

# COMP10002 Workshop Week 11

Representation of integers and floats, Ex. 13.1, 13.2  
File Operations

1. File Operations: Try your hand at Exercise 11.3, **time limit: 10 minutes**
2. Assignment 2:
  1. make progress & ask questions,
  2. submit on Gradescope

# A number can be written using different base

Base 10 (decimal system) uses 10 digits 0..9

10 is written using 2 digits: 10

Base 2 (binary) uses 2 digits 0..1

2 is written using 2 digits: 10

Base 16 (hexadecimal) uses 16 digits 0..9, A..F

16 is written using 2 digits: 10

the number

...  $d_3 \ d_2 \ d_1 \ d_0.$   $d_{-1} \ d_{-2} \dots$  (B) (in base B)

has the value

... +  $d_3 \times B^3 + d_2 \times B^2 + d_1 \times B^1 + d_0 + d_{-1} \times B^{-1} + d_{-2} \times B^{-2} \dots$

Examples:

$$1011.101_{(2)} = ?_{(10)}$$

$$3A.2_{(16)} = ?_{(10)}$$

Practical advice: for conversions, remember:

|       |       |       |       |       |       |       |       |          |          |          |
|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|
| 128   | 64    | 32    | 16    | 8     | 4     | 2     | 1     | 0.5      | 0.25     | 0.125    |
| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ | $2^{-1}$ | $2^{-2}$ | $2^{-3}$ |

# Note: Converting between bases 2 and 16 is easy!

$16=2^4 \rightarrow$  1 hexadecimal digit is equivalent to 4 binary digits.

|                   |                   |                       |                   |              |
|-------------------|-------------------|-----------------------|-------------------|--------------|
| $110101101_{(2)}$ | $\Leftrightarrow$ | $1\ 1010\ 1101_{(2)}$ | $\Leftrightarrow$ | $1AD_{(16)}$ |
| $11111011011$     | $\Leftrightarrow$ |                       | $\Leftrightarrow$ | ?            |
| $111.110001$      | $\Leftrightarrow$ |                       | $\Leftrightarrow$ | ?            |
| ?                 | $\Leftrightarrow$ |                       | $\Leftrightarrow$ | 5F.B6        |

Notes: hexadecimal is normally used for writing binary numbers in a shorter format

# Exercises: Converting Decimal → Binary

Recap:

To change a decimal  $x$  to binary: represent  $x$  as the sum of power of two.

Example:  $23.375 = 16 + 0 \times 8 + 4 + 2 + 1 + 0 \times 0.5 + 0.25 + 0.125$

$$2^4 + 2^2 + 2^1 + 2^0 + + 2^{-1} + 2^{-2}$$

So:  $23.375 =$

$$10111.011_{(2)}$$

Exercises:  $132 \rightarrow_{(2)}$   
 $89.375 \rightarrow_{(2)}$

# Representation of integers (in computers) using $w$ bits

- Note that we use a fixed amount of bits  $w$
- Make difference between `unsigned` and `signed` integers  
(`unsigned int` and `int` in C)

## `unsigned` integers :

Range:  $0 \dots 2^w - 1$       ( $0 \dots 255$  for  $w=8$ )

Representation: Just convert to binary, then add `0` to the front to have enough  $w$  bits.

Examples: represent `89` and `257` as `unsigned int` with  $w=8$

# Representation of signed integers using w bits

**signed integer** range:  $-2^{w-1}$  to  $2^{w-1}-1$   
 $(-128 \dots +127 \text{ for } w=8)$   
 $(-2^{31} \dots +2^{31}-1 \text{ for } w=32)$

- To represent **signed** integers  $x$ :
  - Positive numbers ( $x \geq 0$ ):
    - just like **unsigned**: the binary representation of  $x$  in  $w$  bits,
    - → the first bit will always be  $0$ .
  - Popular Method for negative numbers ( $x < 0$ ):
    - using *twos-complement* of  $|x|$  in  $w$  bit,
    - → the first bit will always be  $1$ .

