

COMP10002 Workshop Week 11

Representation of integers and floats, Ex. 13.1
File Operations: Ex13.2

1. Try your hand at Exercise 11.3, **time limit: 10 minutes**
2. Assignment 2:
 1. make progress & ask questions,
 2. submit on Gradescope

A number can be written using different base

Base **10** (decimal system) uses 10 digits 0..9

10 is written using 2 digits: 10

Base **2** (binary) uses 2 digits 0..1

2 is written using 2 digits: 10

Base **16** (hexadecimal) uses 16 digits 0..9, A..F

16 is written using 2 digits: 10

the number

... $d_3 d_2 d_1 d_0 . d_{-1} d_{-2} \dots$ (in base **B**)

has the value

$$\dots + d_3 \times B^3 + d_2 \times B^2 + d_1 \times B^1 + d_0 + d_{-1} \times B^{-1} + d_{-2} \times B^{-2} \dots$$

Examples:

$$1011.101_{(2)} = ?_{(10)}$$

$$3A.2_{(16)} = ?_{(10)}$$

Practical advice: for conversions, remember:

128	64	32	16	8	4	2	1	0.5	0.25	0.125
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

Note: Converting between bases 2 and 16 is easy!

$16=2^4 \rightarrow$ 1 hexadecimal digit is equivalent to 4 binary digits.

110101101 ₍₂₎	\Leftrightarrow	1 1010 1101 ₍₂₎	\Leftrightarrow	1AD ₍₁₆₎
11111011011	\Leftrightarrow		\Leftrightarrow	?
111.110001	\Leftrightarrow		\Leftrightarrow	?
?	\Leftrightarrow		\Leftrightarrow	5F.B6

Notes: hexadecimal is normally used for writing binary numbers in a shorter format

Exercises: Converting Decimal → Binary

Recap:

To change a decimal x to binary: represent x as the sum of power of two.

Example: $23.375 = 16 + 0 \times 8 + 4 + 2 + 1 + 0 \times 0.5 + 0.25 + 0.125$

$$2^4 + 2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2}$$

So: $23.375 = 10111.011_{(2)}$

Exercises:

132	→	(2)
89.375	→	(2)

Representation of integers (in computers) using w bits

- Note that we use a fixed amount of bits w
- Make difference between `unsigned` and `signed` integers (`unsigned int` and `int` in C)

unsigned integers :

Range: $0 \dots 2^w - 1$ ($0 \dots 255$ for $w=8$)

Representation: Just convert to binary, then add `0` to the front to have enough w bits.

Examples: represent `89` and `257` as unsigned int with $w=8$

signed integer range: -2^{w-1} to $2^{w-1}-1$
(-128 .. +127 for $w=8$)
(-2^{31} .. $+2^{31}-1$ for $w=32$)

- To represent *signed* integers x :
 - Positive numbers ($x \geq 0$):
 - just like *unsigned*: the binary representation of x in w bits,
 - \rightarrow the first bit will always be 0.
 - Popular Method for negative numbers ($x < 0$):
 - using *twos-complement* of $|x|$ in w bit,
 - \rightarrow the first bit will always be 1.

Negative int: Finding twos-complement representation in w bits in 3 steps

Suppose that we need to find the twos-complement representation for $-x$, where x is positive, in $w=16$ bits. Do it in 3 steps:

- 1) Write binary representation of $|x|$ in w bits*
- 2) Find the **rightmost one-bit***
- 3) Inverse (ie. flip 1 to 0, 0 to 1) all bits on the left of that **rightmost one-bit***

<i>find the 2-comp repr of -40</i>	Bit sequence			
1) bin repr of 40 in 16 bits	0000	0000	0010	1000
2) find the rightmost 1	0000	0000	0010	1000
3) inverse its left	1111	1111	1101	1000

Why? Think about finding $(-x)$ so that $x + (-x) = 0$, where x is a bit pattern.

Ex. 13.01

Suppose that a computer uses $w=6$ bits to represent integers. Calculate the two-complement representations for 0, 4, 19, -1, -8, and -31; Verify that $19-8 = 11$;

value	representation
0	
4	
19	
35	
-1	
-8	
-31	

verify!		
+	19	
	-8	
=	11	

Representation of floats

Many different ways! We learnt 2 formats:

- *format-1*: using 16 bits, as described in [lec09.pdf](#) and in the text book
- *format-2* : using 32 bits, which is an [IEEE standard](#), that
 - is employed in most of modern computers for type `int`,
 - is demonstrated in the lecture, and
 - you can experiment with using the program `floatbits.c` ([lec09.pdf](#), around page 26).

