Chapter 2

Time domain methods for speech processing

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Lecture notes can be downloaded from
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- 2.1. Time domain parameters of Speech signal
- 2.2. Methods for extracting the parameters
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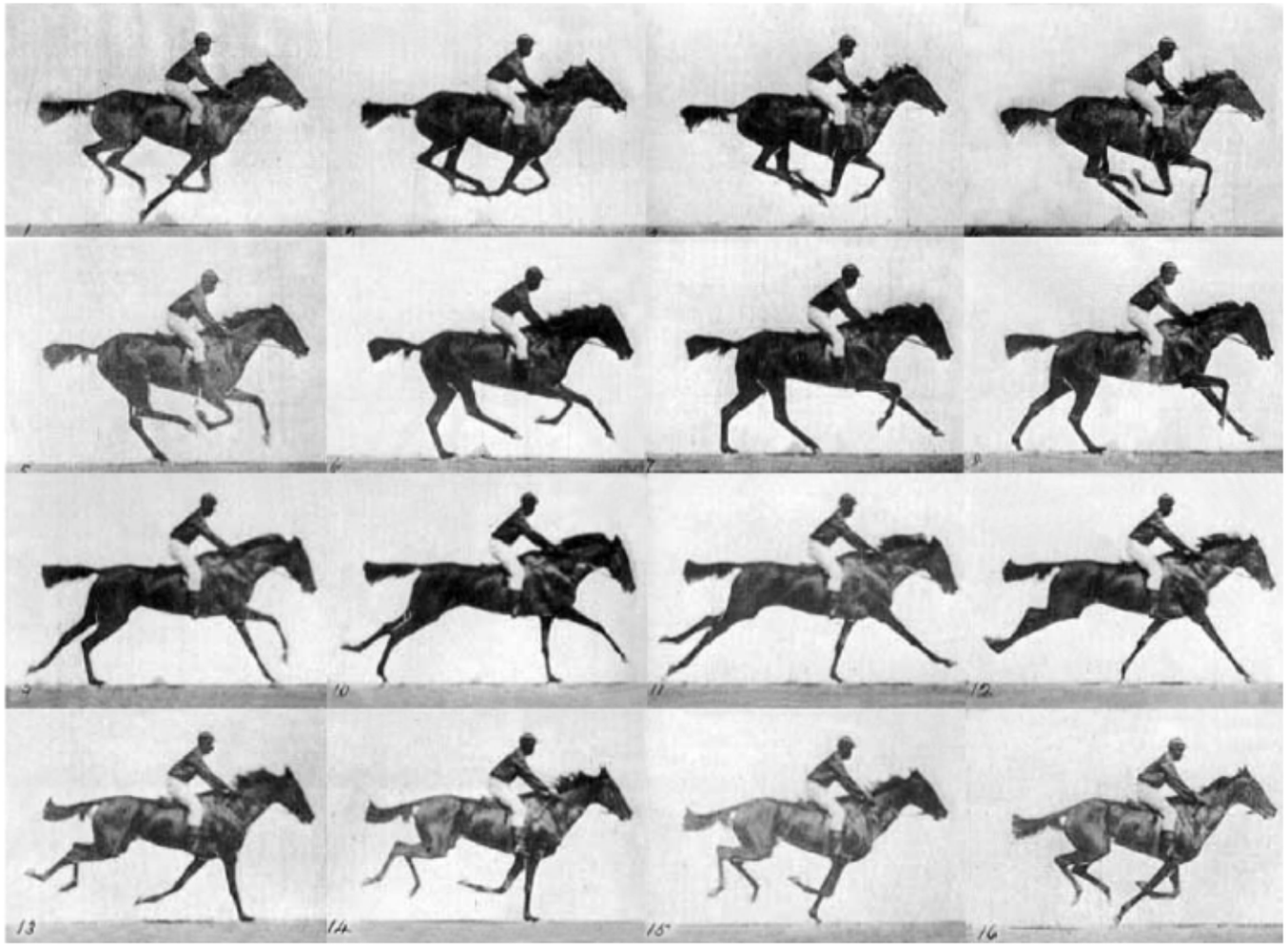
- 2.3. Silence Discrimination using ZCR and energy
- 2.4. Short Time Auto Correlation Function
- 2.5. Pitch period estimation using AutoCorrelation Function

Human eye

Human eye perceives data in blocks

- We see 24 still pictures in one second, then
- we can build up the motion perception in our brain.



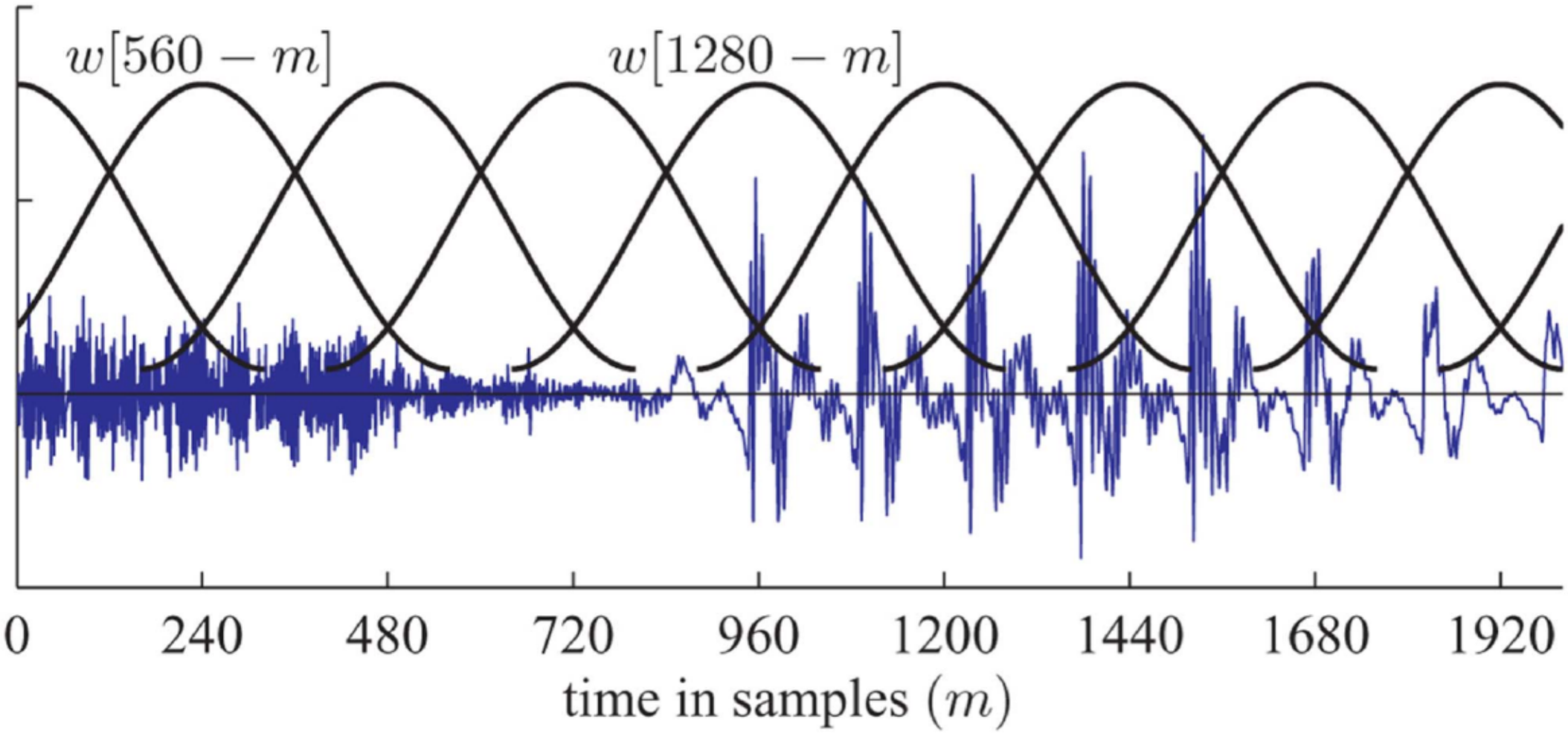
Fundamental Assumptions

- properties of the speech signal change relatively slowly with time (5-10 sounds per second)
  - over very short (5-20 msec) intervals => ***uncertainty*** due to small amount of data, varying pitch, varying amplitude
  - over medium length (20-100 msec) intervals => ***uncertainty*** due to changes in sound quality, transitions between sounds, rapid transients in speech
  - over long (100-500 msec) intervals => ***uncertainty*** due to large amount of sound changes
- there is ***always uncertainty*** in short time measurements and estimates from speech signals

