Hard Disk Drive Read Header Controller Design Project

EE 141 – Principles of Feedback Control
Spring 2015
University of California, Los Angeles

Baixiao Huang 504313981
Weiqian Xu 404297854
Xingjian Yan 104304403

05/21/2015

1.0 OBJECTIVES

The objective of this project is to design a control system for hard disk drive read header. Knowledge of feedback control theories and skills of using Matlab are crucial in designing. The system contains a controller, a motor coil, a reader arm and a sensor forming a closed loop system. In this project, we are going to analyze the relationship between system performance and different compensators or sensors.

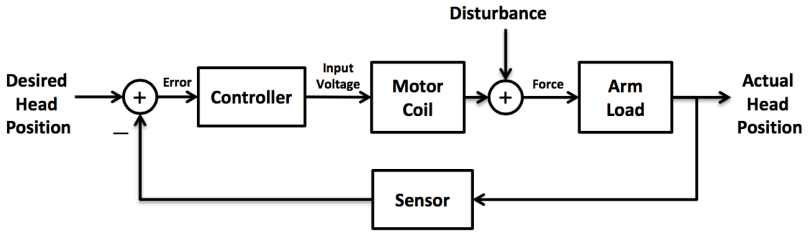


Figure 1

2.0 TASKS

2.1 Task 1

For this part, we are provided with the physical model relating the input torque u to the header position y with the inertia of arm and head J and friction b.

$$J\ddot{y} + b\dot{y} = u(t) \tag{1}$$

Simply apply Laplace transform to get the transfer function:

$$G_2(s) = \frac{Y(s)}{U(s)} = \frac{1}{Js^2+bs} \tag{2}$$

2.2 Task 2

The transfer function of the motor coil that relates the input voltage to the output torque is related by:

$$G_1(s) = \frac{U(s)}{V(s)} = \frac{K_m}{Ls+R} \tag{3}$$

In this task, we are going to obtain the open-loop transfer function of the cascaded HDD head reader assembly and then use Matlab to generate the plot of the system's step response.

$$G_1(s)G_2(s) = \frac{K_m}{(Ls+R)*(Js^2+bs)} = \frac{K_m}{LJs^3+Lbs^2+RJs^2+Rbs} \tag{4}$$

The Matlab code with the constants plugged in and the plot obtained are presented below:

```
s = tf('s');
J = 1;
b = 20;
R = 1;
L = 0.001;
Km = 5;
G1 = Km/(L*s+R)
G2 = 1/(J*s^2+b*s)
openTF = G1*G2
t=[0:0.005:0.5];
y = step(openTF,t);
plot(t,y);
```

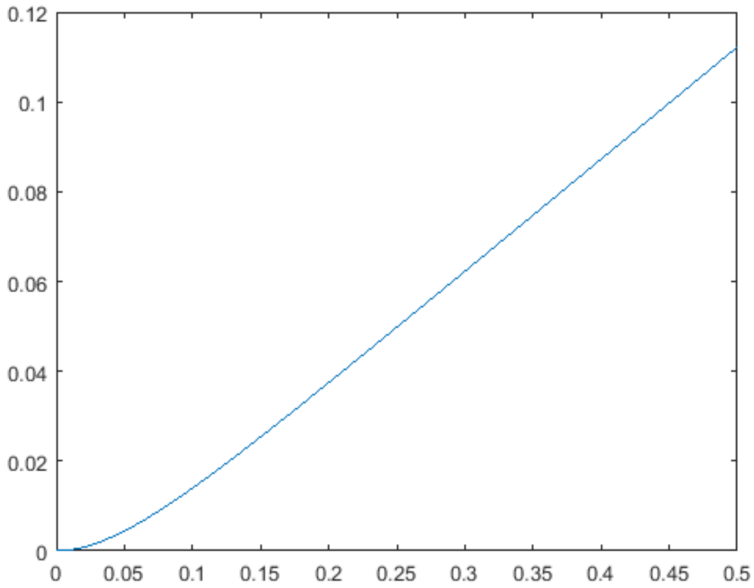


Figure 2

As we can see, if a constant voltage is applied, then the read head will moves with a constant speed. However, at the beginning when the voltage is applied, there is a curvature indicating the read head is accelerating.

2.3 Task 3 – Proportional Compensator

In this part, we are going to make the closed-loop system satisfies several transient response performance specifications: Percent overshoot less than 5%; Settling time (2% deviation) less than 250ms; Maximum value of response to a unit step disturbance less than 5×10^{-3}. Assume the feedback sensor's transfer function is H(s) = 1 and the candidate controller is $F(s) = K_a$.

Part A

We are going to determine and evaluate the responses of the closed-loop system to the unit step reference input without disturbance. If the proportional compensator applied, the transfer function is given by:

$$K_a G_1(s) G_2(s) \tag{5}$$

Then the closed-loop transfer function is given by:

$$\frac{K_a G_1(s) G_2(s)}{1+K_a G_1(s) G_2(s)} \tag{6}$$

After plugging in (2) and (3), the equation becomes:

$$\frac{KaKm}{LJs^3+(Lb+RJ)s^2+Rbs+KaKm} \tag{7}$$

Try $Ka = 50$, the Matlab code (continuous after the previous code) and plot are shown below:

```
Ka = 50;
t=[0:0.005:1.5];
ProportionalTF =(Ka*openTF)/(1+Ka*openTF)
y = step(ProportionalTF, t);
plot(t,y);
```

