

# COMP10002 Workshop Week 11

Representation of integers and floats, Ex. 13.1  
File Operations: Ex13.2

1. Try your hand at Exercise 11.3, **time limit: 10 minutes**
2. Assignment 2:
  1. make progress & ask questions,
  2. submit on Gradescope

# A number can be written using different base

Base **10** (decimal system) uses 10 digits 0..9

10 is written using 2 digits: 10

Base **2** (binary) uses 2 digits 0..1

2 is written using 2 digits: 10

Base **16** (hexadecimal) uses 16 digits 0..9, A..F

16 is written using 2 digits: 10

the number

...  $d_3 d_2 d_1 d_0 . d_{-1} d_{-2} \dots$  (in base **B**)

has the value

$$\dots + d_3 \times B^3 + d_2 \times B^2 + d_1 \times B^1 + d_0 + d_{-1} \times B^{-1} + d_{-2} \times B^{-2} \dots$$

Examples:

$$1011.101_{(2)} = ?_{(10)}$$

$$3A.2_{(16)} = ?_{(10)}$$

*Practical advice: for conversions, remember:*

128	64	32	16	8	4	2	1	0.5	0.25	0.125
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$	$2^{-1}$	$2^{-2}$	$2^{-3}$

# Note: Converting between bases 2 and 16 is easy!

$16=2^4 \rightarrow$  1 hexadecimal digit is equivalent to 4 binary digits.

110101101 <sub>(2)</sub>	$\Leftrightarrow$	1 1010 1101 <sub>(2)</sub>	$\Leftrightarrow$	1AD <sub>(16)</sub>
11111011011	$\Leftrightarrow$		$\Leftrightarrow$	?
111.110001	$\Leftrightarrow$		$\Leftrightarrow$	?
?	$\Leftrightarrow$		$\Leftrightarrow$	5F.B6

Notes: hexadecimal is normally used for writing binary numbers in a shorter format

# Exercises: Converting Decimal → Binary

*Recap:*

*To change a decimal  $x$  to binary: represent  $x$  as the sum of power of two.*

*Example:*  $23.375 = 16 + 0 \times 8 + 4 + 2 + 1 + 0 \times 0.5 + 0.25 + 0.125$

$$2^4 + 2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2}$$

*So:*  $23.375 = 10111.011_{(2)}$

Exercises:

132	→	(2)
89.375	→	(2)

# Representation of integers (in computers) using $w$ bits

- Note that we use a fixed amount of bits  $w$
- Make difference between `unsigned` and `signed` integers (`unsigned int` and `int` in C)

## **unsigned** integers :

Range:  $0 \dots 2^w - 1$  ( $0 \dots 255$  for  $w=8$ )

Representation: Just convert to binary, then add `0` to the front to have enough  $w$  bits.

Examples: represent `89` and `257` as unsigned int with  $w=8$

**signed integer** range:  $-2^{w-1}$  to  $2^{w-1}-1$   
(-128 .. +127 for  $w=8$ )  
( $-2^{31}$  ..  $+2^{31}-1$  for  $w=32$ )

- To represent *signed* integers  $x$ :
  - Positive numbers ( $x \geq 0$ ):
    - just like *unsigned*: the binary representation of  $x$  in  $w$  bits,
    - $\rightarrow$  the first bit will always be 0.
  - Popular Method for negative numbers ( $x < 0$ ):
    - using *twos-complement* of  $|x|$  in  $w$  bit,
    - $\rightarrow$  the first bit will always be 1.

