3D Scanning & Motion Capture

Exercise - 3

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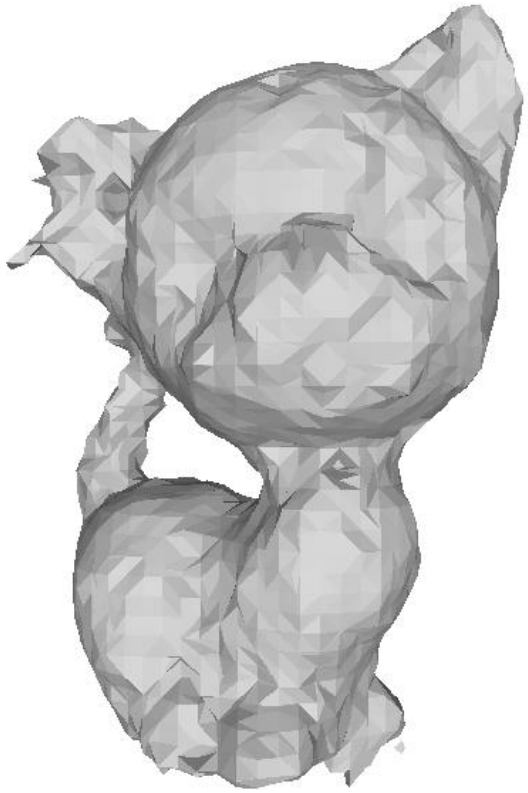
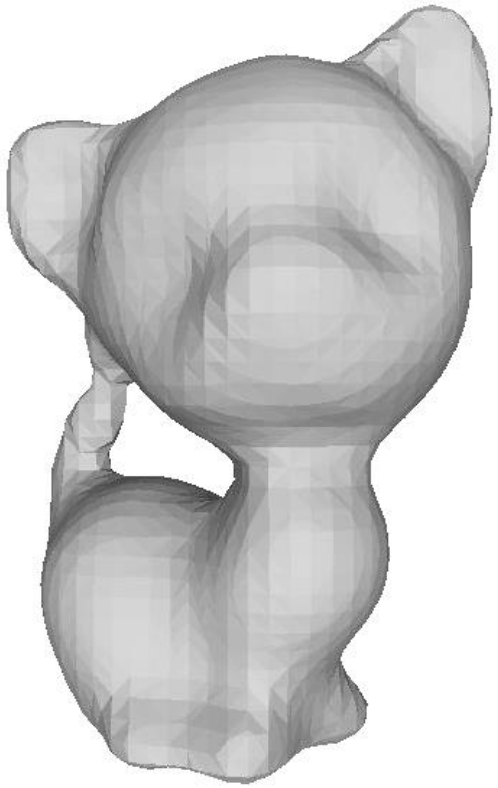
Exercises 2 – Solution





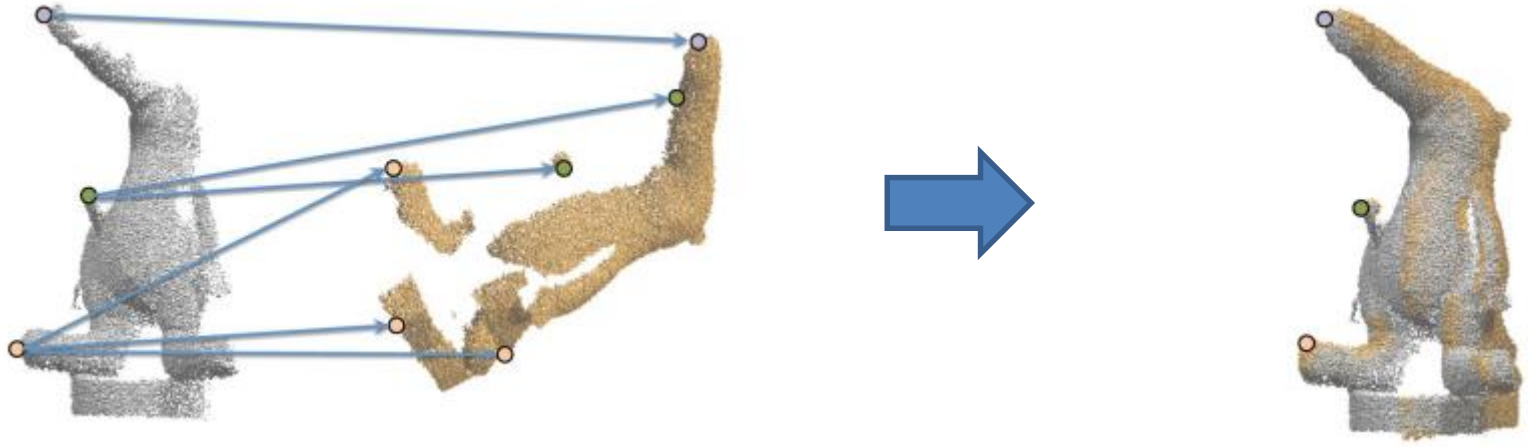
Exercises – Overview

1. Exercise → Camera Intrinsics, Back-projection, Meshes
2. Exercise → Surface Representations
3. **Exercise → Coarse Alignment (Procrustes)**
4. Exercise → Optimization
5. Exercise → Object Alignment, ICP

Procrustes

- Problem: Align two objects using known correspondences

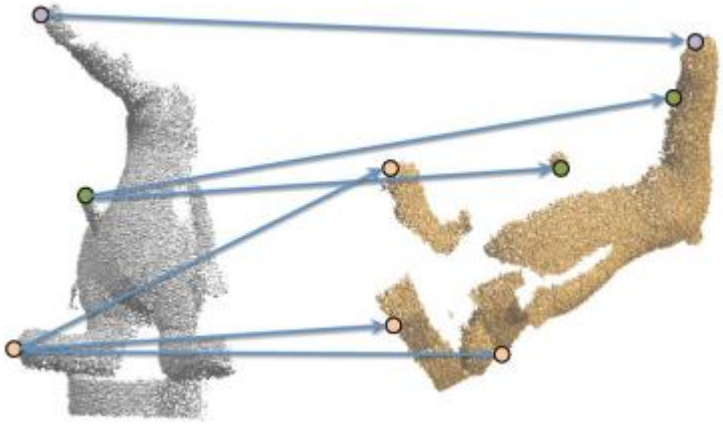
  →scaling, translation, rotation

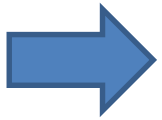


Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation





$$X = R\hat{X} + t$$

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

  - Compute center of gravity of both objects
  - Scale one object to match the avg. distance from all vertices to the center of gravity

