Chapter 3

Search techniques

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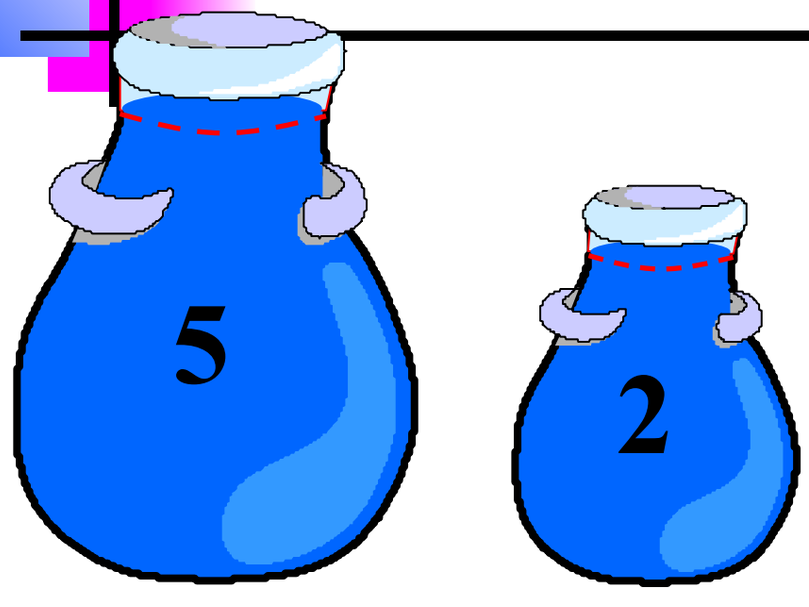
Water Jug Problem

Given a full 5-gallon jug and a full 2-gallon jug, fill the 2-gallon jug with exactly one gallon of water.

- **State:** ?
- **Initial State:** ?
- **Operators:** ?
- **Goal State:** ?

Water Jug Problem



- State = (x,y), where x is the number of gallons of water in the 5-gallon jug and y is # of gallons in the 2-gallon jug
- Initial State = (5,2)
- Goal State = (*,1), where * means any amount

Operator table

Name	Cond.	Transition	Effect
Empty5	–	(x,y)→(0,y)	Empty 5-gal. jug
Empty2	–	(x,y)→(x,0)	Empty 2-gal. jug
2to5	$x \leq 3$	(x,2)→(x+2,0)	Pour 2-gal. into 5-gal.
5to2	$x \geq 2$	(x,0)→(x-2,2)	Pour 5-gal. into 2-gal.
5to2part	$y < 2$	(1,y)→(0,y+1)	Pour partial 5-gal. into 2-gal.

Water jug state space

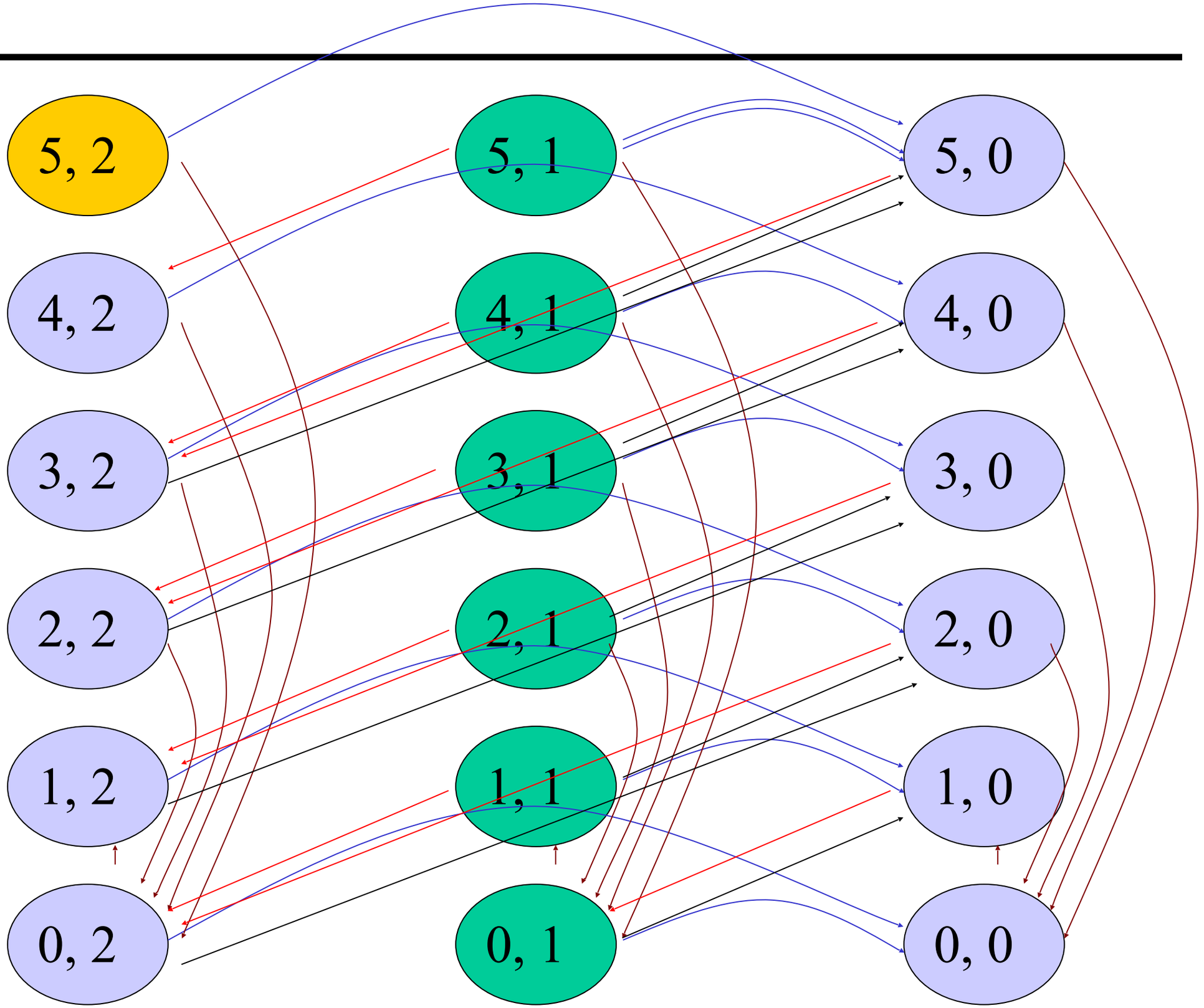
Empty5

Empty2

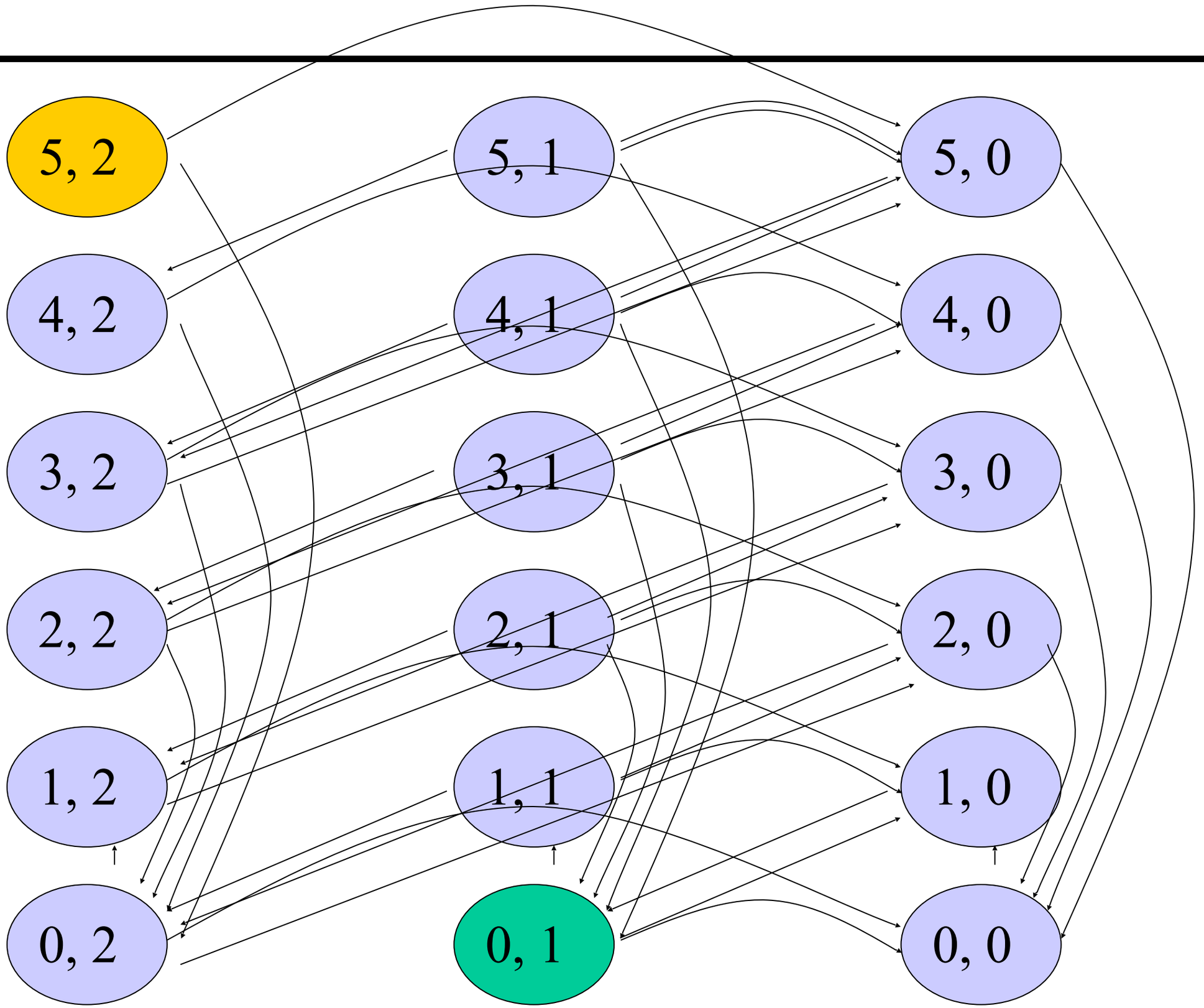
2to5

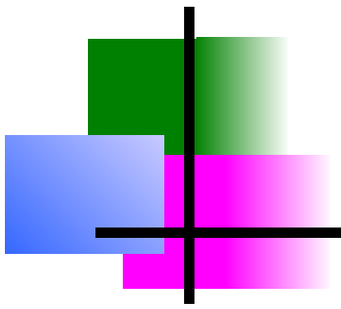
5to2

5to2part



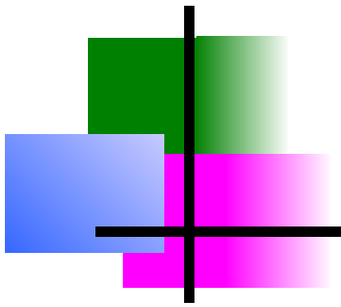
Water jug solution





Formalizing search III

- A **solution** is a sequence of operators that is associated with a path in a state space from a start node to a goal node.
- The **cost of a solution** is the sum of the arc costs on the solution path.
  - If all arcs have the same (unit) cost, then the solution cost is just the length of the solution (number of steps / state transitions)

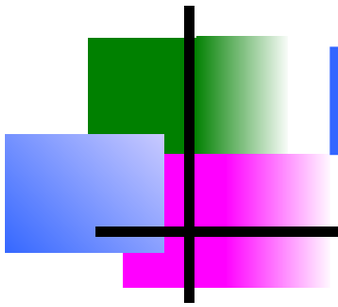


Formalizing search IV

- **State-space search** is the process of searching through a state space for a solution by **making explicit** a sufficient portion of an **implicit** state-space graph to find a goal node.
  - For large state spaces, it isn't practical to represent the whole space.
  - Initially V={S}, where S is the start node; when S is expanded, its successors are generated and those nodes are added to V and the associated arcs are added to E. This process continues until a goal node is found.
- Each node implicitly or explicitly represents a partial solution path (and cost of the partial solution path) from the start node to the given node.
  - In general, from this node there are many possible paths (and therefore solutions) that have this partial path as a prefix.

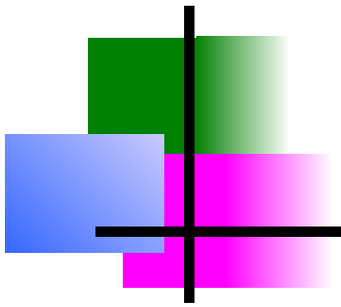
State-space search algorithm

```
function general-search (problem, QUEUEING-FUNCTION)
;; problem describes the start state, operators, goal test, and operator costs
;; queueing-function is a comparator function that ranks two states
;; general-search returns either a goal node or failure
nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  loop
     if EMPTY(nodes) then return "failure"
     node = REMOVE-FRONT(nodes)
     if problem.GOAL-TEST(node.STATE) succeeds
        then return node
     nodes = QUEUEING-FUNCTION(nodes, EXPAND(node,
             problem.OPERATORS))
end
  ;; Note: The goal test is NOT done when nodes are generated
  ;; Note: This algorithm does not detect loops
```



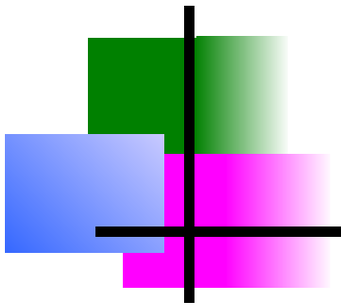
Key procedures to be defined

- EXPAND
  - Generate all successor nodes of a given node
- GOAL-TEST
  - Test if state satisfies all goal conditions
- QUEUEING-FUNCTION
  - Used to maintain a ranked list of nodes that are candidates for expansion



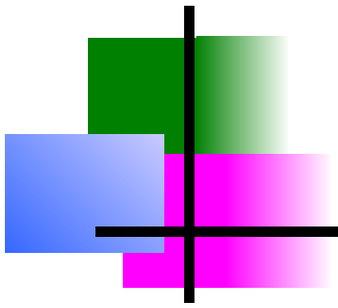
Bookkeeping

- Typical node data structure includes:
  - State at this node
  - Parent node
  - Operator applied to get to this node
  - Depth of this node (number of operator applications since initial state)
  - Cost of the path (sum of each operator application so far)



Some issues

- Search process constructs a search tree, where
  - **root** is the initial state and
  - **leaf nodes** are nodes
    - not yet expanded (i.e., they are in the list “nodes”) or
    - having no successors (i.e., they’re “deadends” because no operators were applicable and yet they are not goals)
- Search tree may be infinite because of loops even if state space is small
- Return a path or a node depending on problem.
  - E.g., in cryptarithmetic return a node; in 8-puzzle return a path
- Changing definition of the QUEUEING-FUNCTION leads to different search strategies



Evaluating Search Strategies

- **Completeness**
  - Guarantees finding a solution whenever one exists
- **Time complexity**
  - How long (worst or average case) does it take to find a solution? Usually measured in terms of the number of nodes expanded
- **Space complexity**
  - How much space is used by the algorithm? Usually measured in terms of the maximum size of the "nodes" list during the search
- **Optimality/Admissibility**
  - If a solution is found, is it guaranteed to be an optimal one? That is, is it the one with minimum cost?