Compromise Solution

- "short-time" processing methods => short segments of the speech signal are "isolated" and "processed" as if they were short segments from a "sustained" sound with fixed (non-time-varying) properties
  - this short-time processing is periodically repeated for the duration of the waveform
  - these short analysis segments, or "analysis frames" almost always overlap one another
  - the results of short-time processing can be a single number (e.g., an estimate of the pitch period within the frame), or a set of numbers (an estimate of the formant frequencies for the analysis frame)
  - the end result of the processing is a new, time-varying sequence that serves as a new representation of the speech signal

Frames and Windows



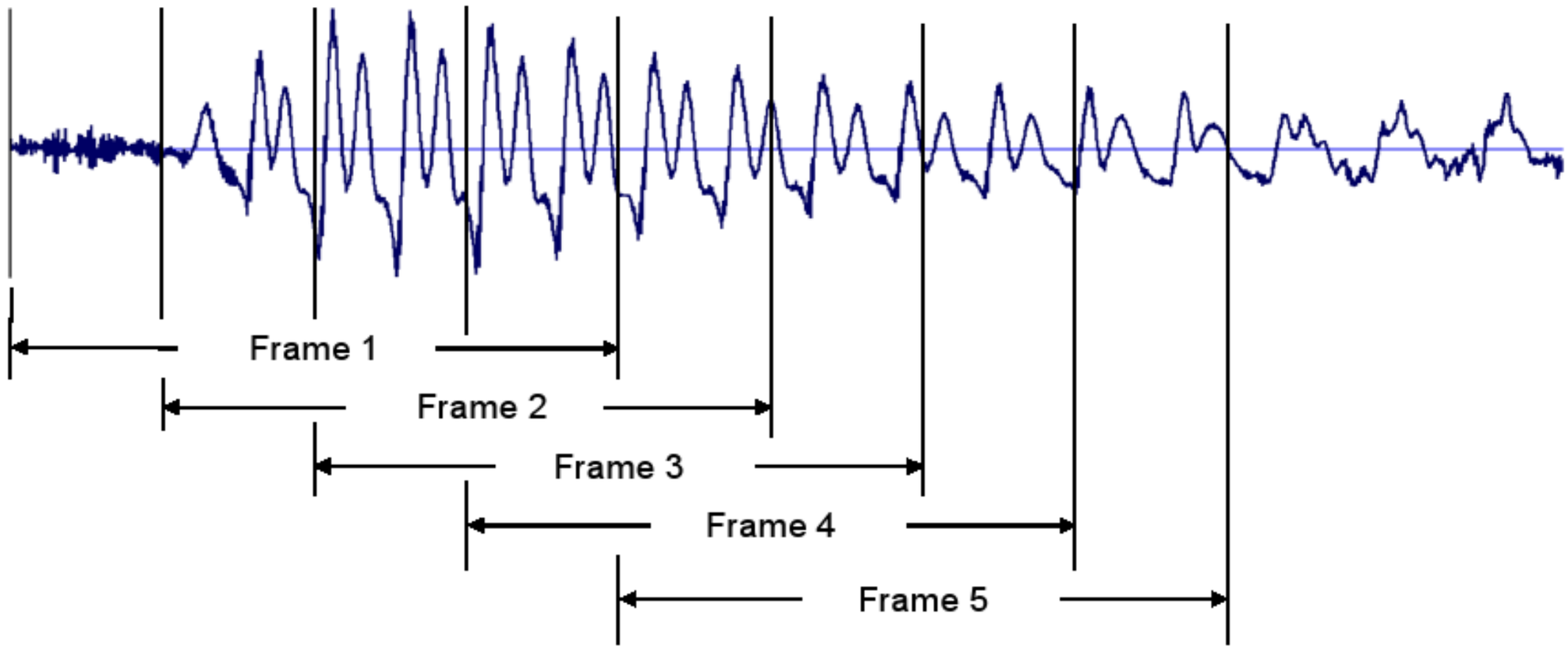
$F_S = 16{,}000$ samples/second

$L = 640$ samples (equivalent to 40 msec frame (window) length)

$R = 240$ samples (equivalent to 15 msec frame (window) shift)

Frame rate of 66.7 frames/second

Frame-by-Frame Processing in Successive Windows



75% frame overlap => frame length=L, frame shift=R=L/4

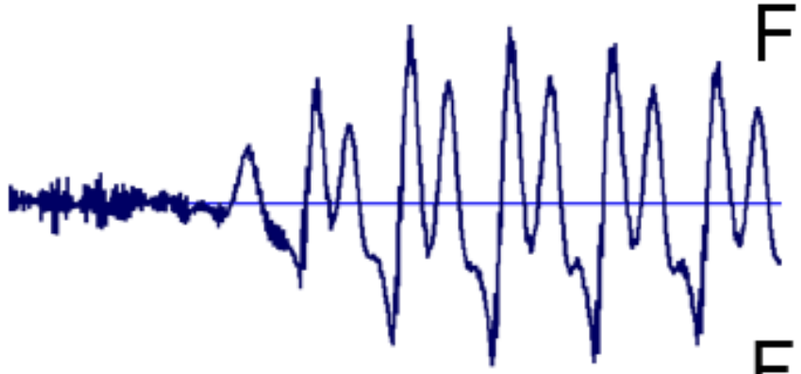
Frame1={x[0],x[1],…,x[L-1]}

Frame2={x[R],x[R+1],…,x[R+L-1]}

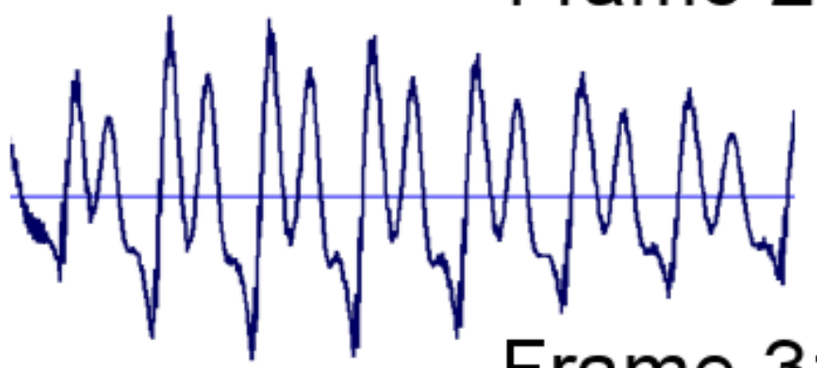
Frame3={x[2R],x[2R+1],…,x[2R+L-1]}

…

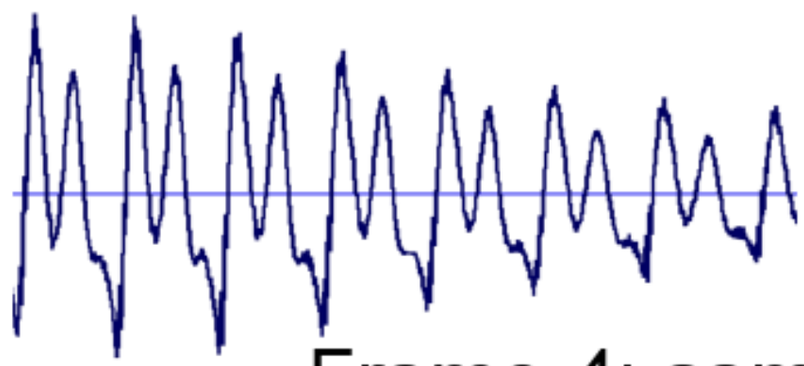
Frame 1: samples $0, 1, \ldots, L-1$



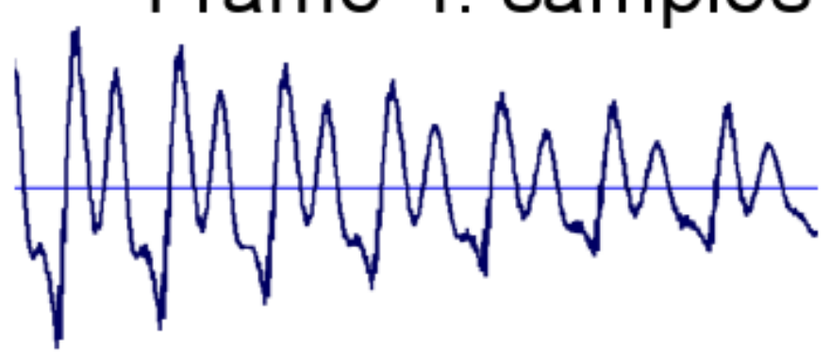
Frame 2: samples $R, R+1, \ldots, R+L-1$