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

  - The translation vector will depend on the rotation, so we will analyse it later

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

  - Assume objects that are zero-centered
    - Target object: $\{x_0, .. x_{n-1}\}$
    - Moving object: $\{\hat{x}_0 , .. \hat{x}_{n-1}\}$

$$\sum_i \|x_i - R \cdot \hat{x}_i\|_2^2 \rightarrow min$$

$$\left\|X - \hat{X}R^T\right\|_F^2 \rightarrow min$$

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

$$\left\| X - \hat{X}R^T \right\|_F^2 \rightarrow min$$

$\|A\|_F^2 = trace(A^T A)$

Cyclic invariance of trace:

$trace(ABC) = trace(CAB)$

$$\left\| X - \hat{X}R^T \right\|_F^2 = trace(X^T X - X^T \hat{X} R^T - \left(\hat{X}R^T\right)^T X + \left(\hat{X}R^T\right)^T (\hat{X}R^T)) \rightarrow min$$

$$trace(-X^T \hat{X} R^T - \left(\hat{X}R^T\right)^T X + \left(\hat{X}R^T\right)^T (\hat{X}R^T)) \rightarrow min$$

$$-2 \cdot trace(X^T \hat{X} R^T) \rightarrow min$$

$$trace(X^T \hat{X} R^T) \rightarrow max$$

SVD: $X^T \hat{X} = USV^T$

$$trace(USV^T R^T) \rightarrow max$$

$$trace(SV^T R^T U) \rightarrow max$$

Singular values →positive

Product of orthogonal matrices →max if equal to Identity

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

$$\left\| X - \hat{X} R^T \right\|_F^2 \rightarrow min$$

  - Compute SVD of the Cross-Covariance Matrix

$$X^T \hat{X} = USV^T$$

  - Compute the rotation

$$R = UV^T$$

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

  - The computed rotation might be a mirroring!
  - The determinant of a rotation matrix must be 1
  - If $\det(UV^T) = -1$, compute the rotation as:

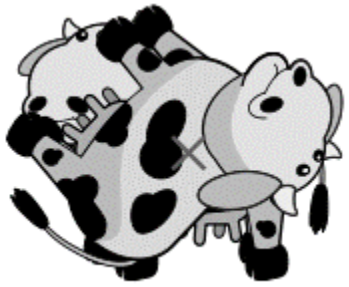
$$R = U \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} V^T$$

Procrustes

- Problem: Align two objects using known correspondences

→scaling, translation, rotation

- Assume objects that are zero-centered
  - Target object: $\{x_0, .. x_{n-1}\}$
  - Moving object: $\{\hat{x}_0 , .. \hat{x}_{n-1}\}$