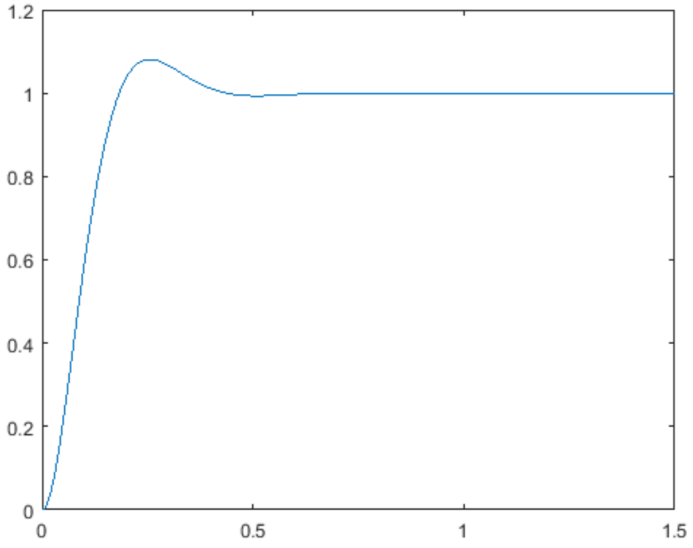


Figure 3

If $Ka = 400$, the Matlab code and plot are shown below:

```
Ka = 400;
ProportionalTF = (Ka*openTF)/(1+Ka*openTF)
y = step(ProportionalTF, t);
plot(t,y);
```

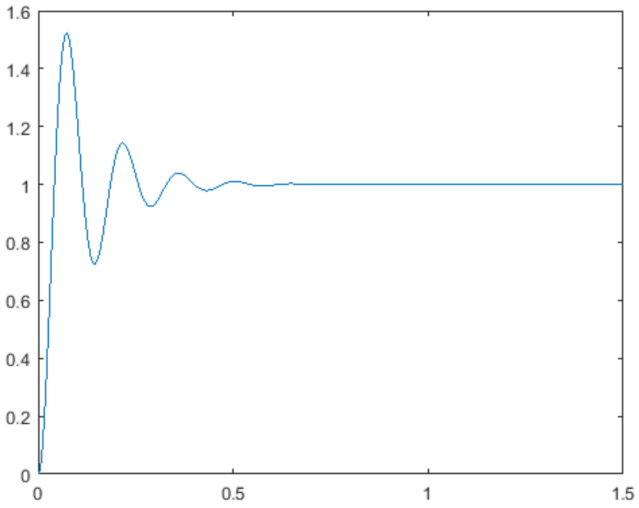


Figure 4

It is clear that the system overshoot when $Ka = 50$ is much lesser than that when $Ka = 400$. If the feedback amplification is too big, the system will be too sensitive.

Part B

In this part, we are going to determine and evaluate the responses of the closed-loop system to the unit step disturbance, assuming zero reference input. The transfer function for disturbance is:

$$T_\omega = \frac{G_2(s)}{1+K_a G_1(s) G_2(s)} \quad (8)$$

As above, we first plot the response with $K_a = 50$.

```
Ka = 50;
Tw = G2/(1+Ka*G1*G2)
y = step(Tw, t);
plot(t,y);
```

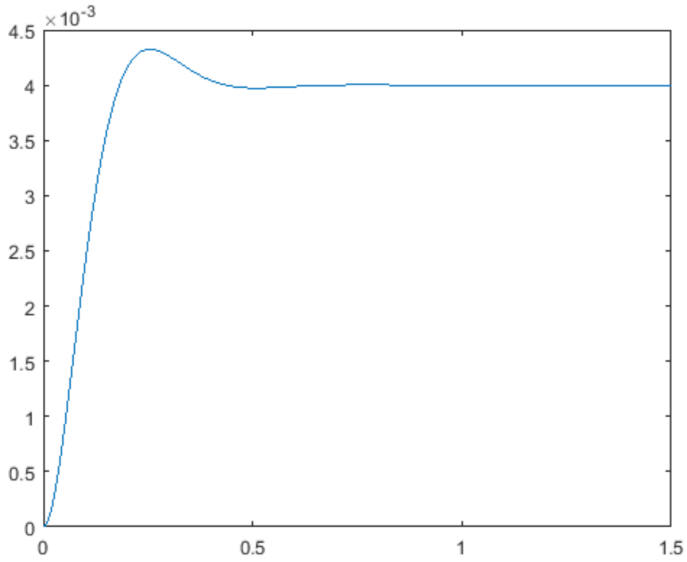


Figure 5

Similarly, we then try $K_a = 400$.

```
Ka = 400;
Tw = G2/(1+Ka*G1*G2)
y = step(Tw, t);
plot(t,y);
```

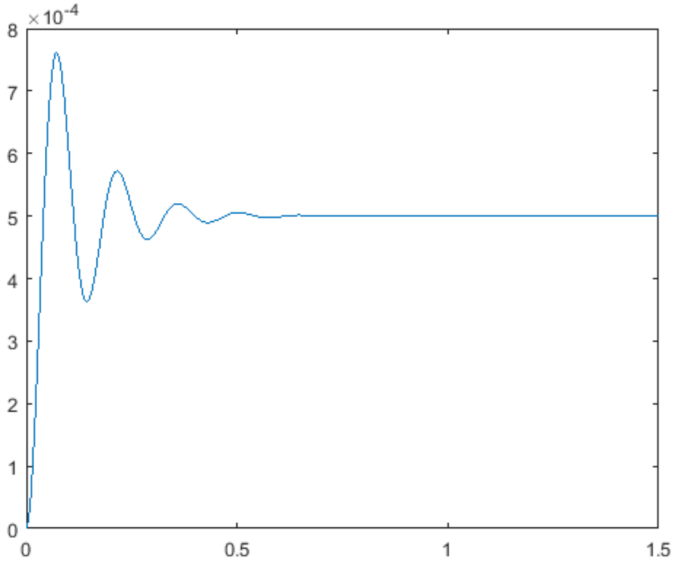


Figure 6

Same as Part A, $Ka = 50$ has less overshoot than $Ka = 400$.