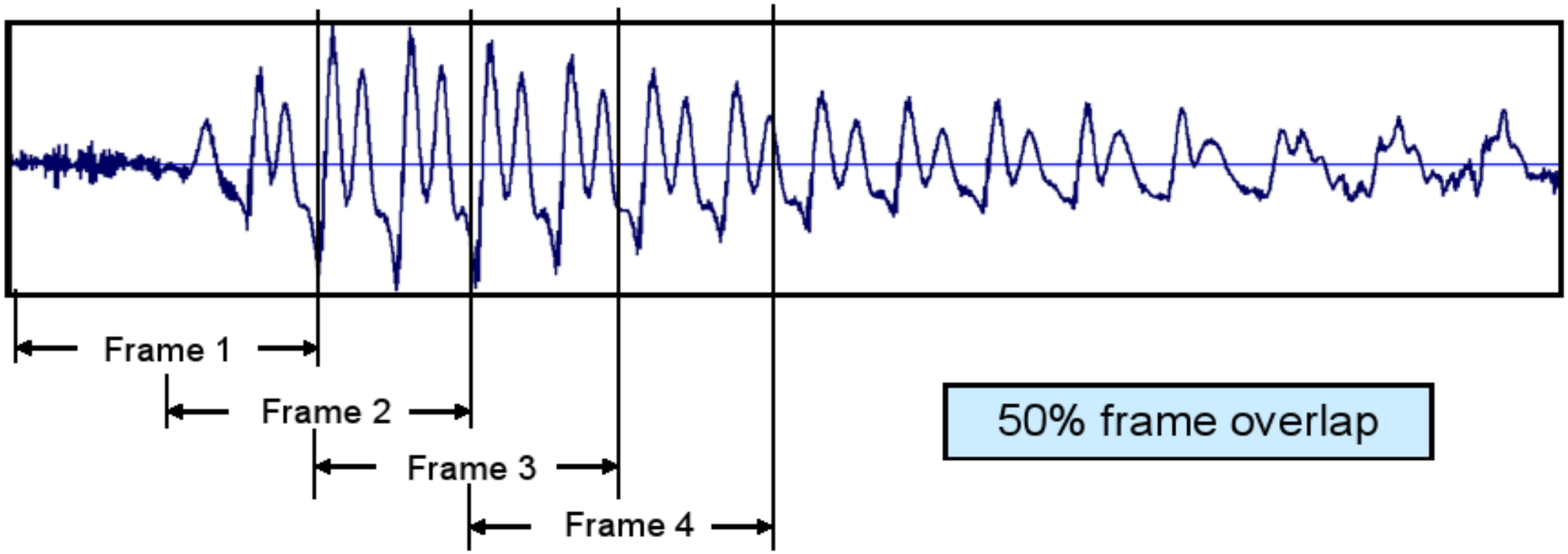
Frame 3: samples $2R, 2R+1, \ldots, 2R+L-1$



Frame 4: samples $3R, 3R+1, \ldots, 3R+L-1$

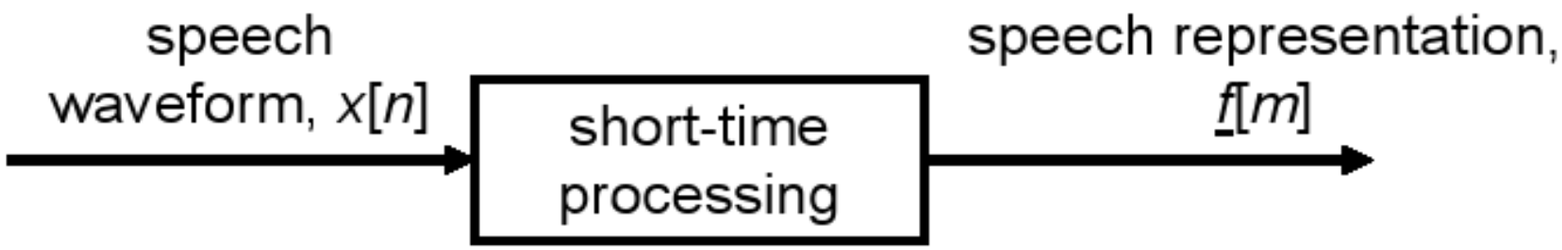


Frame-by-Frame Processing in Successive Windows



- Speech is processed frame-by-frame in overlapping intervals until entire region of speech is covered by at least one such frame
- Results of analysis of individual frames used to derive model parameters in some manner
- Representation goes from time sample $x[n], n = \cdots, 0, 1, 2, \cdots$ to parameter vector $\mathbf{f}[m], m = 0, 1, 2, \cdots$ where n is the time index and m is the frame index.

Short-Time Processing

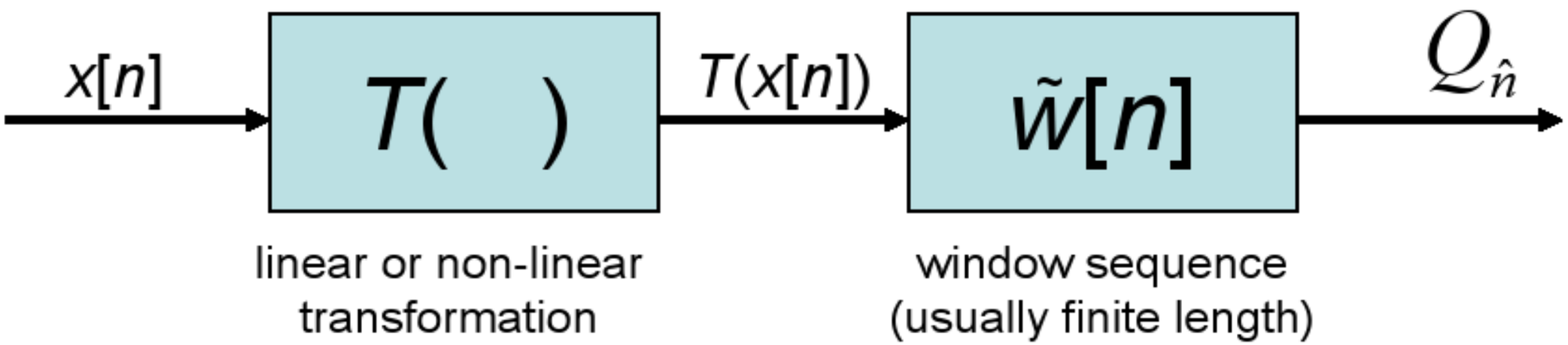


- $x[n]$ = samples at 8000/sec rate; (e.g. 2 seconds of 4 kHz bandlimited speech, $x[n]$, $0 \le n \le 16000$)

- $\vec{f}[m] = \{f_1[m], f_2[m], \ldots, f_L[m]\}$ = vectors at 100/sec rate, $1 \le m \le 200$, L is the size of the analysis vector (e.g., 1 for pitch period estimate, 12 for autocorrelation estimates, etc)

Generic Short-Time Processing

$$Q_{\hat{n}} = \left(\sum_{m=-\infty}^{\infty} T(x[m])\, \tilde{w}[n-m] \right)\Bigg|_{n=\hat{n}}$$



- $Q_{\hat{n}}$ is a sequence of ***local weighted average values*** of the sequence $T(x[n])$ at time $n = \hat{n}$

Short-Time Energy

$$E = \sum_{m=-\infty}^{\infty} x^2[m]$$

-- this is the long term definition of signal energy

-- there is little or no utility of this definition for time-varying signals

$$E_{\hat{n}} = \sum_{m=\hat{n}-L+1}^{\hat{n}} x^2[m] = x^2[\hat{n}-L+1] + \ldots + x^2[\hat{n}]$$