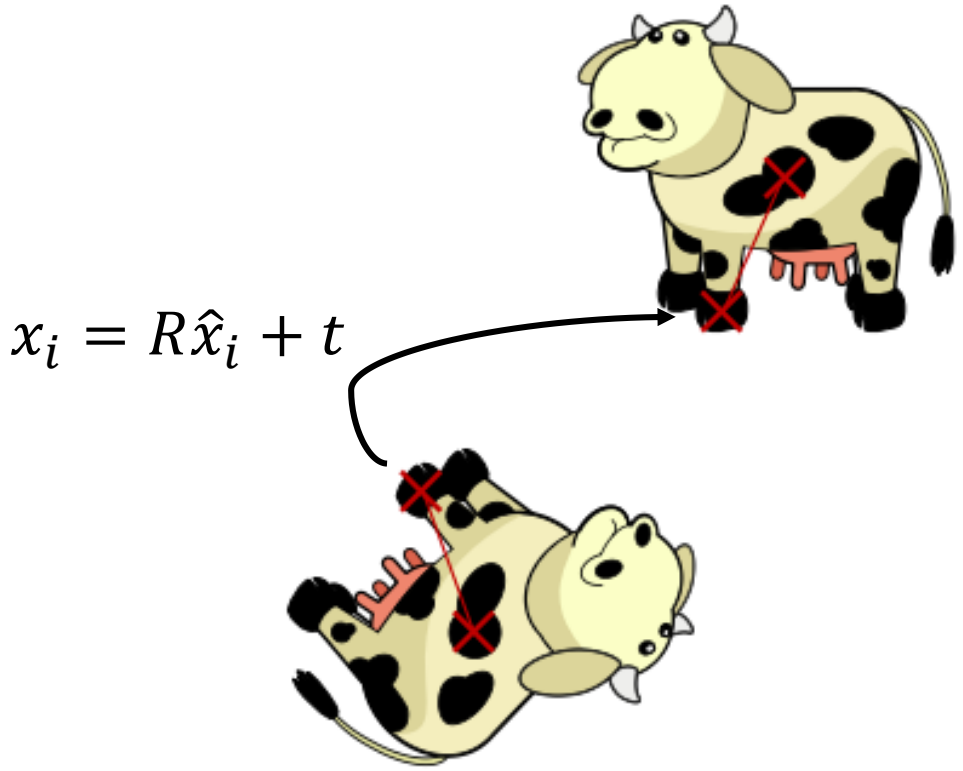


$$\sum_i \|x_i - R \cdot \hat{x}_i\|_2^2 \rightarrow min$$

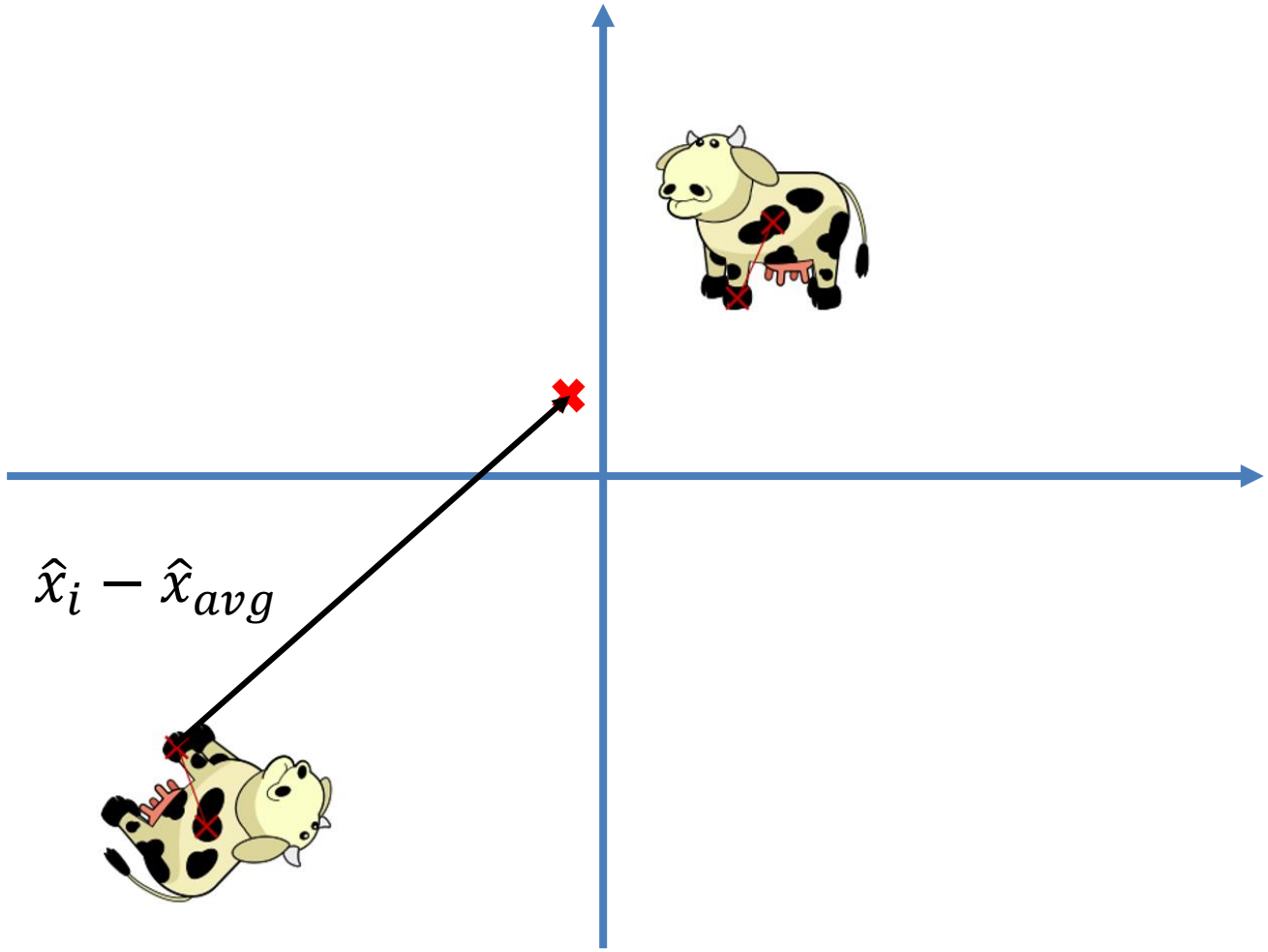
$$\left\|X - \hat{X} R^T\right\|_F^2 \rightarrow min$$

Procrustes

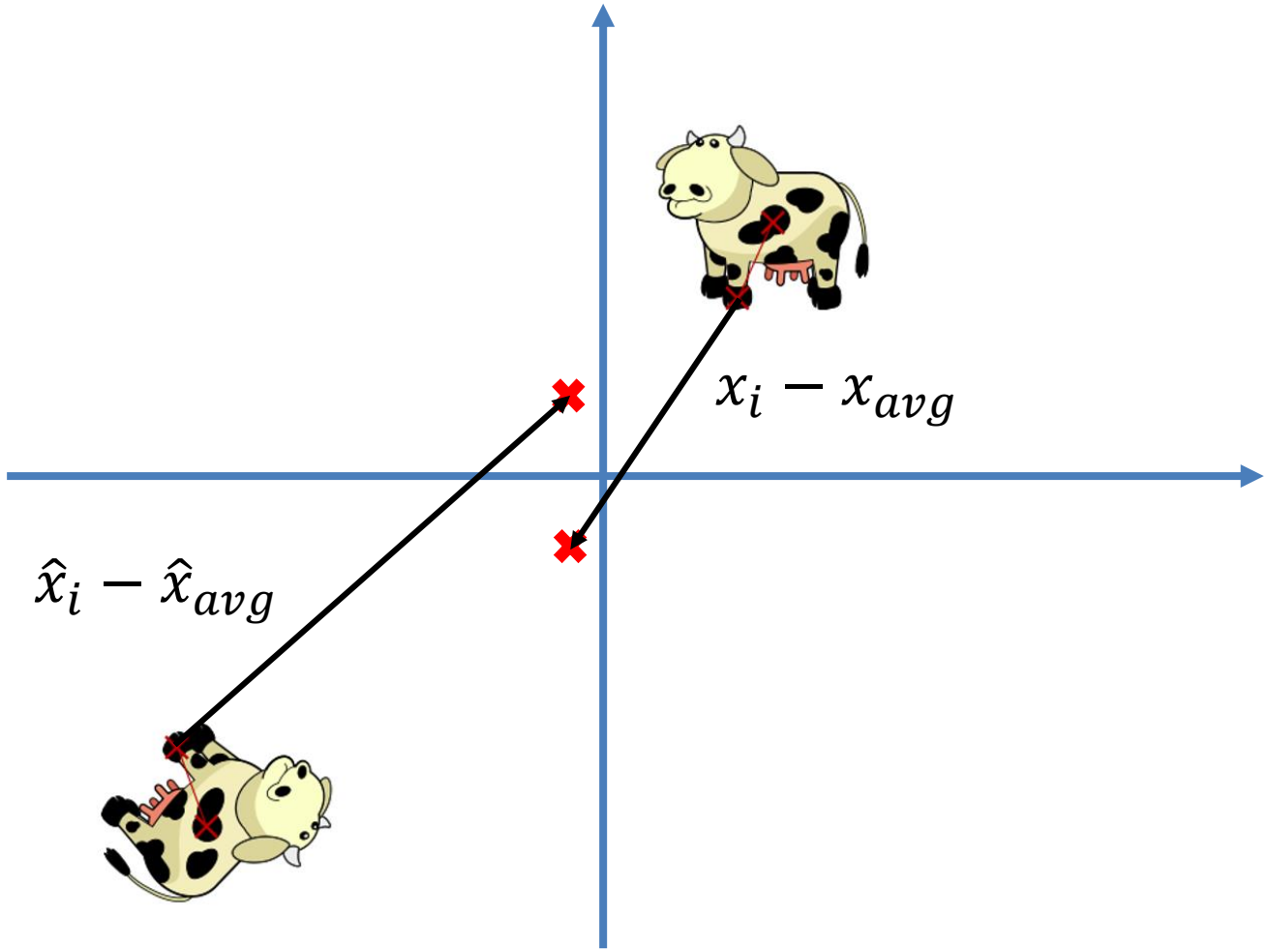
- Problem: Align two objects using known correspondences