General Principle for representing a `float x`? (for example, $x = -20.75$) :

- to get rid of the sign (+ or -) and the dot (.) by replacing `x` with 3 *integers*: sign `s`, mantissa `m`, exponent `e` so that:
 - `s` is 0 (for positive) or 1 (for negative)
 - $|x|$ is equivalent to $0.m \times 2^e$ (*format-1*) or $1.m \times 2^e$ (*format-2*)

Representation of floats (format-1: w=16, as described in lec09.pdf)

$x = (\text{sign}, e, m)$



1 bit
sign

w_e bits
two-compl of e

w_m bits
representation of first n bits of mantissa m

Convert $|x|$ to binary form, and transform so that:

$$|x| = 0.b_0b_1b_2... \times 2^e \text{ where } b_0 = 1$$

e is called exponent, $m = b_0b_1b_2...$ is called mantissa

x is represented as the triple (sign, e, m) as shown in the diagram.

$$|x| \text{ is equivalent to } 0.m \times 2^e$$

Ex. 13.02

Suppose $w_s=1$, $w_e=3$, $w_m=12$, what's the representation of 2.0, -2.5, 7.875 ?

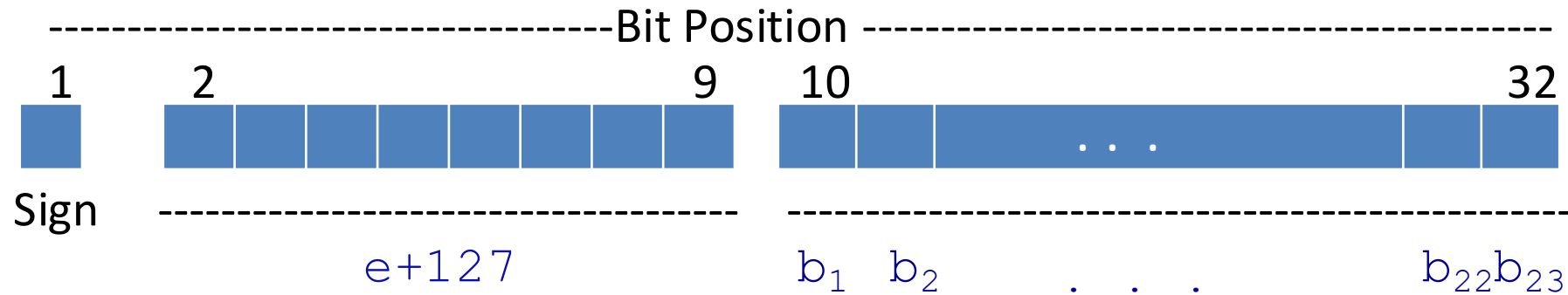
value	binary form	S= ?	m= ?	e= ?	representation
2.0	10.0				
-0.375					
7.875					

Representation of 32-bit float: (format-2: IEEE 754, as in floatbits.c)

$$w_s=1, w_e=8, w_m=23$$

$$|x| = 1.b_1b_2... \times 2^e$$

$$|x| \text{ is equivalent to } 1.m \times 2^e$$



Here:

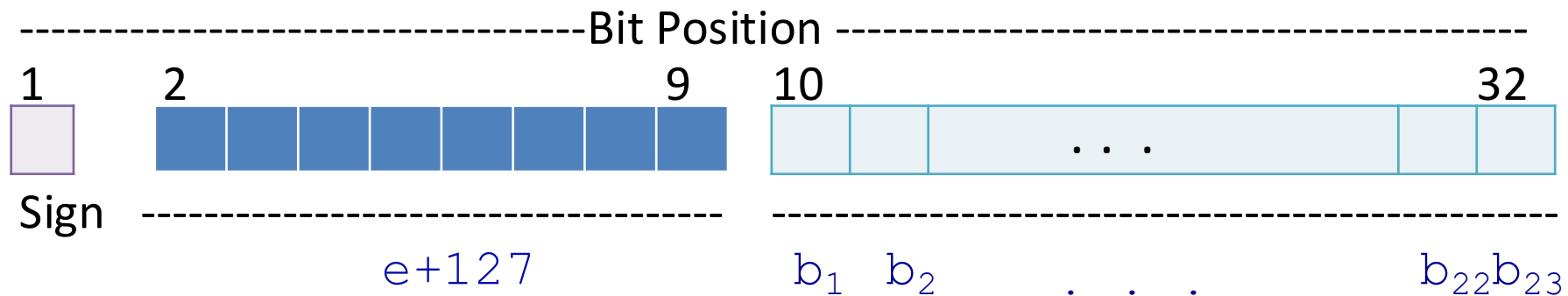
- The sign bit is 0 or 1 as in the previous case
- e is represented in the *excess-127* format: as the unsigned value $e+127$ in $w_e=8$ bits
- The integer part 1 is omitted from the representation, and the mantissa is represented as just $b_1b_2...b_{23}$