-- short-time energy in vicinity of time $\hat{n}$

$$T(x) = x^2$$

$$\tilde{w}[n] = 1 \qquad 0 \le n \le L-1$$

$$= 0 \qquad \text{otherwise}$$

Computation of Short-Time Energy

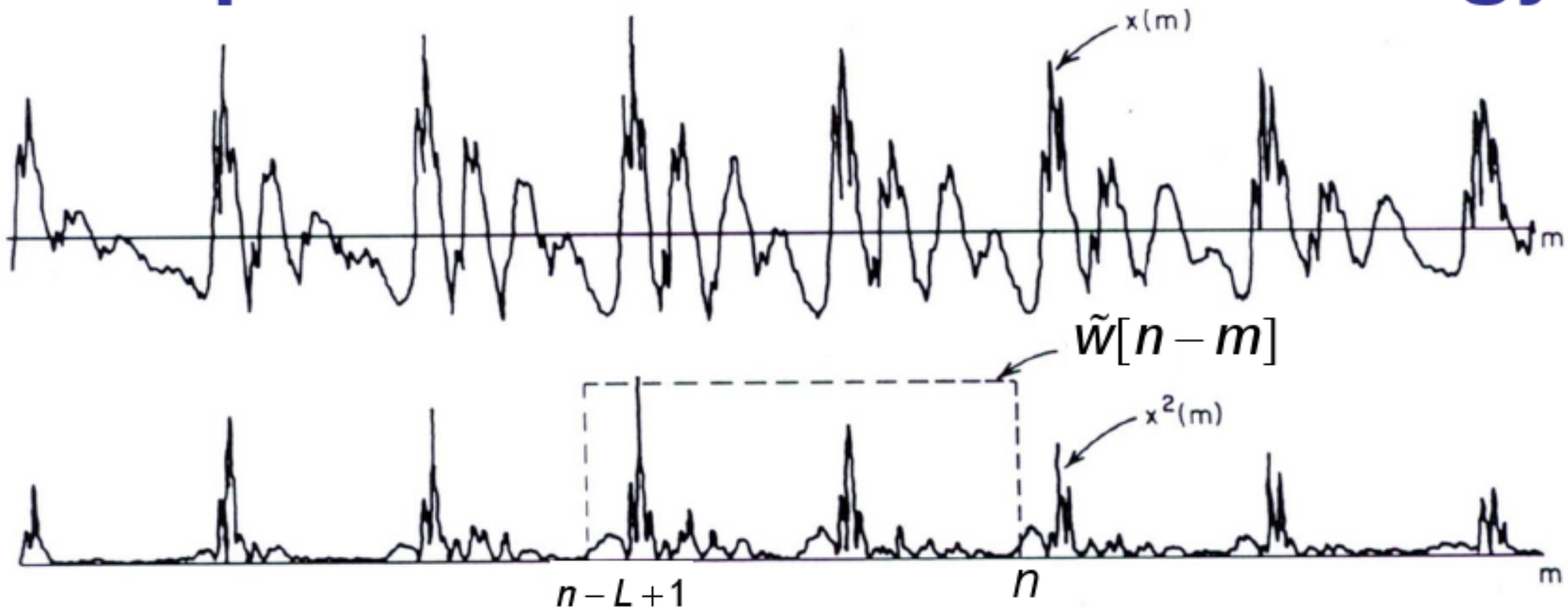


Fig. 4.2 Illustration of the computation of snort-time energy.

- window jumps/slides across sequence of squared values, selecting interval for processing
- what happens to $E_{\hat{n}}$ as sequence jumps by 2,4,8,...,*L* samples ($E_{\hat{n}}$ is a lowpass function—so it can be decimated without lost of information; why is $E_{\hat{n}}$ lowpass?)
- effects of decimation depend on *L*; if *L* is small, then $E_{\hat{n}}$ is a lot more variable than if *L* is large (window bandwidth changes with *L*!)

Short-Time Energy

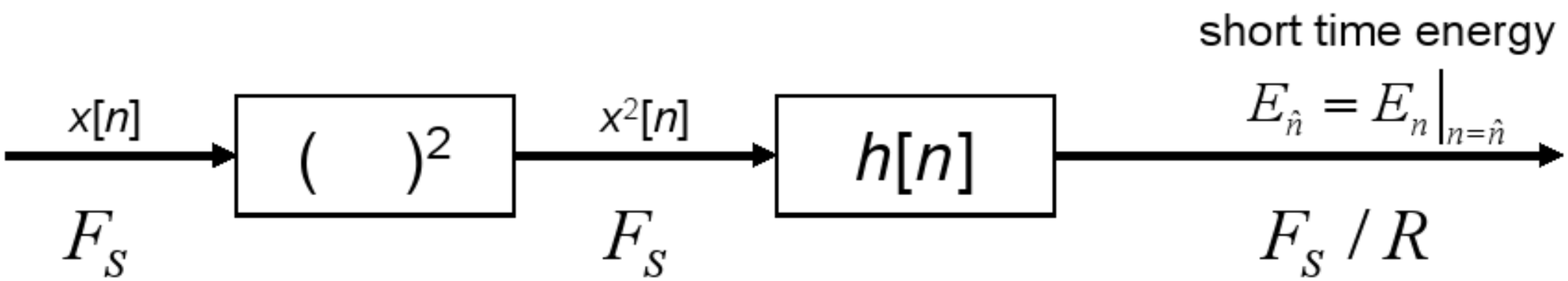
- serves to **differentiate voiced and unvoiced sounds** in speech from silence (background signal)
- natural definition of energy of weighted signal is:

$$E_{\hat{n}} = \sum_{m=-\infty}^{\infty} \left[x[m]\tilde{w}[\hat{n}-m] \right]^2 \text{ (sum or squares of portion of signal)}$$

-- concentrates measurement at sample $\hat{n}$, using weighting $\tilde{w}[\hat{n} - m]$

$$E_{\hat{n}} = \sum_{m=-\infty}^{\infty} x^2[m]\, \tilde{w}^2[\hat{n}-m] = \sum_{m=-\infty}^{\infty} x^2[m]\, h[\hat{n}-m]$$