Procrustes

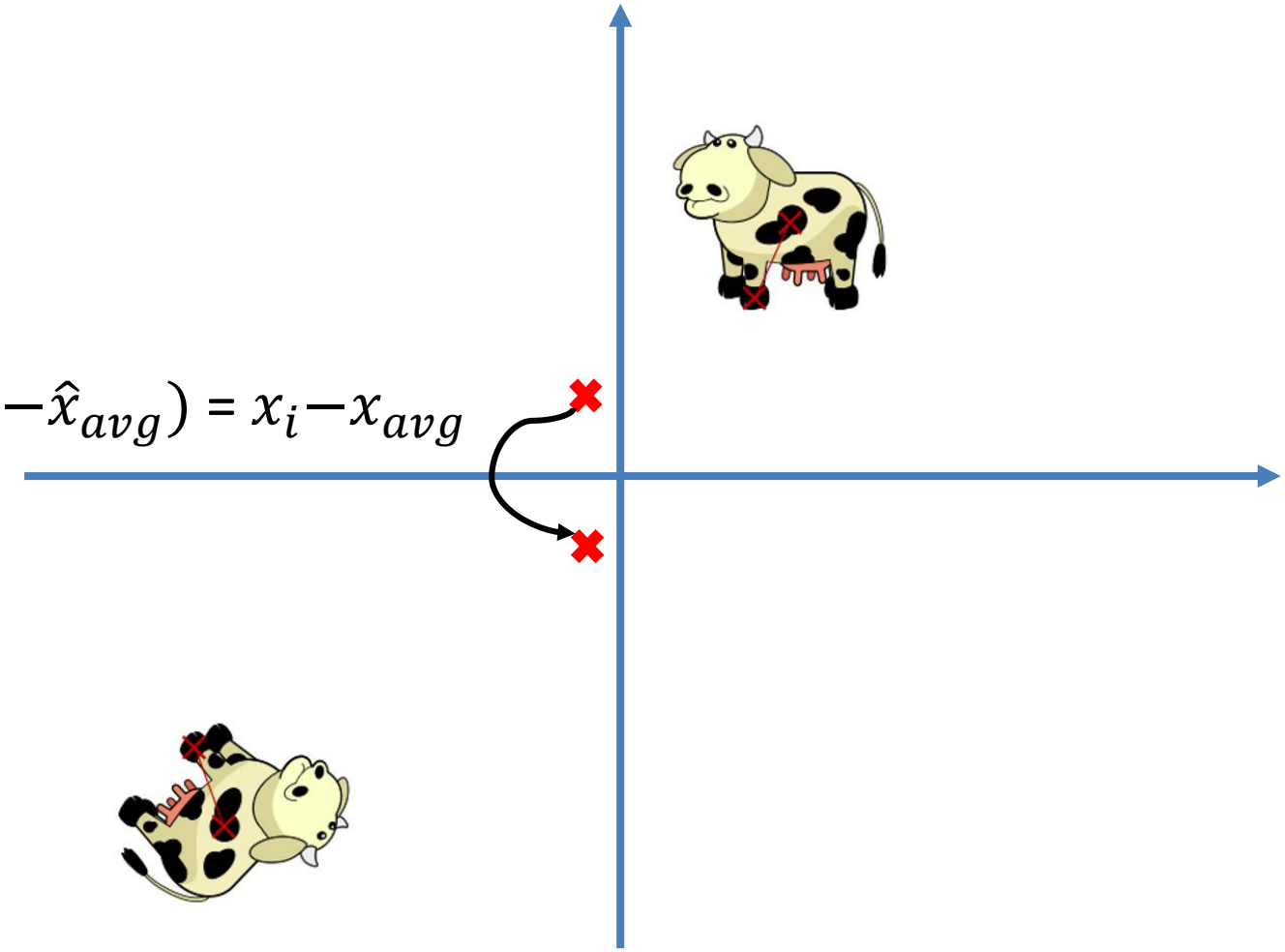


Procrustes



Procrustes

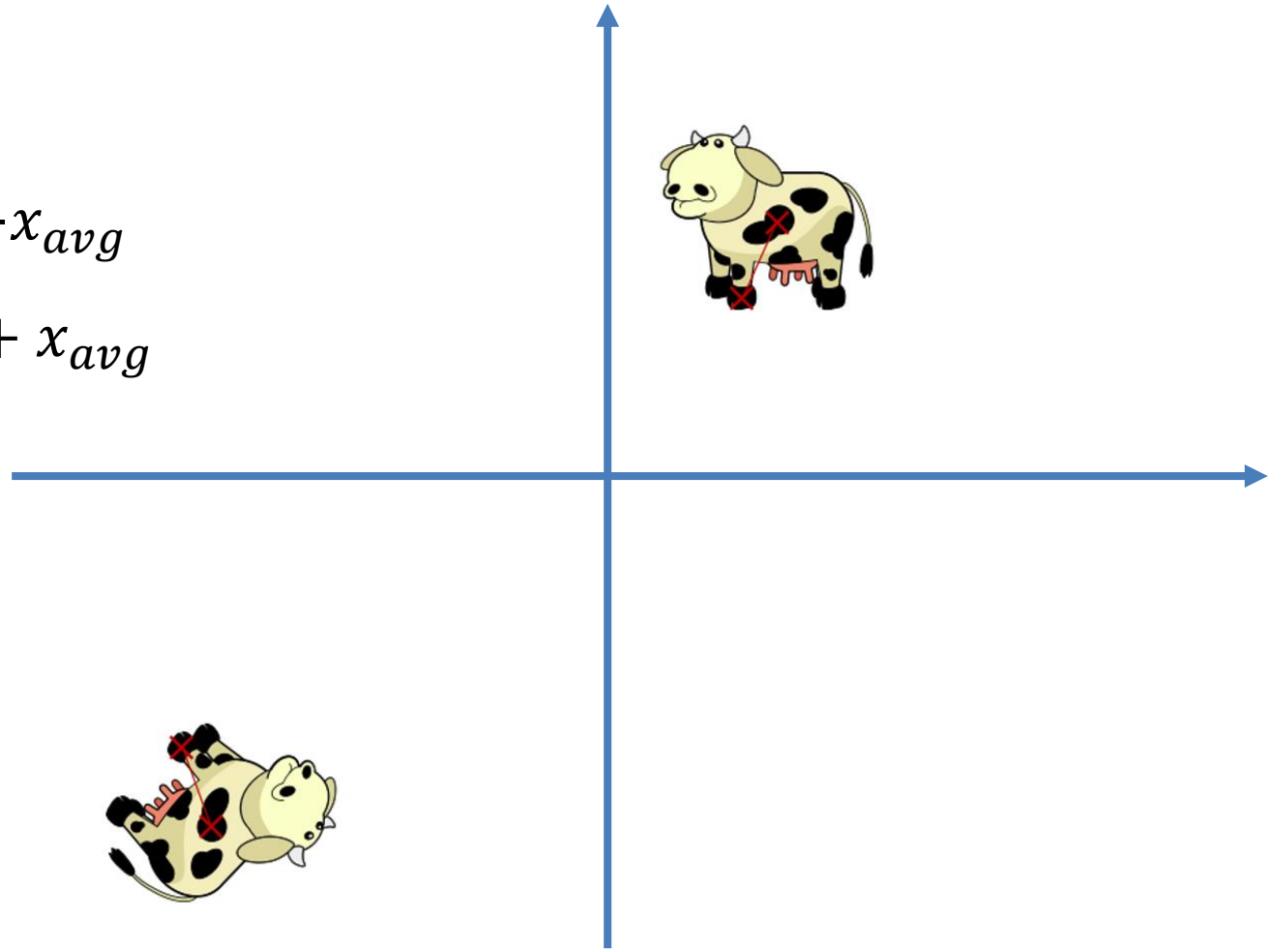
$$R(\hat{x}_i - \hat{x}_{avg}) = x_i - x_{avg}$$



