Vector space :-

① Determine the subspace of $\mathbb{R}^3$ spanned by vectors $\alpha = (1,2,3)$ & $\beta = (3,1,0)$. Examine if $\gamma = (2,1,3)$, $\delta = (-1,3,6)$ are the subspace.

ans $L(\alpha, \beta)$ is the set of ~~all~~ vectors $\{c\alpha + d\beta\}$ where c, d are real no.

$c(1,2,3) + d(3,1,0)$

$(c+3d, 2c+d, 3c) = (2,1,3)$.

$c+3d=2$	$2c+d=1$	$3c=3$
$3d = 2-1 = 1$		$c=1$
$d = 1/3$	$\times$	

$(c+3d, 2c+d, 3c) = (-1,3,6)$

$c+3d=-1$	$2c+d=3$	$3c=6$
$3d = -1-2$	$2\times2+(-1)=3$	$c=2$
$d=-1$	$\checkmark$	

δ is

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$c = 2,\ d = -1$

$\delta = 2(1, 2, 3) + (-1)(3, 1, 0)$

$\therefore \delta \in L(\alpha, \beta)$

2) Let, $S = \{\alpha, \beta\}$ and $T = \{\alpha, \beta, \alpha + \beta\}$ where $\alpha, \beta \in \mathbb{R}^n$. show that $L(S) = L(T)$.

Ans Since, $S \subseteq T$, $L(S) \subseteq L(T)$ ... (1)

Again each element of T can be expressed as a linear combination of α, β.

So, by previous result, $L(T) \subseteq L(S)$

So, by (1), we can say $L(S) = L(T)$

3) Let, $T : \mathbb{R}^3 \to \mathbb{R}^2$ be derived by

$T(a_1, a_2, a_3) = (a_1, a_2)$

Show that T is a linear transform

ans Let, $\alpha = (a_1, a_2, a_3)$

$\beta = (b_1, b_2, b_3) \in \mathbb{R}^3$

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$$T(\alpha+\beta) = T(a_1+b_1, a_2+b_2, a_3+b_3)$$

$$= (a_1+b_1, a_2+b_2)$$

$$= (a_1, a_2) + (b_1, b_2)$$

$$= T(\alpha) + T(\beta)$$

$\forall$ real c, $T(c\alpha) = T(ca_1, ca_2, ca_3)$

$$= (ca_1, ca_2)$$

$$= c(a_1, a_2)$$

$$= c \cdot T(\alpha)$$

$\therefore$ T is a linear transformation.

(4) $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$T(a_1, a_2, a_3) = (a_1+1, a_2+1)$

S.T. T is not a linear mapping.

Let, $\alpha = (a_1, a_2, a_3)$

$\beta = (b_1, b_2, b_3) \subseteq \mathbb{R}^3$.

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$T(\alpha+\beta) = (a_1+b_1, a_2+b_2, a_3+b_3)$

$= \cancel{(a_1+1, a_2+1)}$

$= (a_1+b_1+1, a_2+b_2+1)$

$T(\alpha)+T(\beta) = (a_1+1, (a_2+1) + (b_1+1, b_2+1)$

$= \cancel{T(\alpha)+T(\beta)}$

$= (a_1+b_1+2, a_2+b_2+2).$

$T(\alpha+\beta) \neq T(\alpha) + T(\beta).$

T is not a linear mapping

5) $(1,0,0), (0,1,0), (0,0,1)$ to $(1,1,1); (0,1,-1), (1,2,0).$

$(x_1, x_2, x_3) = x_1(1,0,0) + x_2(0,1,0) + x_3(0,0,1)$

$T(x_1, x_2, x_3) = x_1 T(1,0,0) + x_2 T(0,1,0) + x_3 T(0,0,1)$

$x_1(1,1,1) + x_2(0,1,-1) + x_3(1,2,0)$

$= (x_1 + x_3, x + x_2 + 2x_3, x_1 - x_2)$

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$T(1,1,-1) = (0,0,0)$

$T(2,2,-2) = (0,0,0)$.

(many to one mapping)