Modeling NPV Uncertainty in Long-term Care Insurance:

A Monte Carlo Approach

ST474/674 - Monte Carlo and Simulation Methods

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Abstract

This report presents a quantitative analysis of the financial risk profile of a representative Long-Term Care (LTC) insurance product using a Monte Carlo simulation framework. The study first determines the break-even premium under baseline assumptions by modeling key biometric and cost factors, including mortality, dependency onset, duration of care, severity level, and associated claim costs. Using the calibrated break-even premium, the model generates a distribution of Net Present Value (NPV) outcomes from the insurer's perspective, enabling the calculation of key risk metrics such as expected NPV, standard deviation, probability of loss, and Value-at-Risk at the 95% confidence level. A one-factor-at-a-time sensitivity analysis examines how moderate changes in claim costs, care duration, and discount rates influence pricing and risk exposure, highlighting the parameters with the greatest financial impact. The findings underscore the importance of including pricing margins to buffer against adverse trends, maintaining sufficient capital reserves to absorb tail losses, and regularly reviewing assumptions to adapt to evolving demographic, economic, and cost conditions.

Contents

Chapter 1

Introduction

We evaluated the net present value (NPV) of long-term care (LTC) insurance under uncertainty from the insurer's perspective. LTC insurance protects policy owners from the high costs of care when they cannot perform activities of daily living (ADLs), while the insurer collects fixed premiums and assumes the risk of paying unpredictable benefits; however, it presents the possibility of risk and uncertainty for insurers due to the long duration and uncertain, high-cost claims.

To address these challenges, we use the stochastic simulation-based approach, where our model includes key input variables commonly identified in the Long-Term Care Insurance literature, including the individual's age at entry (typically between 50 and 65), health status at purchase, policy features such as maximum daily benefit, benefit period, and waiting period, as well as economic factors such as discount rates and inflation of care costs. The structure and parameter ranges of these inputs are informed by actuarial and behavioral research, including Bodily and Furman (2016) using SOA based distributions, and Lazoğlu and Büyükyazici (2023) that described longevity risk, dependency levels, and dynamic mortality considerations, along with adopting their use of Weibull-distributed claim durations and multilevel care severity in our modeling framework.

Our ultimate objective is to model different policyholder lifetimes to estimate the distribution of NPV and find the break-even premiums along with identifying the risk-adjusted pricing strategies. We also identify key risk metrics, such as the probability of a negative NPV along with VaR to evaluate the financial stability of an insurer.

Chapter 2

Pricing: Break-Even Premium Estimation via Monte Carlo

The break-even premium is the price at which the insurer's expected Net Present Value (NPV) is zero. Figure 2.1 outlines the computational process: we begin by specifying all model inputs, feed them into a Monte Carlo simulation engine, compute the mean NPV from simulated outcomes, adjust the trial premium accordingly, and iterate until the break-even condition is met.

Figure 2.1: Algorithm outflow for pricing process

2.1 Assumptions and Model Inputs

Our pricing model requires a complete specification of the stochastic environment in which a policy operates. The parameters are calibrated from publicly available sources—namely the *Society of Actuaries' Long-Term Care Intercompany Study (2011)*, the *Genworth 2014 Cost of Care Survey*, and *U.S. Social Security Administration* period life tables. We fit parsimonious parametric forms (Normal for lifetime and entry age, Weibull for claim duration, and a discrete tier for severity) so that simulated moments align with the published means, spreads, and utilization mixes in these sources.

Lifetime (T). We model the age at death as a normal random variable with mean $\mu_T = 85$ years and standard deviation $\sigma_T = 5$ years. The mean life expectancy is drawn from the *U.S. Social Security Administration (SSA) Period Life Table, 2010*, which reports an average life expectancy in the mid-80s for individuals aged 65 or older. The normal form is a simplifying assumption that facilitates simulation and root-finding. While the SOA Intercompany Study provides mortality rates for both active and disabled lives, we use a single unconditional distribution for tractability.

Entry Age to Dependency (A). The age at which an individual first requires long-term care is modeled as normal with mean $\mu_A = 75$ years and standard deviation $\sigma_A = 3$ years. These parameters are calibrated to the *SOA LTC Intercompany Study*, which reports incidence rates by age cohort, gender, and coverage type. The tight spread of ± 3 years reflects the empirical clustering of onset around the mid-70s.

Care Duration (D). We assume that once an individual enters care, the duration follows a Weibull distribution with shape $k = 2$ and scale $\lambda = 3$ years. This choice is based on claim persistence patterns in the *SOA LTC Intercompany Study*, which show right-skewed durations with a significant minority persisting beyond five years. A Weibull with $k > 1$ captures the initially increasing hazard rate followed by a decline. The chosen parameters imply a mean duration of $\lambda\Gamma(1 + 1/k) \approx 2.66$ years, close to the SOA averages.

Annual Severity Level (C). Care cost severity is represented by one of four fixed annual amounts: \$1,500, \$1,000, \$600, or \$400. These tiers approximate the cost distribution implied by the *SOA LTC Intercompany Study* (distribution of type of care at onset) combined with the *Genworth 2014 Cost of Care Survey* median daily costs. The probabilities are chosen so that the expected annual cost matches the weighted average from these sources.

Discount Rate. All cash flows are discounted at a constant rate of 0.2% per month (approximately 2.43% annual effective), reflecting a low interest rate environment. This constant rate simplifies the valuation process and avoids the need to model stochastic interest rates.

2.2 Cash Flow Structure

We consider a single up-front premium P paid at policy inception ($t = 0$), followed by potential claim outflows during months in which the insured is in a dependent state. The present value of claim payments for each life is calculated using the monthly discount factor $v^t = (1 + 0.002)^{-t}$.

