Hard Disk Design

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Task 1

The differential equation of input voltage and head position is given, so it is pretty easy to get the open loop transfer function by taking the lapalce transform of both side and can easily get

$$\frac{Y(s)}{U(s)} = \frac{1}{Js^2 + bs}$$

Task 2

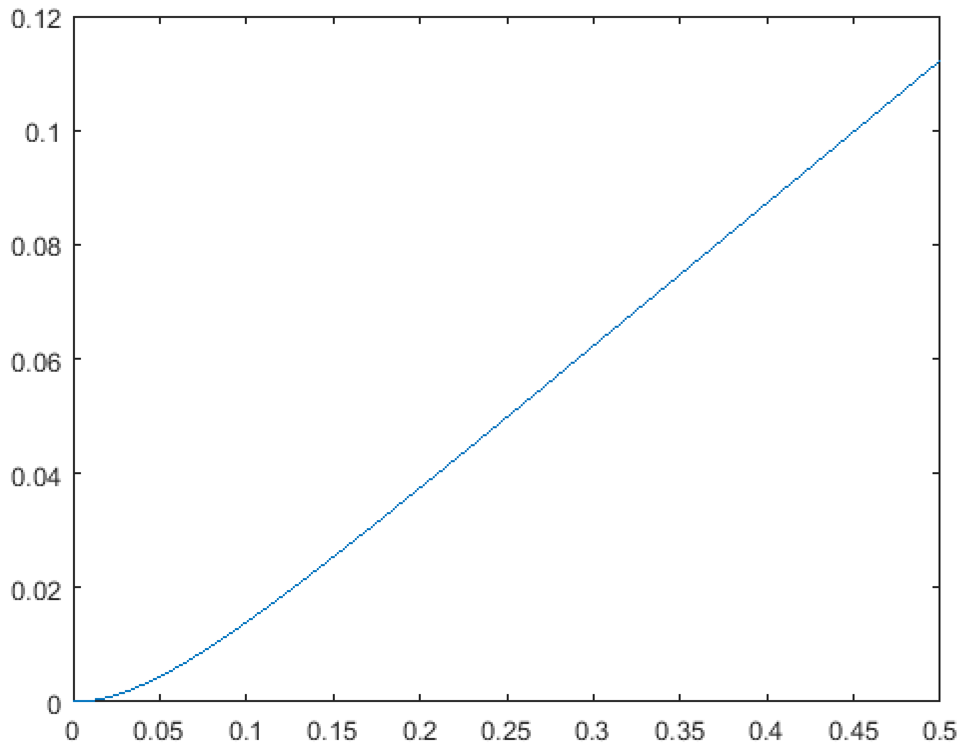
open loop transfer function is given by multiplying both G1 and G2

$$G1(s)G2(s) = \frac{Km}{(Ls+R)*(Js^2+bs)} = \frac{Km}{LJs^3+Lbs^2+RJs^2+Rbs}$$

```
s = tf('s');
J = 1;
b = 20;
R = 1;
L = 0.001;
Km = 5;
G1 = Km/(L*s+R)
G2 = 1/(J*s^2+b*s)
openTF = G1*G2
t=[0:0.005:0.5];
y = step(openTF,t);
plot(t,y);
```

```
G1 =

       5
  -----------
  0.001 s + 1

Continuous-time transfer function.
```

```
G2 =

      1
  ----------
  s^2 + 20 s

Continuous-time transfer function.


openTF =

               5
  ---------------------------
  0.001 s^3 + 1.02 s^2 + 20 s

Continuous-time transfer function.
```



You can see that if a constant voltage is applied, then the read head will moves in a constant speed. And at the begining when the voltage is applied, there is a curve, which indicates that the read head is accelerating under applied voltage.

Task 3a

The proportional compensator is applied, so that the open loop transfer function is given by