Part C

In this part, the objective is to find a value for Ka that satisfies the performance specifications above. We can plot overshoot percentage and settling time as a function of Ka, and analyze the optimal Ka for the system.

```
overshoot = [];
settlingTime = [];
maxError = [];
n=[1:100];
for  Ka=n
    ProportionalTF = (Ka*openTF)/(1+Ka*openTF);
    y = step(ProportionalTF, t);
    info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02);
    overshoot = [overshoot, info.Overshoot];
    settlingTime = [settlingTime info.SettlingTime];
    Tw = G2/(1+Ka*G1*G2);
    y = step(Tw, t);
    maxError = [maxError max(y)];
end

subplot(3,1,1);

plot(n,overshoot);xlabel('Ka');ylabel('Overshoot Percentage');

subplot(3,1,2);
plot(n,settlingTime);xlabel('Ka');ylabel('Settling Time');
subplot(3,1,3);
plot(n,maxError);xlabel('Ka');ylabel('Maximum Error of Disturbance');
```

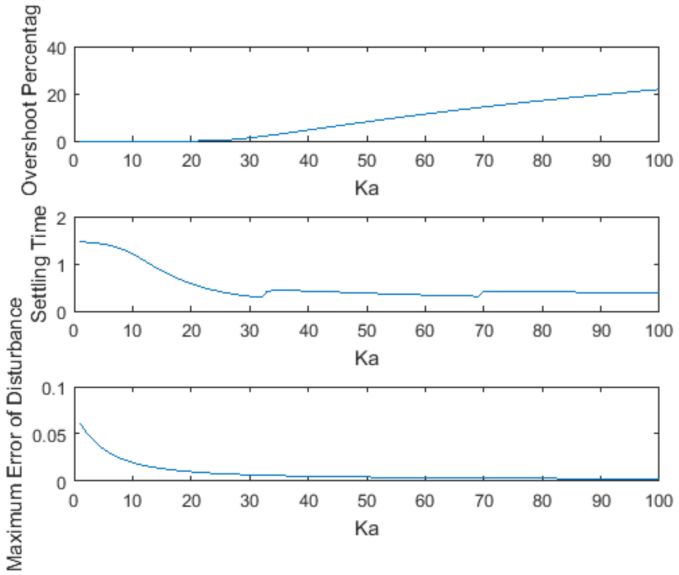


Figure 7

For overshoot less than 5%, Ka is required to be equal or less than 41. The value of Ka that satisfies settling time requirement is more than 100 and is not in the graph. For disturbance less than 0.005, Ka is required to be equal or bigger than 41. As a result, the three requirements cannot be satisfied at the same time.

2.4 Task 4 – Positional and Velocity Sensor

In this part, we substitute the sensor with $H(s) = 1 + K_H s$ that measures the position as well as the velocity of the head reader. Then we need to find K_a and K_H such that the system is stable and satisfies the performance specifications mentioned above. The closed loop transfer function is:

$$\frac{K_a G_1(s) G_2(s)}{1+K_a H(s) G_1(s) G_2(s)} \tag{9}$$

The fundamental idea for finding the solution is that since both K_a and K_h are varying, we can plot a 3-D graph in which x-axis is K_a, y-axis is K_h, and z-axis is the property under examination including settling time, overshoot and disturbance. Then we can find out all of the combinations of K_a and K_h that satisfy all performance requirements.

```
[KaRange, KhRange] = meshgrid(55:65, 0:0.01:0.1);
overshootMatrix = [];
settlingTimeMatrix = [];
candidatePairs = [];
t=[0:0.05:0.8];
G12= G1*G2;
for Kh = KhRange(:,1)'
    %overshootArr=[];
    %settlingTimeArr=[];
    for Ka = KaRange(1,:)
        CLTF = (Ka*G12)/(1+Ka*(1+Kh*s)*G12);
        y = step(CLTF, t);
        info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02);
        %overshootArr = [overshootArr, info.Overshoot];
        %settlingTimeArr = [settlingTimeArr, info.SettlingTime];
        Tw = G2/(1+Ka*(1+Kh*s)*G12);
        y = step(Tw,t);
        maxDisturbance = max(y);
        if(info.Overshoot <= 5 & info.SettlingTime <0.25 & y<0.005)
            candidatePairs = [candidatePairs; Ka, Kh];
        end

    end

    %overshootMatrix = [overshootMatrix; overshootArr];
    %settlingTimeMatrix = [settlingTimeMatrix; settlingTimeArr];
end

%mesh(KaRange, KhRange, settlingTimeMatrix);
```

```
%mesh(KaRange, KhRange, overshootMatrix);
candidatePairs

candidatePairs =
   56.0000    0.0300
   57.0000    0.0300
   58.0000    0.0300
   59.0000    0.0300
   60.0000    0.0300
   61.0000    0.0300
   62.0000    0.0300
   63.0000    0.0300
   64.0000    0.0300
   65.0000    0.0300
```