Procrustes

$$R(\hat{x}_i - \hat{x}_{avg}) = x_i - x_{avg}$$

$$x_i = R(\hat{x}_i - \hat{x}_{avg}) + x_{avg}$$

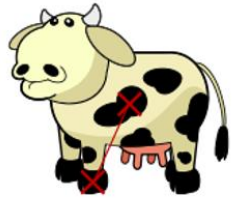


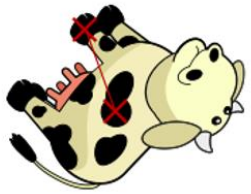
Procrustes

$$R(\hat{x}_i - \hat{x}_{avg}) = x_i - x_{avg}$$

$$x_i = R(\hat{x}_i - \hat{x}_{avg}) + x_{avg}$$

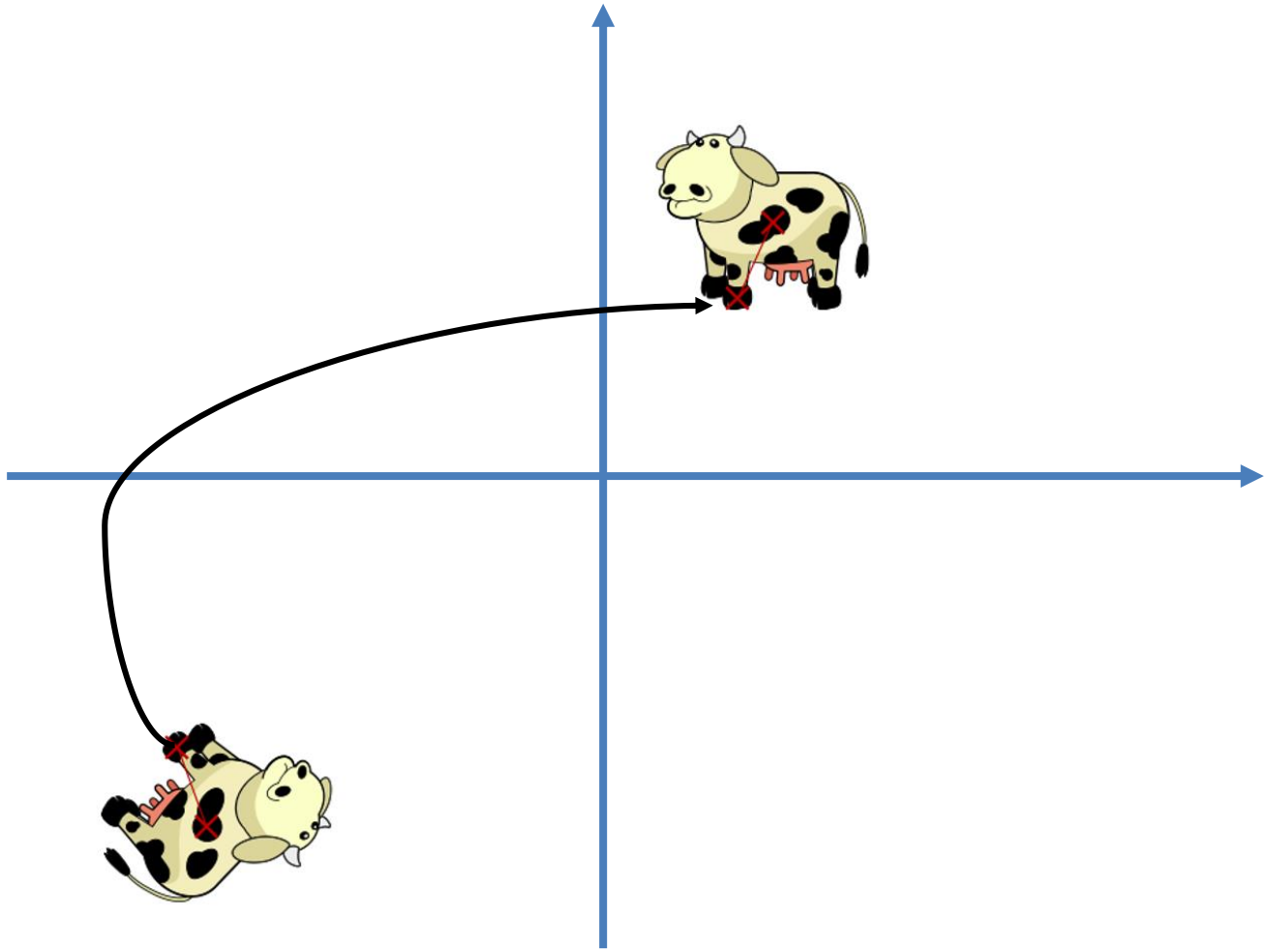
$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$