For an individual life path ω, let $T(\omega)$ be the age at death, $A(\omega)$ the age at dependency entry, $D(\omega)$ the duration of care in years, and $C(\omega)$ the annual care cost. Months are counted from the policy issue date. The first month of dependency is t_{start}, and the last payable month is

$$t_{\text{end}} = \min\{t_{\text{start}} + \lfloor 12D \rfloor, \text{ death month}\}.$$

Monthly claim payments equal $C/12$ during this interval and zero otherwise. If $A \geq T$, the policyholder never enters care and no claims are paid.

The pathwise NPV is therefore:

$$\text{NPV}(P; \omega) = P - \sum_{t=t_{\text{start}}}^{t_{\text{end}}} \frac{C}{12} \cdot v^t$$

This formula captures both inflows (premium) and outflows (discounted monthly claims), producing one simulated NPV for each policyholder life.

2.3 Monte Carlo Estimation and Root-Finding

The break-even premium is found via stochastic root-finding using *common random numbers* (CRN) to ensure smooth convergence. We start with a trial premium P_0, simulate $N = 10{,}000$ independent policyholder lives, and compute the sample mean NPV:

$$\bar{X}(P_k) = \frac{1}{N} \sum_{i=1}^{N} \text{NPV}_i(P_k),$$

together with its standard error $\text{SE} = \hat{\sigma}(\text{NPV})/\sqrt{N}$.

If the mean NPV is positive, the premium is too high and we step it down; if negative, we step it up.

The same set of simulated lifetimes is reused at each iteration to avoid stochastic noise obscuring the trend. Iterations continue until the absolute mean NPV is within $1.95 \times \mathrm{SE}$, corresponding to a 95% confidence interval containing zero.

The implementation is summarized in Algorithm 1(in Appendcix A), which formalizes the break-even search process.

Chapter 3

Risk Quantification and Sensitivity Analysis

3.1 Risk Quantification

Following the break-even premium estimation procedure described in Section 2, we evaluate the financial risk profile of a representative Long-Term Care (LTC) insurance policy using a Monte Carlo simulation framework. The goal of this stage is to measure the distribution of possible financial outcomes from the insurer's perspective and to quantify the probability and severity of losses under baseline assumptions.

For each simulated policyholder i, the Net Present Value (NPV) is computed as

$$\mathrm{NPV}_i \;=\; \mathrm{PV}_{\mathrm{Premiums},\, i} \;-\; \mathrm{PV}_{\mathrm{Claims},\, i}. \tag{3.1}$$

where

- $\mathrm{PV}_{\mathrm{Premiums},\, i}$ is the present value of premium inflows received from policyholder i, discounted at the assumed monthly rate; and
- $\mathrm{PV}_{\mathrm{Claims},\, i}$ is the present value of LTC claim outflows for policyholder i, discounted on the same basis.

This definition captures both the stochastic nature of inflows (due to mortality and lapse patterns) and outflows (due to timing, duration, and severity of dependency).

3.1.1 Simulation Framework

The simulation framework consists of two sequential algorithms (pseudocodes are in Appendix A).

Algorithm 1 — Break-Even Premium Calculation

1. Define a trial premium level.
2. For each of the N simulated policyholders, simulate the following variables:
   - **Mortality**: age at death, drawn from a normal distribution.
   - **Dependency Onset**: age at which LTC services begin, drawn from a normal distribution.
   - **Dependency Duration**: length of time receiving LTC services, drawn from a Weibull distribution to capture increasing hazard rates.

- **Care Severity and Costs**: severity level sampled probabilistically, with higher severity associated with higher monthly costs.
- **Premium Payment Pattern**: annual premium payments until the minimum death,contractual end of payment obligations (e.g., retirement age).

3. Calculate NPV_i for each policyholder, then compute the average NPV across all simulations.
4. Adjust the trial premium iteratively until the mean NPV is approximately zero (within tolerance ϵ). This premium is the break-even premium.

Algorithm 2 — Risk Metrics Calculation Using the break-even premium from Algorithm 1, run a fresh set of N simulations to produce an NPV distribution. Calculate the following risk metrics:

- **Mean NPV:** $E[\text{NPV}]$ — expected profit or loss at break-even premium.
- **Standard Deviation of NPV:** $\sigma(\text{NPV})$ — measures variability in outcomes.
- **Probability of Loss:** $P(\text{NPV} < 0)$ — probability the insurer experiences a negative NPV.
- **Value-at-Risk (**VaR_{95}**):** the 5th percentile of the NPV distribution, representing a worst-case outcome at a 95% confidence level.

3.1.2 Model Inputs and Rationale

- **Number of Simulations** ($N = 100$): This value balances computational efficiency with statistical stability for demonstration purposes. While industry practice often uses thousands of simulations (e.g., Bodily & Furman, 2016), 100 simulations are sufficient for illustrative analysis in this context.
- **Entry Age (60)**: Represents the typical age at which individuals purchase LTC insurance before retirement, consistent with industry sales data.
- **Maximum Age (100)**: Serves as an upper bound on human longevity to cap projections.
- **Discount Rate (0.002 monthly, approximately** 2.4% **annually)**: Chosen to reflect conservative fixed-income returns for insurers.
- **Age at Death** $\sim \mathcal{N}(85, 5^2)$: Matches developed-country life expectancy based on WHO data, incorporating realistic variance in lifespan.
- **Age at Dependency Entry** $\sim \mathcal{N}(75, 3^2)$: Based on empirical data from LTC industry reports, capturing the typical onset age of long-term care needs.
- **Dependency Duration (Weibull, shape** $= 2$**, scaled to ~3 years)**: The Weibull distribution captures increasing hazard rates with age, while the scaling aligns with actuarial literature (Lazoğlu & Büyükyazici, 2023).
- **Care Severity Probability** [0.2, 0.3, 0.3, 0.2]: Reflects empirical severity distributions observed in LTC claim datasets.

