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Inner Product Space.

An inner product space is a real or complex vector space together with a specified inner product on that space.

If V is an inner product space, then for every vector α, β in V and any scalar c in F

(i) $\|c\alpha\| = |c|\,\|\alpha\|$.

(ii) $\|\alpha\| > 0$ for $\alpha \neq 0$

(iii) $|(\alpha, \beta)| \leq \|\alpha\|\,\|\beta\|$

(iv) $\|\alpha+\beta\| \leq \|\alpha\| + \|\beta\|$

Proof :-

statement (i) and (ii) follow immediately from definition

Inequality (iii) is clearly valid for $\alpha = \beta$

when $\alpha = 0$ if $\alpha \neq 0$

put $\gamma = \beta$.

$$\gamma = \beta - \frac{(\beta\alpha)\alpha}{\|\alpha\|^2}$$

then $(\gamma, \alpha) = 0$ and $0 \leq \|\gamma\|^2 = \left(\beta - \frac{(\beta\alpha)\alpha}{\|\alpha\|^2}, \beta - \frac{(\beta\alpha)\alpha}{\|\alpha\|^2}\right)$

$$= \left(\beta, \beta - \frac{(\beta,\alpha)\alpha}{\|\alpha\|^2}\right) - \frac{(\beta,\alpha)}{\|\alpha\|^2}\left(\alpha, \beta - \frac{(\beta,\alpha)\alpha}{\|\alpha\|^2}\right)$$

$$= \|\beta\|^2 - \frac{\overline{(\beta,\alpha)}(\alpha,\beta)}{\|\alpha\|^2} - \cancel{\frac{(\beta,\alpha)\overline{(\beta,\alpha)}}{\|\alpha\|^2}} + \frac{(\beta,\alpha)\overline{(\beta,\alpha)}}{\|\alpha\|^2}$$

$$= \|\beta\|^2 - \frac{(\alpha,\beta)(\beta,\alpha)}{\|\alpha\|^2} = \|\beta\|^2 - \frac{(\alpha,\beta)\overline{(\alpha,\beta)}}{\|\alpha\|^2}$$

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$$= \|\beta\|^2 - \frac{|(\alpha,\beta)|^2}{\|\alpha\|^2} \geq 0$$

$$|(\alpha,\beta)|^2 \leq \|\alpha\|^2 \|\beta\|^2$$

$$\Rightarrow |(\alpha,\beta)| \leq \|\alpha\| \|\beta\|$$

iv) $\|\alpha+\beta\|^2 = (\alpha+\beta, \alpha+\beta)$

$= \|\alpha\|^2 + (\alpha,\beta) + (\beta,\alpha) + \|\beta\|^2$

$= \|\alpha\|^2 + (\alpha,\beta) + \overline{(\alpha,\beta)} + \|\beta\|^2$

$= \|\alpha\|^2 + 2\,Re(\alpha,\beta) + \|\beta\|^2$

$\leq \|\alpha\|^2 + 2\|\alpha\|\|\beta\| + \|\beta\|^2$

$= (\|\alpha\| + \|\beta\|)^2 \leq \|\alpha\| + \|\beta\|$

(proved)

Result (An orthogonal set of non-zero vector is linearly independent) (Prove.).

Statistics

Purposive sampling is one in which samples are selected with definite purpose in view. For example, we want to give a picture that standard of living has increased in the city of Delhi. We may take individual in the sample from rich and posh localities like the French colony, etc. and ignore the localities where low income groups and middle class families live.