There are a bunch of valid pairs of K_a and K_h that satisfies the design. We can select $K_a = 60$ and $K_h = 0.03$ to check the result.

```
Ka = 60; Kh = 0.03;
CLTF = (Ka*G12)/(1+Ka*(1+Kh*s)*G12);
y = step(CLTF, t);
figure(6);
plot(t,y);title('System Step Response');
info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02)

Tw = G2/(1+Ka*(1+Kh*s)*G12);
y = step(Tw,t);
figure(7);
plot(t,y);title('System Step Disturbance Response');
maxDisturbance = max(y)
info =

  RiseTime: 0.1621
  SettlingTime: 0.2380
  SettlingMin: 0.9401
  SettlingMax: 1.0084
  Overshoot: 0.8390
  Undershoot: 0
  Peak: 1.0084
  PeakTime: 0.3500

maxDisturbance = 0.0034
```

The graphs of responses are presented below.

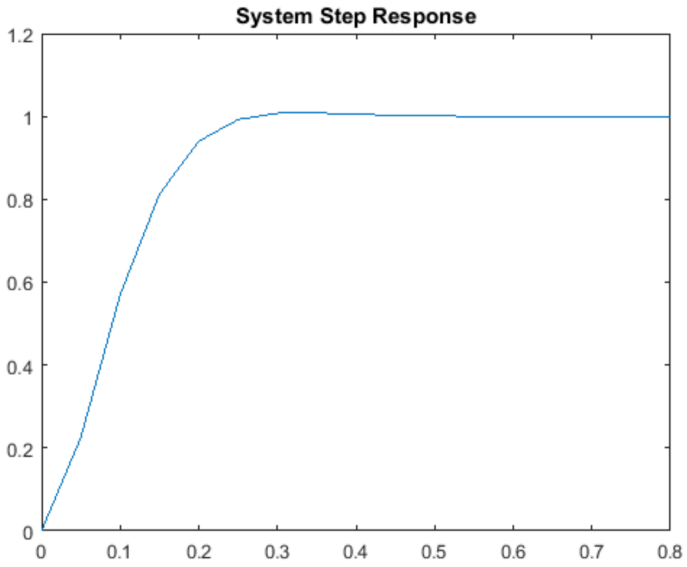


Figure 8

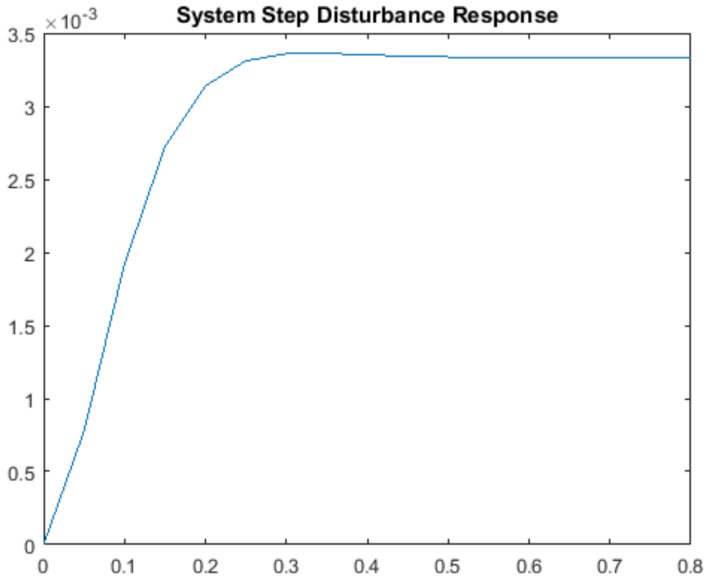


Figure 9

We can then conclude that the pairs of results listed above are valid for the system to meet all requirements.

2.5 Task 5 – PID Compensator

In this part, we revert the sensor back to H(s) = 1 but substitute the controller with $F(s) = K_1 + \frac{K_2}{s} + K_3 s$. Since the arm inherently has an integrating term, we assume $K_2 = 0$. The closed-loop transfer function then becomes:

$$\frac{(K_1+K_3 s)G_1(s)G_2(s)}{1+(K_1+K_3 s)G_1(s)G_2(s)} \quad (10)$$

We can apply the same procedure to generate 3-D graph of constraints.

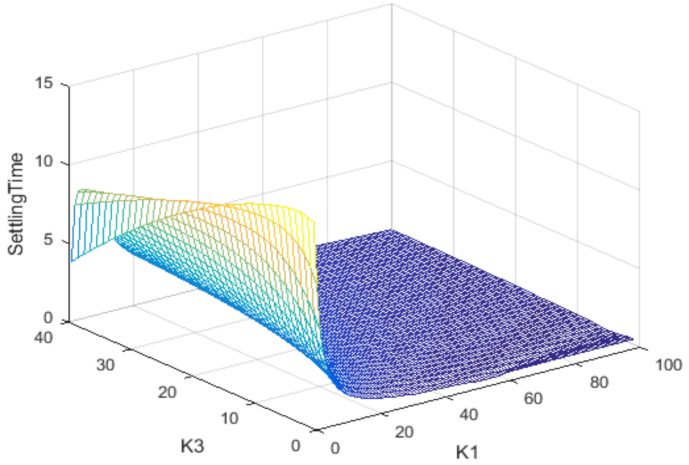


Figure 10 – Settling time

When K_1 is around 100 and K_3 is around 5 to 10, the settling time requirement can be satisfied. With the range of values, we are able to further derive detailed solution. If we take a closer look at the range of values that satisfy the constraints, the graph becomes the one shown by Figure 11.

