

1. What is binary relation?
2. What is binary operation?
3. Number of binary relations on a set.
4. Definition of a groupoid.
5. Definition of a semigroup.
6. Definition of monoid.
 - I. Identity element of $(\mathbb{Z}, +)$
 - II. Identity element of (\mathbb{Z}, \cdot)
 - III. $(\mathbb{R} - \{0\}, \cdot)$
7. Definition of a Group.
 - a) $(\mathbb{R}, +)$, $(\mathbb{R} - \{0\}, \cdot)$, $(\mathbb{Q}, +)$, $(\mathbb{C}, +)$, $(\mathbb{Q} - \{0\}, \cdot)$, $(\mathbb{C} - \{0\}, \cdot)$ is a group.
 - b) (\mathbb{Q}, \cdot) , (\mathbb{C}, \cdot) is not a group
8. Definition of a finite group.
9. Order of finite group.
10. Equivalence classes. Show that set of residues of integer modulo n forms a group.
11. Commutative(abelian) Group and Non Commutative Group.
12. Show that $GL(n, \mathbb{R})$ forms a non commutative Group.
13. Show that $SL(n, \mathbb{R})$ forms a Group.
14. Definition of permutation.
15. Show that set of permutation of n elements forms a group.(Symmetric class).
16. A_n set of all even permutation forms a group.
17. Definition of subgroup.
18. Necessary and sufficient condition for a subgroup
 - a) Let (G, \cdot) be a group and H be a non-empty subset of G then H is a subgroup of G if and only if $a \cdot b^{-1} \in H \quad \forall a, b \in H$.
 - b) Let (G, \cdot) be a group and H be a non-empty finite subset of G then H is a subgroup of G if and only if $a \in H, b \in H \Rightarrow a \cdot b \in H$
19. Show that $SL(n, \mathbb{R})$ is a subgroup of $GL(n, \mathbb{R})$.
20. Show that alternating permutation A_n is a subgroup of S_n
21. $(3\mathbb{Z}, +)$ is a subgroup of $(\mathbb{Z}, +)$
22. Every group is a subgroup of itself(trivial subgroup)
23. Let G be a group and H and K be two subgroups of G
 - a) Show that $H \cap K$ is a subgroup of G
 - b) Show that $H \cup K$ may not be a subgroup
 - c) $H \cup K$ forms a subgroup if and only if $H \subset K$ or $K \subset H$.

24. Definition of cyclic group. Additive and multiplicative notation.
Eg kelin's 4 group.
25. Generator of a cyclic group.
- Generator of $\{1, -1, i, -i\}$
 - Generator of $\{1, w, w^2\}$
26. A finite group G has a unique generator if and only if $\text{oredr}(G) = 2$.
27. Every cyclic group is a commutative group.
28. Prove that if G is a cyclic group then $G = \langle a \rangle$ for some $a \in G$.
29. Prove that subgroup of a cyclic group is cyclic.
- Eg. Kelin's 4 group, subgroup and its generators.
30. How many subgroups of a cyclic group of order 36 ?
31. Order of an element is a group G .
32. Let a be an element of a group (G, \cdot) then if $\text{order}(a) = n$, then for a positive integer m ,
 $\text{order}(a^m) = n / \gcd(m, n)$
33. Cosets. Definition of left cosets and right cosets.
34. Let G be a group and H be a subgroup of G . any two left cosets of H in G are either identical or they have no common element.
35. Let G be a group and H be a subgroup of G . then the set of all left cosets of H in G and the set of all right cosets of H in G have same cardinality.
36. Lagrange Theorem and converse of it. Proof of them.
37. Normal subgroup
- A_n is a normal subgroup of S_n
 - $SL(n, R)$ is a normal subgroup of $GL(n, R)$
38. Necessary and sufficient condition of being a normal subgroup
- Let G be a group and H be a subgroup of G then H is a Normal Subgroup of G if and only if
 $ghg^{-1} \in H \quad \forall g \in G \text{ and } h \in H$
 - Proof $SL(n, R)$ is a normal subgroup of $GL(n, R)$ using this theorem
39. Definition of center of a group denoted by $Z(G)$
- If it is commutative the $Z(G)$ is a subgroup of G
40. Quotient of a set.
41. What is Quotient Group. Examples.