- **Annual Care Costs per Level ({1: 1500, 2: 1000, 3: 600, 4: 400})**: Higher severity levels are associated with greater monthly care costs.

- **Premium Payment Pattern (annual until** $\min(\textbf{death}, 65)$**)**: Models limited-pay LTC contracts where premium payments stop at retirement age or upon death, whichever occurs first.

3.1.3 Key Assumptions

The modelling framework is based on several simplifying assumptions that are intended to balance analytical clarity with computational efficiency. Mortality, dependency onset, and dependency duration are treated as statistically independent variables. While empirical evidence suggests these factors may be correlated (e.g., earlier dependency onset often coinciding with shorter life expectancy), independence is assumed to reduce the complexity of calibration and to limit the amount of required input data. Care severity and associated costs are fixed for each policyholder once dependency begins, with the assigned severity level remaining constant over the duration of care. This approach removes the need to simulate severity progression while still reflecting differences in cost levels across the insured population. A constant monthly discount rate of 0.002 is applied to all projected cash flows. This assumption isolates the analysis from interest-rate volatility and ensures that the results reflect biometric and cost-related risk rather than investment risk. The analysis also assumes that all policyholders retain their coverage until death or maturity, with no lapses, surrenders, or other behavioural changes, thereby eliminating the influence of voluntary terminations on the simulated outcomes.

3.2 Sensitivity Analysis

We performed a One-Factor-at-a-Time (OFAT) sensitivity analysis to examine how changes in key parameters affect the break-even premium and associated risk metrics.

3.2.1 Methodology

1. Start from the baseline parameter set defined in Section 3.1.

2. Adjust one parameter at a time while holding all others constant.

3. For each parameter adjustment:

   - Re-run Algorithm 1 to recompute the break-even premium.
   - Re-run Algorithm 2 to recompute $E[\text{NPV}]$, $\sigma(\text{NPV})$, $P(\text{NPV} < 0)$, and VaR_{95}.

4. Compare the results to the baseline to measure both the magnitude and direction of changes.

3.2.2 Parameters Tested

- **Claim Costs** ($\pm 10\%$) — Represents moderate care cost inflation or deflation scenarios, consistent with historical variability in LTC costs.

- **Dependency Duration** ($\pm 10\%$) — Captures plausible changes in the average length of care that could arise from demographic shifts or evolving health trends.

- **Discount Rate (±50 basis points annually)** — Reflects a moderate change in economic conditions that would impact the discounting of future cash flows.

3.2.3 Implementation in Code Context

In the sensitivity analysis, changes to key parameters were implemented directly within the simulation framework to assess their impact on the break-even premium and associated risk metrics. For claim cost adjustments, the simulated output from the cost function was scaled by a factor of 0.90 to represent a 10 percent decrease or by 1.10 to represent a 10 percent increase. Changes to dependency duration were applied in a similar manner, with the simulated duration values multiplied by 0.90 or 1.10 to model shorter or longer average care periods. Adjustments to the discount rate were implemented by modifying the monthly discount rate so that the equivalent annual rate shifted by ±0.50 percentage points. The chosen magnitudes for these perturbations reflect a balance between practical relevance and analytical clarity. They are sufficiently large to illustrate the sensitivity of results, yet moderate enough to avoid extreme and implausible stress scenarios. These parameter ranges align with recommendations in the long-term care insurance literature, including Bodily and Furman (2016) and Lazoğlu and Büyükyazici (2023), which suggest similar scales for realistic stress-testing exercises.

3.3 Interpretation of Results

The Monte Carlo simulation and OFAT sensitivity analysis provide two complementary perspectives: (1) the baseline financial risk profile of the LTC product, and (2) the relative importance of individual assumptions in driving that risk.

3.3.1 Baseline Risk Profile

Under the calibrated break-even premium, the mean NPV is near zero ($E[\text{NPV}] \approx 0$), as expected by design. However, the distribution of NPVs is wide, reflecting substantial uncertainty in claim timing, duration, and severity. The standard deviation of NPV is materially larger than the mean, indicating that variability dominates over systematic bias. The probability of loss is non-trivial, highlighting that "break-even" pricing still entails significant downside risk. VaR_{95} quantifies the worst 5% of cases, which is crucial for solvency and capital planning.

3.3.2 Sensitivity Insights

The sensitivity analysis revealed that changes in claim costs had the most significant impact on the model's results. Increasing claim costs by 10 percent substantially raised the break-even premium and worsened key loss metrics, including the probability of loss and the Value-at-Risk, reflecting the direct effect of higher outflows on insurer profitability. Adjustments to dependency duration produced a similar pattern, with longer average care periods increasing the required premium and deepening losses in the lower tail of the NPV distribution. In contrast, changes to the discount rate produced smaller effects. This is because discounting influences both premium inflows and claim outflows in the same direction, partially offsetting its net impact on the overall NPV.

3.3.3 Strategic Implications

The results of the risk quantification and sensitivity analysis highlight several important strategic considerations for long-term care insurance product design and risk management. Pricing strategies should incorporate sufficient margins to protect against potential adverse shifts in morbidity rates and care cost trends, ensuring that the insurer can maintain profitability even under less favorable conditions. In addition, capital reserves or reinsurance arrangements should be structured to absorb the tail losses identified by the 95 percent Value-at-Risk metric, thereby safeguarding solvency in extreme but plausible scenarios. Finally, it is critical to maintain an ongoing process of monitoring key assumptions, such as mortality rates, dependency onset patterns, care durations, and discount rates. Regular updates to these inputs will allow the insurer to adjust pricing and risk strategies proactively, minimizing exposure to emerging trends and market volatility over time.