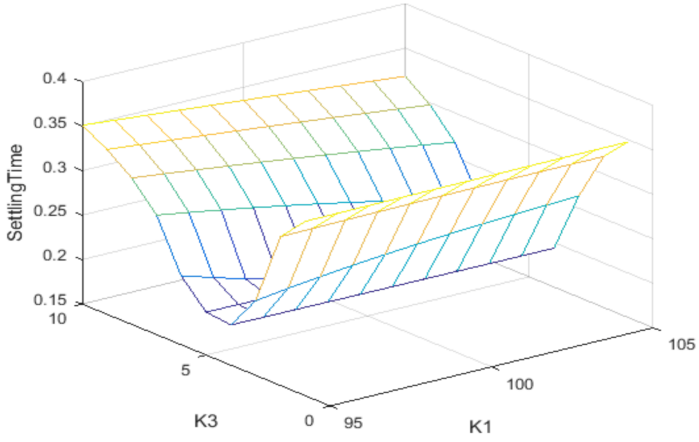


Figure 11 – Settling time

As shown in Figure 11, there are couples of K_1 and K_3 pairs that meet the settling time requirement. If we exam $K_1 = 100$ and $K_3 = 5$ for example, the settling time will under 250ms. The next step is to evaluate the overshoot of the system.

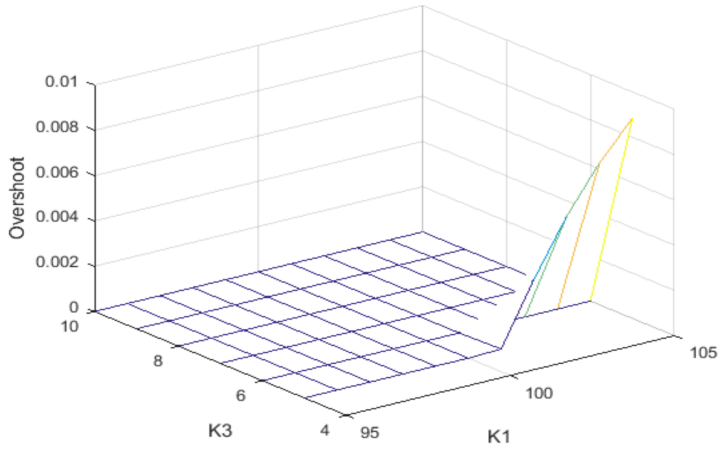


Figure 12 – Overshoot

As we can see, for $K_1 = 100$ and $K_3 = 5$, there is no apparent overshoot. We can then examine the disturbance of this range.

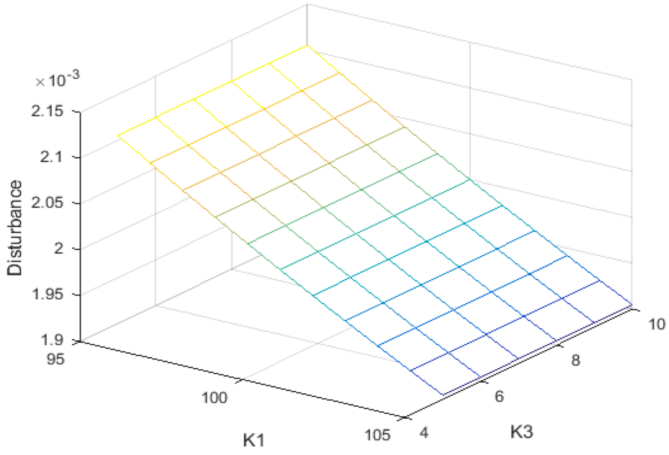


Figure 13 – Disturbance

The disturbance of $K_1 = 95$ and $K_3 = 100$ meets the requirement. The code for generates graphs above are presented below:

```
[K1Range K3Range] = meshgrid(95:105, 5:10);
overshootMatrix = [];
settlingTimeMatrix = [];
disturbanceMatrix=[];
candidatePairs = [];
t=[0:0.2:15];
G12= G1*G2;
```

```
for K3 = K3Range(:,1)'
   overshootArr=[];
   settlingTimeArr=[];
   disturbanceArr=[];
          for K1 = K1Range(1,:)
              CLTF = ((K1+K3*s)*G12)/(1+(K1+K3*s)*G12);
              y = step(CLTF, t);
              info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02);
              overshootArr = [overshootArr, info.Overshoot];

              settlingTimeArr = [settlingTimeArr, info.SettlingTime];
              Tw = G2/(1+(K1+K3*s)*G12);
              y = step(Tw,t);
              maxDisturbance = max(y);
              disturbanceArr = [disturbanceArr, maxDisturbance];
              if(info.Overshoot <= 5 & info.SettlingTime <0.25 & y<0.005)
                   candidatePairs = [candidatePairs; Ka, Kh];
              end
          end

   overshootMatrix = [overshootMatrix; overshootArr];
   settlingTimeMatrix = [settlingTimeMatrix; settlingTimeArr];
   disturbanceMatrix = [disturbanceMatrix; disturbanceArr];
end

mesh(K1Range, K3Range,
  overshootMatrix);xlabel('K1');ylabel('K3');zlabel('Overshoot')
mesh(K1Range, K3Range,
  settlingTimeMatrix);xlabel('K1');ylabel('K3');zlabel('SettlingTime')
mesh(K1Range,
  K3Range,disturbanceMatrix);xlabel('K1');ylabel('K3');zlabel('Disturban
      ce');view(3
candidatePairs
```