# Negative int: Finding twos-complement representation in w bits in 3 steps

*Suppose that we need to find the twos-complement representation for  $-x$ , where  $x$  is positive, in  $w=16$  bits. Do it in 3 steps:*

- 1) Write binary representation of  $|x|$  in  $w$  bits*
- 2) Find the **rightmost one-bit***
- 3) Inverse (ie. flip 1 to 0, 0 to 1) all bits on the left of that **rightmost one-bit***

<i>find the 2-comp repr of -40</i>	Bit sequence			
1) bin repr of 40 in 16 bits	0000	0000	0010	1000
2) find the <b>rightmost 1</b>	0000	0000	0010	<b>1000</b>
3) inverse its left	1111	1111	1101	<b>1000</b>

**Why?** Think about finding  $(-x)$  so that  $x + (-x) = 0$ , where  $x$  is a bit pattern.

## Ex. 13.01

*Suppose that a computer uses  $w=6$  bits to represent integers. Calculate the two-complement representations for 0, 4, 19, -1, -8, and -31; Verify that  $19-8 = 11$ ;*

value	representation
0	
4	
19	
35	
-1	
-8	
-31	

verify!		
+	19	
	-8	
=	11	



Many different ways! We learnt 2 formats:

- *format-1*: using 16 bits, as described in [lec09.pdf](#) and in the text book
- *format-2* : using 32 bits, which is an [IEEE standard](#), that
  - is employed in most of modern computers for type `int`,
  - is demonstrated in the lecture, and
  - you can experiment with using the program `floatbits.c` ([lec09.pdf](#), around page 26).

**General Principle** for representing a `float x`? (for example,  $x = -20.75$ ) :

- to get rid of the sign (+ or -) and the dot (.) by replacing `x` with 3 *integers*: sign `s`, mantissa `m`, exponent `e` so that:
  - `s` is 0 (for positive) or 1 (for negative)
  - $|x|$  is equivalent to  $0.m \times 2^e$  (*format-1*) or  $1.m \times 2^e$  (*format-2*)

# Representation of floats (format-1: w=16, as described in lec09.pdf)

$x = (\text{sign}, e, m)$



1 bit  
sign

$w_e$  bits  
two-compl of  $e$

$w_m$  bits  
representation of first  $n$  bits of mantissa  $m$

Convert  $|x|$  to binary form, and transform so that:

$$|x| = 0.b_0b_1b_2... \times 2^e \text{ where } b_0 = 1$$

$e$  is called exponent,  $m = b_0b_1b_2...$  is called mantissa

$x$  is represented as the triple  $(\text{sign}, e, m)$  as shown in the diagram.

$$|x| \text{ is equivalent to } 0.m \times 2^e$$

## Ex. 13.02

Suppose  $w_s=1$ ,  $w_e=3$ ,  $w_m=12$ , what's the representation of 2.0, -2.5, 7.875 ?

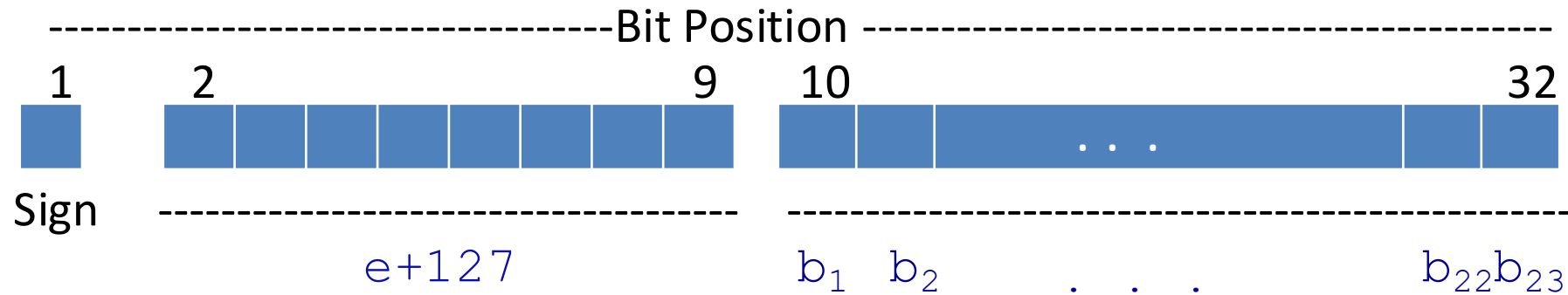
value	binary form	S= ?	m= ?	e= ?	representation
2.0	10.0				
-0.375					
7.875					