Chapter 4

Results

4.1 Summary of Analysis

All statistics below are computed at the estimated break–even premium. Throughout, positive NPV denotes profit to the insurer and negative NPV denotes loss. The following analysis is based on 10 simulations of output of our code.

Metric	**Mean**	**Range**
Break–even premium ($)	14,280	13,900 – 14,700
Mean NPV ($)	−707.67	−2,779.68 – 2,925.69
Std. NPV ($)	18,020.38	14,121.31 – 20,952.20
Pr(NPV < 0)	41.9%	31% – 49%
$\mathrm{VaR}_{0.95}$ ($)[1]	−35,098.30	−47,417.92 – −26,347.20

Table 4.1: Simulation summary at the estimated break–even premium.

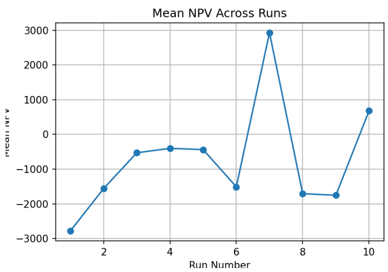
```
Risk Metrics Summary (10 runs):
   Break-Even Premium  Mean NPV   Std NPV  Prob(NPV < 0)  VaR (95%)
0             14100.0  -2779.68  17801.32           0.48  -37466.62
1             14100.0  -1561.46  19037.75           0.47  -32560.59
2             14700.0   -530.04  17618.76           0.42  -36141.86
3             13900.0   -402.18  19140.20           0.38  -32828.97
4             14300.0   -437.27  14121.31           0.39  -32292.21
5             14100.0  -1509.87  20952.20           0.40  -47417.92
6             14300.0   2925.69  16349.90           0.31  -26347.20
7             14300.0  -1706.42  19460.40           0.45  -37277.00
8             14700.0  -1753.65  17682.37           0.49  -36141.86
9             14300.0    678.18  18041.57           0.40  -32511.53
```

Figure 4.1: Table of output of risk metrics for 10 simulations

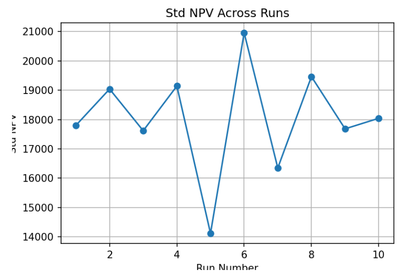
4.1.1 Break–Even Premium

The premium that equates expected present–value premiums and claims concentrates around $14,280, with a tight iteration range of roughly $800 (13,900–14,700). This narrow band, despite stochastic lifetimes

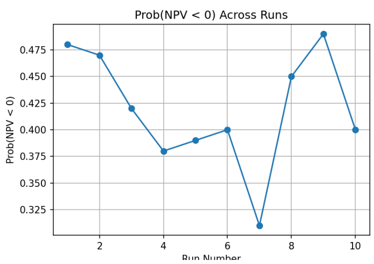
[1] $\mathrm{VaR}_{0.95} = \mathrm{quantile}_{0.05}(\mathrm{NPV})$; negative values indicate insurer loss in the 5% worst outcomes.



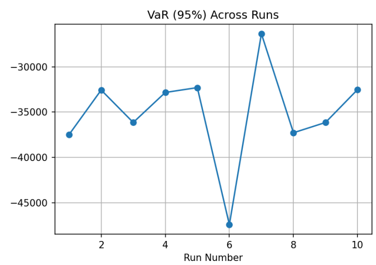
(a) Mean NPV Across Runs



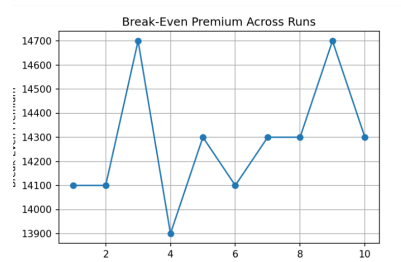
(b) Std NPV Across Runs



(c) Prob(NPV<0) Across Runs



(d) VaR(95%) Across Runs



(e) Break-even Premium Across Runs

and claim severities, reflects a stable solver and consistent model dynamics. Operationally, it indicates the premium level at which the insurer expects neither gain nor loss across a large, diversified portfolio.

4.1.2 Mean NPV at the Estimated Premium

At the reported break–even premium, the sample mean NPV is slightly negative ($-707.67), which is statistically close to zero given Monte Carlo error and the CI–based stopping rule. Individual iteration means span approximately $-2.78k to $+2.93k, a spread driven by randomness in dependency onset, care duration, and severity. Small deviations from zero therefore reflect sampling variability rather than systematic mispricing.

4.1.3 Dispersion: Standard Deviation of NPV

The standard deviation of individual–policy NPV is large (about $18k on average, ranging $14.1k–$21.0k), evidencing substantial idiosyncratic risk. This is economically intuitive: many policyholders never claim or claim briefly, whereas a minority incur prolonged, high–cost care episodes, producing wide NPV swings policy by policy.

4.1.4 Probability of Loss

Even at the break–even premium, the probability that a randomly selected policy produces a loss is material: $\Pr(\text{NPV} < 0) \approx 41.9\%$ (range 31%–49%). This underscores a key property of break–even pricing: it targets zero *expected* profit, not zero loss frequency. From a risk management perspective, such loss probability must be offset by portfolio diversification, capital, and/or a risk margin.

4.1.5 Downside Tail Risk (VaR)

The 95% Value–at–Risk is approximately $35.1k (loss), with iteration ranges between $26.3k and $47.4k. Thus, in the worst 5% of outcomes per policy, the insurer stands to lose on the order of $26–47k. This heavy–tail behavior is characteristic of LTC severity and duration risk and highlights the potential for adverse aggregation in stress scenarios.