$$h[n] = \tilde{w}^2[n]$$



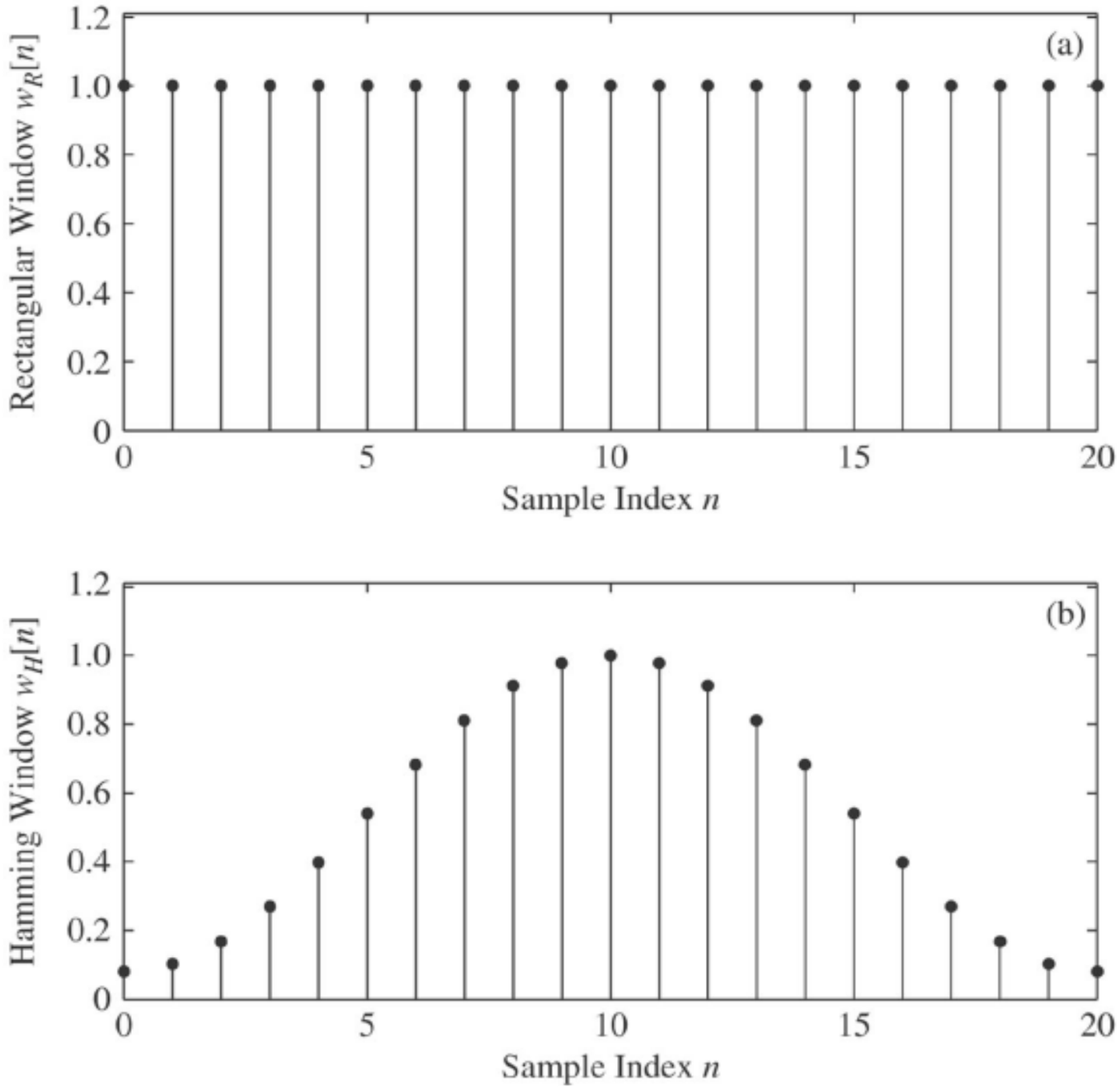
Short-Time Energy Properties

- depends on choice of $h[n]$, or equivalently, window $\tilde{w}[n]$
  - if $w[n]$ duration very long and constant amplitude ($\tilde{w}[n]=1,\ n=0,1,\ldots,L-1$), E_n would not change much over time, and would not reflect the short-time amplitudes of the sounds of the speech
  - very long duration windows correspond to **narrowband lowpass filters**
  - want E_n to change at a rate comparable to the changing sounds of the speech => this is the essential ***conflict*** in all speech processing, namely we need short duration window to be responsive to rapid sound changes, but short windows will not provide sufficient averaging to give smooth and reliable energy function

Windows

- consider two windows, $\tilde{w}[n]$
  - rectangular window:
    - $h[n]=1,\ 0 \le n \le L-1$ and 0 otherwise
  - Hamming window (raised cosine window):
    - $h[n]=0.54-0.46\cos(2\pi n/(L-1)),\ 0 \le n \le L-1$ and 0 otherwise
  - rectangular window gives ***equal weight*** to all L samples in the window $(n,\ldots,n-L+1)$
  - Hamming window gives ***most weight*** to middle samples and ***tapers off*** strongly at the beginning and the end of the window

Rectangular and Hamming Windows



$L = 21$ samples

Window Frequency Responses

- rectangular window

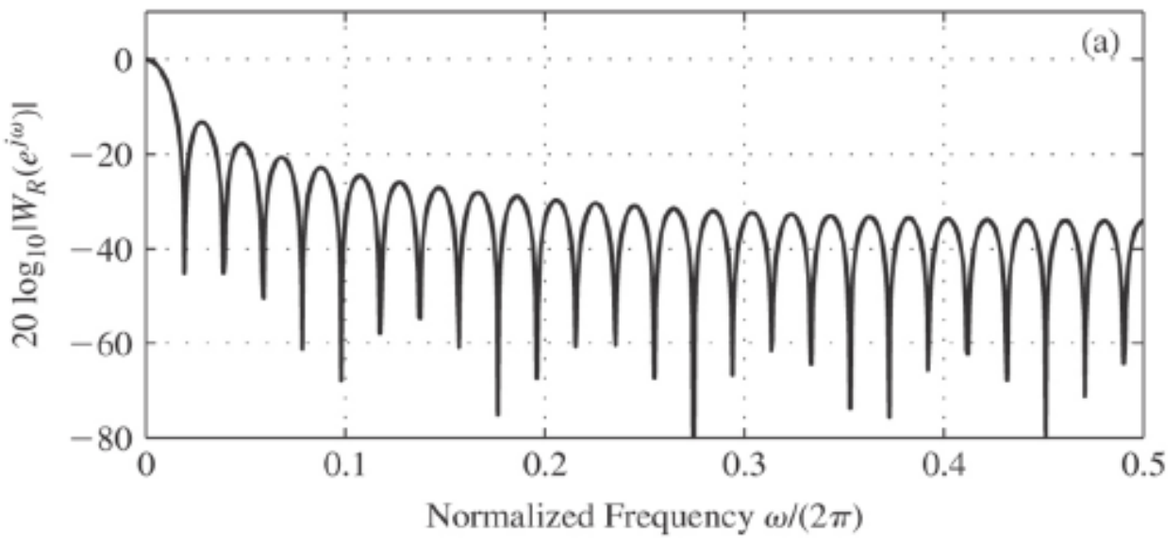
$$H(e^{j\Omega T}) = \frac{\sin(\Omega L T / 2)}{\sin(\Omega T / 2)} e^{-j\Omega T (L-1)/2}$$

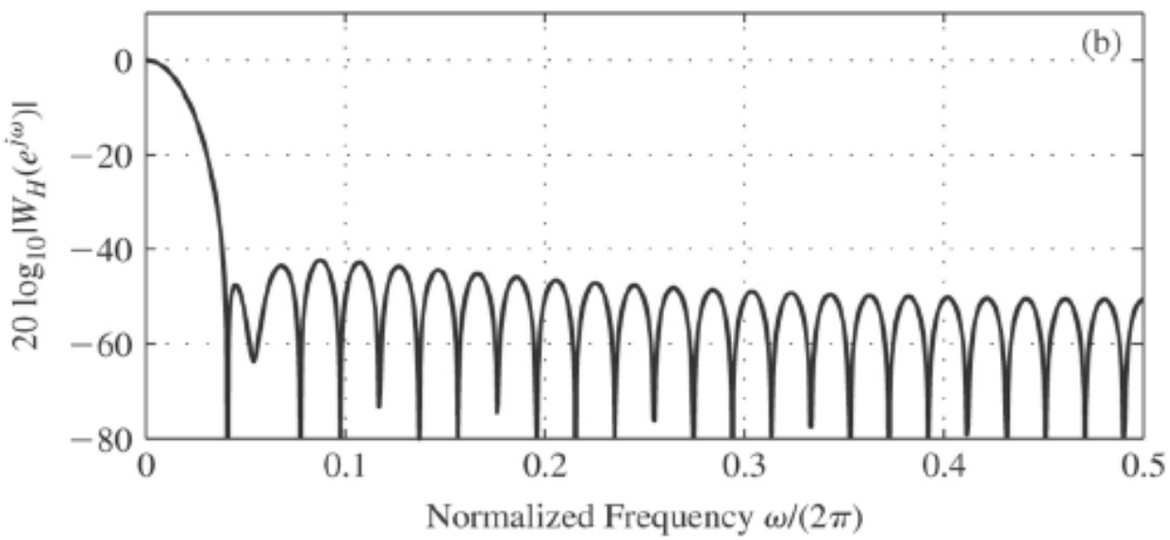
- first zero occurs at $f=F_s/L=1/(LT)$ (or $\Omega=(2\pi)/(LT)$) => nominal cutoff frequency of the equivalent "lowpass" filter
- Hamming window

$$\tilde{w}_H[n] = 0.54\tilde{w}_R[n] - 0.46 * \cos(2\pi n / (L-1))\tilde{w}_R[n]$$