Class Exercise: IEEE representation for -3.375, 0.125

Note: There are 256 possible values for e . But valid e is $-126 \rightarrow +127$, corresponding to bit patterns values 0000 0001 \rightarrow 1111 1110

Representation of 32-bit float: (IEEE 754, as in floatbits.c)

$$w_e=8$$



Un-important note on the exponent part:

- Valid e is $-126 \rightarrow +127$, corresponding to bit patterns $0000\ 0001 \rightarrow 1111\ 1110$
- Pattern $0000\ 0000$ used for representing 0.0 ,
- Pattern $1111\ 1111$ used to represent *infinity*,
- And, 0.0 is all 32 *zero-bit*, and *infinity* is all 32 *one-bit*.

Class Exercise: IEEE representation for 3.5

Representation of 32-bit float: (IEEE 754, as in floatbits.c)

- $w_s=1, w_e=8, w_m=23$
- $|x| = 1.m \times 2^e$

Example: $x = -3.5$

In binary: $x = -11.1 \rightarrow |x| = 1.11 \times 2^1$

→ sign bit: 1

→ $e=1$ is represented as $e+127=128$ in 8 bits

→ e is represented as

→ $m=23$ -bit mantissa: 110 0000 0000 0000 0000 0000

→ Final representation:

0100 0000 0110 0000 0000 0000 0000 0000

or 4 0 6 0 0 0 0 0 (16)

LAB TIME: Assignment 2 + exercise 11.3

- **2b|~2b** : do exercise 11.3 in less than 10 minutes
- Work on assignment 2, ask questions

Ex 11.3: The Unix `tee` command writes its `stdin` through to `stdout` in the same way that the `cat` command does. But it also creates an additional copy of the file into each of the filenames listed on the command-line when it is executed.

Implement a simple version of this command.

Hint: you will need an array of files all opened for writing.

```
./program file1 file2
```

```
Hello world!
```

```
[^D]
```

```
Hello world!
```

[The program will also create two files named `file1` and `file2`, both containing "Hello world!\n" as the file content.]

```
// here is an implementation of command cat
// change it to that for tee

int main(int argc, char *argv[]) {
    char c;

    while ( (c=getchar()) != EOF) {
        putchar(c);
    }

    return 0;
}
```


Decimal → Binary: A procedure for converting Integer Part

*Changing integer x to binary: Just divide x and the subsequent quotients by 2 until getting zero. The sequence of remainders, **in reverse order** of appearance, is the binary form of x .*

Example: 23

operation	quotient	remainder
23 :2	11	1
11:2	5	1
5:2	2	1
2:2	1	0
1:2	0	1

So: $23 = 10111_{(2)}$ $11 = 1011_{(2)}$ $46 = 101110_{(2)}$

Decimal → Binary: General Method for the Fraction Part

Problem: Converting fraction could be complicated

$$0.1 = 0 \times 0.5 + 0 \times 0.25 + 0 \times 0.125 + 0.0625 + 0.0375 (= \dots)$$

Easy method: Multiply it, and subsequent fractions, by 2 until getting zero.
Result= sequence of integer parts of results, in appearance order. Examples:

0.375			0.1		
operation	int	fraction	operation	int	fraction
.375 x 2	0	.75	.1 x 2	0	.2
.75 x 2	1	.5	.2 x 2	0	.4
.5 x 2	1	.0	.4 x 2	0	.8
			.8 x 2	1	.6
			.6 x 2	1	.2

So: $0.375 = 0.011_{(2)}$

$0.1 = 0.00011(0011)_{(2)}$

part inside () is repeated indefinitely