To examine the correctness of the answer, we select $K_1 = 100$ and $K_3 = 5$ to evaluate more specifically

```
K1=100; K3=5;
CLTF = ((K1+K3*s)*G12)/(1+(K1+K3*s)*G12);
y = step(CLTF, t);
info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02)
plot(t,y);
```

```
info =    RiseTime: 0.0879
          SettlingTime: 0.1560
          SettlingMin: 0.9211
          SettlingMax: 1.0000
          Overshoot: 0
          Undershoot: 0
          Peak: 1.0000
          PeakTime: 0.8000
```

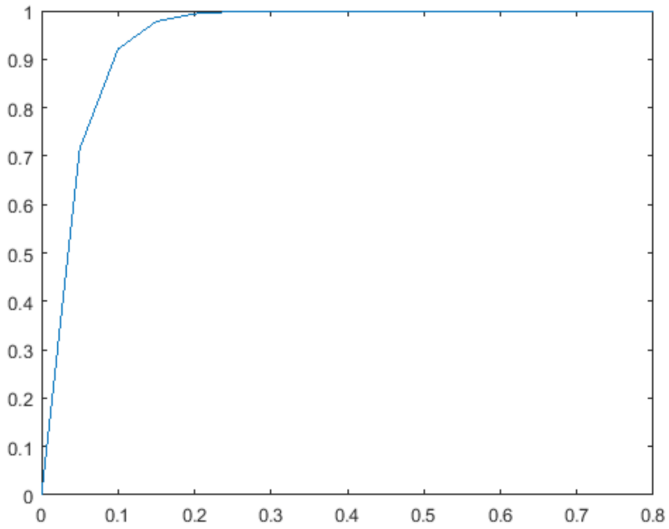


Figure 14 – System overshoot

With the information generated and the curve shown in Figure 14, we can discover that at the same time satisfying other system requirements, the system has no overshoot. The reason is that K_3 acts as a damping factor that prevents the response from overshooting. We can then analyze the disturbance.

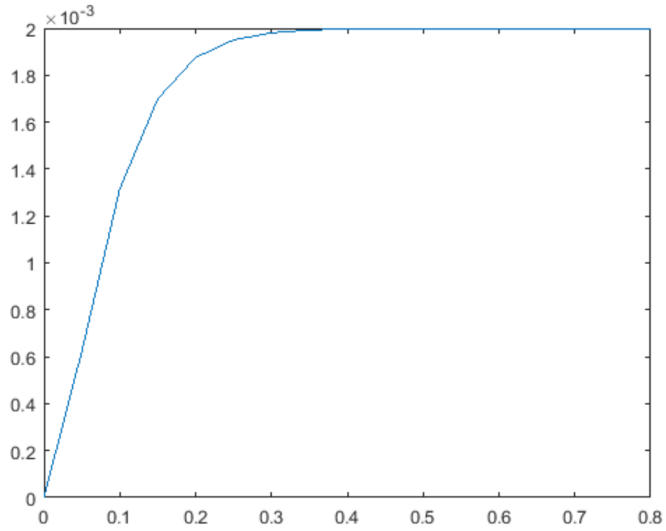


Figure 15 – System disturbance

The maximum disturbance only reaches 2×10^{-3}, within the range of requirement. We can conclude for this task that, compared to the proportional compensator, the PID compensator can be much more effective in controlling the system especially reducing the overshoot.

2.6 Tvask 6 – PID Compensator

In this part, we no longer assume the mathematical model between the flexure spring and the reader to be rigid but has been refined and revised instead. The transfer function from the force from the reader arm to the head position is given by:

$$G_3(s) = \frac{{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2} = \frac{1}{1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{{\omega_n}^2}} \tag{11}$$

where ζ = 0.3 and ω_n = 18850 rad. At the same time, we assume $H(s) = 1$.

Part A

The process of generating the step response is shown below.

```
Ze = 0.3;
Wn = 18850;
G3 = 1/(1+(2*Ze*s)/Wn+(s/Wn)^2);
t = [0:0.00001:0.002];
y = step(G3,t);
plot(t,y);
```

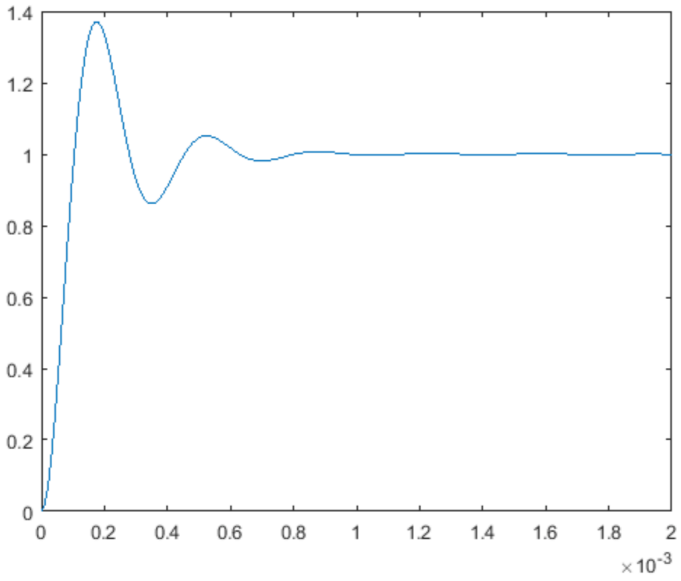


Figure 16

The output is desirable, since when the second order system is introduced, the oscillation of the response is due to the spring.