$$Ka * G1(s) * G2(s)$$

and closed loop transfer function is given by

$$\frac{Ka * G1(s) * G2(s)}{1 + Ka * G1(s) * G2(s)}$$

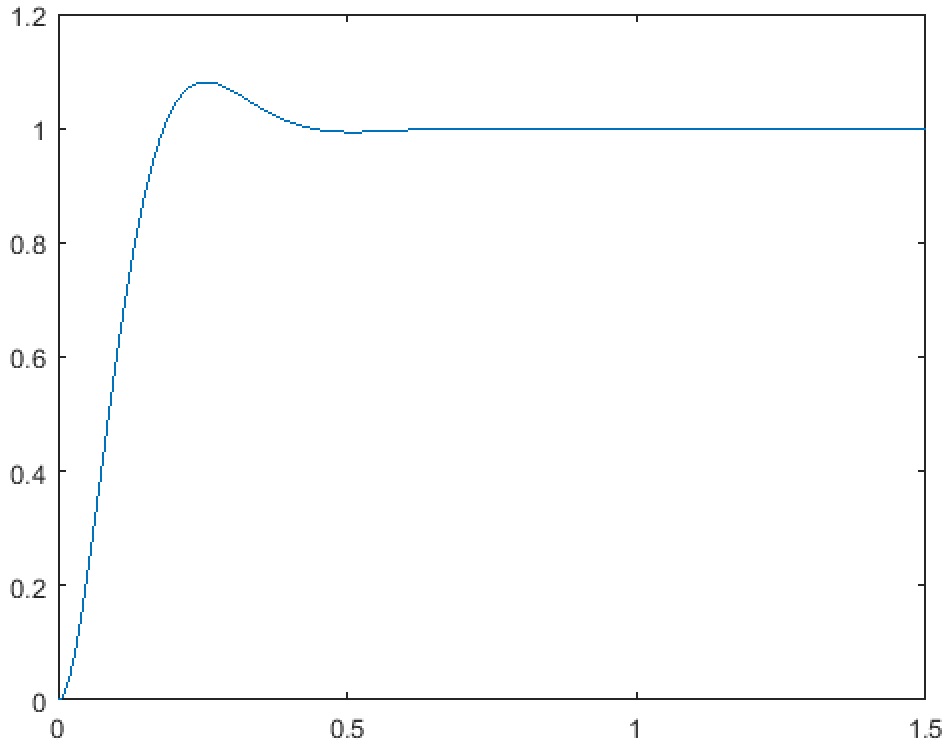
after plug in G1(s)G2(s) from previous computation, we have

$$\frac{Ka * Km}{LJs^3 + (Lb + RJ)s^2 + Rbs + KaKm}$$

Try plugging in Ka = 50, we can get the following plot

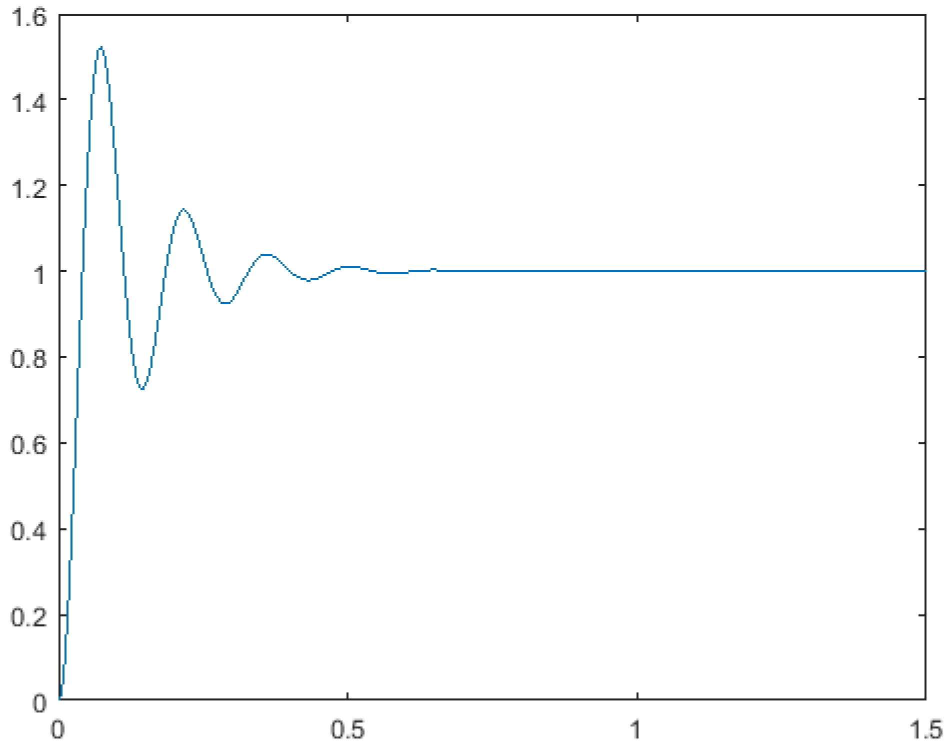
```
Ka = 50;
t=[0:0.005:1.5];
ProportionalTF =(Ka*G1*G2)/(1+Ka*G1*G2)
y = step(ProportionalTF, t);
plot(t,y);
```

```
ProportionalTF =

                     0.25 s^3 + 255 s^2 + 5000 s
  -----------------------------------------------------------------
  1e-06 s^6 + 0.00204 s^5 + 1.08 s^4 + 41.05 s^3 + 655 s^2 + 5000 s

Continuous-time transfer function.
```



