\Delta = \begin{vmatrix} a^2 & (b-a)^2 & (b-a)^2 \\ 0 & b^2 & 0 \\ (b-c)^2 & (b-c)^2 & c^2 \end{vmatrix} = 0$$

$\therefore$ s^2 is a factor

$s-a$ " "

$s-b$ " "

$s-c$ " "

and as it is symmetric with a, b, c $\therefore$ $a+b+c$ is also a factor. $\therefore$ s is also a factor.

Finding the constant

$a = 2, b = 1, c = 0 \quad \therefore s = \cancel{3}\ 3/2$

$k s^3 (s-a)(s-b)(s-c)$ is a factor.

$$\Delta = \begin{vmatrix} 4 & 1/4 & 1/4 \\ 1/4 & 1 & 1/4 \\ 9/4 & 9/4 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 1/4 & 1/4 \\ -\frac{15}{4} & 3/4 & 0 \\ 9/4 & 9/4 & 0 \end{vmatrix} \qquad R_2' \to R_2 - R_1$$

$$= 1/4 \left[\frac{9}{4} \left(-\frac{15}{4} - 3/4 \right) \right]$$

$$= \frac{9}{16} \left(-\frac{18}{4} \right)$$

$$= -\frac{162}{64}$$

$$k s^3 (s-a)(s-b)(s-c)$$

$$= k \left(\frac{27}{8} \times \left(-\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{3}{2}\right) \right)$$

$$= k \left(\frac{27 \times 3}{64} \right)$$

$$k \left(\frac{27 \times 3}{64} \right) = \frac{-162}{64}$$

$$\boxed{k = 2}$$

Symmetric Matrix

$\Delta = |a_{ij}|_{n\times n}$

$a_{ij} = a_{ji}$

$$\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}$$

Skew symmetric matrix

$a_{ij} = -a_{ji}$

when, $i = j$ $\quad a_{ii} = -a_{ii} = 0.$

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

Determinant multiplication

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1 l_1 + b_1 m_1 + c_1 n_1 \\ \\ \\ \end{vmatrix}$$