STOR415: Introduction to Optimization

Department of Statistics and Operations Research

———————— Fall 2022 ————————

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Homework 8: Linear Programming Applications

Each problem should be solved in a separate Jupyter notebook.

- Create an ipynb with exactly the same name as is required in the problem. In the Julia code, declare variables with the name given in the problem. Then, after solving the problem, in the last cell of your notebook print (or use @show) all the values of all of the variables in your optimization problem as well as the value of the objective function.
- Please comment add comments to your code describing the variables, constraints and objective function of your model.
- The homework assignment will have two parts. For the first part (with this outline), you should submit a PDF of your Jupyter notebook for each homework problem (You should be able to submit multiple PDF's for the assignment. If you submit everything in a single pdf, please assign pages to make it easier to grade). For the second part, which is Homework 8 IPYNB, please submit your Jupyter notebook files (.ipynb). We won't directly grade these, however they may be used if there are issues with your PDF submissions.
- Ensure that your notebook runs properly before submitting it. In the main bar, perform **Clear Outputs of All Cells** then **Run All**. to ensure that there are no errors.
- To generate a PDF of your notebook:
  - In the main bar, click **Export** (may be hidden behind a 3 dots dropdown menu)
  - Choose Export as PDF (may require additional extensions).
  - If Export as PDF fails or does not give proper output, export the file as HTML, open this HTML file in a web browser and save the HTML file as a PDF.
  - If you cannot get exporting to work in VSCode, you may use a Jupyter Notebook to PDF converter online.
- Remember to submit the .pdf versions to the normal Homework 8 assignment and the .ipynb files to the Homework 8 IPYNB assignment.

Question 1. (**30 points**): Let's revisit the salaries problem from an earlier homework. This problem was:

The director of Awesome Tech startup needs to decide what salaries to offer its employees for the coming year. In order to keep the employees satisfied, she needs to satisfy the following constraints:

- Tom wants at least $30,000 or he will quit;
- Peter, Nina, and Samir each want to be paid at least $8000 more than Tom;
- Gary wants his salary to be at least as high as the combined salary of Tom and Peter;
- Linda wants her salary to be $500 more than Gary;
- The combined salary of Nina and Samir should be at least twice the combined salary of Tom and Peter;
- Bob's salary is at least as high as that of Peter and at least as high as that of Samir;
- The combined salary of Bob and Peter should be at least $75,000;
- Linda should not make more money than the combined salary of Bob and Tom.

Suppose that we divide the employees into two groups:

- IT professionals: This group consists of Tom, Peter, Nina, and Samir.
- Customer service representatives: This is the rest of the employees.

Instead of minimizing the original objective, we instead want to minimize:

- the largest IT professional salary + the largest customer service representative salary.

The employees demands for salaries must still be satisfied. Note that this problem is a min-max problem and should be reformulated using the epigraph form discussed in class.

Create a Jupyter notebook titled *salaries.ipynb*. In the Julia code, declare variables for each person's salary as well as the variables needed to reformulate the min-max problem into a linear program. Model and solve this problem as a linear program. To check your solution, the optimal objective is 136,500.

Question 2. (**30 points**): Hackensack Blended Whisky Company uses three grades of whiskey to make blended varieties. They do not produce their own, whiskey and instead purchase three varieties to make their blends: Standard, Choice, and Prime. These unblended grades can be used to make the following two brands of whiskey, with associated characteristics:

- Scottish Club: Must contain at least 60% Prime and at most 20% Standard. Scottish Club sells for $6.80 per liter.
- Johnny Gold: Must contain at least 15% Prime and at most 60% Standard. Johnny Gold sells for $5.70 per liter.

The amount of available raw whiskey and their associated costs are:

Whiskey	Available (no. of liters)	Cost per liter (in dollars)
Standard	1,200	4.00
Choice	2,500	5.00
Prime	2,000	7.00

Hackensack doesn't want to produce more whiskey than it knows it can sell. It estimates its current demand for each type of blended whiskey as:

Blended Whiskey	Demand (in liters)
Scottish Club	1,000
Johnny Gold	600

Hackensack can increase its demand by advertising its blended whiskeys. For each dollar spent on advertising any type of its blended whiskeys, that type of whiskey's demand will increase by 1.25 liters.

Formulate an LP to maximize the total profit (revenue minus cost). To check correctness of your solution: the maximum profit is ≈ 1613.33.

Create a Jupyter notebook named *whiskey.ipynb*. In the Julia code, declare a vector for each type of raw whiskey (Standard, Choice, and Prime) as well as a vector for each type of blended whiskey (Scottish Club and Johnny Gold). Declare a variable x which is indexed over the raw and blended whiskeys. In addition, add a variable to represent the amount of money spent by Hackensack on advertising. Formulate and solve the linear program. Make sure to print out the optimal objective value and the value of all decision variables.

Question 3. (**40 points**):

The CRUD chemical plant produces as part of its production process a noxious compound called chemical X. Chemical X is highly toxic and needs to be disposed of properly. Fortunately, CRUD is linked by a pipe system to the FRESHAIR recycling plant that can safely reprocess chemical X. On any give day, the CRUD plant will produce the following amount of Chemical X (the plant operates between 9am and 3pm only):

Time	9-10 AM	10-11 AM	11 AM-12 PM	12-1 PM	1-2 PM	2-3 PM
Chemical X (in liters)	300	240	600	200	300	600

Because of environmental regulation, at no point in time is the CRUD plant allowed to keep more than 1000 litres on site and no chemical X is allowed to be kept overnight. At the top of every hour, at most 650 liters of chemical X can be sent to the FRESHAIR recycling plant. The cost of recycling chemical X is different for every hour:

Time	10 AM	11 AM	12 PM	1 PM	2 PM	3 PM
Chemical X (in liters)	30	40	35	45	38	50

You need to decide how much chemical to send from the CRUD plant to the FRESHAIR recycling plant at the top of each hour, so that you can minimize the total recycling cost but also meet all the environmental constraints. Formulate this problem as an LP. To check correctness, the optimal objective value is 88,050.

Create a Jupyter notebook named *crud.ipynb*. In the Julia code, for each hour of the day, declare a variable to represent the amount of chemical X currently held at CRUD at the end of that hour as well as a variable to represent the amount of chemical X sent to FRESHAIR at the end of that hour. Formulate and solve the linear program. Make sure to print out the optimal objective value and the value of all decision variables.

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