4.1.6 Implications for Pricing and Capital

Taken together, these results show that a break–even premium does not eliminate economic risk. With a around 42% loss probability and a 95% tail loss near $35k per policy, prudent practice is to incorporate a *risk margin* above the break–even level (or equivalently, hold economic capital) to achieve target solvency, earnings volatility, and loss–frequency objectives. Sensitivity analysis and stress testing should be used to determine the magnitude of this margin under alternative morbidity, longevity, and discount–rate assumptions.

4.2 Sensitivity analysis

We ran the code an additional 5 times to focus on sensitivity analysis (outputs are in Appendix A), examining how variations in key variable assumptions, such as claim cost, duration of dependency, and discount rate, impact the break-even premium, net present value statistics, and risk metrics for LTC insurance.

Increasing costs of claims by 10% consistently increased the break-even premium across simulations, reflecting the need for higher premiums to account for higher expected costs of care. All other metrics varied, with some iterations increasing values and others decreasing values. This suggests that premium adjustment is stable and directly correlated with increases in cost of claims, while other metrics fluctuate due to sampling variability and the mix of claims realized in each case.

Decreasing costs of claims by 10% consistently decreased the break-even premium across simulations, highlighting how lower expected costs of care reduce the funding needed for the insurer to cover expected LTC claim payouts. The standard deviation of NPVs and value at risk also decreased consistently across simulations showing how lower claim costs reduce magnitude of variation in each policy outcome as high-cost claims decrease in value, lowering the volatility in NPV, and since claim amounts are reduced on average, the largest loss cases are lower in turn which works to shrink the value at risk. Mean NPV and Probability of loss fluctuate between iterations, showing the continued impact of random sampling on mean NPV and the chance of NPV falling below 0.

Through increasing dependency duration by 10%, we saw a consistent increase in break even premium across simulations, as longer care durations on average means policyholders stay in a dependent state for longer periods, which in turn leads to higher cummulative claim payouts, leading to insurer needing to raise premium prices to break even when accounting for additional expected payouts. All other metrics fluctuated based on the iteration due to the stochastic sampling of the model.

From decreasing dependency duration by 10%, we observed a constant decrease in break-even premium across iterations as policyholders staying dependent for shorter periods leads to shorter claim durations, which in turn reduces the cumulative cost of claims. This allows insurers to charge a lower premium while still breaking even. For most iterations, the standard deviation of NPV decreased as long-duration claims became less likely, reducing the range of outcomes and stabilizing the volatility in turn; however, one run showed a slight increase as random sampling is still at play. Other metrics varied within iterations, while we can still note the significant decreases in value at risk in 2 iterations, as value at risk is roughly cut in half as shorter duration periods reduce worst-case scenarios in lower tails, reducing the occurrence of extreme claims, thereby impacting the value at risk in some cases.

Through increasing the discount rate by 50 basis points, we observed a consistent increase in break-even premiums as the present value of future premiums is reduced; insurers must charge a higher premium upfront to compensate and still break even against expected claim payouts when future payments are worth less. Through the diminished present value of future premiums, the spread of outcomes is reduced, resulting in a lowered standard deviation of NPVs in most runs, with a minor increase in one run due to sampling variance. Other metrics varied throughout iterations due to distribution tails and high cost cases, as well as sampling variance still being a factor.

From decreasing the discount rate by 50 basis points, we saw a stable decrease in break-even premiums throughout simulations as lower rates increase the present value of future premiums, allowing insurers to charge lower premiums to break even against expected payouts due to future payments being worth more at the present value. The probability of loss increases for most cases; this could be due to insurers being more vulnerable to loss due to the lower premiums set, as premiums received may not cover high-cost claim cases. We also saw the standard deviation of NPVs mostly decrease as the present value of different timed payments becomes closer, which can lower the difference in outcomes and, in turn, work to lower the standard deviation in most cases. Other metrics fluctuated within simulations as worst-case tail outcomes and model volatility still occur due to random sampling.

4.3 Uncertainty and Risks

The long term nature of LTC insurance makes it very effective against uncertainty and risks. Modeling future claims requires consideration of factors and changes such as longevity, economic volatility and heavy -tailed costs. These can each significantly influence the financial outcomes from the insurers perspective. Net Present value uses fixed averages, not taking account of these factors, thus, we used Monte Carlos simulations to incorporate these.

With the health care and living conditions constantly improving, the life expectancy of an individual increases. With the increase in lifespan, the likelihood of experiencing disabilities and being unable to perform daily acts rises, unable to perform daily activities, resulting in longer need for LTC. This is a very important risk as the probability for long term care increases significantly at advanced ages, and other than increasing the duration of LTC, it also increases the distribution of claims to a higher dependency level, raising the overall cost. Because the overall outflows of funds increases, it results in the rise for the overall cost of present value of liabilities. Since the claim payments are for years after LTC insurance is initially bought and as the value of money is higher than in the future, once longevity increases, the claims get larger than expected, and as future liabilities increase, the present value of it also increases. Additionally, the risk of longevity also reduces the effectiveness of the initial assumptions of pricing at the time of making. Since the premium prices are decided decades in advance, those prices may not represent the actual claim trends due to the mortality improvements, this then results in premiums that are too low to cover the actual payout.