Procrustes

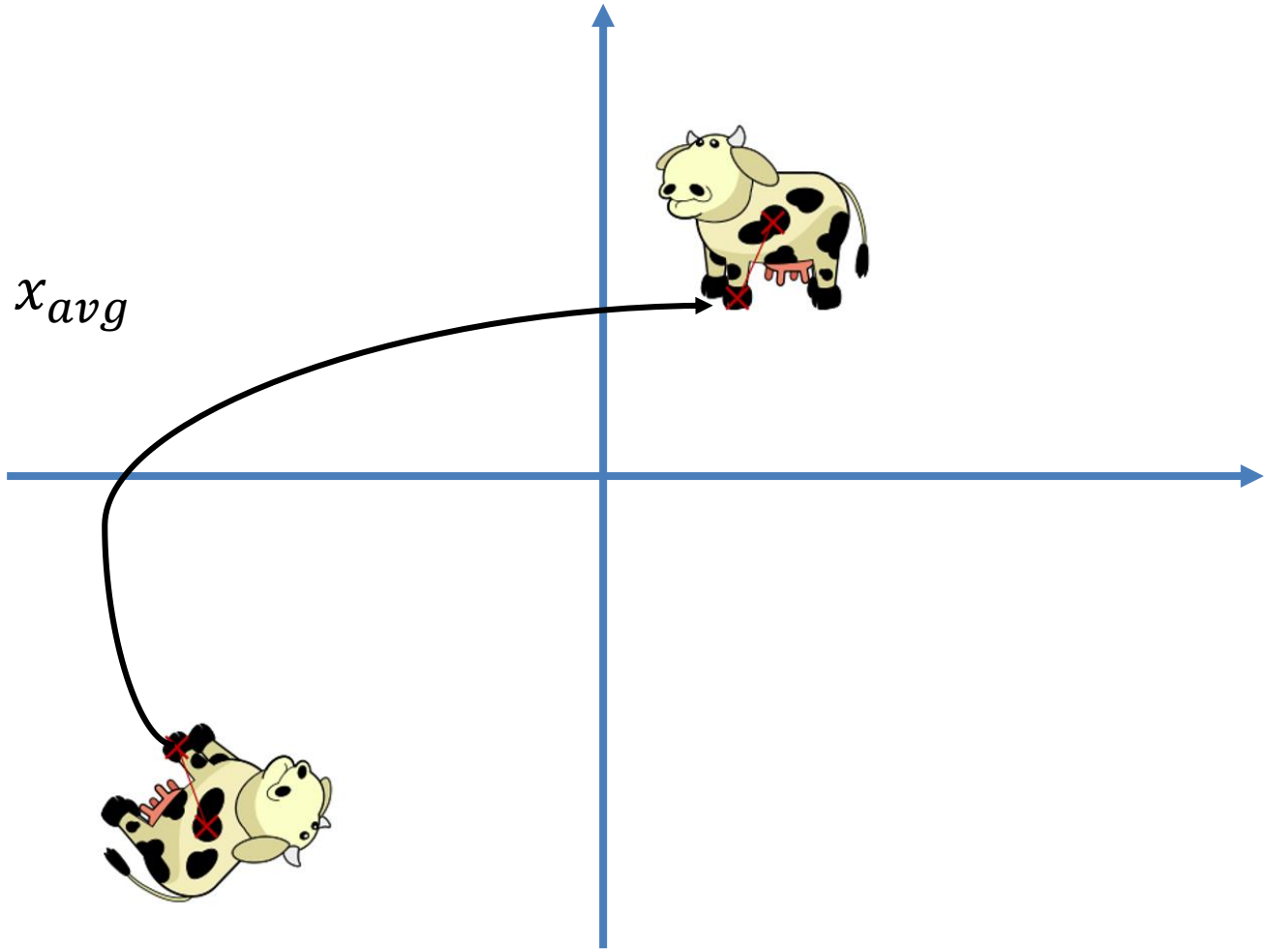
$$x_i = R\hat{x}_i + t$$



Procrustes

$$x_i = R\hat{x}_i + t$$

$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$