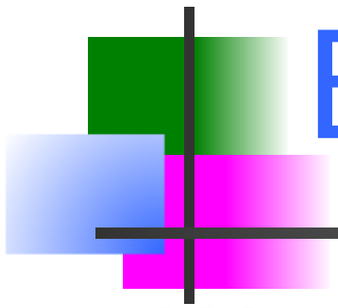
Uninformed vs. informed search

- **Uninformed search strategies**
  - Also known as "blind search," uninformed search strategies use no information about the likely "direction" of the goal node(s)
  - Uninformed search methods: Breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional
- **Informed search strategies**
  - Also known as "heuristic search," informed search strategies use information about the domain to (try to) (usually) head in the general direction of the goal node(s)
  - Informed search methods: Hill climbing, best-first, greedy search, beam search, A, A*

Uninformed Search Methods

Uninformed search strategies

- Uninformed: While searching you have no clue whether one non-goal state is better than any other. Your search is blind. You don't know if your current exploration is likely to be fruitful.
- Various blind strategies:
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Iterative deepening search



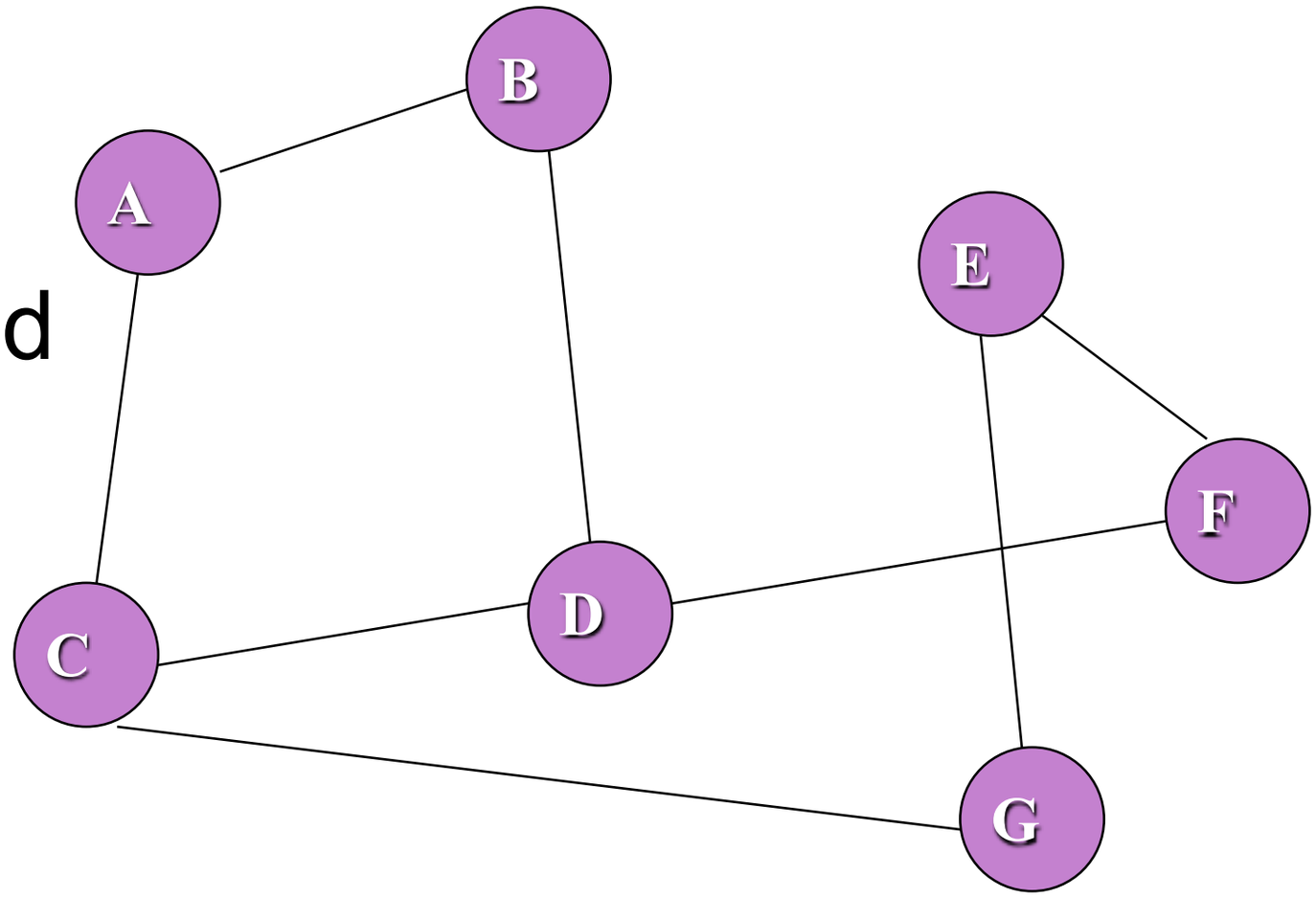
Blind Searching

- We call search algorithms that do not use any extra information regarding the problem or our representation of the problem, “blind”.

- These searches typically visit all of the nodes of a tree in a certain order looking for a pre-specified goal.
  - No cleverness is used to decide where to look next.
  - Searches may never find the goal state.
  - In some cases blind search is the right approach.

Path Finding Problems

- One class of search problems which best illustrates the important algorithms involves the search through a graph to find a path between two nodes.



Find a path from A to E

Graph Searching Problems

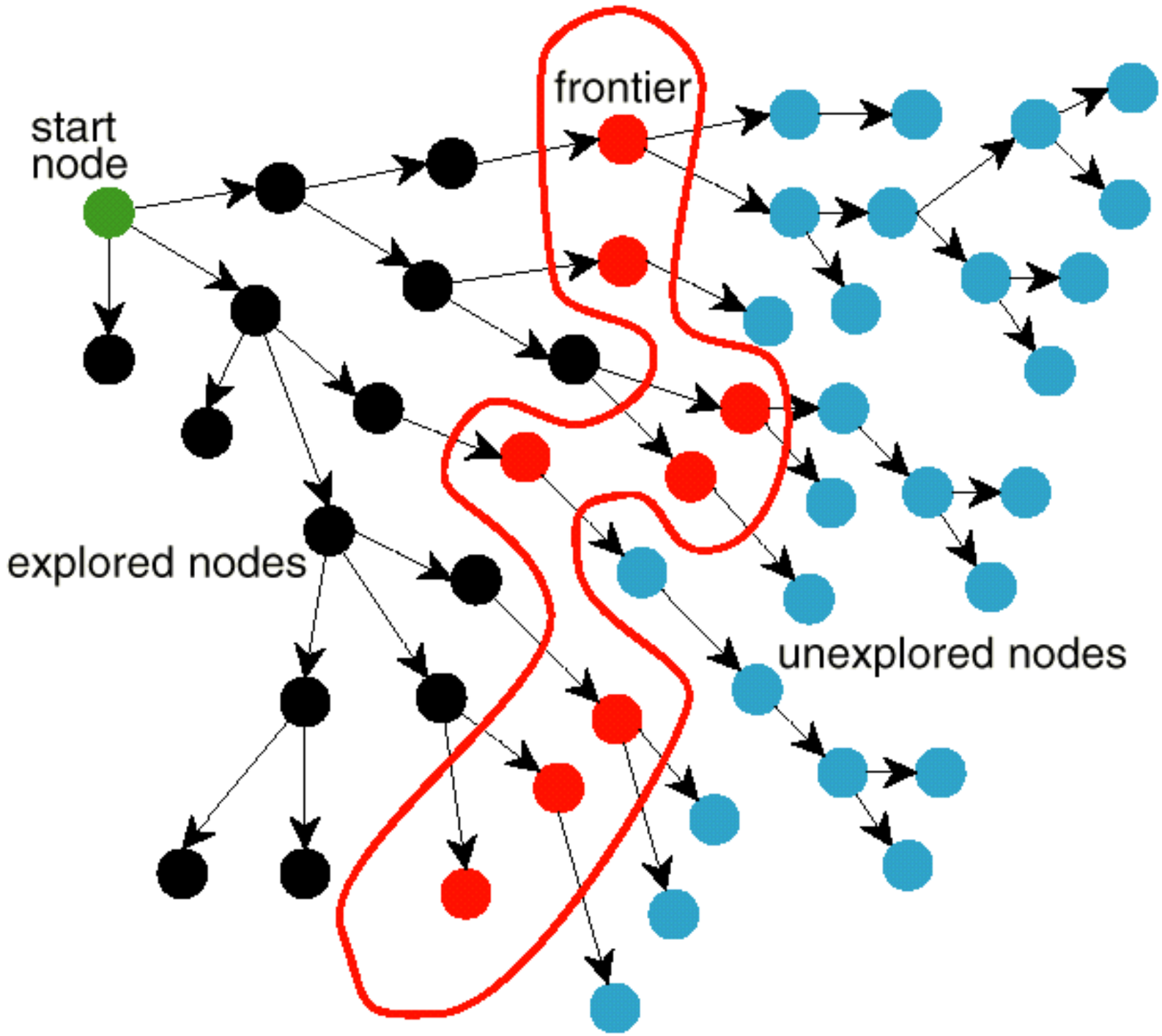
- Generic search algorithm: given a graph, start nodes, and goal nodes, incrementally explore paths from the start nodes.

- Maintain a ***frontier*** of paths from the start node that have been explored.

- As search proceeds, the frontier expands into the unexplored nodes until a goal node is encountered.

- The way in which the frontier is expanded defines the ***search strategy***.

Example



Breadth First Search

- PROCESS: Slow descent method

Place the start node at the end of a queue

Examine the node at the front of the queue

- if the queue is empty, stop

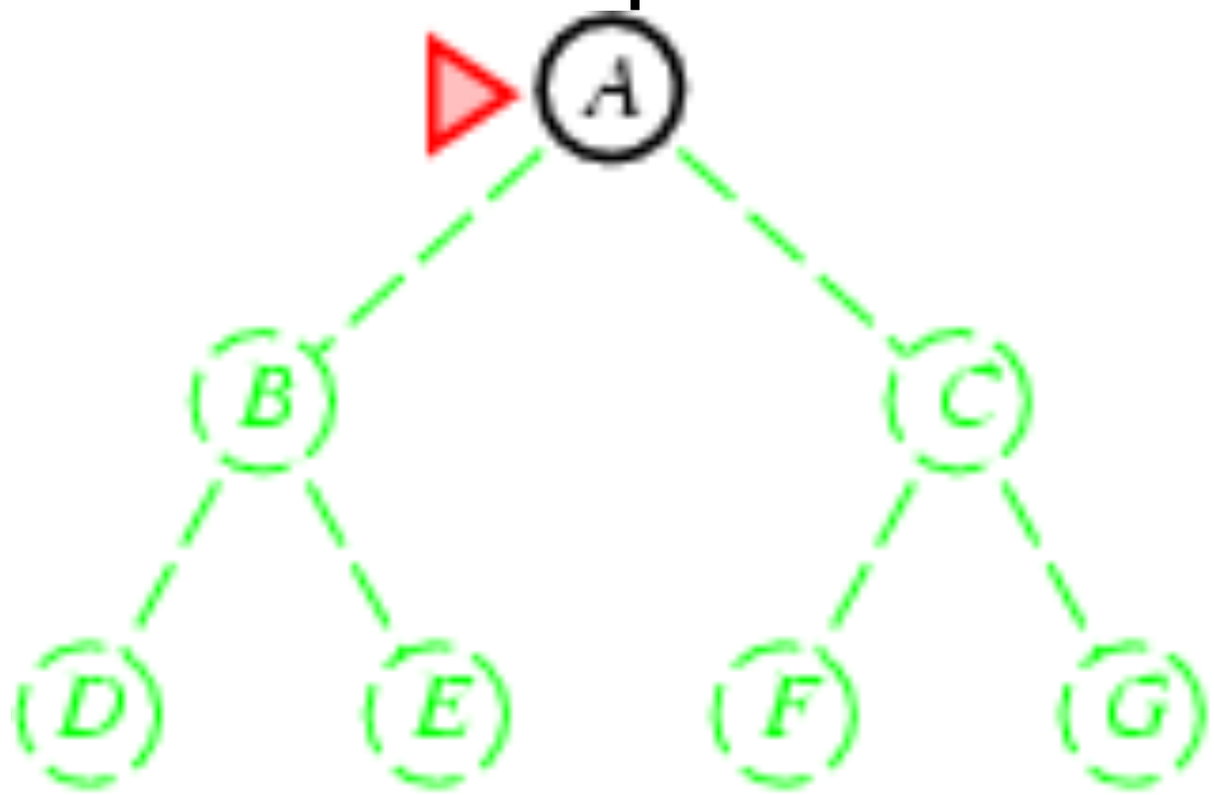
- if the node is the goal, stop

- otherwise, add the children of the node to the end of the queue

Breadth-first search

- Expand shallowest unexpanded node
- Fringe: nodes waiting in a queue to be explored
- Implementation:
  - *fringe* is a first-in-first-out (FIFO) queue, i.e., new successors go at end of the queue.

Is A a goal state?

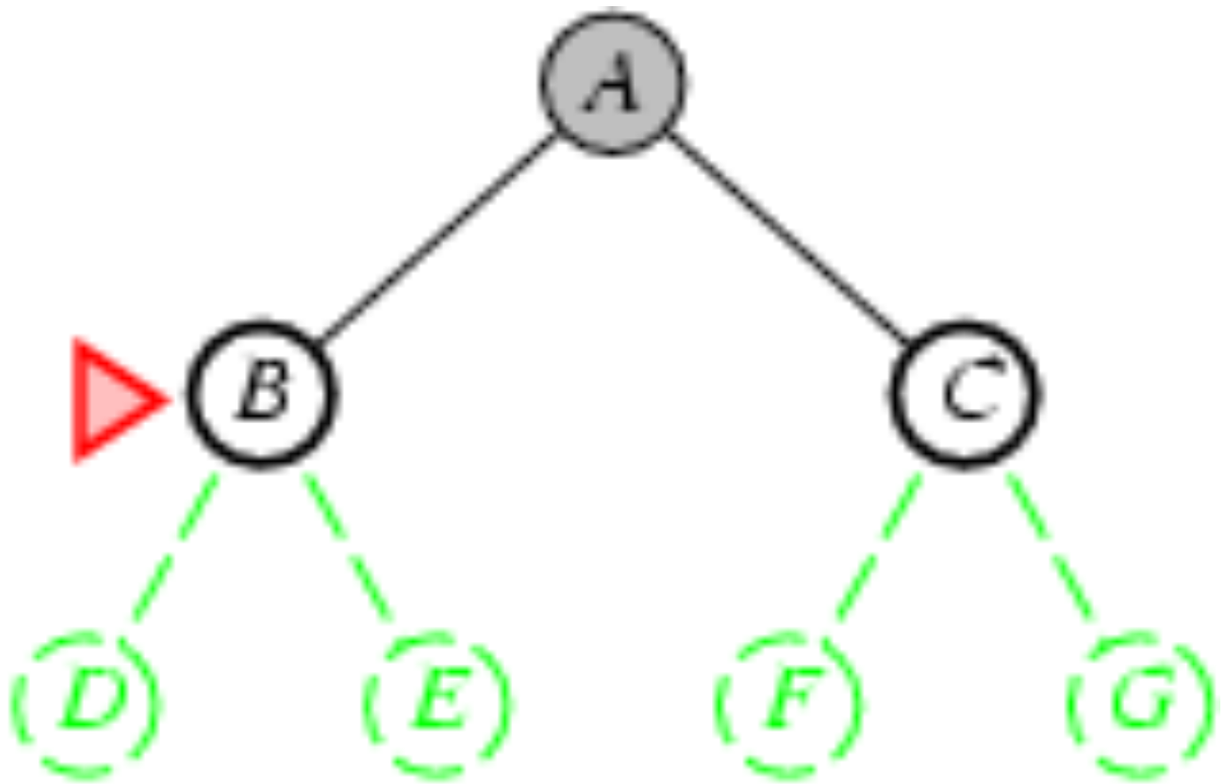


Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:
fringe = [B,C]

Is B a goal state?



Breadth-first search

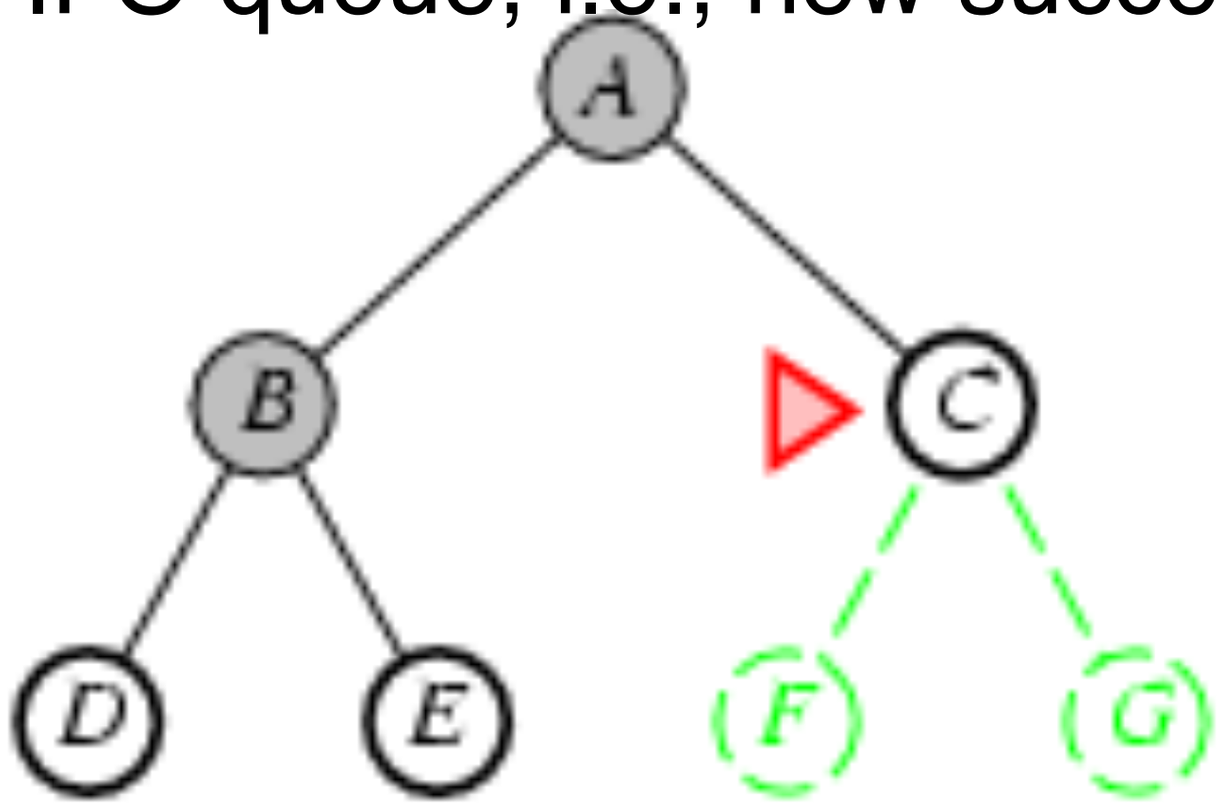
- Expand shallowest unexpanded node

- Implementation:

  fringe is a FIFO queue, i.e., new successors go at end

Expand:
fringe=[C,D,E]

Is C a goal state?