Part B

We are then going to determine the values of the PID parameters K_1 and K_3. The closed loop transfer function of the system becomes:

$$\frac{F(s)G_1(s)G_2(s)G_3(s)}{1+F(s)G_1(s)G_2(s)G_3(s)} \tag{12}$$

Using similar procedure as the part above, it is found that the resonant frequency is met in region where $K_1 = 40$ and $K_3 = 1$.

```
t = [0:0.05:1];
G123 = G12*G3;

[K1Range K3Range] = meshgrid(35:1:45, 0:0.1:1);
overshootMatrix = [];
settlingTimeMatrix = [];
disturbanceMatrix=[];
candidatePairs = [];
t=[0:0.1:10];
G12= G1*G2;
for K3 = K3Range(:,1)'
    overshootArr=[];
    settlingTimeArr=[];
    disturbanceArr=[];
    for K1 = K1Range(1,:)
        CLTF = ((K1+K3*s)*G123)/(1+(K1+K3*s)*G123);
        y = step(CLTF, t);
        info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02);
        overshootArr = [overshootArr, info.Overshoot];
        settlingTimeArr = [settlingTimeArr, info.SettlingTime];
        if(info.Overshoot <= 5 & info.SettlingTime <0.25)
            candidatePairs = [candidatePairs; K1, K3];
        end

        end

    overshootMatrix = [overshootMatrix; overshootArr];
    settlingTimeMatrix = [settlingTimeMatrix; settlingTimeArr];
end
```

The overshoot plot and the settling time plot around $K_1 = 40$ and $K_3 = 1$ is presented below.

```
figure();
mesh(K1Range, K3Range,
 overshootMatrix);xlabel('K1');ylabel('K3');zlabel('Overshoot')
```

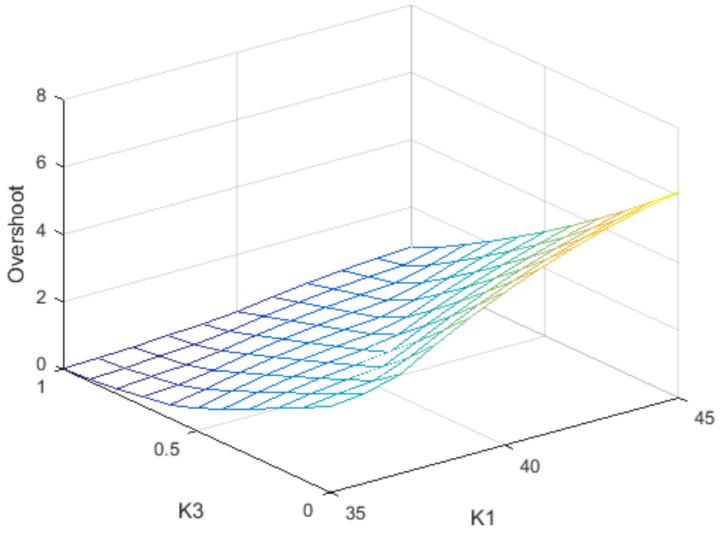


Figure 17 – overshoot

```
figure();

mesh(K1Range, K3Range,
 settlingTimeMatrix);xlabel('K1');ylabel('K3');zlabel('SettlingTime')
```

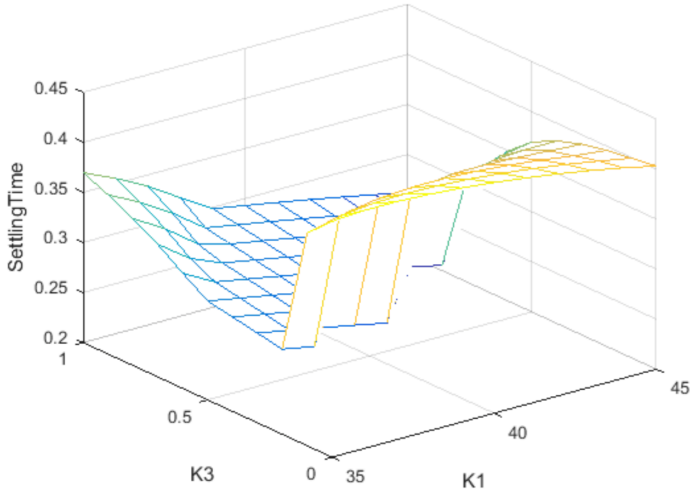


Figure 18 – Settling Time

We are able to find one pair of K_1 and K_3 that are 45.0000 and 0.8000.

```
K1 = 45;
K3 = 0.8;
t=[0:0.01:1];
CLTF = ((K1+K3*s)*G123)/(1+(K1+K3*s)*G123);
figure();
bode(CLTF);
```