Furthermore, we factor in economic volatility which are the fluctuations in the economic conditions, including interest rates, inflation and investment returns. The fluctuation in these affect the financial outcomes, making them difficult to predict. Due to low interest rates or any change in the economies' markets, the possibility of investment returns dropping results in earning less from the assets, meaning using the premiums or saving that money to help pay for claims, but it becomes difficult to stay profitable if the cost of claims is more than anticipated. If interest rates decrease, the expected value of the money insurers are supposed to receive in the future would be worth less currently, making the present value of those claims more expensive. Inflation also makes paying for items more expensive, especially healthcare costs. Since premiums are the same, if the insurers don't increase them, and medical expenses are rising, the insurers would not have enough premiums to cover these increased costs.

Most policyholders have moderate care needs, those who need normal assistance with daily activities, but a small group requires a higher need for care. Ones that are suffering from cognitive decline and long term conditions such as Alzheimer's, Dementia and Parkinson's disease. Individuals that are suffering from these conditions often get worse over time and need more care and increase the need for supervision and dependency. Even though this group is quite rare and with few individuals, they still result in higher payouts that exceed the average expectations, increasing financial risk and pricing difficulties for insurers, which is the risk of heavy tailed costs.

Chapter 5

Conclusion

This analysis applied Monte Carlo simulation to the net present value (NPV) of long-term care insurance (LTC), taking into account uncertainty and risk, such as the effect of longevity, economic volatility, and heavy-tailed care costs. Using different outcomes instead of relying on fixed averages, we were able to come up with an analysis that helped us approximate the financial exposures insurers faced.

From our simulations, we demonstrated break-even premiums, calculated to yield a mean NPV close to zero, but even with that, it did not eliminate the financial volatility. With a break-even premium of approximately $14,280, there was a loss with probability of 42% per policy, showing that the uncertainty cannot be fully ignored by pricing itself and a 95% VaR of approximately $35,000, proving the downside for insurers. The heavy-tailed risk volatility was reflected through the high standard deviation of the NPV outcomes, specifically the high-cost claims related to those with cognitive decline or long-term illnesses.

The sensitivity analysis made it clear that the duration of care along with claim costs has the highest influence on the pricing and the potential losses. If there is an increase of 10% in duration of care or claim costs, the insurer has to charge higher premiums to break even. Unfortunately, because of this, the worst-case losses also increase, meaning the VaR value gets worse. But when the claim costs decrease by 10%, insurers are able to charge lower premiums, improving the VaR and volatility. Changing the discount rate also affects our results, but it has a small effect as it impacts the outflows and inflows of premiums in the same direction, partly canceling out. The analysis shows that, for long-term sustainability, it is essential to maintain accurate, risk-adjusted premiums, implement effective risk-reduction strategies, and continually monitor economic and demographic conditions.

For premiums to be risk adjusted, we need to include the margins in the break-even level as it provides protection for any negative changes from the original assumptions, especially with rising longevity and increasing healthcare costs. Apart from this, insurers have to maintain strong capital reserves to absorb the rare but severe losses without sacrificing their stability. Lastly, to enable the pricing and the financial plans to take into consideration the changing economic conditions and demographics, it's important to continually readdress and update the key assumptions, which include life expectancy, the age one needs care, inflation and interest rates.

In summary, the Monte Carlo simulation for LTC insurance pricing gave us an analytical approach to evaluate and manage the financial risk. Although the uncertainty cannot fully be ignored, it still allows us to prepare for the possible outcomes, making sure insurers stay financially stable.

Chapter 6

Reference

- Bodily, P., & Furman, J. (2016). Long-term care insurance decisions: Assessing the value of coverage. *Society of Actuaries Research Report*.
- Lazoğlu, M. A., & Büyükyazici, M. (2023). Pricing for longevity risk in long-term care insurance using semi-Markov models. *Insurance: Mathematics and Economics*, 111, 102942.

Chapter 7

Appendix A

Algorithm 1 Estimate Break-Even LTC Premium via Monte Carlo

```
Inputs: Assumptions, parameter distributions, convergence tolerance ε, number of simulations N
Initialize: premium ← initial guess
repeat
    sumNPV ← 0
    for i = 1 to N do
        Simulate one policyholder's lifetime path
        Compute cash inflows: total premiums paid
        Compute cash outflows: claims + administrative costs
        Discount all cash flows to time 0 and compute NPV_i
        sumNPV ← sumNPV + NPV_i
    end for
    NPV‾ ← sumNPV/N
    if NPV‾ > 0 then
        Decrease premium
    else if NPV‾ < 0 then
        Increase premium
    end if
until |NPV‾| ≤ ε
Output: break-even premium ← premium
```