## Negative int: Finding twos-complement representation in w bits in 3 steps

Suppose that we need to find the twos-complement representation for  $-x$ , where  $x$  is positive, in  $w=16$  bits. Do it in 3 steps:

- 1) Write binary representation of  $|x|$  in  $w$  bits
- 2) Find the **rightmost one-bit**
- 3) Inverse (ie. flip 1 to 0, 0 to 1) all bits on the left of that **rightmost one-bit**

| <i>find the 2-comp repr of -40</i> | Bit sequence |      |      |             |
|------------------------------------|--------------|------|------|-------------|
| 1) bin repr of 40 in 16 bits       | 0000         | 0000 | 0010 | 1000        |
| 2) find the <b>rightmost 1</b>     | 0000         | 0000 | 0010 | <b>1000</b> |
| 3) inverse its left                | 1111         | 1111 | 1101 | <b>1000</b> |

**Why?** Think about finding  $(-x)$  so that  $x + (-x) = 0$ , where  $x$  is a bit pattern.

## Ex. 13.01

Suppose that a computer uses  $w= 6$  bits to represent integers. Calculate the two-complement representations for 0, 4, 19, -1, -8, and -31; Verify that  $19-8 = 11$ ;

| value | representation |
|-------|----------------|
| 0     |                |
| 4     |                |
| 19    |                |
| 35    |                |
| -1    |                |
| -8    |                |
| -31   |                |

| verify! |    |    |
|---------|----|----|
|         | 19 |    |
| +       |    | -8 |
| =       | 11 |    |

# Representation of floats

Many different ways! We learnt 2 formats:

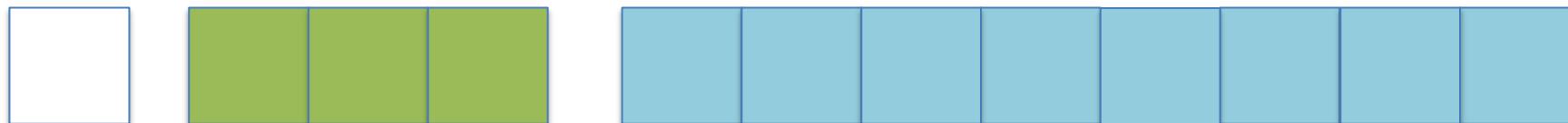
- *format-1*: using 16 bits, as described in [lec09.pdf](#) and in the text book
- *format-2* : using 32 bits, which is an [IEEE standard](#), that
  - is employed in most of modern computers for type `int`,
  - is demonstrated in the lecture, and
  - you can experiment with using the program `floatbits.c` ([lec09.pdf](#), around page 26).

**General Principle** for representing a float  $x$ ? (for example,  $x = -20.75$ ) :

- to get rid of the sign (+ or -) and the dot (.) by replacing  $x$  with 3 integers: sign  $s$ , mantissa  $m$ , exponent  $e$  so that:
  - $s$  is 0 (for positive) or 1 (for negative)
  - $|x|$  is equivalent to  $0.m \times 2^e$  (*format-1*) or  $1.m \times 2^e$  (*format-2*)

## Representation of floats (format-1: w=16, as described in lec09.pdf)

$x = (\text{sign}, \quad e \quad , \quad m \quad )$



1 bit

sign

$w_e$  bits

two-compl of  $e$

$w_m$  bits

representation of first  $n$  bits of mantissa  $m$

Convert  $|x|$  to binary form, and transform so that:

$$|x| = 0.b_0b_1b_2\dots \times 2^e \text{ where } b_0 = 1$$

$e$  is called exponent,  $m = b_0b_1b_2\dots$  is called mantissa

$x$  is represented as the triple  $(\text{sign}, \ e, \ m)$  as shown in the diagram.

$|x|$  is equivalent to  $0.m \times 2^e$

## Ex. 13.02

Suppose  $w_s=1$ ,  $w_e=3$ ,  $w_m=12$ , what's the representation of 2.0, -2.5, 7.875 ?