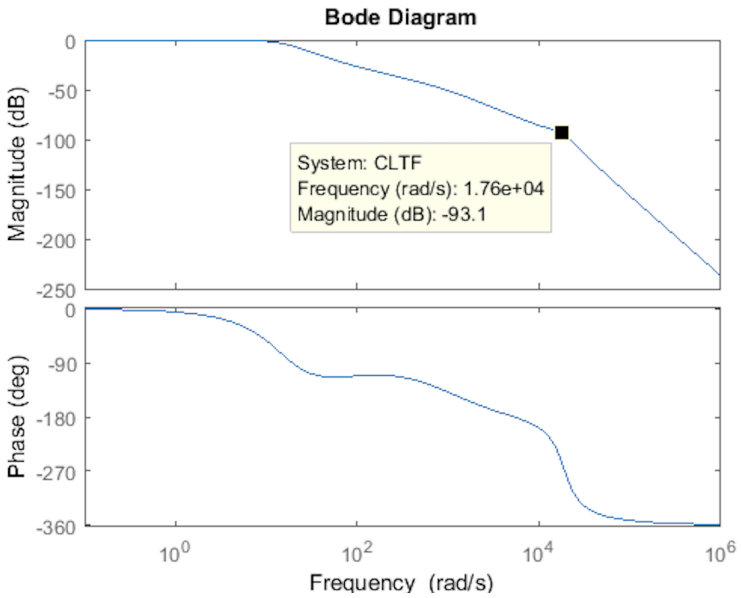


Figure 19

We can then show the -3dB frequency and then plot the unit step response.

```
% The system fulfils the transient requirements
info = stepinfo(y, t, 'SettlingTimeThreshold', 0.02)
bandWidth = 13.6610
info =

    RiseTime: 0.0173
    SettlingTime: 0.0262
    SettlingMin: 0.9356
    SettlingMax: 1.0082
    Overshoot: 0.8169
    Undershoot: 0
    Peak: 1.0082
    PeakTime: 0.0400

y = step(CLTF,t);
figure();
plot(t,y);
```

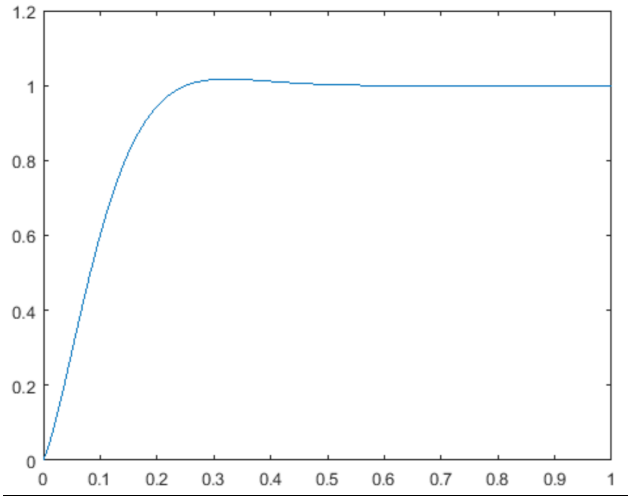


Figure 20

Part C

We can then determine the gain and phase margin of the system determined from Task B.

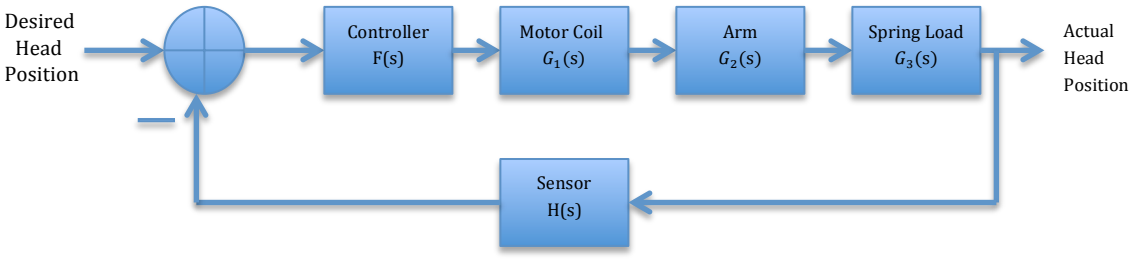
```
[GainMargin, PhaseMargin, Wgm, Wpm] = margin(CLTF);
GainMargin
PhaseMargin
```

```
GainMargin =
6.6390e+03

PhaseMargin = -180
```

3.0 CONCLUTION

The final architecture of the system that satisfies all of the performance requirements is shown below:



Transfer function of each block are presented below:

$$F(s) = K_1 + \frac{K_2}{s} + K_3 s \tag{13}$$

$$G_1(s) = \frac{K_m}{Ls+R} \tag{14}$$

$$G_2(s) = \frac{1}{Js^2+bs} \tag{15}$$

$$G_3(s) = \frac{{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2} = \frac{1}{1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{{\omega_n}^2}} \tag{16}$$

$$H(s) = 1 \tag{17}$$

The closed-loop transfer function for the entire system is:

$$\frac{F(s)G_1(s)G_2(s)G_3(s)}{1+F(s)G_1(s)G_2(s)G_3(s)} \tag{18}$$

Parameters are shown in the table below:

Table 1

Parameters	Value
Inertia of arm and head J	1 $N \cdot m \cdot s^2$/rad
Friction b	20 Kg/m/s
Field resistance R	1 Ω
Field inductance L	0.001 H
Motor constant K_m	5 $N \cdot m/A$
ζ	0.3
ω_n	18850 rad
K_1	45
K_2	0
K_3	0.8

We compared the system performance with the given specifications in **Table 2**

Table 2

Specification Variable Name	Specification Variable Value	Designed System Value
Percent Overshoot	< 5%	0.8169
Settling time(2% deviation)	< 0.250s	0.0262
Maximum value of response to a unit step disturbance	< 0.005	0.0045
Resonant Frequency	-	1.76×10^4
Gain Margin	-	6.639×10^3
Phase Margin	-	-180

In conclusion, this project is quite successful. The final system satisfied all of the performance requirements with desirable resonant frequency, gain margin and phase margin. After comparing the system behavior with proportional compensator or the PID compensator, it is not hard to find that the PID compensator is more effective in controlling a system. It can effectively reduce the over shoot and quickly response.