Permutation

$|S| = n \quad f : S \longrightarrow S$

$$\begin{pmatrix} 1, & 2, & 3, & \dots, & n \\ 2 & 3 & 4 & \dots & 1 \end{pmatrix}$$

3 cycle

$$(1 \quad 2 \quad 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

$$= (1 \quad 3)(1 \quad 2)$$

2 cycle (transposition)

$S = \{1, 2, 3\}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = (1, 2\ 3$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = 3\ 1\ 2$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = (2, 3)$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = (1 \quad 3)$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = (1 \quad 2)$$

A permutation $\sigma \in S_n$ is called even permutation if the no of transposition is even.

$A_n \to$ Collection of all even permutation.

$(A_n, \circ)$ satisfies all the 4 properties of group.

$$(1\ 2\ 3\ 4\ 5) \to (1\ 5)(1\ 4)(1\ 3)(1\ 2)$$

$S_n \to$ Permutation group $n!$ $\longrightarrow$ Non commutative

$f \circ g \neq g \circ f$

$f : S \to S$

$B = \{f : S \to S\}$

$(B, \circ) \to$ Infinite group.

Subgroup

$(G, *)$ $\quad * : G \times G \to G$

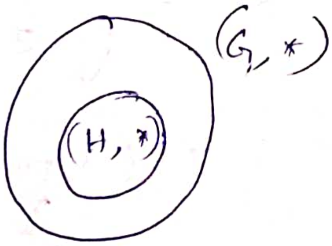
$H \neq \phi \subseteq G$

$* : H \times H \to H$

$(H, *) \to$ Subgroup

$f : A \to B$

$(X \neq \phi) \subseteq A$ $\quad g : X \to B$



Necessary and sufficient condition

Let $(G, *)$ be a group and H be a non empty subset of G then H is a subgroup of G if and only if $a * b^{-1} \in H \ \forall \ a, b \in H$.

$G \in SL(n, \mathbb{R})$

$A, B \in SL(n, \mathbb{R})$

$AB^{-1} \in SL(n, \mathbb{R})$

$\det A = \det B = 1$

$GL(n, \mathbb{R}) = \{ A \in M_{n \times n}(\mathbb{R}) : \det A$

$SL(n, \mathbb{R}) = \{ A \in GL(n, \mathbb{R}) : \det A = 1 \}$

$\det(AB^{-1}) = \det A \det B^{-1} = \det A (\det B)^{-1}$

$= 1 \times 1^{-1}$

$= 1.$

$\forall A, B \quad A, B \in SL(n, \mathbb{R})$

$AB^{-1} \in SL(n, \mathbb{R})$

$(\mathbb{Z}, +) \subseteq (\mathbb{Q}, +) \subseteq (\mathbb{R}, +) \subseteq (\mathbb{C}, +)$

A_n is a subgroup of S_n.

$\sigma_1, \sigma_2 \in A_n \mid \sigma_1 \circ \sigma_2^{-1} \in A_n$

If H is a finite set, $ab \in H \quad \forall a, b \in H$.

$(\mathbb{Z}, +) \longrightarrow$ Subgroup $(\mathbb{Z}, +)$

every group is a subgroup of itself. (Trivial subgroup).

Let G be a group and H_1, H_2 be two subgroups of G.

$G = (\mathbb{Z}, +) \quad H_1 = (2\mathbb{Z}, +) \quad H_2 = (3\mathbb{Z}, +)$

H_1, H_2 are subgroups of G.

$\underline{H_1 \cup H_2}$

$2 \in H_1 \subseteq H_1 \cup H_2$

$3 \in H_2 \subseteq H_1 \cup H_2$

$5 = 2 + 3 \notin H_1 \cup H_2$

Let $a, b \in H_1 \cap H_2$

$\Rightarrow a, b \in H_1$ and $a, b \in H_2$

$ab^{-1} \in H_1$, $ab^{-1} \in H_2$

$\Rightarrow ab^{-1} \in H_1 \cap H_2$

Intersection of two subgroups always forms a subgroup but union does not.

If $H_1 \subseteq H_2$ and $H_2 \subseteq H_1$ then $H_1 \cup H_2$ will form subgroup.

Cyclic group

$G \rightarrow$ group

$H \rightarrow$ subgroup

$K_n = \{e, a, b, c\}$ $\rightarrow$ kelin 4 gr

$\langle H \rangle = \bigcap H_i$

$H \subseteq H_i$

$a^2 = e$

$b^2 = e$

$c^2 = e$

$ab = ba = c$

$ac = ca = b$

$bc = cb = a$

$G = \langle a \rangle = \{a^n : n \in z\}$

$= \{na : n \in z\}$

$(Z, +)$ $Z = \langle 1 \rangle$

$(Z_2, +)$ $\quad Z_2 = \{[0], [1]\}$

$Z_2 = \langle [1] \rangle$

A finite group G has unique generator if and only if $|G| = 2$.

$G = \{1, -1, i, -i\}$ Generator $(i, -i)$

$G = \{1, \omega, \omega^2\}$ | $\omega^3 = 1$

Theorem

Every ~~ab~~ cyclic group is a commutative group.
abelian.

proof

Let G be a cyclic group. Then $G = \langle a \rangle$ for some $a \in G$.

Let $x, y \in G = \langle a \rangle$ then $x = a^m$
$y = a^n$

Now $xy = a^m a^n = a^{m+n}$

$yx = a^n a^m = a^{m+n}$

$\therefore$ $xy = yx$

$\therefore$ ~~evey~~ every cyclic group is commutative.

~~bo~~ but ~~evey~~ every commutative group is not cyclic. e.g Kelin's 4 group.

Theorem

Subgroup of a cyclic group is a cyclic group

$K_4 = \{e, a, b, c\}$

$H_0 = \{e\}$	$H_1 = \{e, a\}$	$H_2 = \{e, b\}$	$H_3 = \{e, c\}$
$= \langle e \rangle$	generator $= \langle a \rangle$	generator $= \langle b \rangle$	generator $= \langle c \rangle$

Proper subgroup of a non cyclic group is cyclic.

Q) How many subgroups of a cyclic group of order 36 ?

Ans: No of divisors of 36.

$36 \to \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$.

Order of an element in a group G

$(G, \cdot)$	$(G, +)$	$(\mathbb{Z}, +)$
$\exists\, n \in \mathbb{N}$	$na = 0$	order of $0 = 1$
$a^n = e_G$		order of $1 = \infty$
		" of $2 = \infty$
		$\vdots$

$K_4 = \{e, a, b, c\}$

$o(e) = o(a) = o(b) = o(c) = 1$

$G = \{1, -1, i, -i\}$

$o(G) = 4$

$o(1) = 1$

$o(-1) = 2$

$o(i) = 4$

$o(-i) = 4$] → same as $o(G)$ (always true for cyclic group)

G $\quad o(a) = n$

$$o(a^t) = \frac{o(a)}{GCD(n, t)}$$