- can decompose Hamming Window FR into combination of three terms

RW and HW Frequency Responses

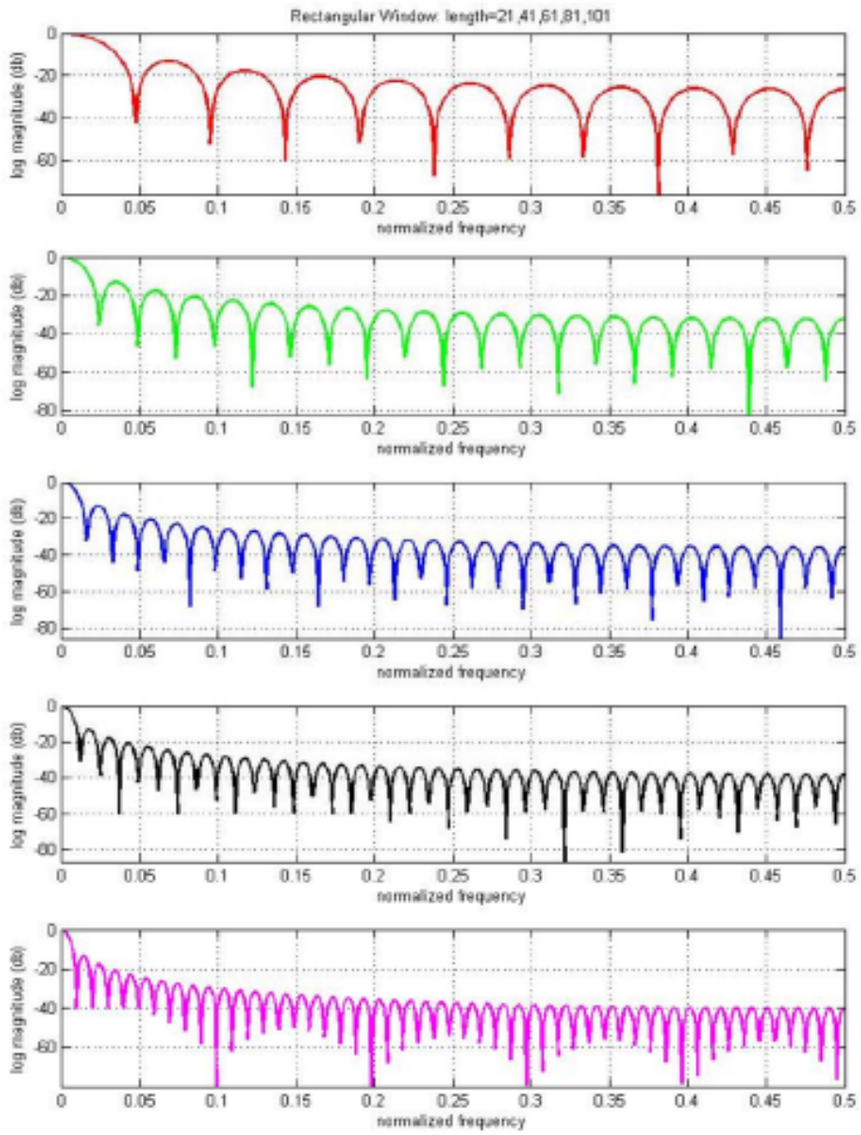


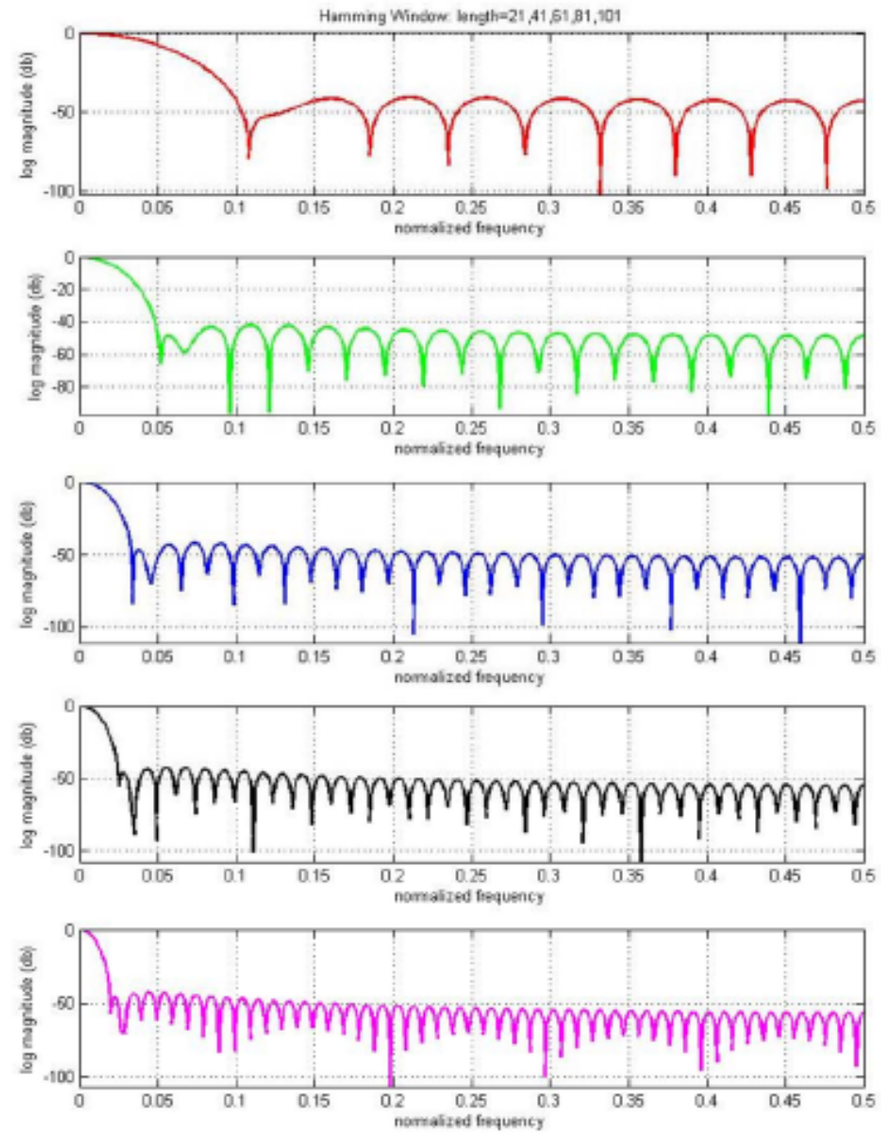


- log magnitude response of RW and HW
- ***bandwidth*** of HW is approximately twice the bandwidth of RW
- ***attenuation*** of more than 40 dB for HW outside passband, versus 14 dB for RW
- stopband attenuation is essentially <u>**independent**</u> of L, the window duration => increasing L simply decreases window bandwidth
- L needs to be larger than a pitch period (or severe fluctuations will occur in E_n), but smaller than a sound duration (or E_n will not adequately reflect the changes in the speech signal)

There is no perfect value of L, since a pitch period can be as short as 20 samples (500 Hz at a 10 kHz sampling rate) for a high pitch child or female, and up to 250 samples (40 Hz pitch at a 10 kHz sampling rate) for a low pitch male; a compromise value of L on the order of 100-200 samples for a 10 kHz sampling rate is often used in practice