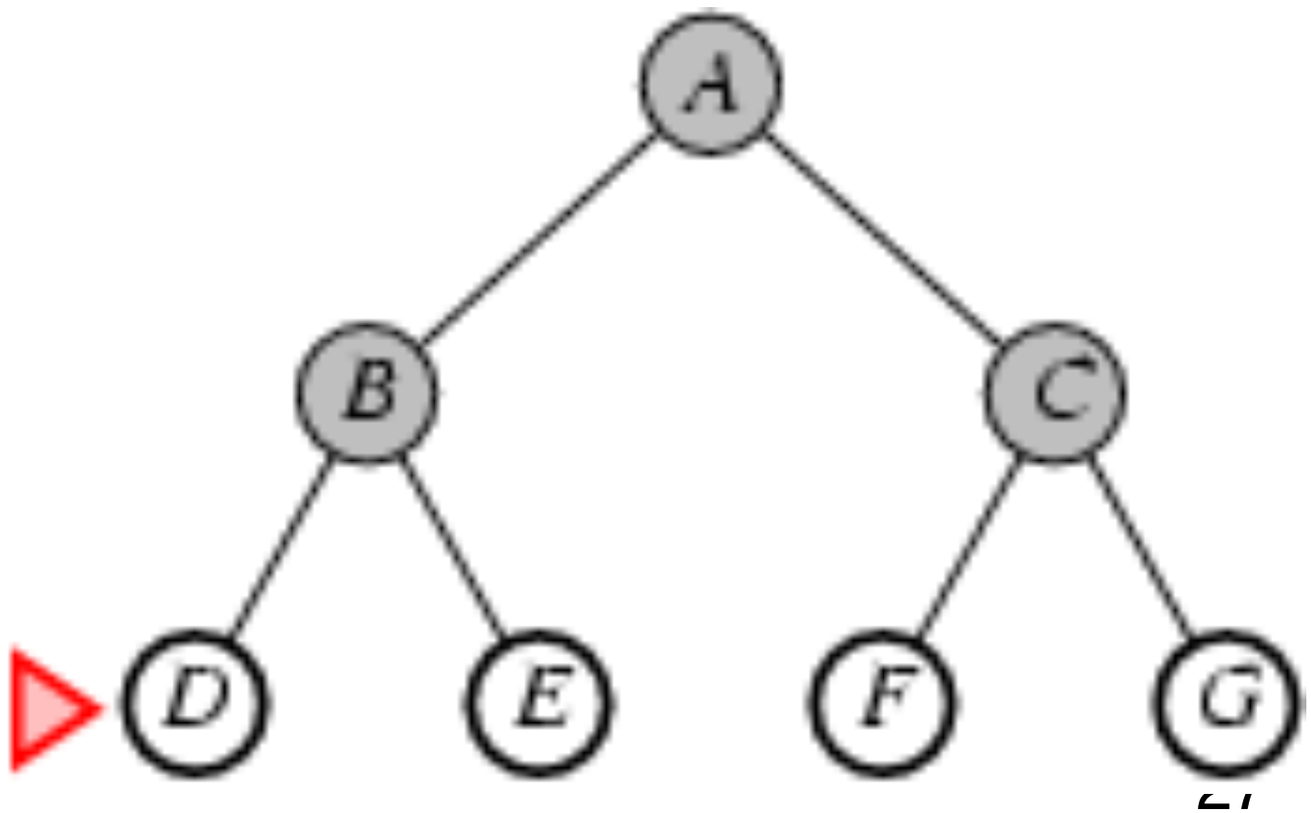


Breadth-first search

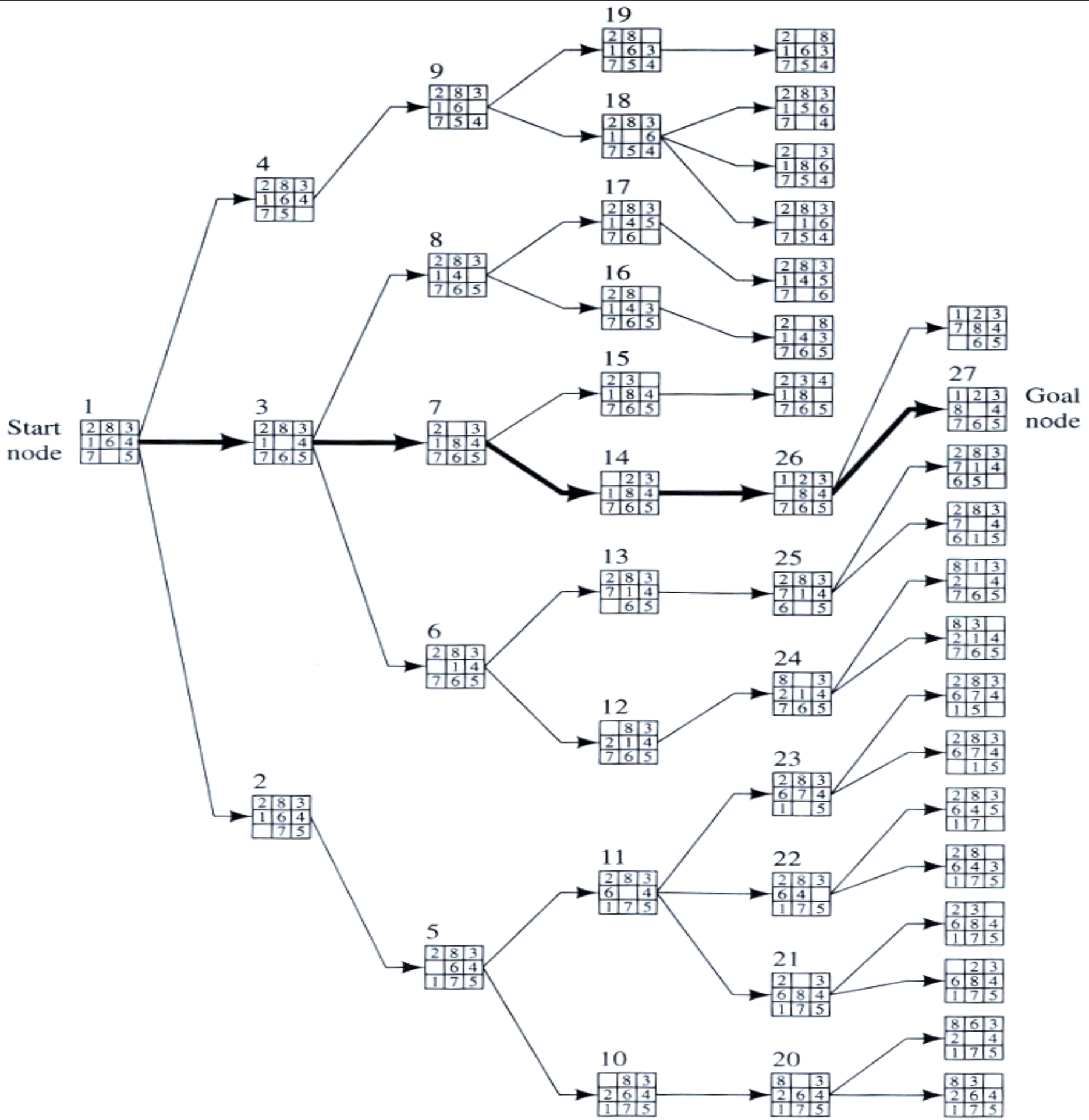
- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:
fringe=[D,E,F,G]

Is D a goal state?



Exai BFS


Start node
Goal node
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27

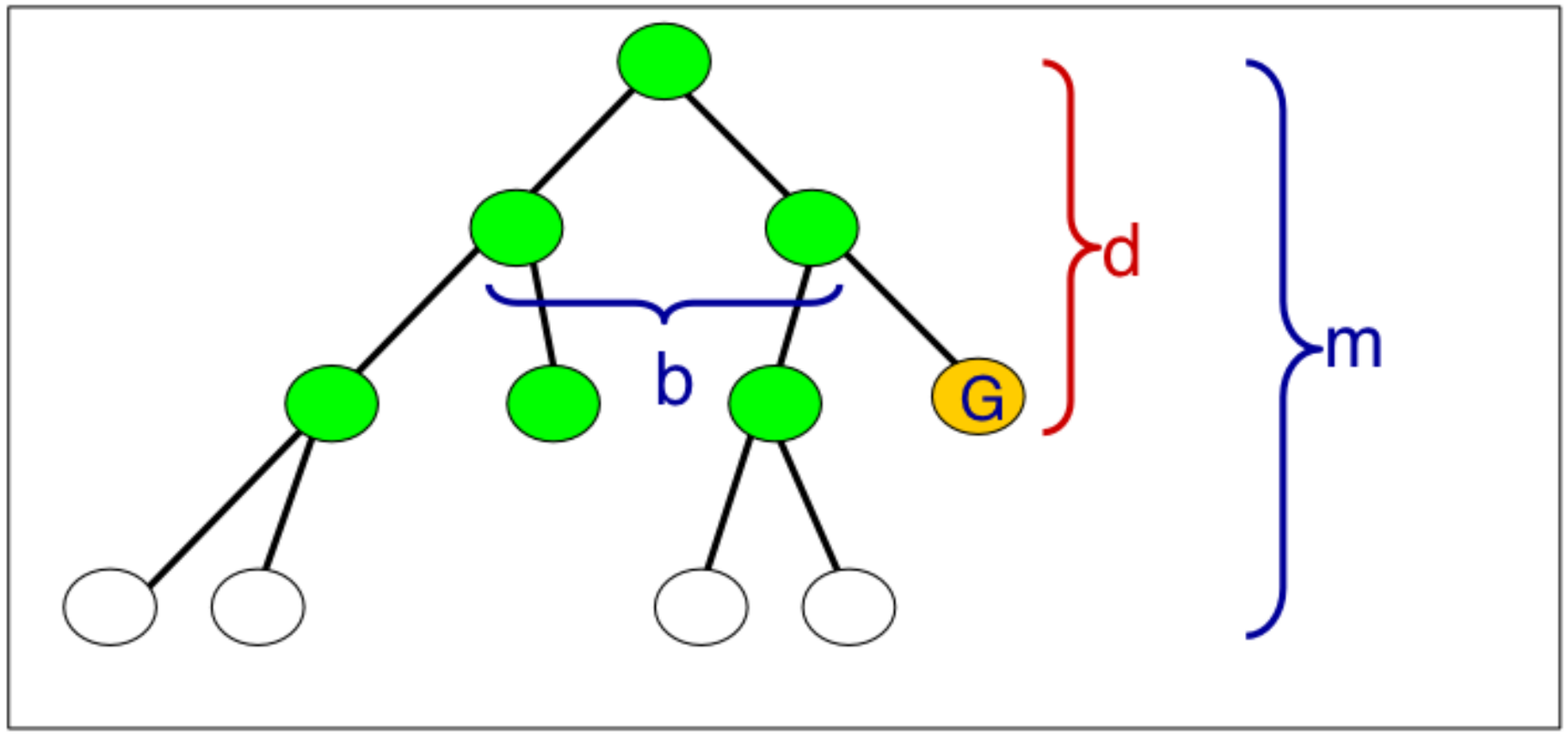
Properties of breadth-first search

- Complete? Yes it always reaches goal (if *b* is finite)
- Time? $1+b+b^2+b^3+\ldots+b^d + (b^{d+1}-b)) = O(b^{d+1})$
  (this is the number of nodes we generate)
- Space? $O(b^{d+1})$ (keeps every node in memory, either in fringe or on a path to fringe).
- Optimal? Yes (if we guarantee that deeper solutions are less optimal, e.g. step-cost=1).

- Space is the bigger problem (more than time)

Time complexity of breadth-first search

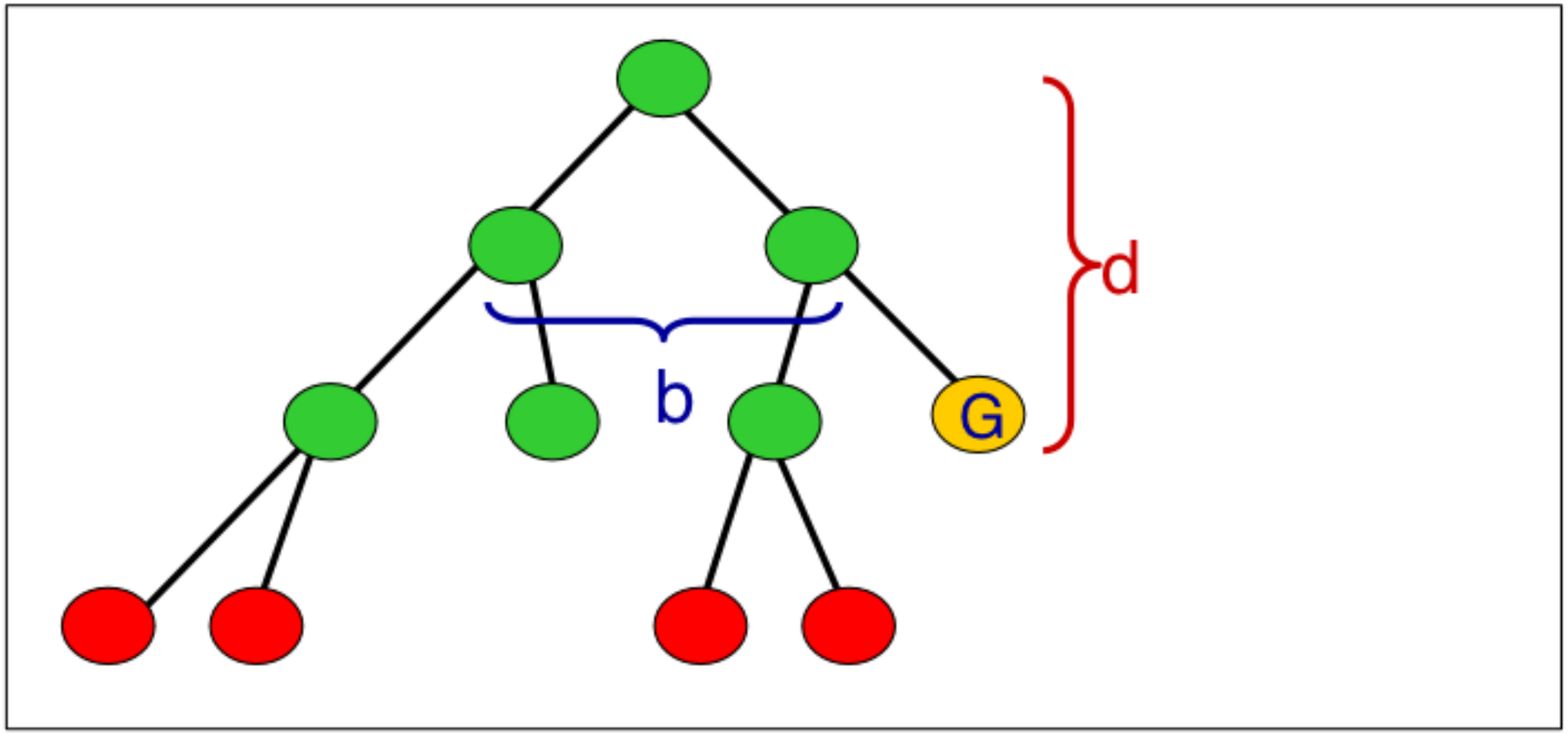
- If a goal node is found on depth d of the tree, all nodes up till that depth are created and examined (note: and the children of nodes at depth d are created and enqueued, but not yet examined).



- Thus: $O(b^d)$

Space complexity of breadth-first search

- Largest number of nodes in QUEUE is reached on the level d+1 just beyond the goal node.