Algorithm 2 Post–Break-Even Analysis and Sensitivity for LTC Premium

```
1: Inputs: Break-even premium p*, assumptions, distributions, number of simulations N, convergence tolerance ε
2: Fixed: premium ← p*
3: Simulation of NPV Distribution:
4:    Initialize sumNPV ← 0, store array NPV[ ]
5: for i = 1 to N do
6:    Simulate one policyholder's lifetime path
7:    Compute cash inflows: total premiums paid (using premium p*)
8:    Compute cash outflows: claims + administrative costs
9:    Discount all cash flows to time 0 and compute NPV_i
10:   Store NPV_i in array
11: end for
12: Compute summary statistics:
       • Mean: N̄PV = (1/N) Σ_i NPV_i
       • Standard deviation: σ_NPV
       • Probability of loss: Pr(NPV < 0)
       • Value-at-Risk at level α: VaR_α
13: Sensitivity Analysis:
14: for each key input variable X do
15:    Perturb X by a small amount ΔX
16:    Rerun steps 3–11 to obtain new summary statistics
17:    Record changes in risk metrics relative to baseline
18: end for
19: Output:
       • Full distribution of simulated NPVs
       • Risk metrics: Pr(NPV < 0), VaR_α
       • Sensitivity results showing impact of each X on key metrics
```

```
Break-even premium: $ 14700.0

Risk Metrics Summary:
 Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
            14700.0  -1740.92 19869.08            0.4  -36661.47

Sensitivity Analysis:
       Scenario  Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
       Baseline             14700.0  -1740.92 19869.08           0.40  -36661.47
      Cost +10%             15900.0   2150.38 18727.03           0.35  -35293.31
      Cost -10%             13100.0    364.91 16827.62           0.34  -28951.74
  Duration +10%             16100.0   2122.73 16996.88           0.36  -29876.77
  Duration -10%             12700.0   -984.98 16987.82           0.44  -34439.27
Discount +50 bp             19300.0  -1417.75 19946.56           0.40  -46263.21
Discount -50 bp             10900.0   -721.33 17222.54           0.46  -32254.41
```

```
Break-even premium: $ 14100.0

Risk Metrics Summary:
 Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
            14100.0   1588.12 16381.68           0.34  -32560.59

Sensitivity Analysis:
       Scenario  Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
       Baseline             14100.0   1588.12 16381.68           0.34  -32560.59
      Cost +10%             16500.0  -2858.48 21974.10           0.53  -39313.22
      Cost -10%             12500.0    431.97 14836.45           0.38  -29756.89
  Duration +10%             16100.0  -1264.18 20619.54           0.39  -44734.10
  Duration -10%             12900.0  -1342.70 17128.13           0.44  -34170.89
Discount +50 bp             19700.0   2094.95 12945.34           0.42  -25308.95
Discount -50 bp             10500.0    748.48 16543.25           0.41  -32752.23
```

```
Break-even premium: $ 14100.0

Risk Metrics Summary:
 Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
            14100.0     946.8 18530.17           0.35  -33378.29

Sensitivity Analysis:
       Scenario  Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
       Baseline             14100.0    946.80 18530.17           0.35  -33378.29
      Cost +10%             15700.0   -719.21 16800.73           0.46  -35561.69
      Cost -10%             12700.0    157.32 13355.87           0.37  -23659.00
  Duration +10%             16100.0    -41.98 15918.27           0.41  -29876.77
  Duration -10%             12900.0   1882.24 17597.50           0.30  -34390.21
Discount +50 bp             19300.0   1081.80 16684.98           0.37  -25702.08
Discount -50 bp             10700.0     71.40 17128.69           0.45  -33219.90
```

```
Break-even premium: $ 14500.0

Risk Metrics Summary:
 Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
            14500.0    113.83 19693.38           0.39  -48377.75

Sensitivity Analysis:
       Scenario  Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
       Baseline             14500.0    113.83 19693.38           0.39  -48377.75
      Cost +10%             15900.0   2289.33 17353.83           0.34  -28753.80
      Cost -10%             12900.0  -3197.12 16193.91           0.52  -32970.51
  Duration +10%             15700.0   -476.34 20904.81           0.42  -46767.45
  Duration -10%             13300.0   2096.87 13703.40           0.38  -16883.36
Discount +50 bp             19700.0  -2449.13 19567.43           0.45  -45870.09
Discount -50 bp             10500.0  -2943.06 17824.40           0.53  -32987.45
```

```
Break-even premium: $ 14100.0

Risk Metrics Summary:
 Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
            14100.0    286.93 15967.69            0.4  -32779.91

Sensitivity Analysis:
       Scenario  Break-Even Premium  Mean NPV  Std NPV  Prob(NPV < 0)  VaR (95%)
       Baseline             14100.0    286.93 15967.69           0.40  -32779.91
      Cost +10%             15900.0   -576.46 19895.68           0.43  -36106.13
      Cost -10%             13100.0   1056.15 13884.02           0.35  -23403.84
  Duration +10%             16100.0    954.48 18243.11           0.38  -44734.10
  Duration -10%             12500.0   2081.46 11980.43           0.42  -17956.89
Discount +50 bp             18500.0    -43.68 14891.82           0.36  -26803.55
Discount -50 bp             10500.0   1326.11 14392.25           0.38  -26543.26
```