Window Frequency Responses

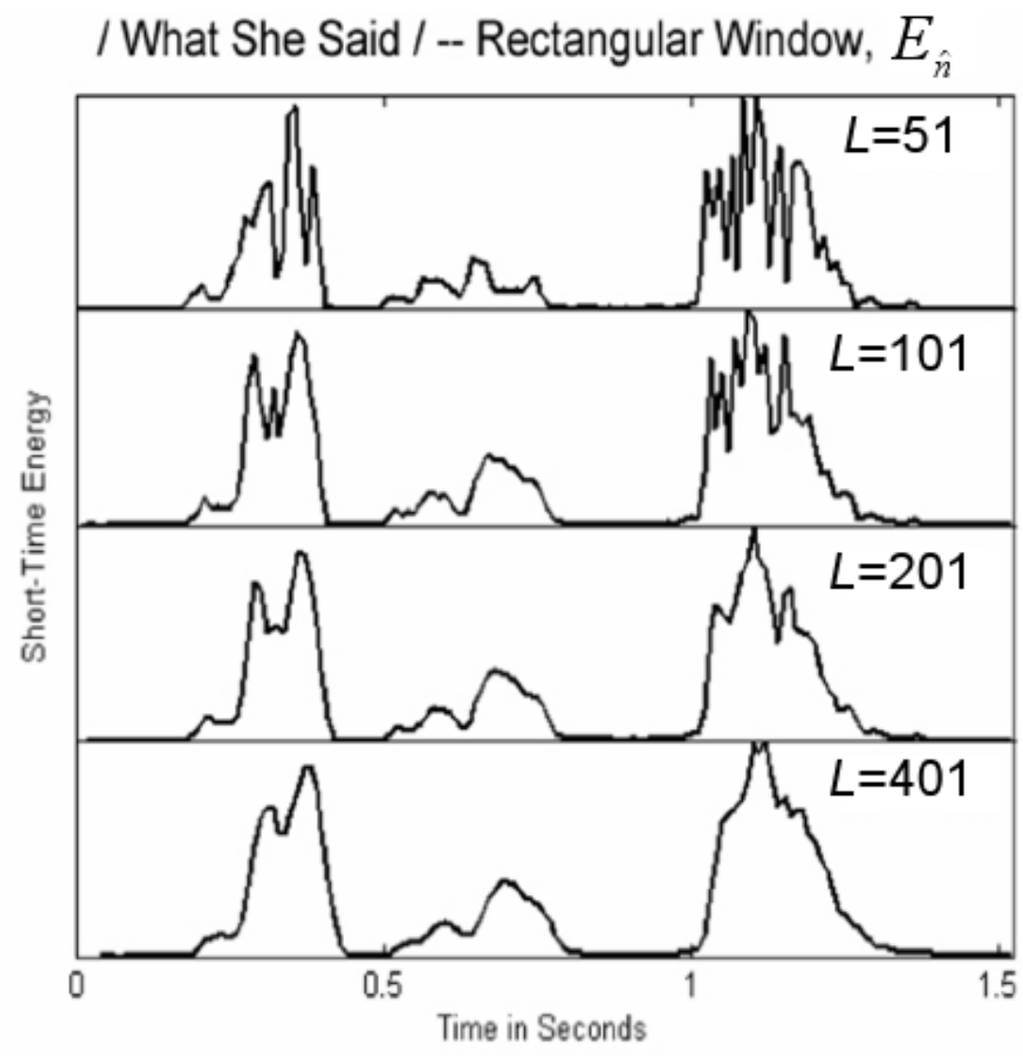


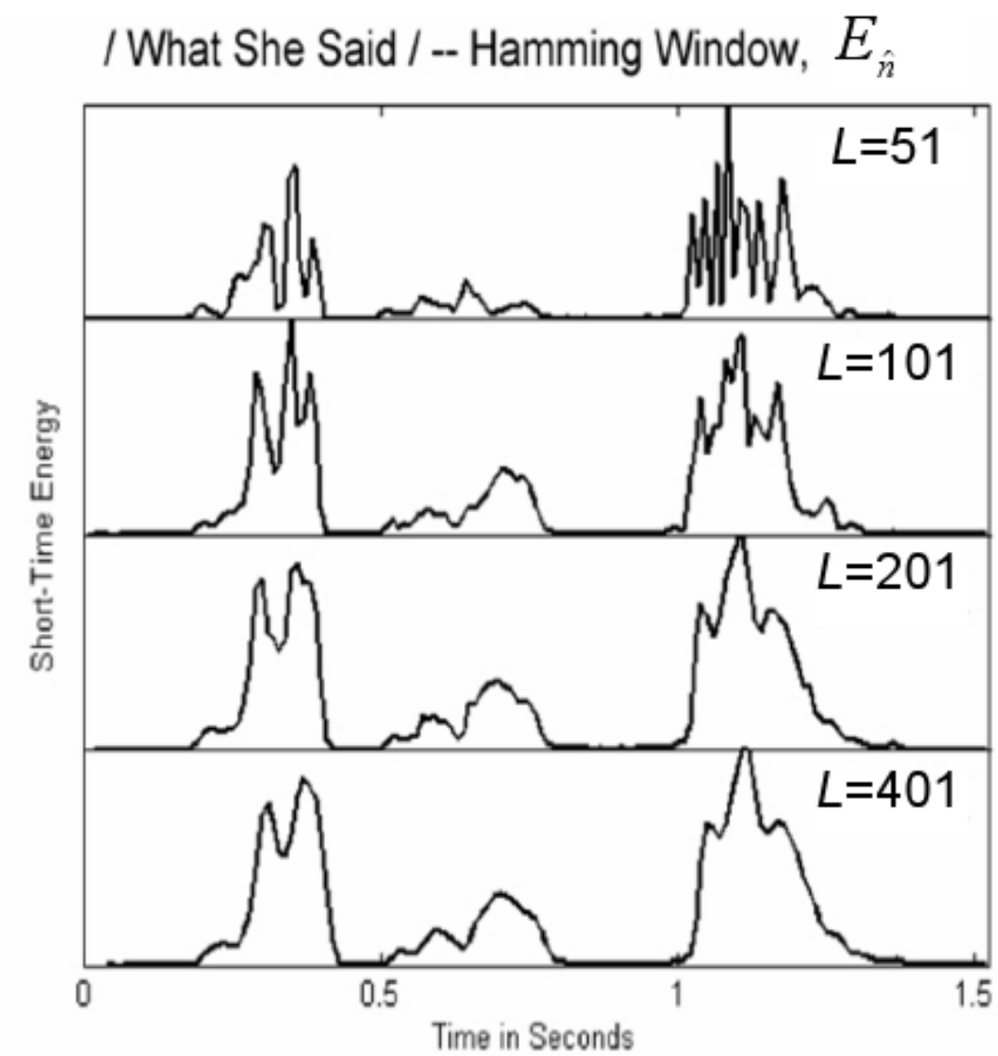


Rectangular Windows, L=21,41,61,81,101

Hamming Windows, L=21,41,61,81,101

Short-Time Energy using RW/HW





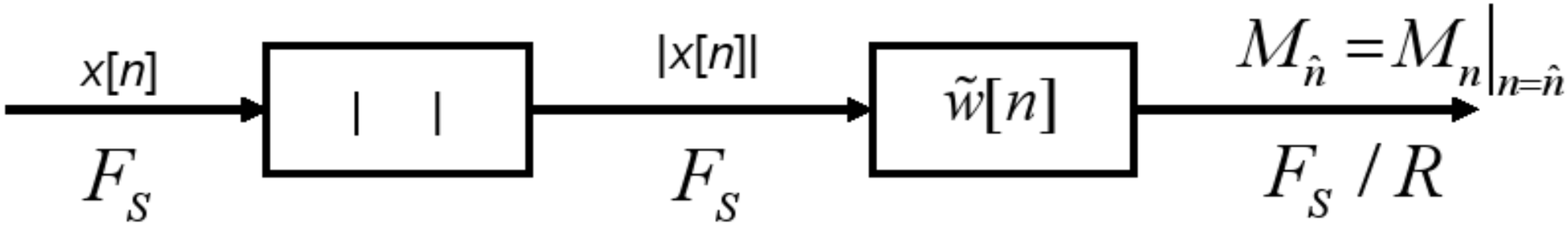
- as *L* increases, the plots tend to converge (however you are smoothing sound energies)
- short-time energy provides the basis for distinguishing voiced from unvoiced speech regions, and for medium-to-high SNR recordings, can even be used to find regions of silence/background signal

Short-Time Magnitude

- short-time energy is very sensitive to large signal levels due to $x^2[n]$ terms
  - consider a new definition of 'pseudo-energy' based on average signal magnitude (rather than energy)

$$M_{\hat{n}} = \sum_{m=-\infty}^{\infty} |x[m]|\, \tilde{w}[\hat{n}-m]$$

  - weighted sum of magnitudes, rather than weighted sum of squares



- computation avoids multiplications of signal with itself (the squared term)