- QUEUE contains all nodes. (Thus: 4) .
- In General: $b^{d+1} - b \sim b^{d}$

BFS Weakness

- Exponential Growth

Time and memory requirements for breadth-first search, assuming a branching factor of 10, 100 bytes per node and searching 1000 nodes/second

Depth	Nodes	Time		Memory	
0	1	1	millisecond	100	kbytes
2	111	0.1	second	11	kilobytes
4	11,111	11	seconds	1	megabyte
6	10^6	18	minutes	111	megabytes
8	10^8	31	hours	11	gigabytes
10	10^{10}	128	days	1	terabyte
12	10^{12}	35	years	111	terabytes
14	10^{14}	3500	years	11,111	terabytes

Uniform-cost search

Breadth-first is only optimal if step costs is increasing with depth (e.g. constant). Can we guarantee optimality for any step cost?

Uniform-cost Search: Expand node with smallest path cost g(n).

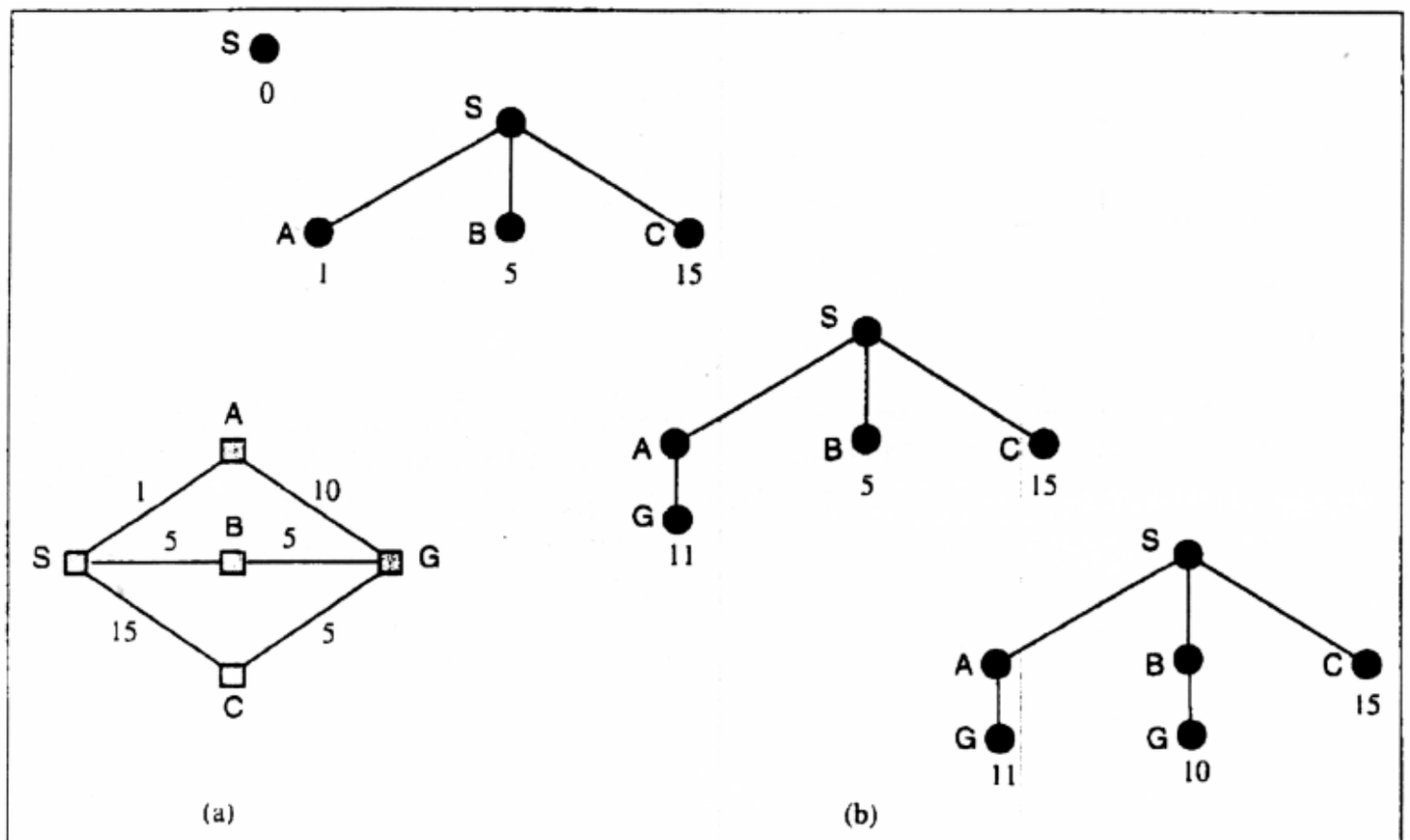


Figure 3.13 A route-finding problem. (a) The state space, showing the cost for each operator. (b) Progression of the search. Each node is labelled with $g(n)$. At the next step, the goal node with $g = 10$ will be selected.

Proof Completeness:

Given that every step will cost more than 0, and assuming a finite branching factor, there is a finite number of expansions required before the total path cost is equal to the path cost of the goal state. Hence, we will reach it.

Proof of optimality given completeness:

Assume UCS is not optimal.
Then there must be a goal state with path cost smaller than the goal state which was found (invoking completeness).
However, this is impossible because UCS would have expanded that node first by definition.
Contradiction.

Uniform-cost search

Implementation: *fringe* = queue ordered by path cost
Equivalent to breadth-first if all step costs all equal.

Complete? Yes, if step cost ≥ ε
(otherwise it can get stuck in infinite loops)

Time? # of nodes with *path cost* ≤ cost of optimal solution.

Space? # of nodes on paths with path cost ≤ cost of optimal solution.

Optimal? Yes, for any step cost ≥ ε

Depth First Search

- PROCESS: Dive into the graph and follow a single path as far as possible
  - Use a stack to keep track of nodes

Put the start node on the stack

Pop the stack

If the stack is empty then stop, there is no path

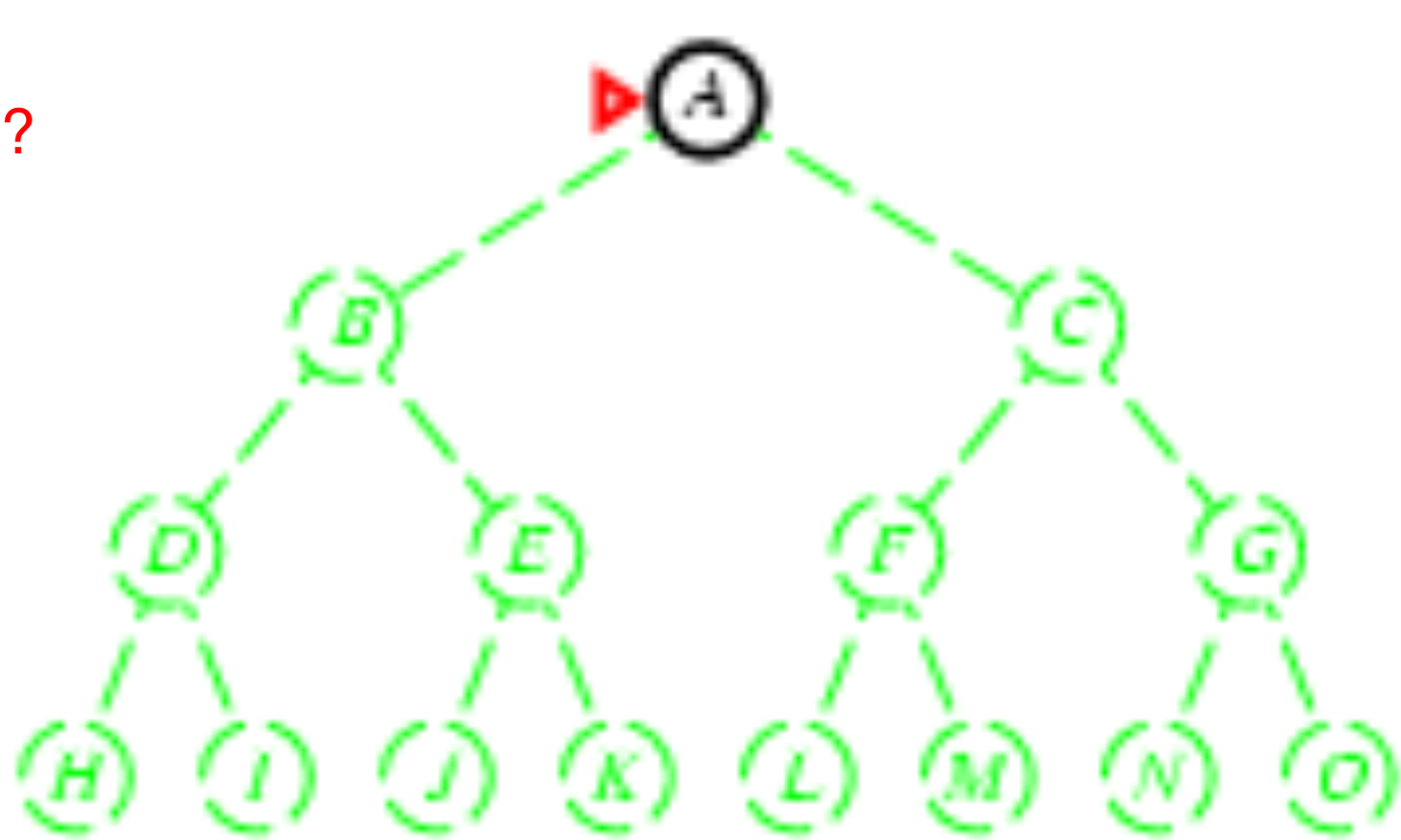
If the top of the stack is the goal, stop

Otherwise, push the nodes connected to the top of the stack on the stack (provided they are not already on the stack)

Depth-first search

- Expand *deepest* unexpanded node
- Implementation:
  - *fringe* = Last In First Out (LIPO) queue, i.e., put successors at front

Is A a goal state?

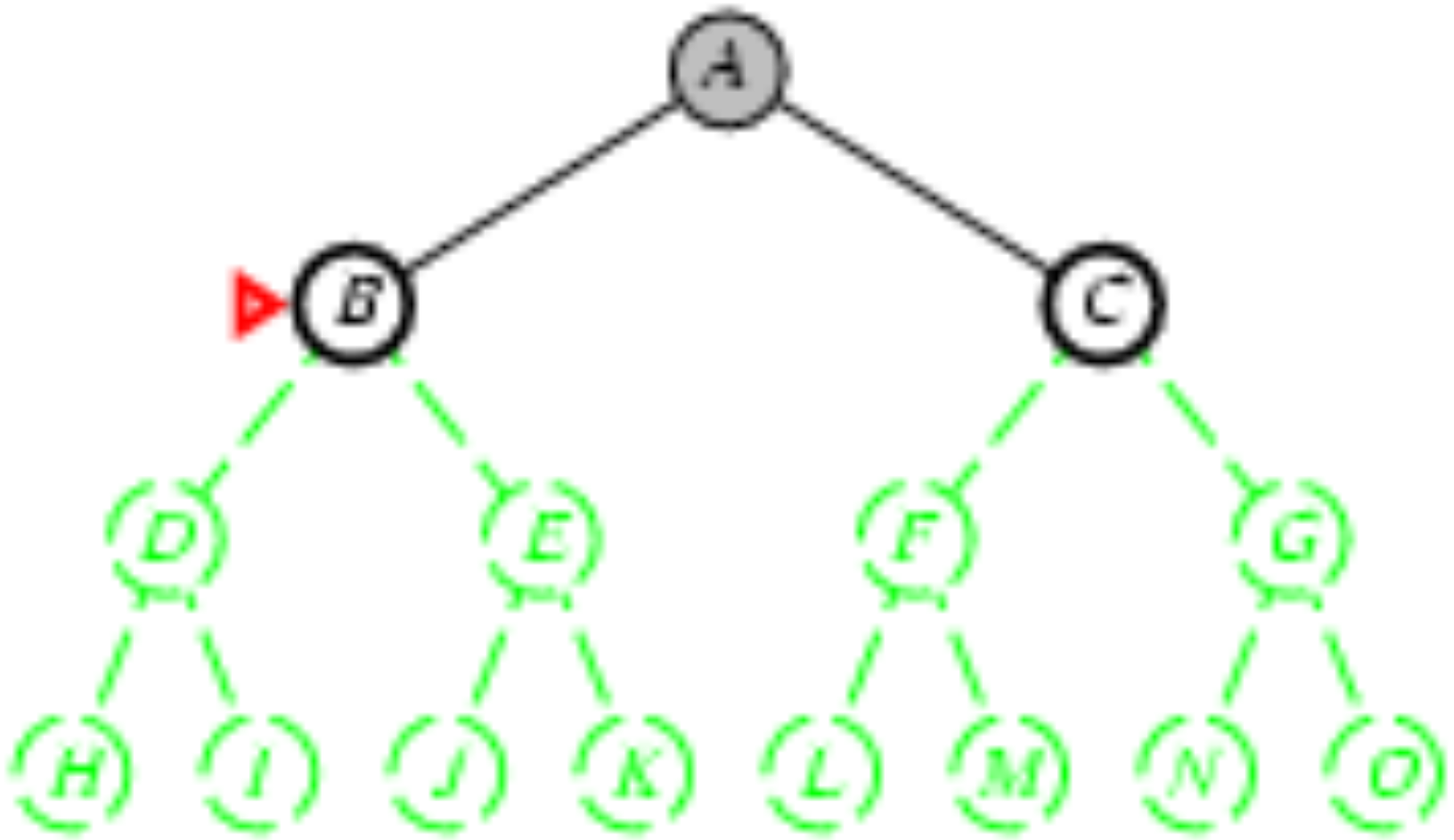


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[B,C]

Is B a goal state?

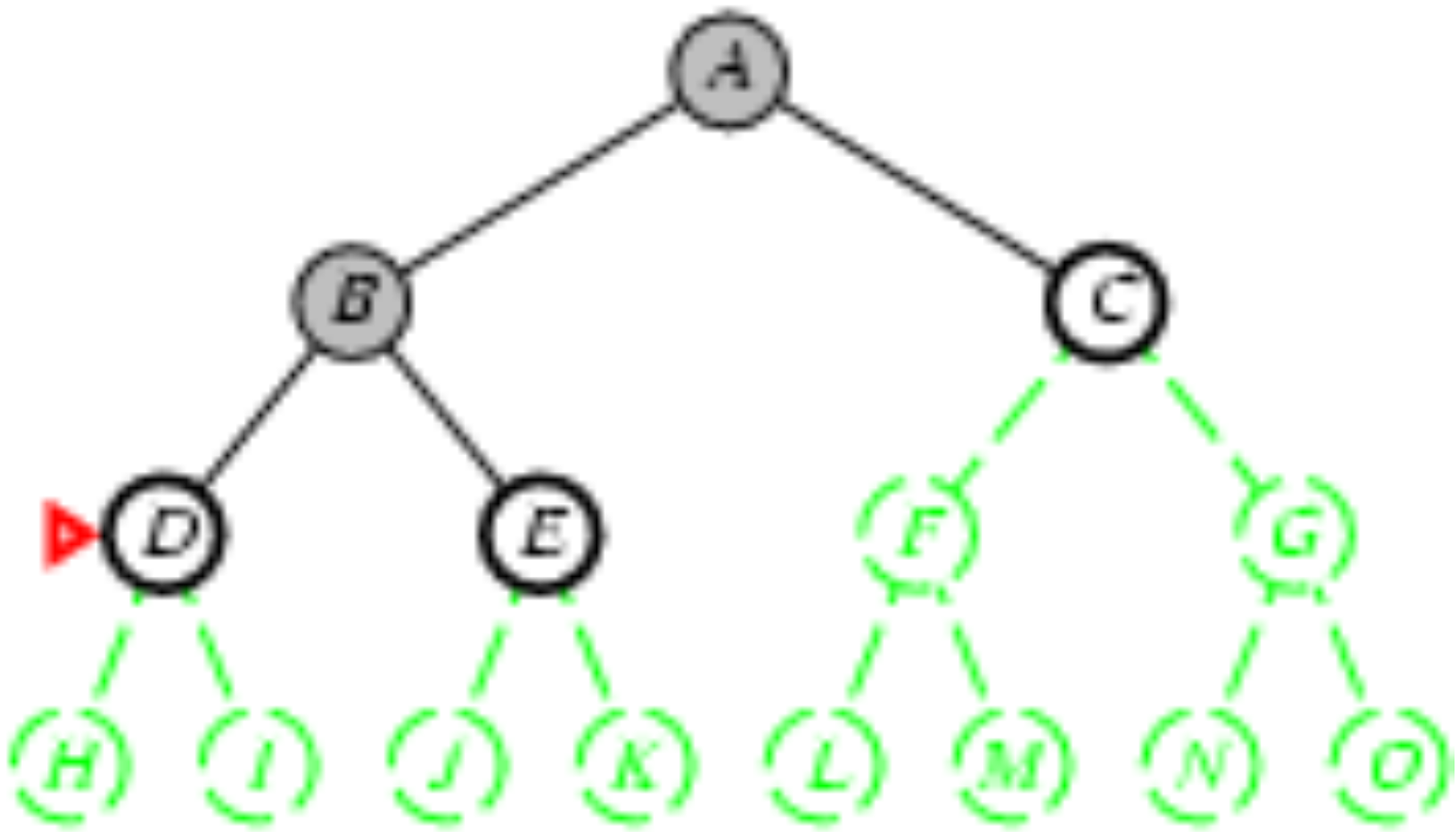


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[D,E,C]

Is D = goal state?