Then applied step when Ka = 400

```
Ka = 400;
ProportionalTF = (Ka*openTF)/(1+Ka*openTF)
y = step(ProportionalTF, t);
plot(t,y);


ProportionalTF =

                        2 s^3 + 2040 s^2 + 40000 s
  ------------------------------------------------------------------
  1e-06 s^6 + 0.00204 s^5 + 1.08 s^4 + 42.8 s^3 + 2440 s^2 + 40000 s

Continuous-time transfer function.
```



Clearly you can see that the Ka=50 has less over shoot than Ka=400 does. This is because for Ka=400, the feedback amplification is too big and make system too sensitive.

Task 3b

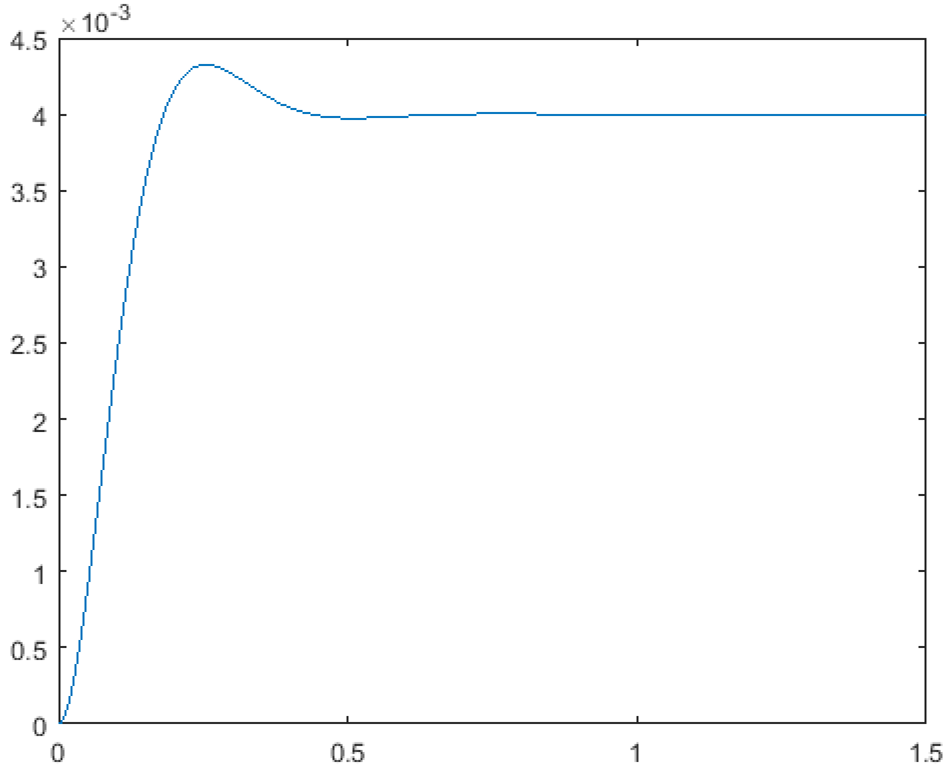
So when disturbance is Introduced to the system, the disturbance input flows into system between G1 and G2. So that we can derive the transfer function for disturbance:

$$Tw = \frac{G2(s)}{1 + Ka * G1(s)}$$

First plot the response of disturbance in case of Ka = 50

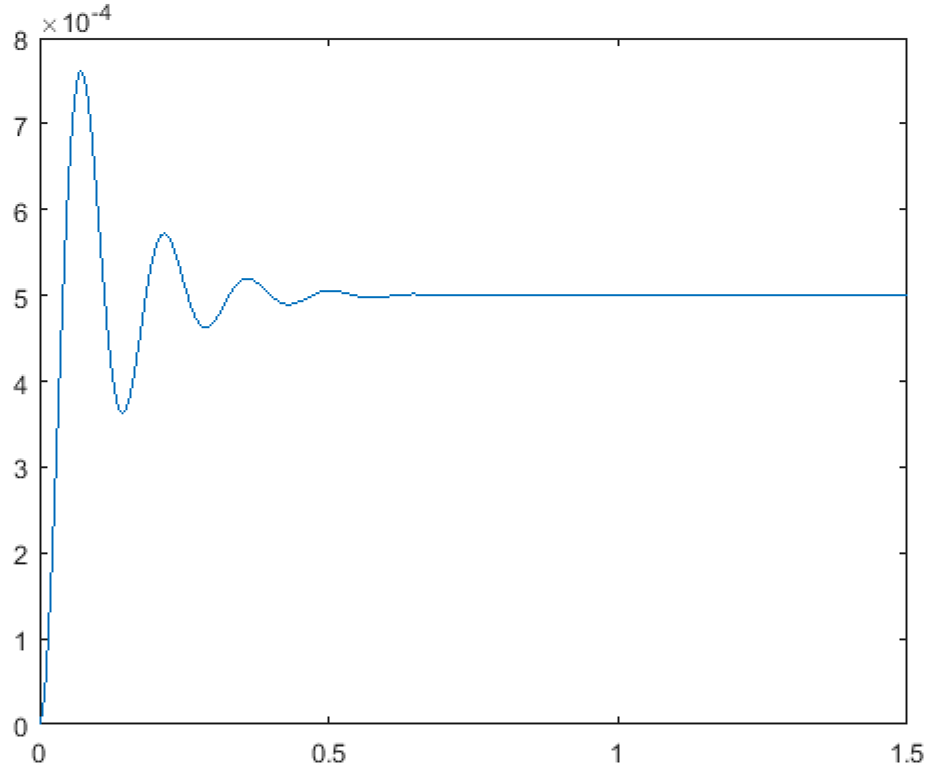
```
Ka = 50;
Tw = G2/(1+Ka*G1*G2)
y = step(Tw, t);
plot(t,y);


Tw =

             0.001 s^3 + 1.02 s^2 + 20 s
  -------------------------------------------------
  0.001 s^5 + 1.04 s^4 + 40.4 s^3 + 650 s^2 + 5000 s
```

```
Continuous-time transfer function.
```



Then plot the response of disturbance in case of Ka = 400

```
Ka = 400;
Tw = G2/(1+Ka*G1*G2)
y = step(Tw, t);
plot(t,y);


Tw =

              0.001 s^3 + 1.02 s^2 + 20 s
  ----------------------------------------------------
  0.001 s^5 + 1.04 s^4 + 40.4 s^3 + 2400 s^2 + 40000 s

Continuous-time transfer function.
```

