

Problem 1: Equilibria in a prey-predator model

Consider the following prey-predator model

$$\begin{cases} \dot{x} &= 3x(1-x) - xy - \lambda(1 - e^{-5x}), \\ \dot{y} &= -y + 3xy, \end{cases}$$

where $\lambda \in [0, 1]$ is a parameter and $x, y \geq 0$.

1. Compute with MATCONT branches of nonnegative equilibria in this system in the (x, y, λ) -space.
2. Classify the equilibria by their stability.
3. Identify branching, limit, and Hopf points and study at least one of them analytically.
4. Simulate the system in MATCONT for various $\lambda \in [0, 1]$ and try to produce all its generic phase portraits.
5. Which global bifurcation(s) you expect to happen in the system ?