# Representation of 32-bit float: (format-2: IEEE 754, as in floatbits.c )

$$w_s=1, w_e=8, w_m=23$$

$$|x| = 1.b_1b_2... \times 2^e$$

$$|x| \text{ is equivalent to } 1.m \times 2^e$$



Here:

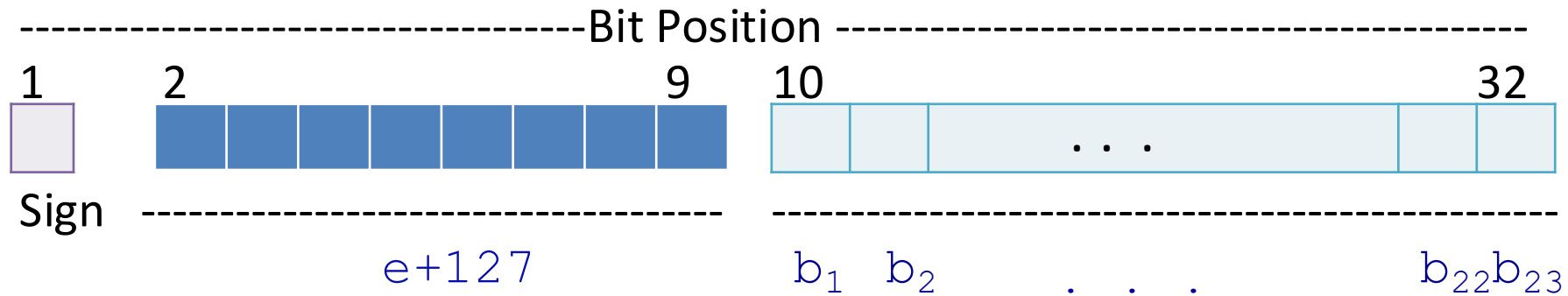
- The sign bit is 0 or 1 as in the previous case
- $e$  is represented in the *excess-127* format: as the unsigned value  $e+127$  in  $w_e=8$  bits
- The integer part 1 is omitted from the representation, and the mantissa is represented as just  $b_1b_2...b_{23}$

**Class Exercise:** IEEE representation for -3.375, 0.125

**Note:** There are 256 possible values for  $e$ . But valid  $e$  is  $-126 \rightarrow +127$ , corresponding to bit patterns values 0000 0001  $\rightarrow$  1111 1110

# Representation of 32-bit float: (IEEE 754, as in floatbits.c )

$$w_e=8$$



Un-important note on the exponent part:

- Valid  $e$  is  $-126 \rightarrow +127$ , corresponding to bit patterns  $0000\ 0001 \rightarrow 1111\ 1110$
- Pattern  $0000\ 0000$  used for representing  $0.0$ ,
- Pattern  $1111\ 1111$  used to represent *infinity*,
- And,  $0.0$  is all 32 *zero-bit*, and *infinity* is all 32 *one-bit*.

**Class Exercise:** IEEE representation for 3.5

# Representation of 32-bit float: (IEEE 754, as in floatbits.c )

- $w_s=1, w_e=8, w_m=23$
- $|x| = 1.m \times 2^e$

Example:  $x = -3.5$

In binary:  $x = -11.1 \rightarrow |x| = 1.11 \times 2^1$

→ sign bit: 1

→  $e=1$  is represented as  $e+127=128$  in 8 bits

→  $e$  is represented as

→  $m=23$ -bit mantissa: 110 0000 0000 0000 0000 0000

→ Final representation:

0100 0000 0110 0000 0000 0000 0000 0000

or 4 0 6 0 0 0 0 0 (16)

## LAB TIME: Assignment 2 + exercise 11.3

- **2b|~2b** : do exercise 11.3 in less than 10 minutes
- Work on assignment 2, ask questions

**Ex 11.3:** The Unix `tee` command writes its `stdin` through to `stdout` in the same way that the `cat` command does. But it also creates an additional copy of the file into each of the filenames listed on the command-line when it is executed.

Implement a simple version of this command.