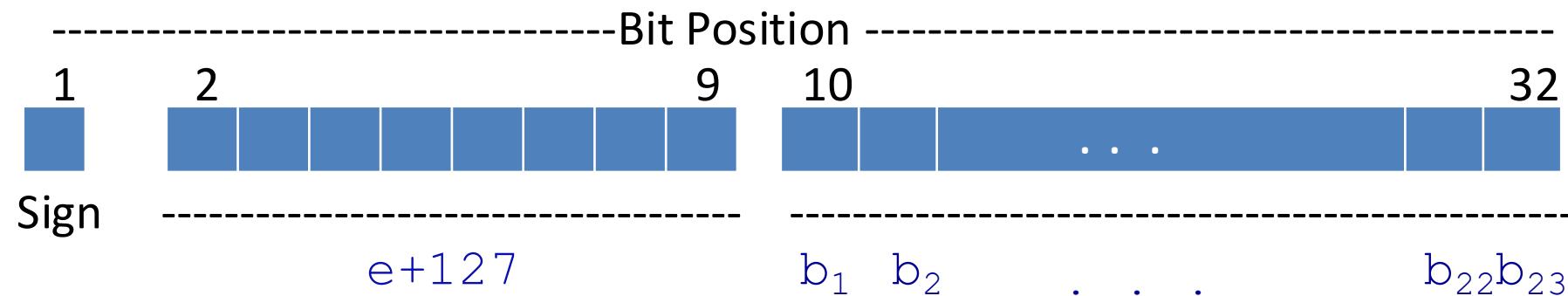
| value  | binary form | s= ? | m= ? | e= ? | representation |
|--------|-------------|------|------|------|----------------|
| 2.0    | 10.0        |      |      |      |                |
| -0.375 |             |      |      |      |                |
| 7.875  |             |      |      |      |                |

# Representation of 32-bit float: (format-2: IEEE 754, as in floatbits.c )

$w_s=1$ ,  $w_e=8$ ,  $w_m=23$

$$|x| = 1.b_1b_2\dots \times 2^e$$

$|x|$  is equivalent to  $1.m \times 2^e$



Here:

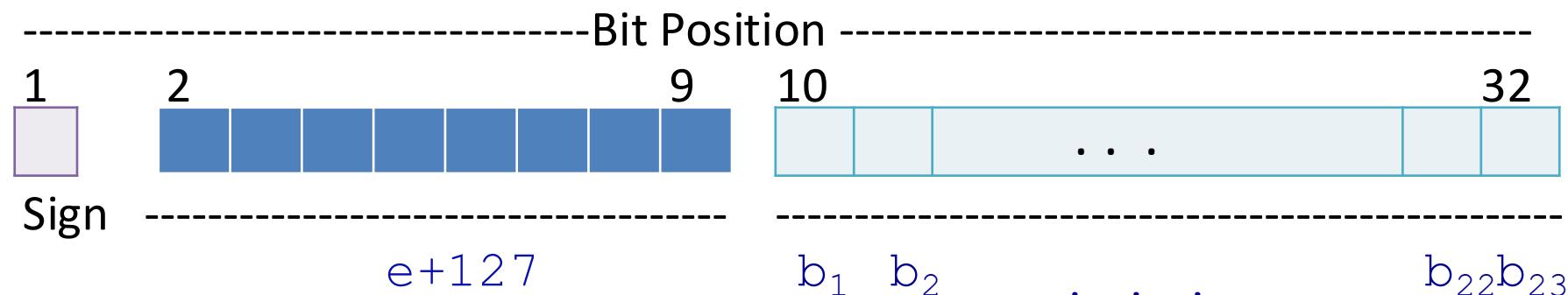
- The sign bit is 0 or 1 as in the previous case
- $e$  is represented in the excess-127 format: as the unsigned value  $e+127$  in  $w_e=8$  bits
- The integer part 1 is omitted from the representation, and the mantissa is represented as just  $b_1b_2\dots b_{23}$

**Class Exercise:** IEEE representation for -3.375, 0.125

**Note:** There are 256 possible values for  $e$ . But valid  $e$  is  $-126 \rightarrow +127$ , corresponding to bit patterns values 0000 0001  $\rightarrow$  1111 1110

# Representation of 32-bit float: (IEEE 754, as in floatbits.c )

$w_e=8$



Un-important note on the exponent part:

- Valid  $e$  is -126  $\rightarrow$  +127, corresponding to bit patterns `0000 0001`  $\rightarrow$  `1111 1110`
- Pattern `0000 0000` used for representing `0.0`,
- Pattern `1111 1111` used to represent `infinity`,
- And, `0.0` is all 32 zero-bit, and `infinity` is all 32 one-bit.