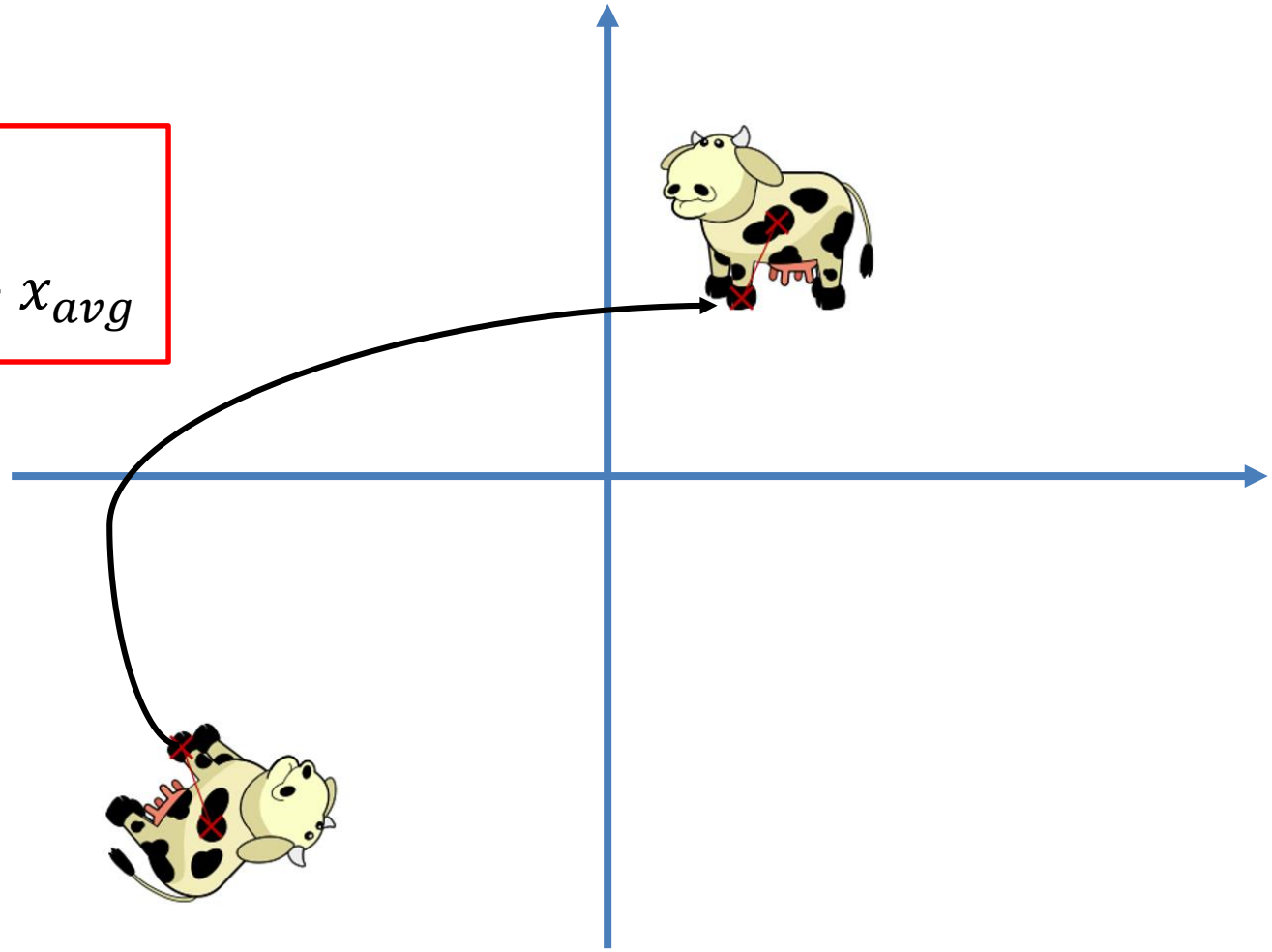


Procrustes

$$x_i = R\hat{x}_i + t$$

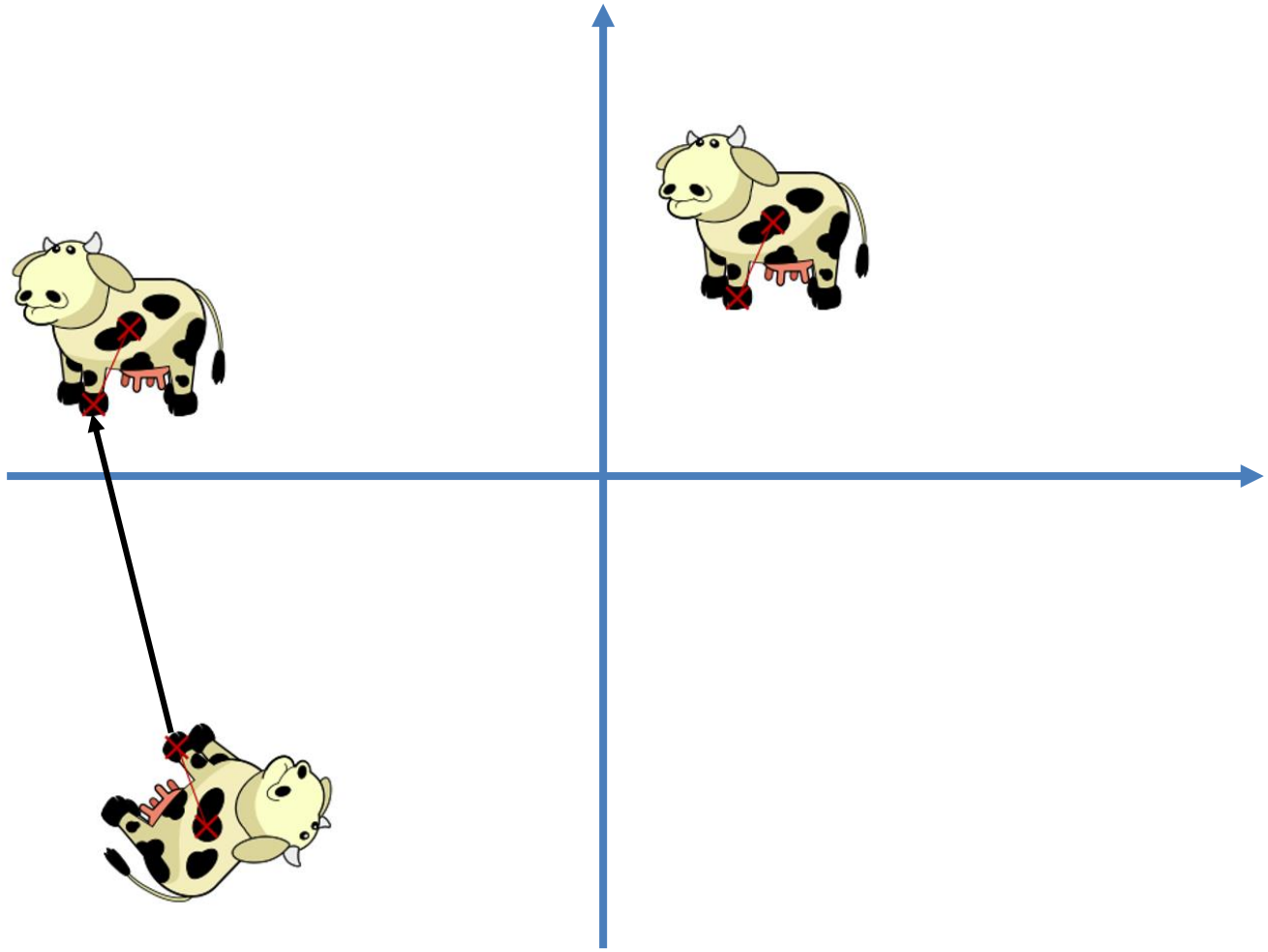
$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$