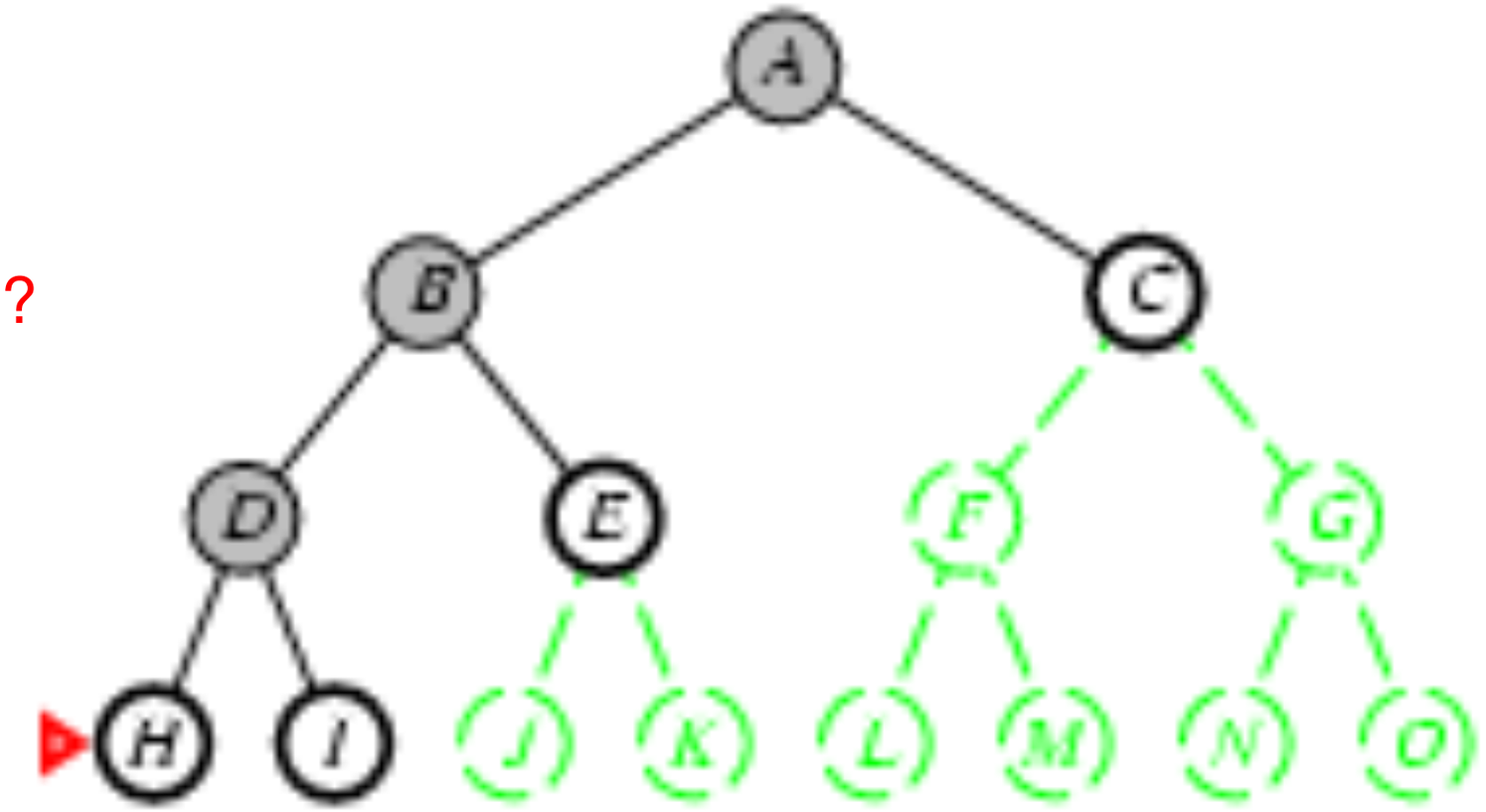


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[H,I,E,C]

Is H = goal state?

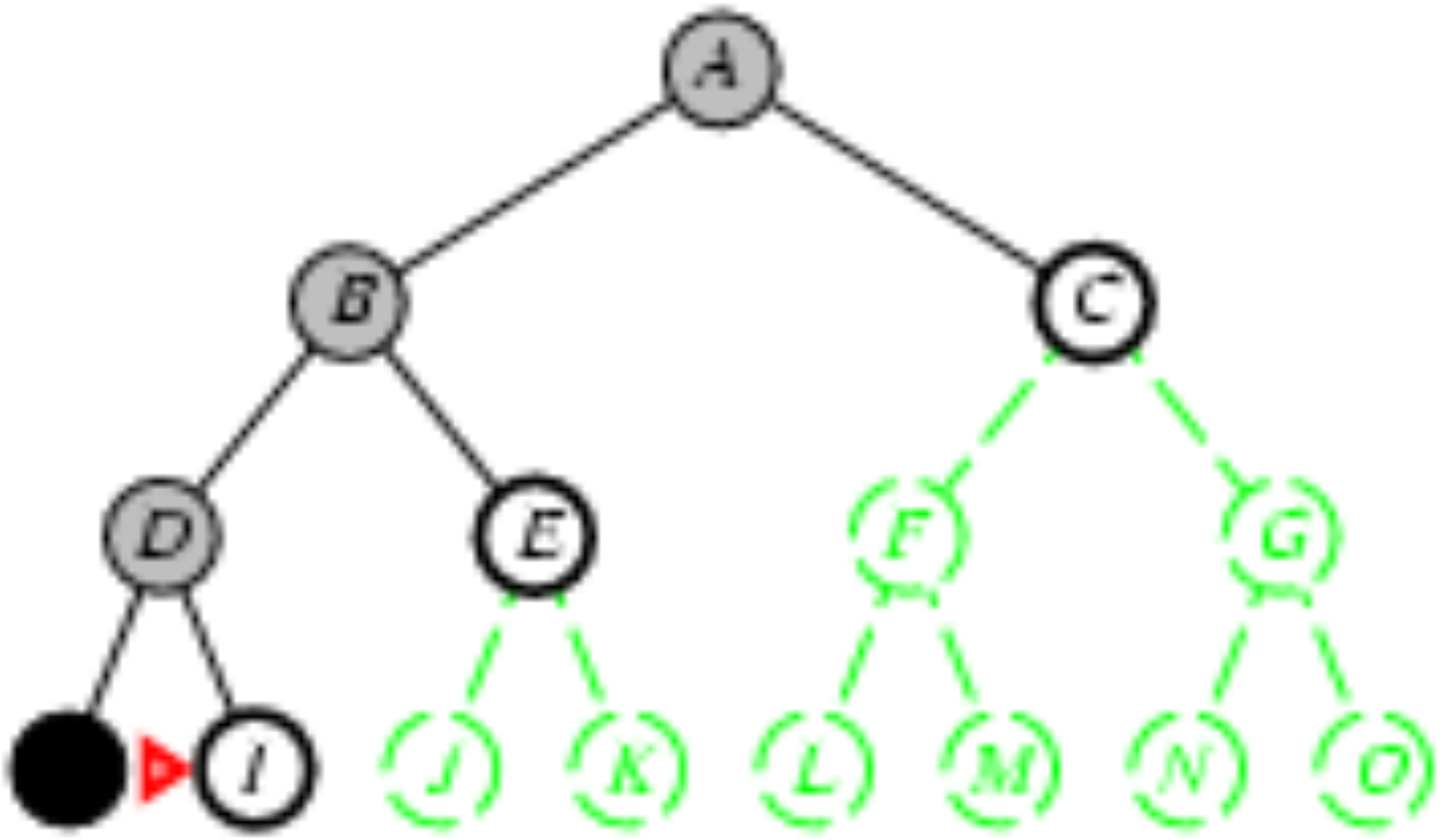


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[I,E,C]

Is I = goal state?

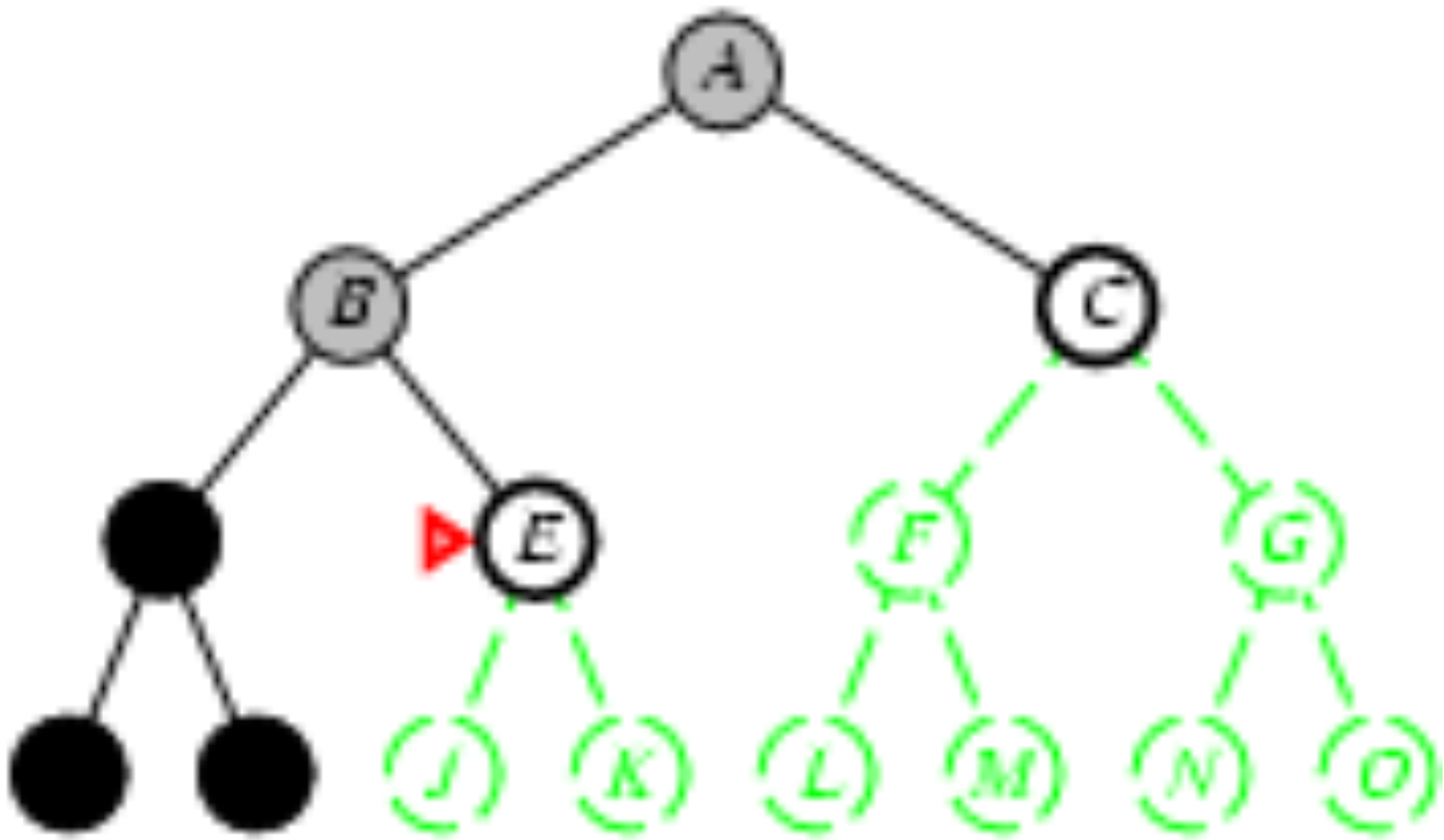


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[E,C]

Is E = goal state?

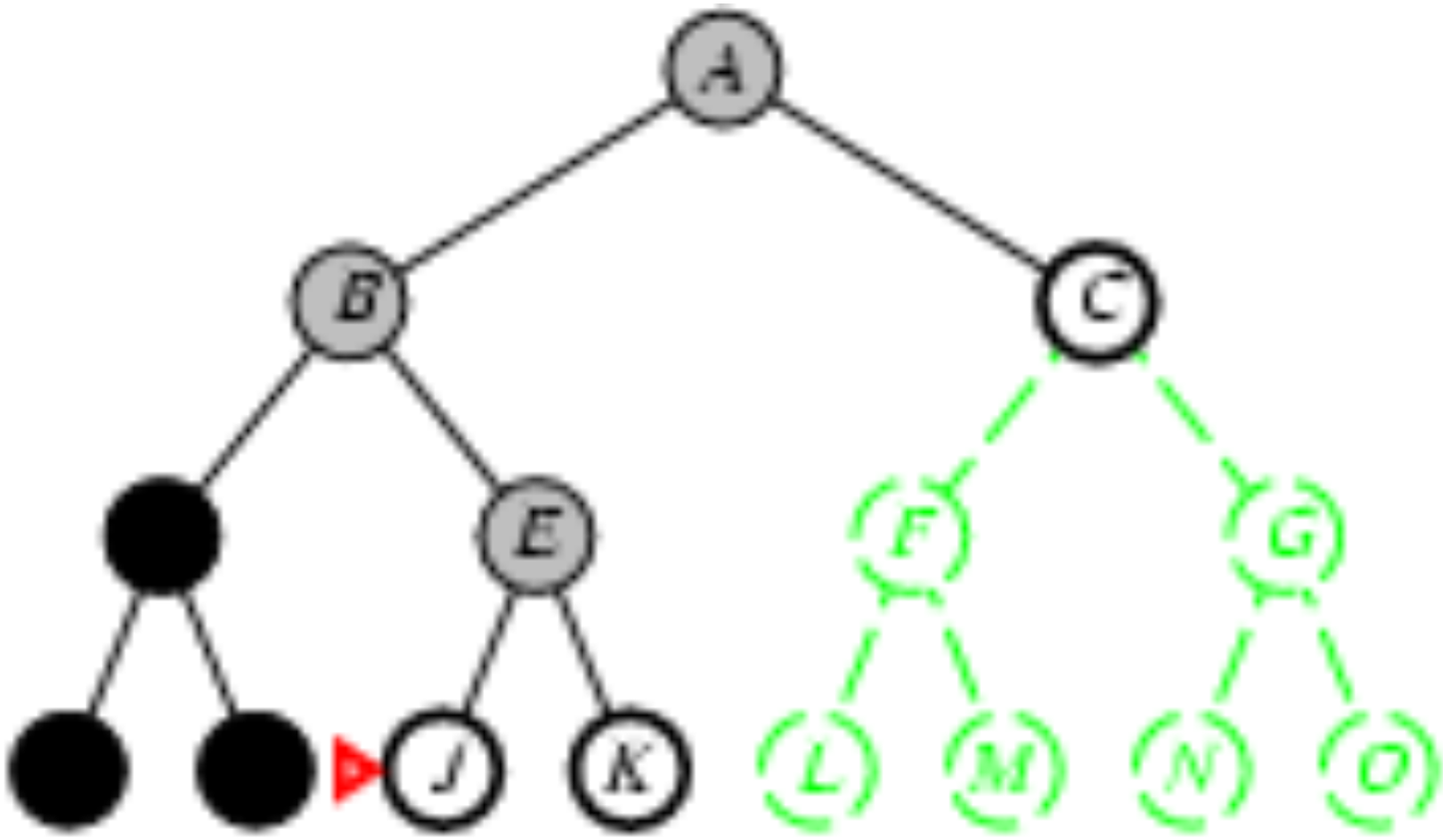


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[J,K,C]

Is J = goal state?