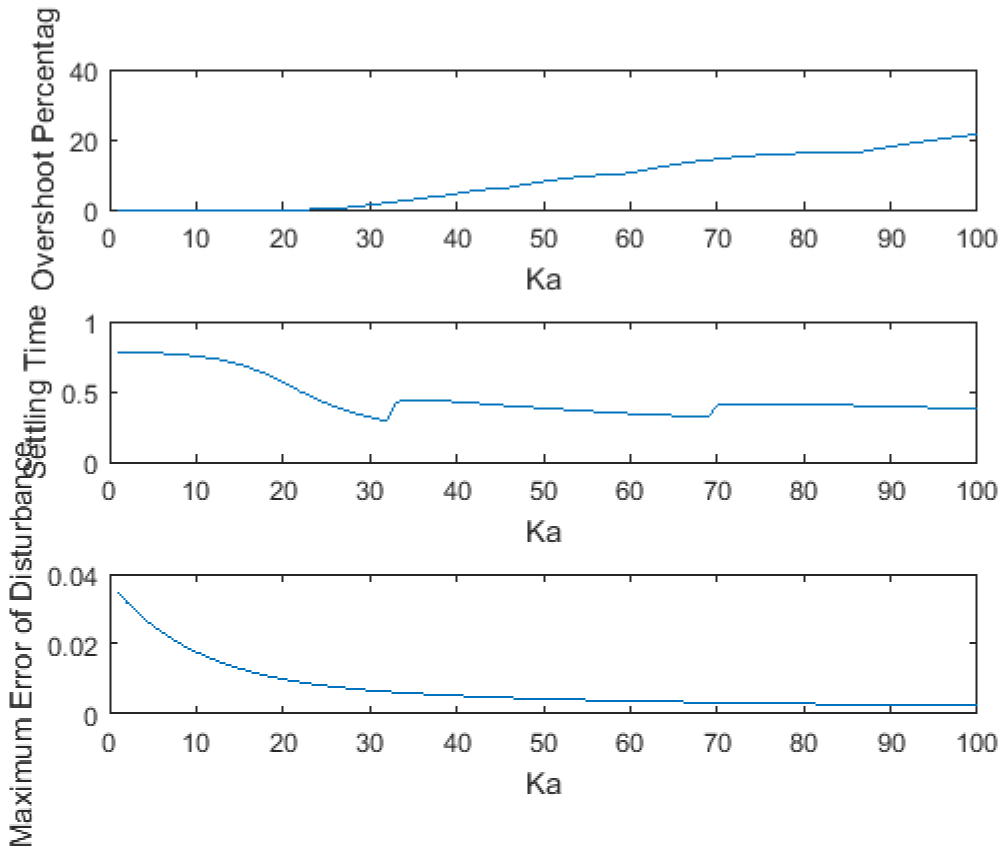


Task 3C

We can plot overshoot percentage and settling time as a function of Ka, and analyze the optiomal Ka for the system

```
overshoot = [];
settlingTime = [];
maxError = [];
n=[1:100];
t=[0:0.05:0.8];
for  Ka=n
    ProportionalTF = (Ka*openTF)/(1+Ka*openTF);
    y = step(ProportionalTF, t);
    info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02);
    overshoot = [overshoot, info.Overshoot];
    settlingTime = [settlingTime info.SettlingTime];

    Tw = G2/(1+Ka*G1*G2);
    y = step(Tw, t);
    maxError = [maxError max(y)];
end
subplot(3,1,1);
plot(n,overshoot);xlabel('Ka');ylabel('Overshoot Percentage');
subplot(3,1,2);
plot(n,settlingTime);xlabel('Ka');ylabel('Settling Time');
subplot(3,1,3);
```

```
plot(n,maxError);xlabel('Ka');ylabel('Maximum Error of Disturbance');
```



For overshoot less than 5%, Ka is required to be equal or less than 41, Ka value that satisfy settling time requirement is more than 100 and is not in the graph. For disturbance less than 0.005, Ka is required to be equal or bigger than 41 Clearly, you cannot satisfy three requirements at the same time.

Task 4

The closed loop transfer function in this case would be

$$\frac{KaG1(s)G2(s)}{1 + KaH(s)G1(s)G2(s)}$$

So both Ka and Kh are varying, so we can plot a 3-D graph in which x aixs is Ka and Y axis is Kh and Z axis is the property under examination like Settling Time, Overshoot and disturbance. We can find the candidate that sastisfies all three constraints by finding the overlap of Ka and Kh values that satisfies three contraints.

```
[KaRange, KhRange] = meshgrid(55:65, 0:0.01:0.1);
overshootMatrix = [];
settlingTimeMatrix = [];
candidatePairs = [];
t=[0:0.05:0.8];
G12= G1*G2;
for Kh = KhRange(:,1)'
```

```
    %overshootArr=[];
    %settlingTimeArr=[];
    for Ka = KaRange(1,:)
        CLTF = (Ka*G12)/(1+Ka*(1+Kh*s)*G12);
        y = step(CLTF, t);
        info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02);
        %overshootArr = [overshootArr, info.Overshoot];
        %settlingTimeArr = [settlingTimeArr, info.SettlingTime];
        Tw = G2/(1+Ka*(1+Kh*s)*G12);
        y = step(Tw,t);
        maxDisturbance = max(y);

        if(info.Overshoot <= 5 & info.SettlingTime <0.25 & y<0.005)
            candidatePairs = [candidatePairs; Ka, Kh];
        end
    end
    %overshootMatrix = [overshootMatrix; overshootArr];
    %settlingTimeMatrix = [settlingTimeMatrix; settlingTimeArr];
end
%mesh(KaRange, KhRange, settlingTimeMatrix);
%mesh(KaRange, KhRange, overshootMatrix);
candidatePairs
```

```
candidatePairs =

   56.0000    0.0300
   57.0000    0.0300
   58.0000    0.0300
   59.0000    0.0300
   60.0000    0.0300
   61.0000    0.0300
   62.0000    0.0300
   63.0000    0.0300
   64.0000    0.0300
   65.0000    0.0300
```

We can see that there are bunch of valid pair of Ka and Kh that satisfies the design, we can select Ka = 60 and Kh = 0.03 to examine.

```
Ka = 60; Kh = 0.03;
CLTF = (Ka*G12)/(1+Ka*(1+Kh*s)*G12);
y = step(CLTF, t);
info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02)
Tw = G2/(1+Ka*(1+Kh*s)*G12);
y = step(Tw,t);
maxDisturbance = max(y)
```

```
info =

        RiseTime: 0.1621
    SettlingTime: 0.2380
     SettlingMin: 0.9401
```

```
     SettlingMax: 1.0084
       Overshoot: 0.8390
      Undershoot: 0
            Peak: 1.0084
        PeakTime: 0.3500


maxDisturbance =

    0.0034
```

So all of the contraints are satisfied.

Task 5

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