$$t = -R\hat{x}_{avg} + x_{avg}$$



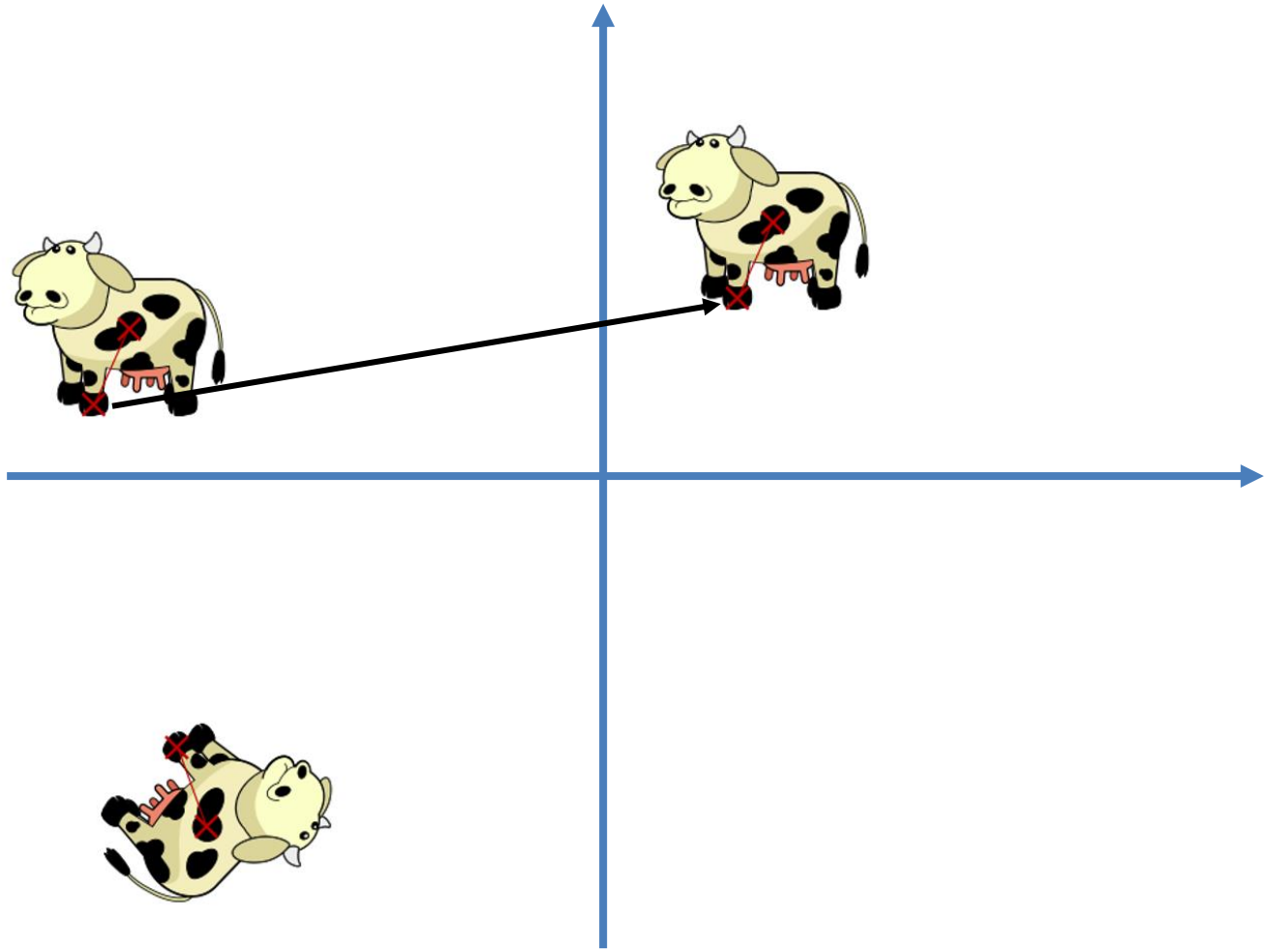
Procrustes

$$x_i = R\hat{x}_i + t$$



Procrustes

$$x_i = R\hat{x}_i + t$$

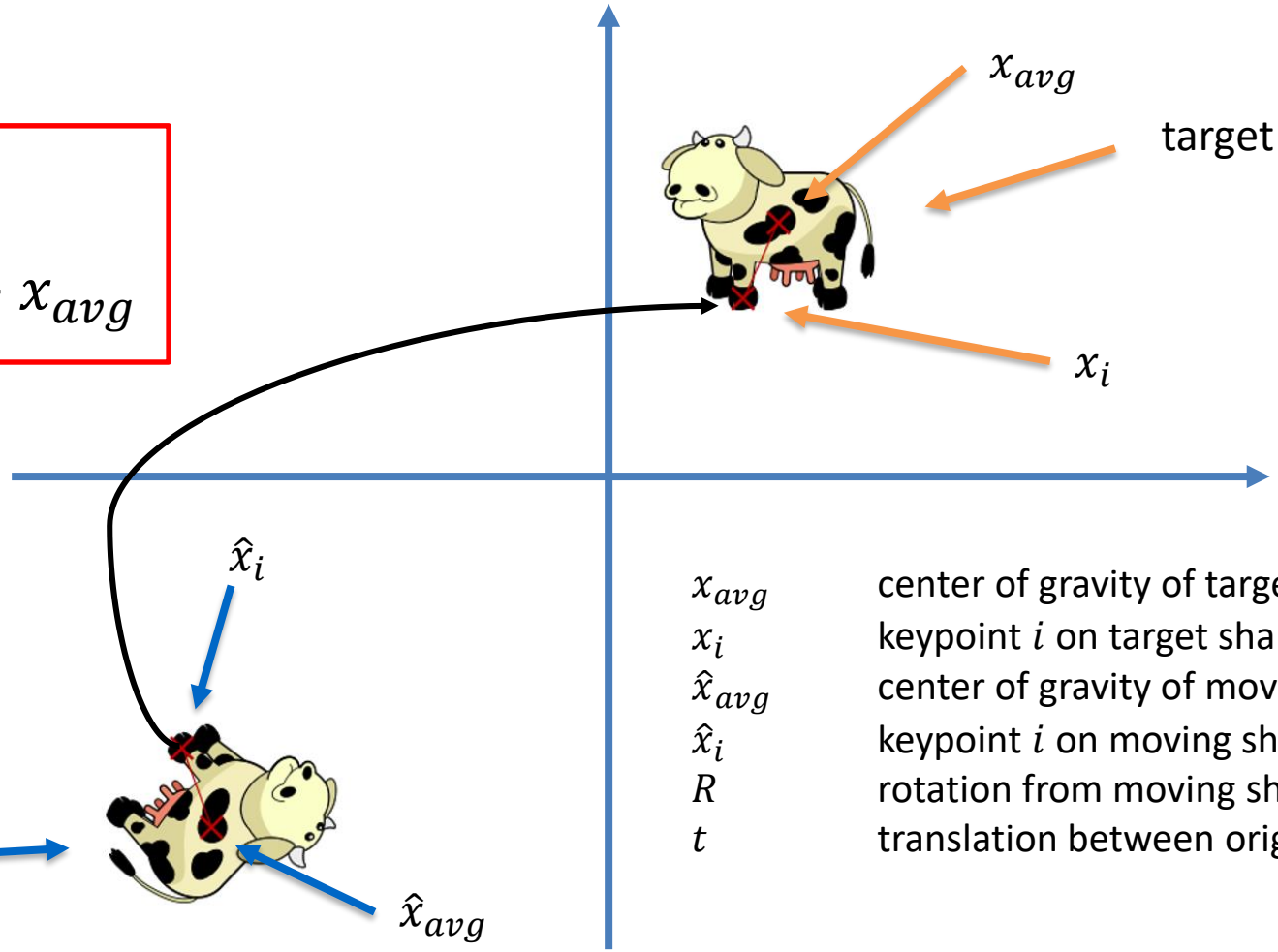


Procrustes

$$x_i = R\hat{x}_i + t$$

$$x_i = R\hat{x}_i - R\hat{x}_{avg} + x_{avg}$$

$$t = -R\hat{x}_{avg} + x_{avg}$$



x_{avg}	center of gravity of target shape
x_i	keypoint i on target shape
$\hat{x}_{avg}$	center of gravity of moving shape
$\hat{x}_i$	keypoint i on moving shape
R	rotation from moving shape to target shape
t	translation between original poses after rotation

See you next time!