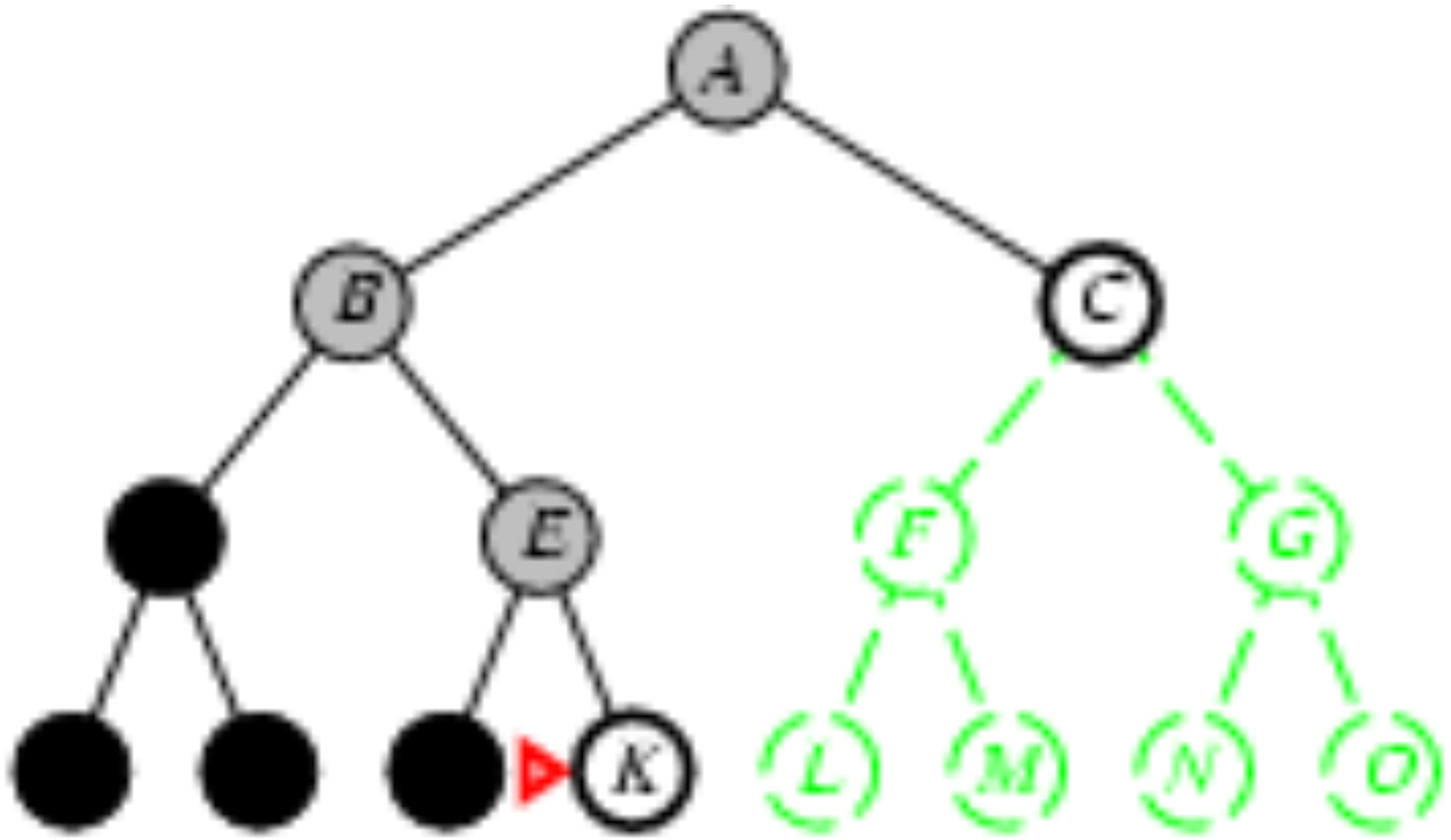


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[K,C]

Is K = goal state?

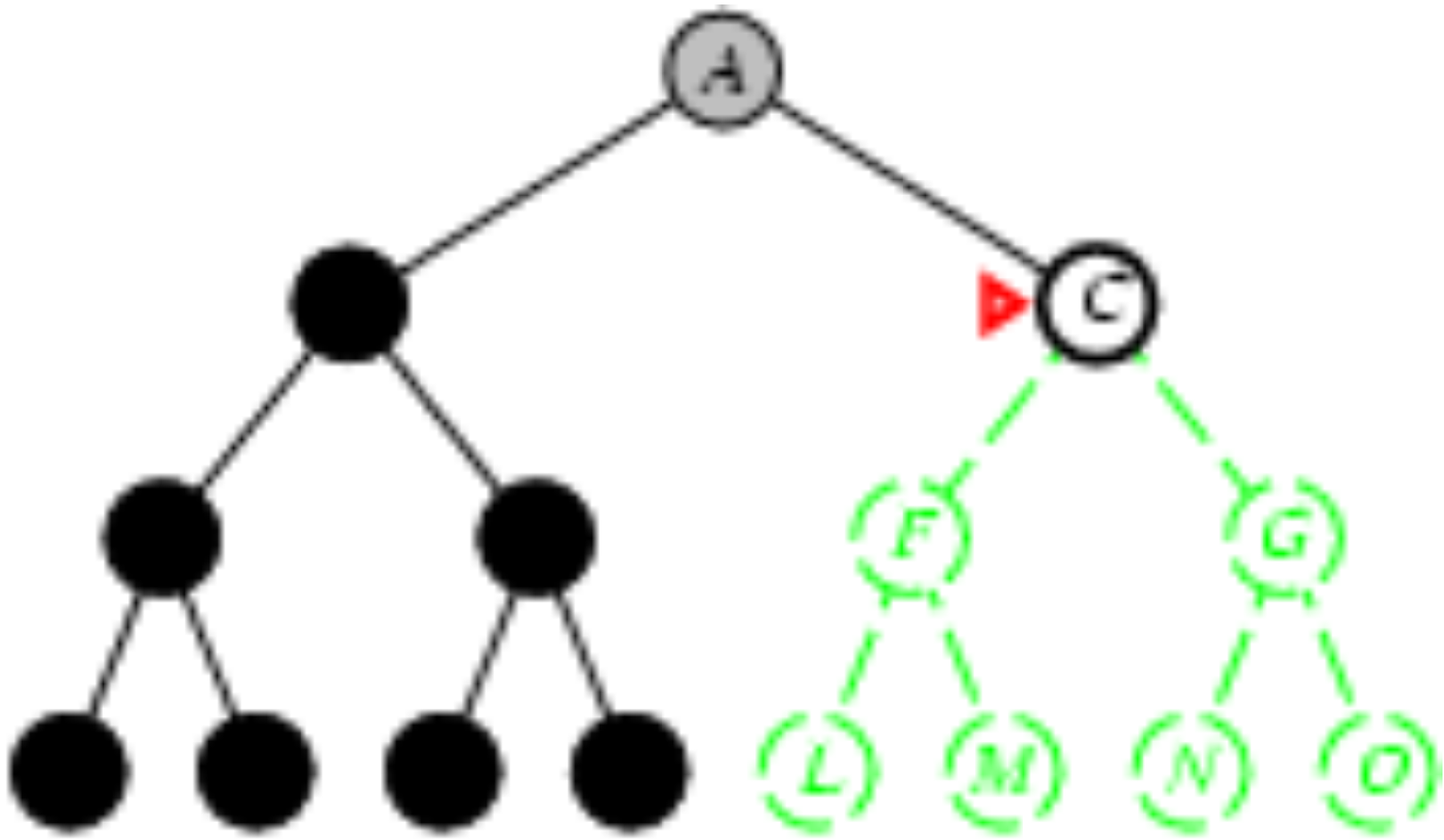


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[C]

Is C = goal state?

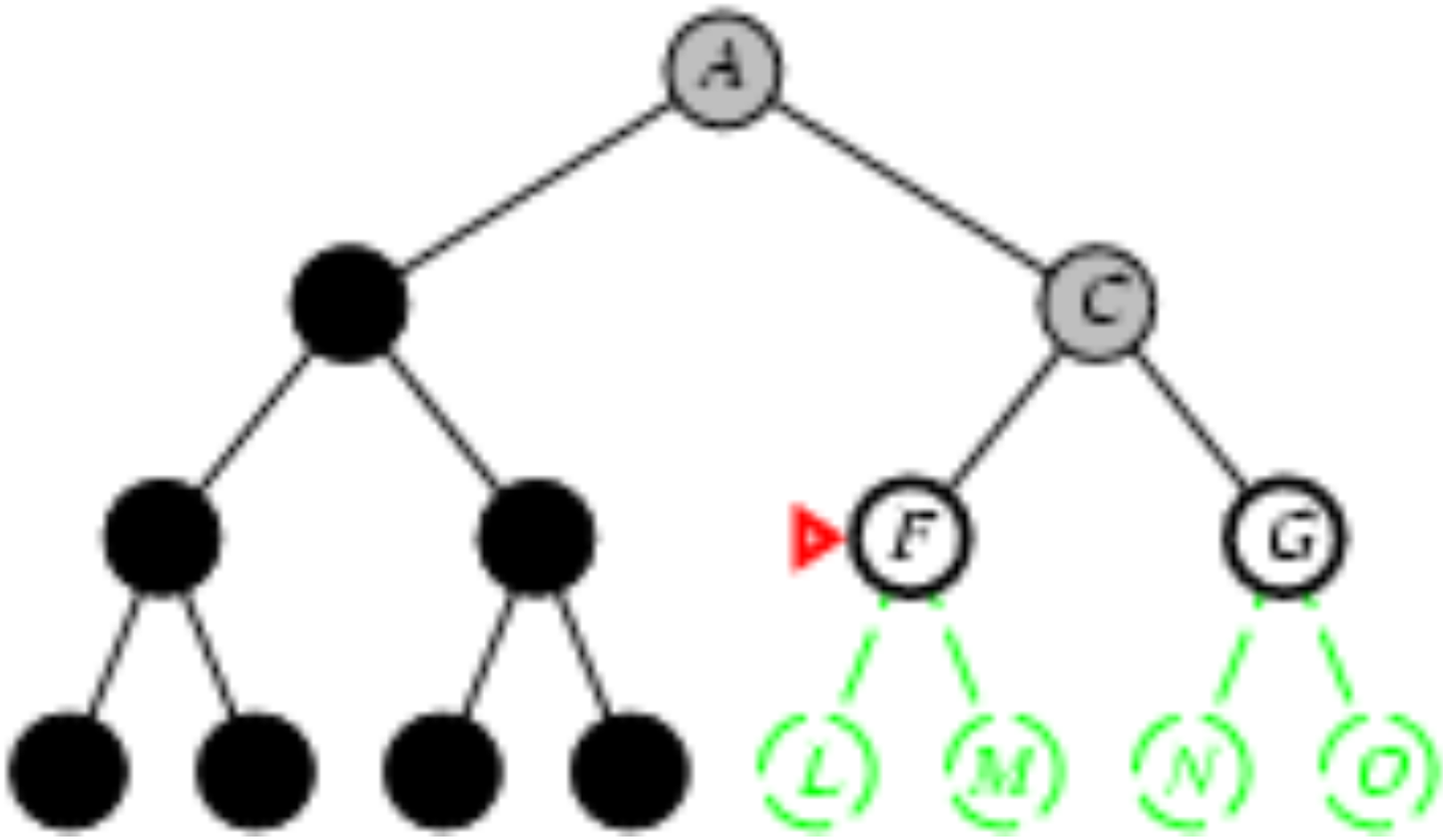


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[F,G]

Is F = goal state?

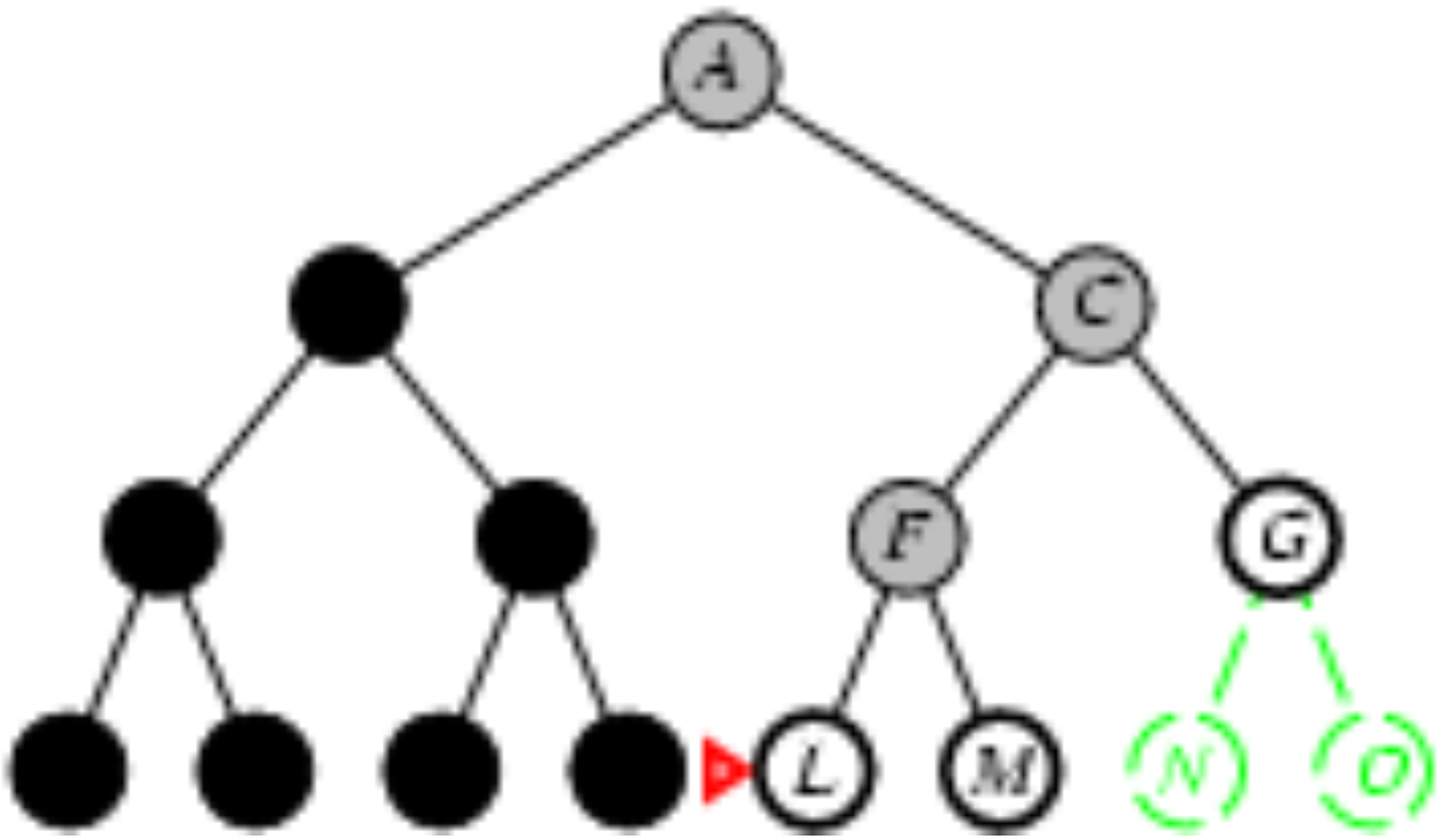


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[L,M,G]

Is L = goal state?

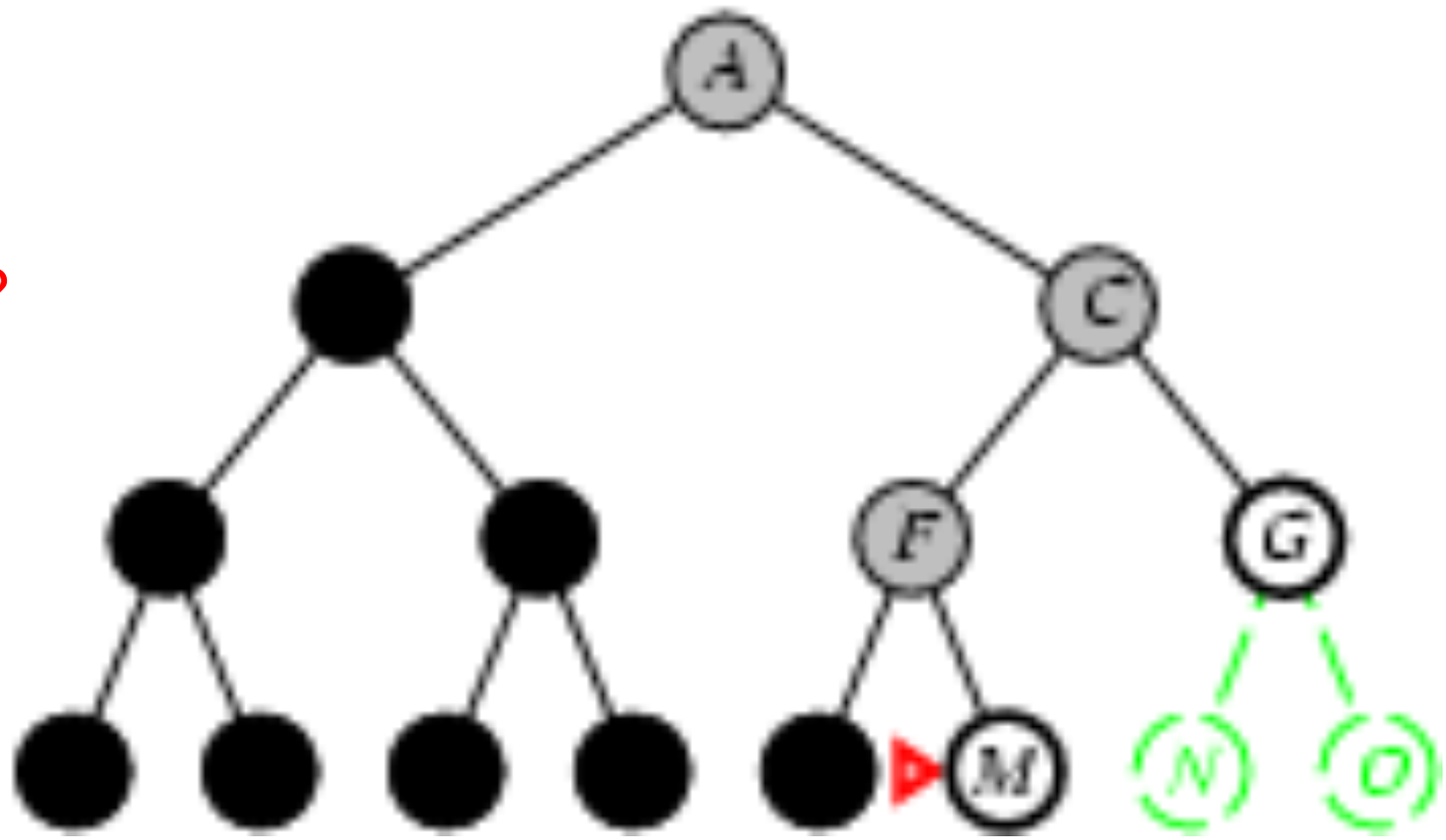


Depth-first search

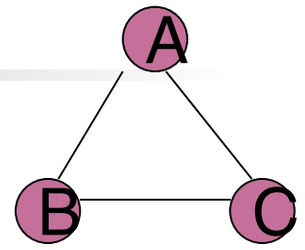
- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

queue=[M,G]

Is M = goal state?