**Class Exercise:** IEEE representation for 3.5

## Representation of 32-bit float: (IEEE 754, as in floatbits.c )

- $w_s=1, w_e=8, w_m=23$
- $|x| = 1.m \times 2^e$

Example:  $x = -3.5$

In binary:  $x = -11.1 \rightarrow |x| = 1.11 \times 2^1$

→ sign bit: 1

→  $e=1$  is represented as  $e+127=128$  in 8 bits

→ e is represented as

→ m= 23-bit mantissa: 110 0000 0000 0000 0000 0000

→ Final representation:

0100 0000 0110 0000 0000 0000 0000

or 4 0 6 0 0 0 0 0 (16)

# LAB TIME: Assignment 2 + exercise 11.3

- **2b|~2b** : do exercise 11.3 in less than 10 minutes
- Work on assignment 2, ask questions

# LAB: (Do Assignment 2 || Do Ex 11.3) && Q&A for Assignment 2

**Ex 11.3:** The Unix `tee` command writes its `stdin` through to `stdout` in the same way that the `cat` command does. But it also creates an additional copy of the file into each of the filenames listed on the command-line when it is executed.

Implement a simple version of this command.

Hint: you will need an array of files all opened for writing.

```
./program file1 file2
```

```
Hello world!
```

```
[^D]
```

```
Hello world!
```

[The program will also create two files named `file1` and `file2`, both containing "Hello world!\n" as the file content.]

```
// here is an implementation of command cat
// change it to that for tee
```

```
int main(int argc, char *argv[]) {
    char c;

    while ( (c=getchar()) != EOF) {
        putchar(c);
    }

    return 0;
}
```

# Additional Slides

# Decimal $\rightarrow$ Binary: A procedure for converting Integer Part

*Changing integer  $x$  to binary: Just divide  $x$  and the subsequent quotients by 2 until getting zero. The sequence of remainders, in reverse order of appearance, is the binary form of  $x$ .*

*Example:* 23

| operation | quoation | remainder |
|-----------|----------|-----------|
| $23 : 2$  | 11       | 1         |
| $11 : 2$  | 5        | 1         |
| $5 : 2$   | 2        | 1         |
| $2 : 2$   | 1        | 0         |
| $1 : 2$   | 0        | 1         |

$$\text{So: } 23 = 10111_{(2)} \quad 11 = ?_{(2)} \quad 46 = ?_{(2)}$$

# Decimal → Binary: General Method for the Fraction Part

*Problem: Converting fraction could be complicated*

$$0.1 = 0 \times 0.5 + 0 \times 0.25 + 0 \times 0.125 + 0.0625 + 0.0375 (= \dots)$$

**Easy method:** Multiply it, and subsequent fractions, by 2 until getting zero.  
Result = sequence of integer parts of results, in appearance order. Examples:

| 0.375     |     |          | 0.1       |     |          |
|-----------|-----|----------|-----------|-----|----------|
| operation | int | fraction | operation | int | fraction |
| .375 × 2  | 0   | .75      | .1 × 2    | 0   | .2       |
| .75 × 2   | 1   | .5       | .2 × 2    | 0   | .4       |
| .5 × 2    | 1   | .0       | .4 × 2    | 0   | .8       |
|           |     |          | .8 × 2    | 1   | .6       |
|           |     |          | .6 × 2    | 1   | .2       |

$$\text{So: } 0.375 = 0.\mathbf{011}_{(2)}$$

$$0.1 = 0.\mathbf{00011(0011)}_{(2)}$$

part inside ( ) is repeated indefinitely