Random Sampling
Suppose we take a sample of size 'n' from a finite population of size N, then there are NCn possible samples. A sampling technique in which each of NCn samples has an equal chance of being selected is known as random sampling.

Simple Sampling. – simple sampling is a random sampling in which each unit of population has an equal chance say 'p' of being ~~selected~~ included in the sample and that probability is independent of the previous drawing.

It must be pointed out that random sampling does not necessarily imply simple sampling though obviously converse is true. for example, if an urn contains 'a' white balls and 'b' black ~~bla~~ balls. Then, the probability of drawing a white ball at the first draw is $\frac{a}{a+b} = P_1$ (say) and

if this white ball is not replaced, the probability of drawing a white ball again ~~will be~~ in second draw is $\frac{a-1}{a+b-1} = P_2$ (say). since

$P_1 \neq P_2$, the sampling is not simple but since in first draw, each white ball has the same ~~sample~~ chance ie $\frac{a}{a+b}$ of being drawn and in second draw again, each white ball has the same chane ie $\frac{a-1}{a+b-1}$

of being drawn, the sampling is a random sampling. Hence, in this case, the sampling, though random, is not simple. To ensure that sampling is simple, it must be done with replacement, if population is finite.

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Unbiased estimate :- A statistic $t = t(x_1, x_2, \ldots, x_n)$, a function on sample values $x_1, x_2, \ldots, x_n$ is an unbiased estimate of the population parameter 'θ' if ~~expectation~~ $E(t) = \theta$ ← expectation of t

Sampling distribution :- If we draw a sample of size n from a given finite population of size N, then total number of possible samples $^NC_n = \frac{N!}{n!(N-n)!} = k$ (say).

For each of the 'k' samples, we can conclude from statistic $t = t(x_1, x_2, \ldots x_n)$, in particular, the mean $\bar{x}$, variance s^2, etc, the set of values of the statistic so obtained for each sample constitutes which is called sampling distribution.

Sampling from a finite population (without replacement)

Let the value $y_1, y_2, \ldots y_N$ constitute a finite population, with mean μ and variance σ^2 so that →

$$\mu = \frac{1}{N}(y_1 + y_2 + y_3 + \ldots + y_N)$$

$$\sigma^2 = \frac{1}{N}\left[(y_1 - \mu)^2 + (y_2 - \mu)^2 + \ldots + (y_N - \mu)^2\right]$$

$$= \frac{1}{N}\left(y_1^2 + y_2^2 + \ldots + y_N^2\right) - \mu^2$$

Let $x_1, x_2, \ldots x_n$ be a random sample drawn without replacement from a population and let the sample mean be $\bar{x}$.

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Then, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$

$$= \frac{1}{n}\sum_{i=1}^{n} E(x_i) \quad \text{--- (1)}$$

Now for ~~fixed~~ fixed value 'i', the ~~variance~~ variate x_i takes values $y_1, y_2, \ldots y_N$ each with probability $\frac{1}{N}$ so that

$$E(x_i) = \frac{1}{N}(y_1 + y_2 + \cdots y_n)$$

$$\mu_i \quad (i = 1, 2, \ldots n)$$

$$E(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n} E(x_i) \quad \text{--- from (1)}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \mu = \mu$$

Thus the expected value of $\bar{x}$, sample mean in random sampling is the population mean μ irrespective of the sample size.

The variance $\sigma_{\bar{x}}^2$ of the sample mean.

$$\sigma_{\bar{x}}^2 = E\left[\bar{x} - E(\bar{x})\right]^2 = E(\bar{x} - \mu)^2$$

$$= \frac{1}{n^2} E\left[(x_1 - \mu) + (x_2 - \mu) + \cdots + (x_n - \mu)\right]^2$$

$$= \frac{1}{n^2}\left[\sum_{i=1}^{n} E(x_i - \mu)^2 + 2\sum_{j=1}^{n}\sum_{i=1}^{n} E\left[(x_i - \mu)(x_j - \mu)\right]\right] \quad (i < j)$$

Since $x_1, x_2, \ldots x_n$ is a random sample, n expected value in the first sum are the same and for same reason for $n(n-1)$ expected values,

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second sum is identical, therefore

$$\sigma_{\bar{x}}^2 = \frac{1}{n^2}\left[n \sum (x_i - \mu)^2 + n(n-1) E\{(x_1-\mu)(x_2-\mu)\}\right]$$

Now $E(x_1-\mu)^2 = \frac{1}{N}\left[(y_1-\mu)^2 + (y_2-\mu)^2 + \dots + (y_N-\mu)^2\right] = \sigma^2$

and the conversion between any two members of the sample is

$$E(x_1-\mu)(x_2-\mu) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} (y_i-\mu)(y_j-\mu)$$

$$= \frac{1}{N(N-1)} \sum_{i=1}^{N} (y_i-\mu)\left[(y_1-\mu)+(y_2-\mu) + \dots + (y_N-\mu) - (y_i-\mu)\right]$$

$$= \frac{1}{N(N-1)}\left[\sum_{i=1}^{N} (y_i-\mu)\{0-(y_i-\mu)\}\right]$$

$$= -\frac{1}{N(N-1)} \sum_{i=1}^{N} (y_i-\mu)^2 = \frac{-1}{N(N-1)} N\sigma^2$$

$$= \frac{-\sigma^2}{(N-1)}$$

$$\operatorname{var}(\bar{x}) = \frac{\sigma^2}{n} + \frac{(n-1)}{n}\left(\frac{-\sigma^2}{N-1}\right) = \frac{\sigma^2}{n}\left[1 - \frac{n-1}{N-1}\right]$$

$$= \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$

Thus variance of the sample mean for random sampling from finite population, is

$$\frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$