8.6 Compute the closure of the following set F of functional dependencies for relation schema $R = (A, B, C, D, E)$.

$$A \rightarrow BC$$
$$CD \rightarrow E$$
$$B \rightarrow D$$
$$E \rightarrow A$$

List the candidate keys for R.

Answer: Note: It is not reasonable to expect students to enumerate all of F^+. Some shorthand representation of the result should be acceptable as long as the nontrivial members of F^+ are found.

Starting with $A \rightarrow BC$, we can conclude: $A \rightarrow B$ and $A \rightarrow C$.

Since $A \rightarrow B$ and $B \rightarrow D$, $A \rightarrow D$	(decomposition, transitive)
Since $A \rightarrow CD$ and $CD \rightarrow E$, $A \rightarrow E$	(union, decomposition, transitive)
Since $A \rightarrow A$, we have	(reflexive)
$A \rightarrow ABCDE$ from the above steps	(union)
Since $E \rightarrow A$, $E \rightarrow ABCDE$	(transitive)
Since $CD \rightarrow E$, $CD \rightarrow ABCDE$	(transitive)
Since $B \rightarrow D$ and $BC \rightarrow CD$, $BC \rightarrow ABCDE$	(augmentative, transitive)
Also, $C \rightarrow C$, $D \rightarrow D$, $BD \rightarrow D$, etc.	

Therefore, any functional dependency with A, E, BC, or CD on the left hand side of the arrow is in F^+, no matter which other attributes appear in the FD. Allow * to represent any set of attributes in R, then F^+ is $BD \rightarrow B$, $BD \rightarrow D$, $C \rightarrow C$, $D \rightarrow D$, $BD \rightarrow BD$, $B \rightarrow D$, $B \rightarrow B$, $B \rightarrow BD$, and all FDs of the form $A* \rightarrow \alpha$, $BC* \rightarrow \alpha$, $CD* \rightarrow \alpha$, $E* \rightarrow \alpha$ where α is any subset of $\{A, B, C, D, E\}$. The candidate keys are A, BC, CD, and E.

8.9 Given the database schema $R(a, b, c)$, and a relation r on the schema R, write an SQL query to test whether the functional dependency $b \rightarrow c$ holds on relation r. Also write an SQL assertion that enforces the functional dependency. Assume that no null values are present. (Although part of the SQL standard, such assertions are not supported by any database implementation currently.)

Answer:

a. The query is given below. Its result is non-empty if and only if $b \rightarrow c$ does not hold on r.

```
select b
from r
group by b
having count(distinct c) > 1
```

b.

```
create assertion b_to_c check
   (not exists
        (select b
         from r
         group by b
         having count(distinct c) > 1
        )
   )
```

8.21 Normalize the following schema, with given constraints, to 4NF.

$books(accessionno, isbn, title, author, publisher)$
$users(userid, name, deptid, deptname)$
$accessionno \rightarrow isbn$
$isbn \rightarrow title$
$isbn \rightarrow publisher$
$isbn \twoheadrightarrow author$
$userid \rightarrow name$
$userid \rightarrow deptid$
$deptid \rightarrow deptname$

Answer: In *books*, we see that

$$isbn \twoheadrightarrow title, publisher, author$$

and yet, *isbn* is not a super key. Thus, we break *books* into

$books\_accno(accessionno, isbn)$
$books\_details(isbn, title, publisher, author)$

After this, we still have

$$isbn \twoheadrightarrow author$$

but neither is *isbn* a primary key of *book_details*, nor are the attributes of *book_details* equal to $\{isbn\} \cup \{author\}$. Therefore we decompose *book_details* again into

$books\_details1(isbn, title, publisher)$
$books\_authors(isbn, author)$

Similarly, in *users*,

$$deptid \rightarrow deptname$$

and yet, *deptid* is not a super key. Hence, we break *users* to

$users(userid, name, deptid)$
$departments(deptid, deptname)$

We verify that there are no further functional or multivalued dependencies that cause violation of 4NF, so the final set of relations are:

$books\_accno(accessionno, isbn)$
$books\_details1(isbn, title, publisher)$
$books\_authors(isbn, author)$ $users(userid, name, deptid)$
$departments(deptid, deptname)$

8.29 Consider the following set F of functional dependencies on the relation schema $r(A, B, C, D, E, F)$:

$$A \rightarrow BCD$$
$$BC \rightarrow DE$$
$$B \rightarrow D$$
$$D \rightarrow A$$

a. Compute B^+.

b. Prove (using Armstrong's axioms) that AF is a superkey.

c. Compute a canonical cover for the above set of functional dependencies F; give each step of your derivation with an explanation.

d. Give a 3NF decomposition of r based on the canonical cover.

e. Give a BCNF decomposition of r using the original set of functional dependencies.

f. Can you get the same BCNF decomposition of r as above, using the canonical cover?

e. We start with

$$r(A, B, C, D, E, F)$$

We see that the relation is not in BCNF because of the first FD. Hence, we decompose it accordingly to get

$$r_1(A, B, C, D)\, r_2(A, E, F)$$

Now we notice that $A \rightarrow E$ is an FD in F^+, and causes r_2 to violate BCNF. Once again, decomposing r_2 gives

$$r_1(A, B, C, D)\, r_2(A, F)\, r_3(A, E)$$

This schema is now in BCNF.