

COMP 550 – Algorithms and Analysis

Spring 2023-Assignment03

Issue Date: February 20, 2023

Due Date: Monday, February 27, 11:55 pm

Marks: 5

Student Name: _____

Student PID: _____

Submission:

- You must upload your submission(s) before the deadline in Gradescope.
- Please ensure that your answers are within the given space allocated after the question.

Rules for ALL HWs:

You are encouraged to discuss the homework assignments and study together in groups, but when it comes to formulating/writing solutions **you must work alone and independently**. If required, you should be able to explain your answer clearly to TAs/LAs. Copying homework solutions from another student, from the Internet, solution sets of friends, or other sources will be considered cheating and treated accordingly.

1 Asymptotic notation

1. Prove that if $f(n) \in O(g(n))$ then $g(n) \in \Omega(f(n))$

2. Prove that $o(f(n)) \cap \omega(f(n)) = \emptyset$

2 Substitution method

1. $T(n) = 3T(\frac{n}{\sqrt{2}}) + O(n^4)$ First use the master method to find the Upper bound (find O), and then use the Substitution method to prove it is correct.

$$T(n) = \begin{cases} 3T(\frac{n}{\sqrt{2}}) + O(n^4) & n \geq 2 \\ 1 & n < 2 \end{cases}$$

3 Probability

1. Consider a jar with white and black balls, where the number of black balls is even. We are given that the probability that two balls, picked at random from the jar, are both white is $\frac{1}{2}$. Calculate the minimum number of balls in the jar.

2. Suppose there are A black balls and B white balls in a jar. We randomly pick a ball from the jar and put it back until we have a white ball. Denote X as the number of balls we have picked. What would be the distribution and expectation of X ? (The distribution is the general formula for $P(x = k)$ for each $k \in \mathbb{N}$). (You should derive the expectation from the definition).