Properties of depth-first search

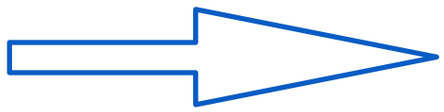


- Complete? No: fails in infinite-depth spaces

  Can modify to avoid repeated states along path
- Time? $O(b^m)$ with m=maximum depth
- terrible if *m* is much larger than *d*
  - but if solutions are dense, may be much faster than breadth-first
- Space? *O(bm),* i.e., linear space! (we only need to remember a single path + expanded unexplored nodes)
- Optimal? No (It may find a non-optimal goal first)

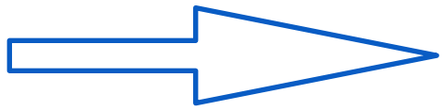
Iterative deepening search

- To avoid the infinite depth problem of DFS, we can decide to only search until depth L, i.e. we don't expand beyond depth L.



Depth-Limited Search

- What of solution is deeper than L? -->Increase L iteratively.



Iterative Deepening Search

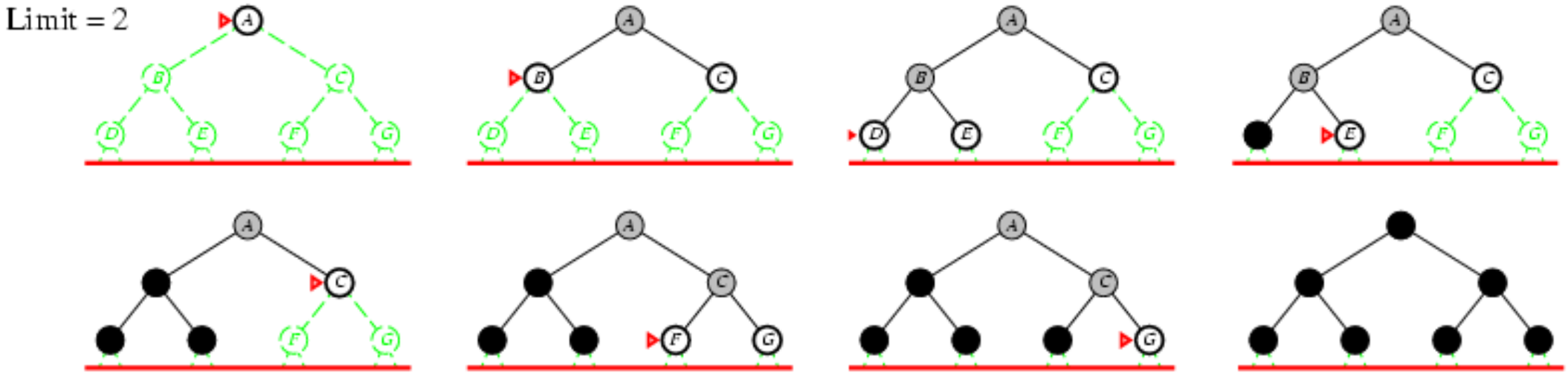
- As we shall see: this inherits the memory advantage of Depth-First search, and is better in terms of time complexity than Breadth first search.

Iterative deepening search *L*=0

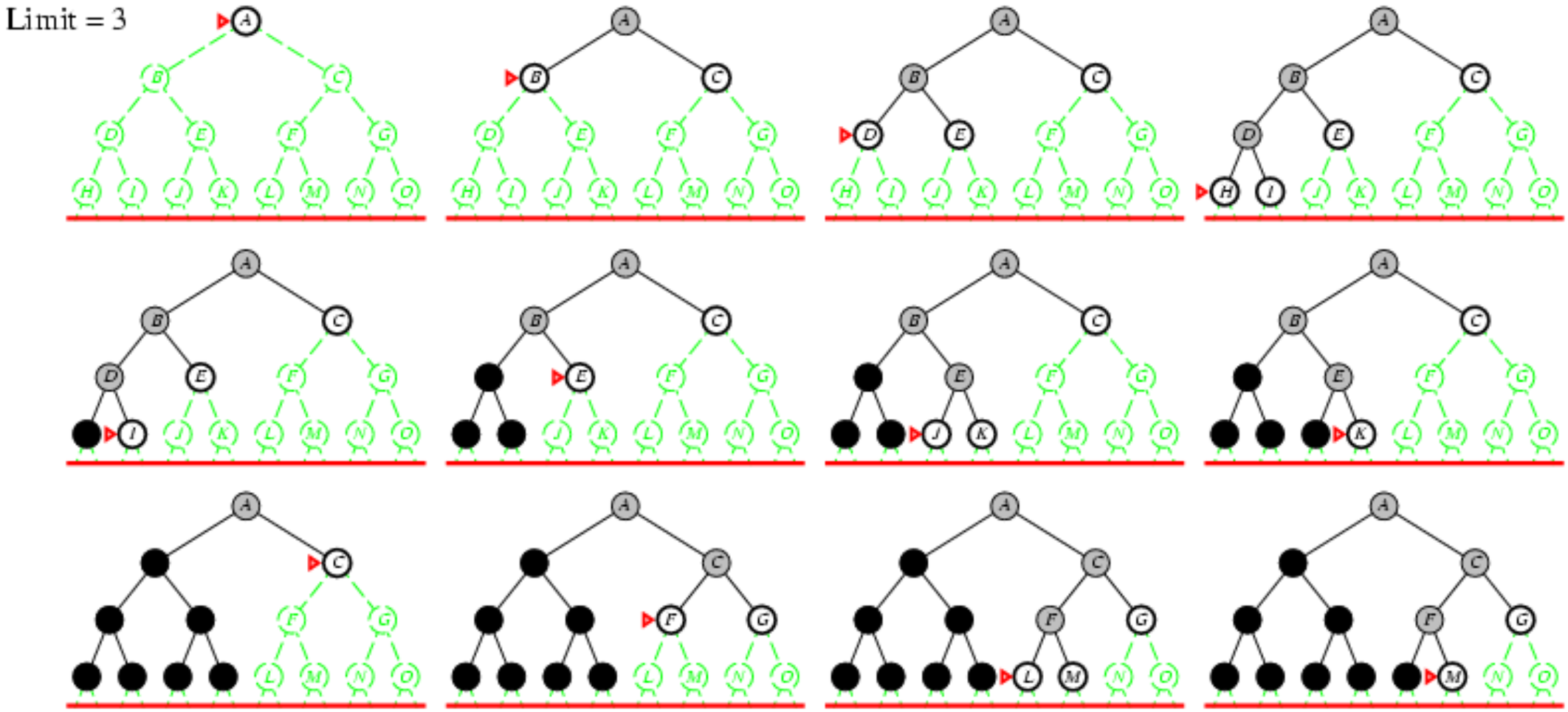
Limit = 0

Iterative deepening search *L*=1

Iterative deepening search *L*=2



Iterative Deepening Search *L*=3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

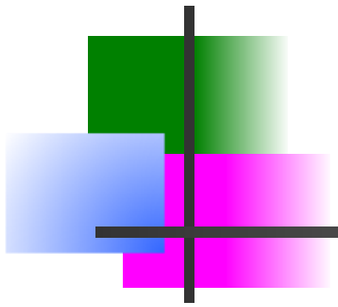
- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\,b^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d =$$

$O(b^d) \neq O(b^{d+1})$

BFS

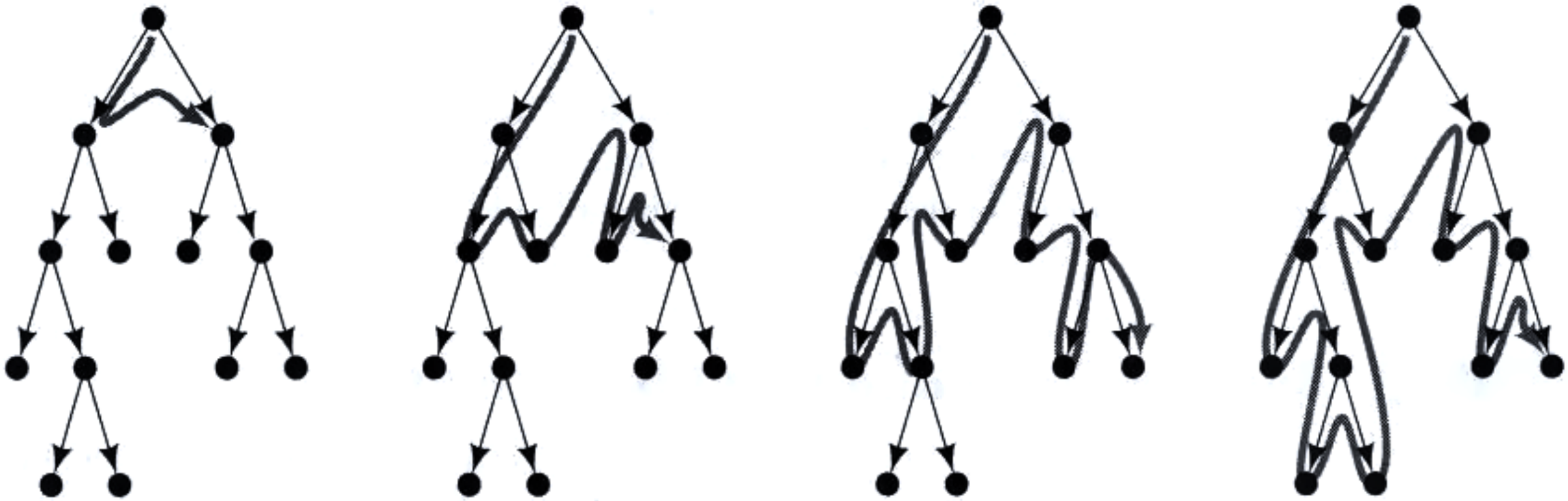
- For *b = 10*, *d = 5*,
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}450$
  - N_{BFS} = ... = 1,111,100



Properties of iterative deepening search

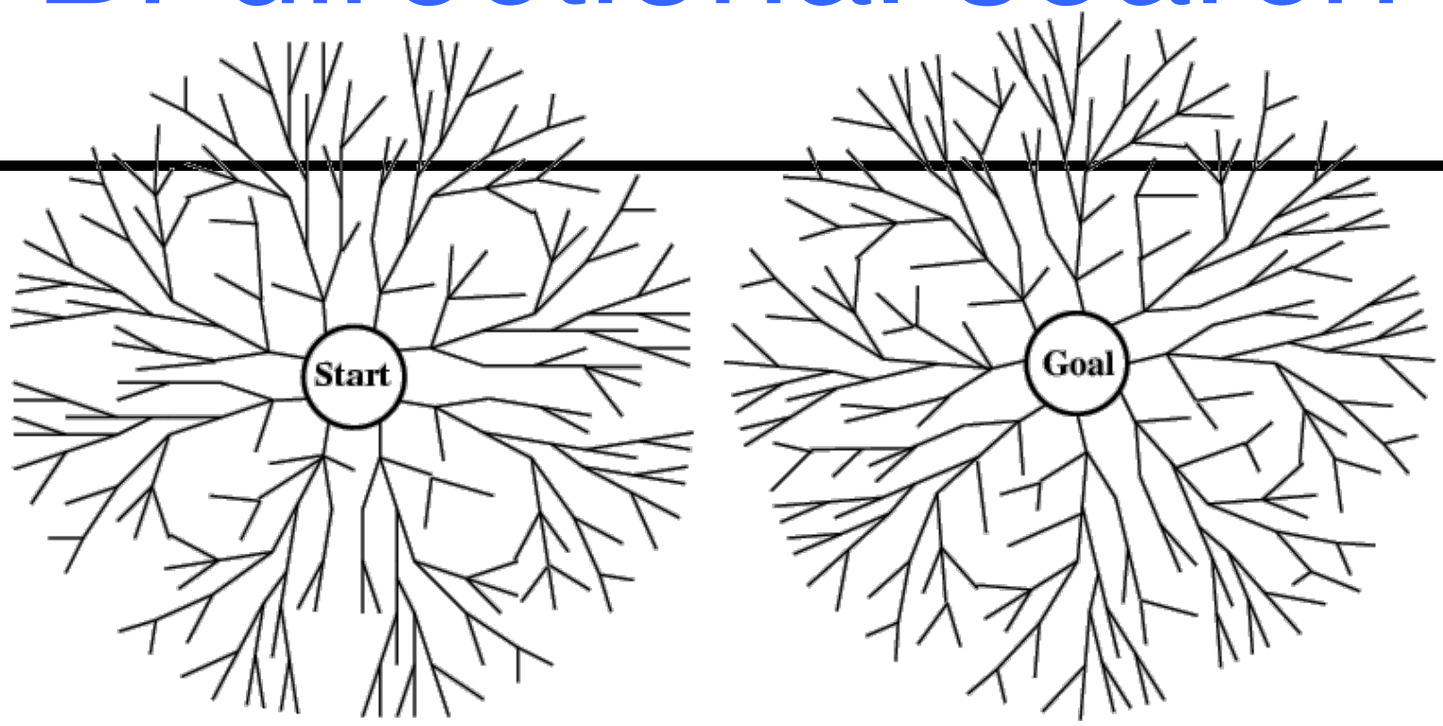
- Complete? Yes
- Time? $O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1 or increasing function of depth.

Example IDS

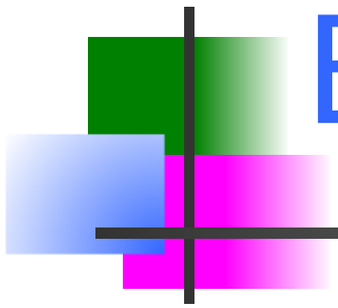


Stages in Iterative-Deepening Search

Bi-directional search



- Alternate searching from the start state toward the goal and from the goal state toward the start.
- Stop when the frontiers intersect.
- Works well only when there are unique start and goal states.
- Requires the ability to generate “predecessor” states.
- Can (sometimes) lead to finding a solution more quickly.
- Time complexity: $O(b^{d/2})$. Space complexity: $O(b^{d/2})$.



Bidirectional Search

- Idea
  - simultaneously search forward from S and backwards from G
  - stop when both “meet in the middle”
  - need to keep track of the intersection of 2 open sets of nodes
- What does searching backwards from G mean
  - need a way to specify the predecessors of G
    - this can be difficult,
    - e.g., predecessors of checkmate in chess?
  - which to take if there are multiple goal states?
  - where to start if there is only a goal test, no explicit list?