Figure 7.1: Output of Sensitivity Analysis for 5 Simulations

Chapter 8

Appendix - Code

monte_carlo_sim.py

```python
import numpy as np
import pandas as pd

# ------------------------------
# PARAMETERS
# ------------------------------
N = 100  # number of policyholder simulations
epsilon = 1e-2  # convergence tolerance
premium_guess = 2000.0  # initial premium guess
premium_step = 100.0  # adjustment step size
discount_rate = 0.002  # monthly rate (e.g., 0.2%)
entry_age = 60
max_age = 100

# ------------------------------
# SIMULATION COMPONENTS
# ------------------------------

def simulate_lifetime():
    return np.random.normal(loc=85, scale=5)  # age at death

def simulate_dependency_entry():
    return np.random.normal(loc=75, scale=3)  # age at dependency

def simulate_dependency_duration():
    return np.random.weibull(2) * 3  # duration in years

def simulate_claim_cost(level=2):
    return {1: 1500, 2: 1000, 3: 600, 4: 400}.get(level, 1000)

def discount_cashflow(amount, t):
    return amount / ((1 + discount_rate) ** (t * 12))  # yearly discounting

# ------------------------------
```

```python
# ALGORITHM 1: BREAK-EVEN PREMIUM
# ------------------------------

def compute_npv(premium):
    npv_sum = 0
    for _ in range(N):
        age_at_death = simulate_lifetime()
        entry = simulate_dependency_entry()
        duration = simulate_dependency_duration()
        level = np.random.choice([1, 2, 3, 4], p=[0.2, 0.3, 0.3, 0.2])
        benefit = simulate_claim_cost(level)

        if entry > age_at_death:
            claims = 0
        else:
            claim_years = min(duration, age_at_death - entry)
            claims = sum(discount_cashflow(benefit * 12, t) for t in range(1, int(claim_years) + 1))

        # Premiums (paid until age 65)
        premium_years = range(entry_age, int(min(age_at_death, 65)) + 1)
        premiums = sum(discount_cashflow(premium, t) for t in premium_years)

        npv_sum += premiums - claims

    return npv_sum / N

# Find break-even premium
premium = premium_guess
iteration = 0
while True:
    avg_npv = compute_npv(premium)
    if abs(avg_npv) <= epsilon or iteration > 200:
        break
    premium += -premium_step if avg_npv > 0 else premium_step
    iteration += 1
    print(f"${avg_npv}")

break_even_premium = round(premium, 2)
print(f"Break-even premium: $ {break_even_premium}")

# ------------------------------
# ALGORITHM 2: RISK METRICS
# ------------------------------

npvs = []
for _ in range(N):
    age_at_death = simulate_lifetime()
    entry = simulate_dependency_entry()
```

```python
    duration = simulate_dependency_duration()
    level = np.random.choice([1, 2, 3, 4], p=[0.2, 0.3, 0.3, 0.2])
    benefit = simulate_claim_cost(level)

    if entry > age_at_death:
        claims = 0
    else:
        claim_years = min(duration, age_at_death - entry)
        claims = sum(discount_cashflow(benefit * 12, t) for t in range(1, int(claim_years) + 1))

    premium_years = range(entry_age, int(min(age_at_death, 65)) + 1)
    premiums = sum(discount_cashflow(break_even_premium, t) for t in premium_years)

    npvs.append(premiums - claims)

npvs = np.array(npvs)

# Risk Metrics
mean_npv = round(np.mean(npvs), 2)
std_npv = round(np.std(npvs), 2)
prob_loss = round(np.mean(npvs < 0), 4)
var_95 = round(np.percentile(npvs, 5), 2)

summary = pd.DataFrame({
    "Break-Even Premium": [break_even_premium],
    "Mean NPV": [mean_npv],
    "Std NPV": [std_npv],
    "Prob(NPV < 0)": [prob_loss],
    "VaR (95%)": [var_95]
})

print("\nRisk Metrics Summary:")
print(summary.to_string(index=False))

# -------------------------------
# SENSITIVITY ANALYSIS
# -------------------------------

def recompute_break_even():
    premium = premium_guess
    iteration = 0
    while True:
        avg_npv = compute_npv(premium)
        if abs(avg_npv) <= epsilon or iteration > 200:
            break
        premium += -premium_step if avg_npv > 0 else premium_step
        iteration += 1
    return round(premium, 2)
```

```python

def risk_metrics_for_premium(prem):
    npvs_local = []
    for _ in range(N):
        age_at_death = simulate_lifetime()
        entry = simulate_dependency_entry()
        duration = simulate_dependency_duration()
        level = np.random.choice([1, 2, 3, 4], p=[0.2, 0.3, 0.3, 0.2])
        benefit = simulate_claim_cost(level)

        if entry > age_at_death:
            claims = 0.0
        else:
            claim_years = min(duration, age_at_death - entry)
            claims = sum(discount_cashflow(benefit * 12, t) for t in range(1, int(claim_years) + 1))

        premium_years = range(entry_age, int(min(age_at_death, 65)) + 1)
        premiums = sum(discount_cashflow(prem, t) for t in premium_years)

        npvs_local.append(premiums - claims)

    npvs_local = np.array(npvs_local)
    return {
        "Mean NPV": round(np.mean(npvs_local), 2),
        "Std NPV": round(np.std(npvs_local), 2),
        "Prob(NPV < 0)": round(np.mean(npvs_local < 0), 4),
        "VaR (95%)": round(np.percentile(npvs_local, 5), 2),
    }

def run_scenario(name, cost_mult=1.0, duration_mult=1.0, dr_bp_annual=0):

    global discount_rate, simulate_dependency_duration, simulate_claim_cost

    # backups
    base_dr = discount_rate
    base_duration_fn = simulate_dependency_duration
    base_cost_fn = simulate_claim_cost

    try:
        discount_rate = base_dr + (dr_bp_annual / 10000.0) / 12.0


        def duration_wrapped():
            return base_duration_fn() * duration_mult

        def cost_wrapped(level=2):
            return base_cost_fn(level) * cost_mult

```

```python
        simulate_dependency_duration = duration_wrapped
        simulate_claim_cost = cost_wrapped

        be = recompute_break_even()
        metrics = risk_metrics_for_premium(be)
        metrics.update({"Scenario": name, "Break-Even Premium": be})
        return metrics

    finally:

        discount_rate = base_dr
        simulate_dependency_duration = base_duration_fn
        simulate_claim_cost = base_cost_fn

# Build scenarios
scenarios = [
    ("Cost +10%",       1.10, 1.00,   0),
    ("Cost -10%",       0.90, 1.00,   0),
    ("Duration +10%",   1.00, 1.10,   0),
    ("Duration -10%",   1.00, 0.90,   0),
    ("Discount +50 bp", 1.00, 1.00,  50),
    ("Discount -50 bp", 1.00, 1.00, -50),
]


baseline_row = {
    "Scenario": "Baseline",
    "Break-Even Premium": break_even_premium,
    "Mean NPV": mean_npv,
    "Std NPV": std_npv,
    "Prob(NPV < 0)": prob_loss,
    "VaR (95%)": var_95
}

# Run sensitivity
sens_results = [baseline_row] + [run_scenario(name, cm, dm, bp) for (name, cm, dm, bp) in scenarios]
sens_df = pd.DataFrame(sens_results, columns=["Scenario","Break-Even Premium","Mean NPV","Std NPV","Prob(NPV < 0)","VaR (95%)"])

print("\nSensitivity Analysis:")
print(sens_df.to_string(index=False))
```