Hint: you will need an array of files all opened for writing.

```
./program file1 file2
```

```
Hello world!
```

```
[^D]
```

```
Hello world!
```

[The program will also create two files named `file1` and `file2`, both containing "Hello world!\n" as the file content.]

```
// here is an implementation of command cat
// change it to that for tee

int main(int argc, char *argv[]) {
    char c;

    while ( (c=getchar()) != EOF) {
        putchar(c);
    }

    return 0;
}
```



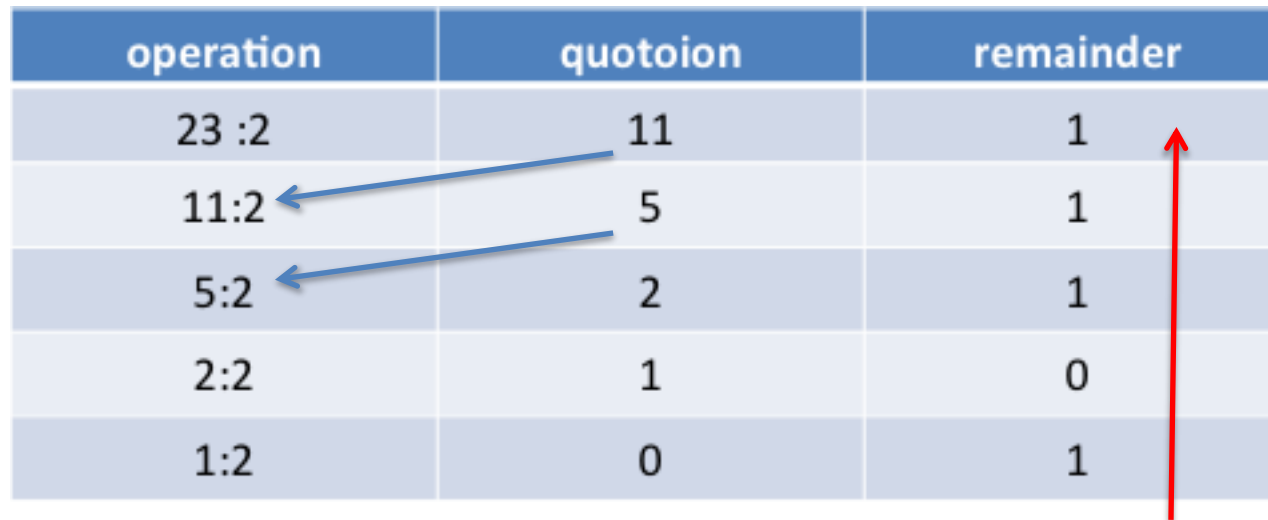


# Decimal → Binary: A procedure for converting Integer Part

*Changing integer  $x$  to binary: Just divide  $x$  and the subsequent quotients by 2 until getting zero. The sequence of remainders, **in reverse order** of appearance, is the binary form of  $x$ .*

*Example: 23*

operation	quotient	remainder
23 :2	11	1
11:2	5	1
5:2	2	1
2:2	1	0
1:2	0	1



So:  $23 = 10111_{(2)}$   $11 = 1011_{(2)}$   $46 = 101110_{(2)}$

# Decimal → Binary: General Method for the Fraction Part

*Problem: Converting fraction could be complicated*

$$0.1 = 0 \times 0.5 + 0 \times 0.25 + 0 \times 0.125 + 0.0625 + 0.0375 (= \dots)$$

**Easy method:** Multiply it, and subsequent fractions, by 2 until getting zero.  
*Result= sequence of integer parts of results, in appearance order. Examples:*

0.375			0.1		
operation	int	fraction	operation	int	fraction
.375 x 2	0	.75	.1 x 2	0	.2
.75 x 2	1	.5	.2 x 2	0	.4
.5 x 2	1	.0	.4 x 2	0	.8
			.8 x 2	1	.6
			.6 x 2	1	.2

So:  $0.375 = 0.011_{(2)}$

$0.1 = 0.00011(0011)_{(2)}$