Bi-Directional Se

Complexity: time and space complexity are:

$O(\ ^{d}/_{b}{}^{2}$

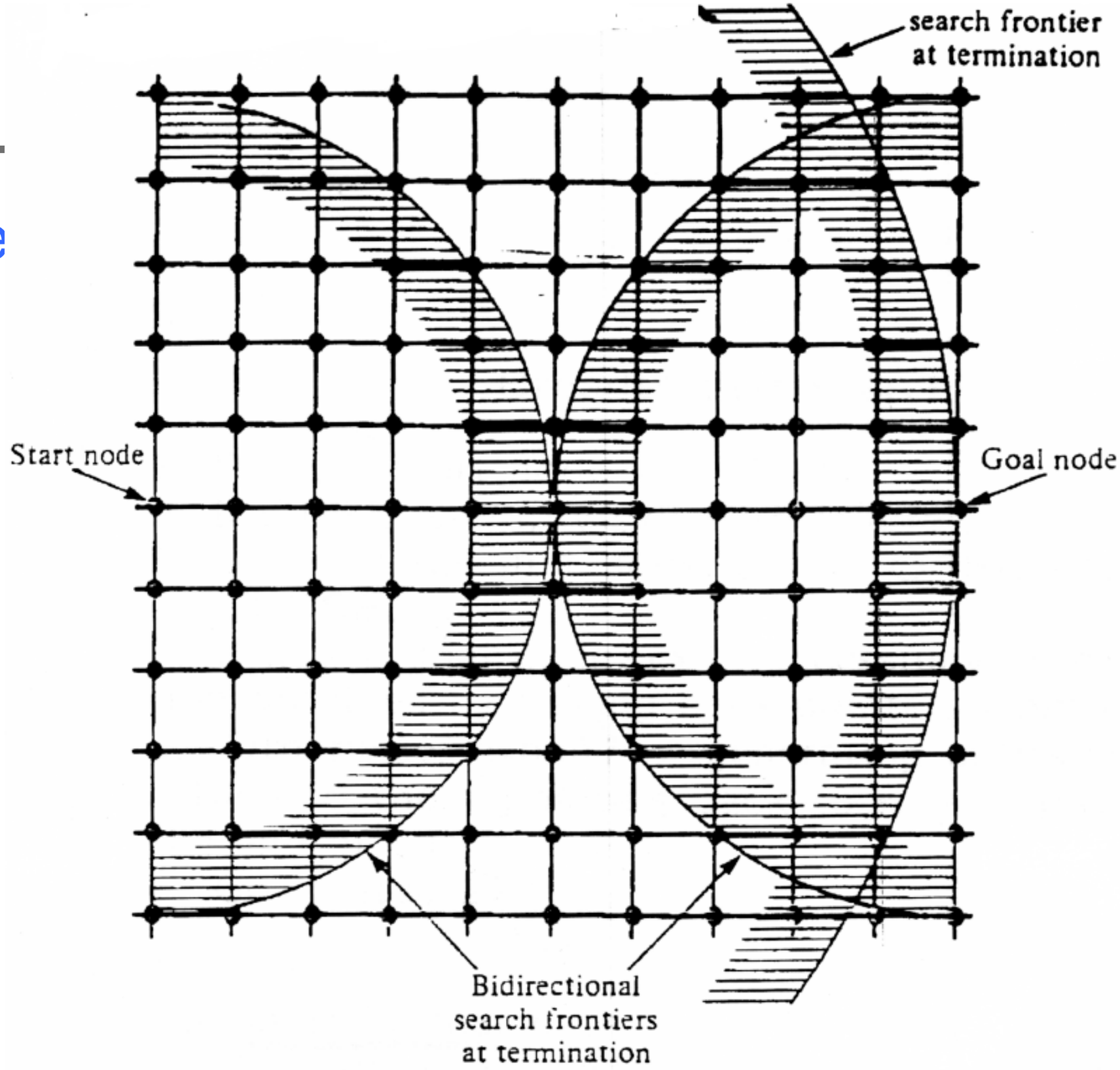


Fig. 2.10 Bidirectional and unidirectional breadth-first searches.

Comparing Search Strategies

b – branching factor
m – maximum depth
d – depth of optimal solution
l – depth limit

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

even complete
if step cost is not
increasing with depth.

preferred
uninformed
search strategy

Summary of algorithms

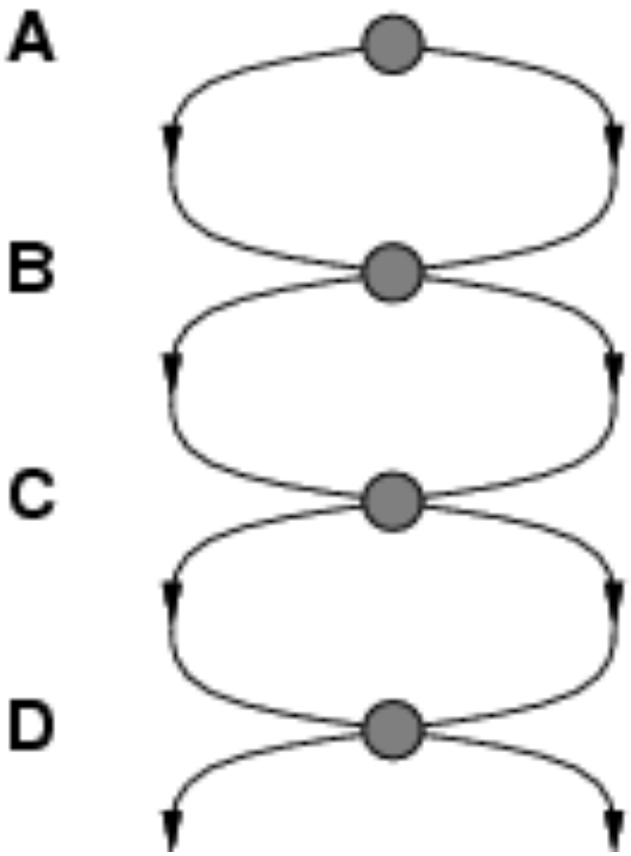
Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

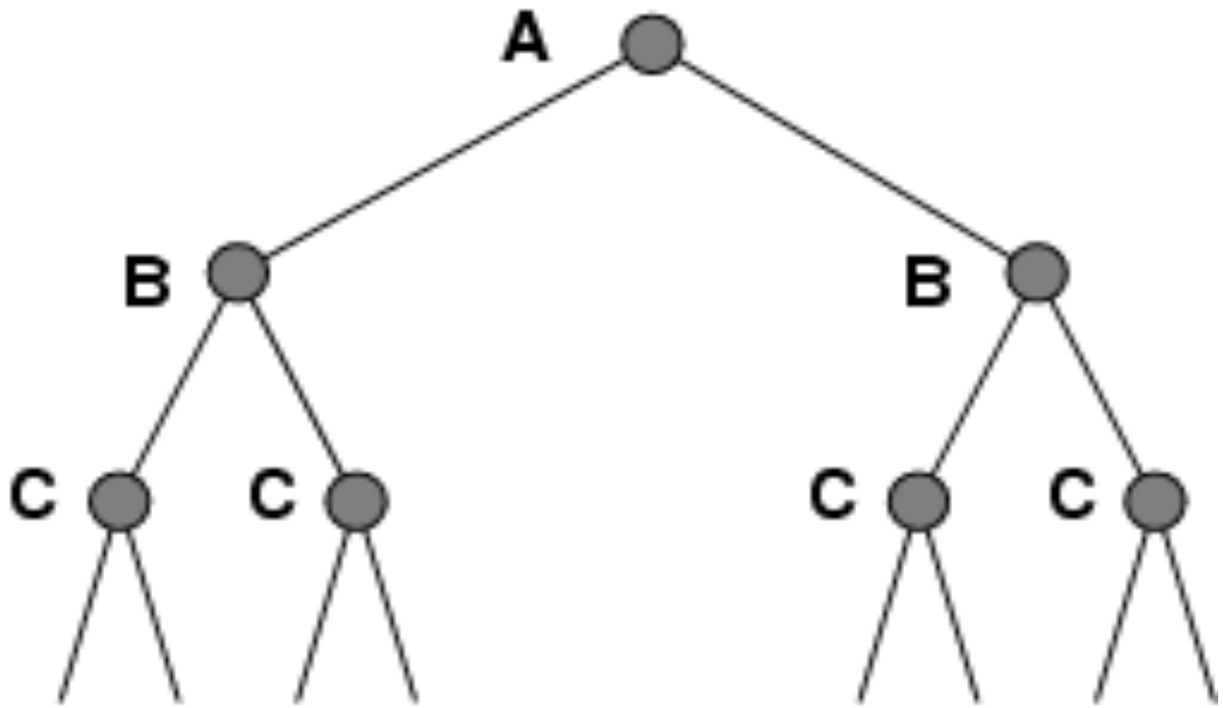
even complete
if step cost is not
increasing with depth.

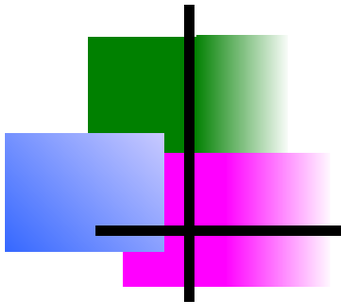
preferred
uninformed
search strategy

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!



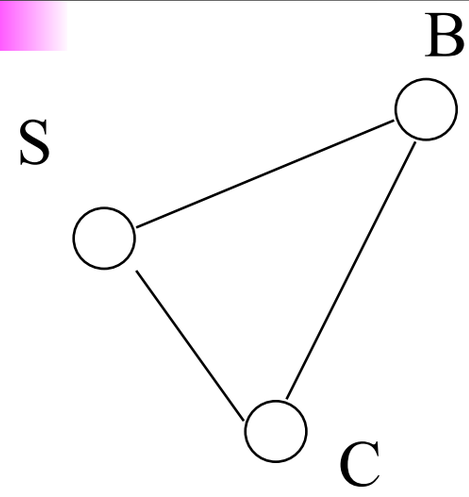




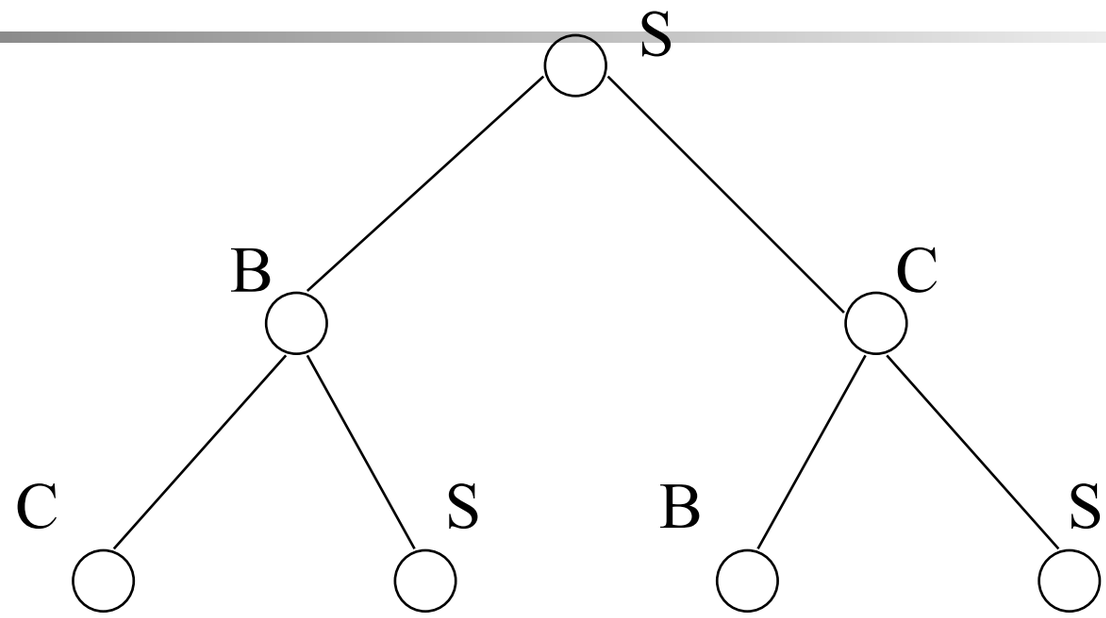
Avoiding Repeated States

- In increasing order of effectiveness in reducing size of state space and with increasing computational costs:
  1. Do not return to the state you just came from.
  2. Do not create paths with cycles in them.
  3. Do not generate any state that was ever created before.
- Net effect depends on frequency of “loops” in state space.

Solutions to Repeated States



State Space

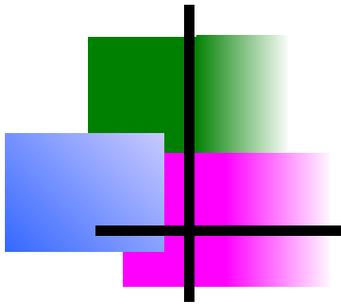


Example of a Search Tree

- Graph search ⟵ optimal but memory inefficient
  - never generate a state generated before
    - must keep track of all possible states (uses a lot of memory)
    - e.g., 8-puzzle problem, we have 9! = 362,880 states
    - approximation for DFS/DLS: only avoid states in its (limited) memory: avoid looping paths.
      (this does not avoid the problem on the previous slide)
    - Graph search optimal for BFS and UCS, not for DFS.

Graph Search

```
function graph-search (problem, QUEUEING-FUNCTION)
;; problem describes the start state, operators, goal test, and operator costs
;; queueing-function is a comparator function that ranks two states
;; graph-search returns either a goal node or failure
nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  closed = {}
  loop
     if EMPTY(nodes) then return "failure"
     node = REMOVE-FRONT(nodes)
     if problem.GOAL-TEST(node.STATE) succeeds
        then return node.SOLUTION
     if node.STATE is not in closed
        then ADD(node, closed)
                nodes = QUEUEING-FUNCTION(nodes,
               EXPAND(node, problem.OPERATORS))
end
  ;; Note: The goal test is NOT done when nodes are generated
  ;; Note: closed should be implemented as a hash table for efficiency
```



Graph Search Strategies

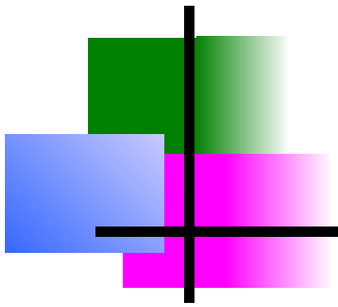
- Breadth-first search and uniform-cost search are optimal graph search strategies.
- Iterative deepening search and depth-first search can follow a non-optimal path to the goal.
- Iterative deepening search can be used with modification:
  - It must check whether a new path to a node is better than the original one
  - If so, IDS must revise the depths and path costs of the node's descendants.

Breadth-First Search

Expanded node	Nodes list
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ B^1 C^8 D^6 E^{10} G^{18} }
B^1	{ C^8 D^6 E^{10} G^{18} G^{21} }
C^8	{ D^6 E^{10} G^{18} G^{21} G^{13} }
D^6	{ E^{10} G^{18} G^{21} G^{13} }
E^{10}	{ G^{18} G^{21} G^{13} }
G^{18}	{ G^{21} G^{13} }

Solution path found is S A G , cost 18

Number of nodes expanded (including goal node) = 7



Depth-First Search

Expanded node	**Nodes list**
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ D^6 E^{10} G^{18} B^1 C^8 }
D^6	{ E^{10} G^{18} B^1 C^8 }
E^{10}	{ G^{18} B^1 C^8 }
G^{18}	{ B^1 C^8 }

Solution path found is S A G, cost 18

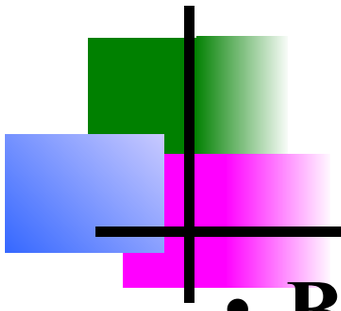
Number of nodes expanded (including goal node) = 5

Uniform-Cost Search

Expanded node	Nodes list
	{ S^0 }
S^0	{ B^1 A^3 C^8 }
B^1	{ A^3 C^8 G^{21} }
A^3	{ D^6 C^8 E^{10} G^{18} G^{21} }
D^6	{ C^8 E^{10} G^{18} G^1 }
C^8	{ E^{10} G^{13} G^{18} G^{21} }
E^{10}	{ G^{13} G^{18} G^{21} }
G^{13}	{ G^{18} G^{21} }

Solution path found is S B G, cost 13

Number of nodes expanded (including goal node) = 7



How they perform

- **Breadth-First Search**:
  - Expanded nodes: S A B C D E G
  - Solution found: S A G (cost 18)
- **Depth-First Search:**
  - Expanded nodes: S A D E G
  - Solution found: S A G (cost 18)
- **Uniform-Cost Search**:
  - Expanded nodes: S A D B C E G
  - Solution found: S B G (cost 13)

  This is the only uninformed search that worries about costs.
- **Iterative-Deepening Search**:
  - nodes expanded: S S A B C S A D E G
  - Solution found: S A G (cost 18)

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

- Variety of uninformed search strategies

- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

http://www.cs.rmit.edu.au/AI-Search/Product/
http://aima.cs.berkeley.edu/demos.html (for more demos)