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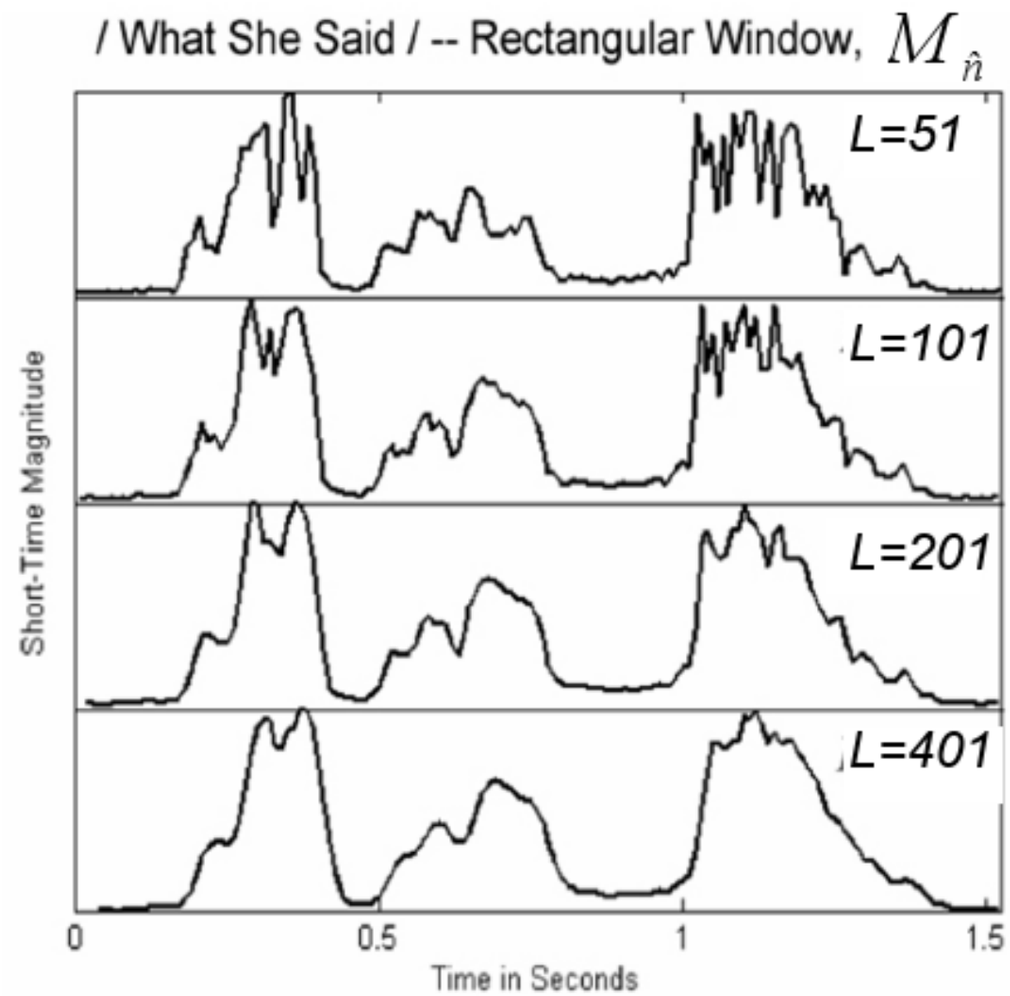
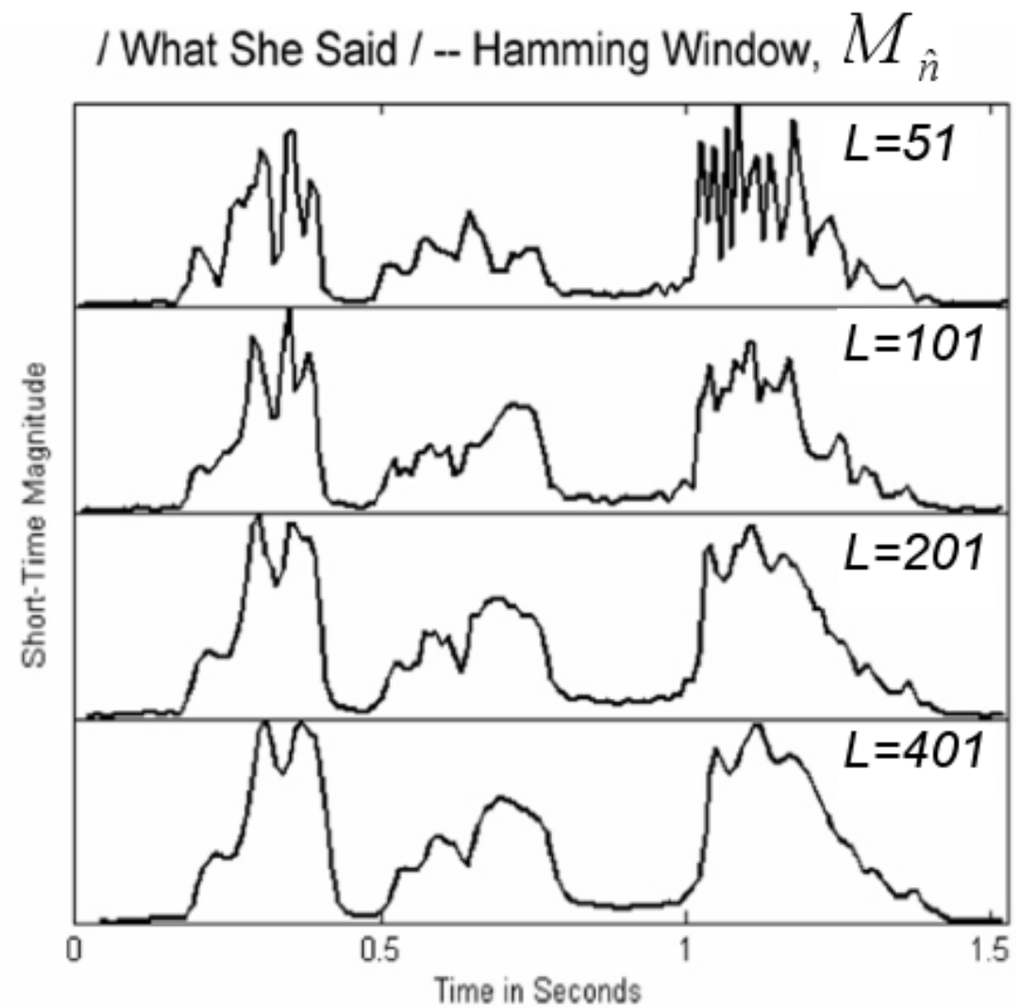
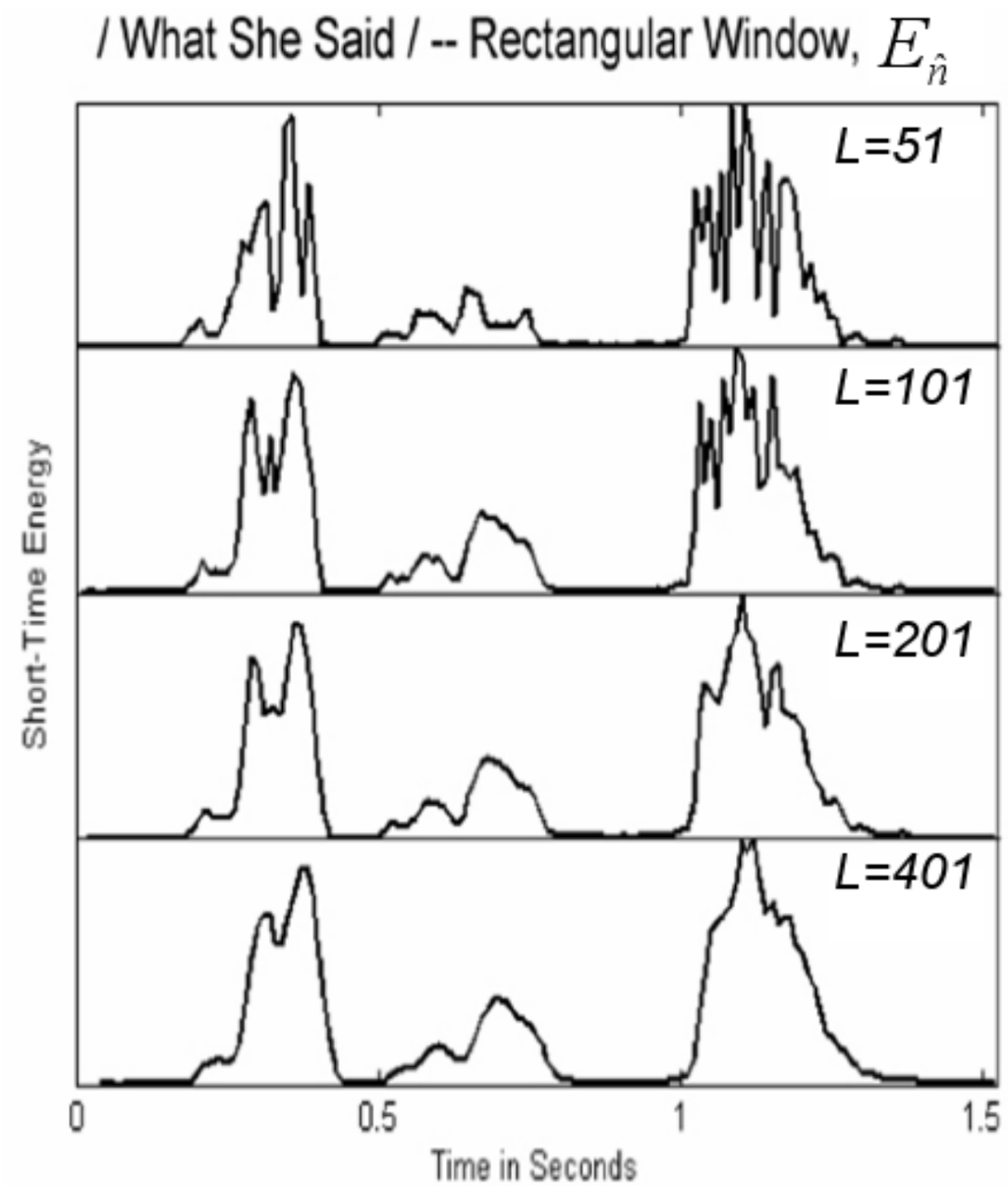
Short-Time Magnitudes

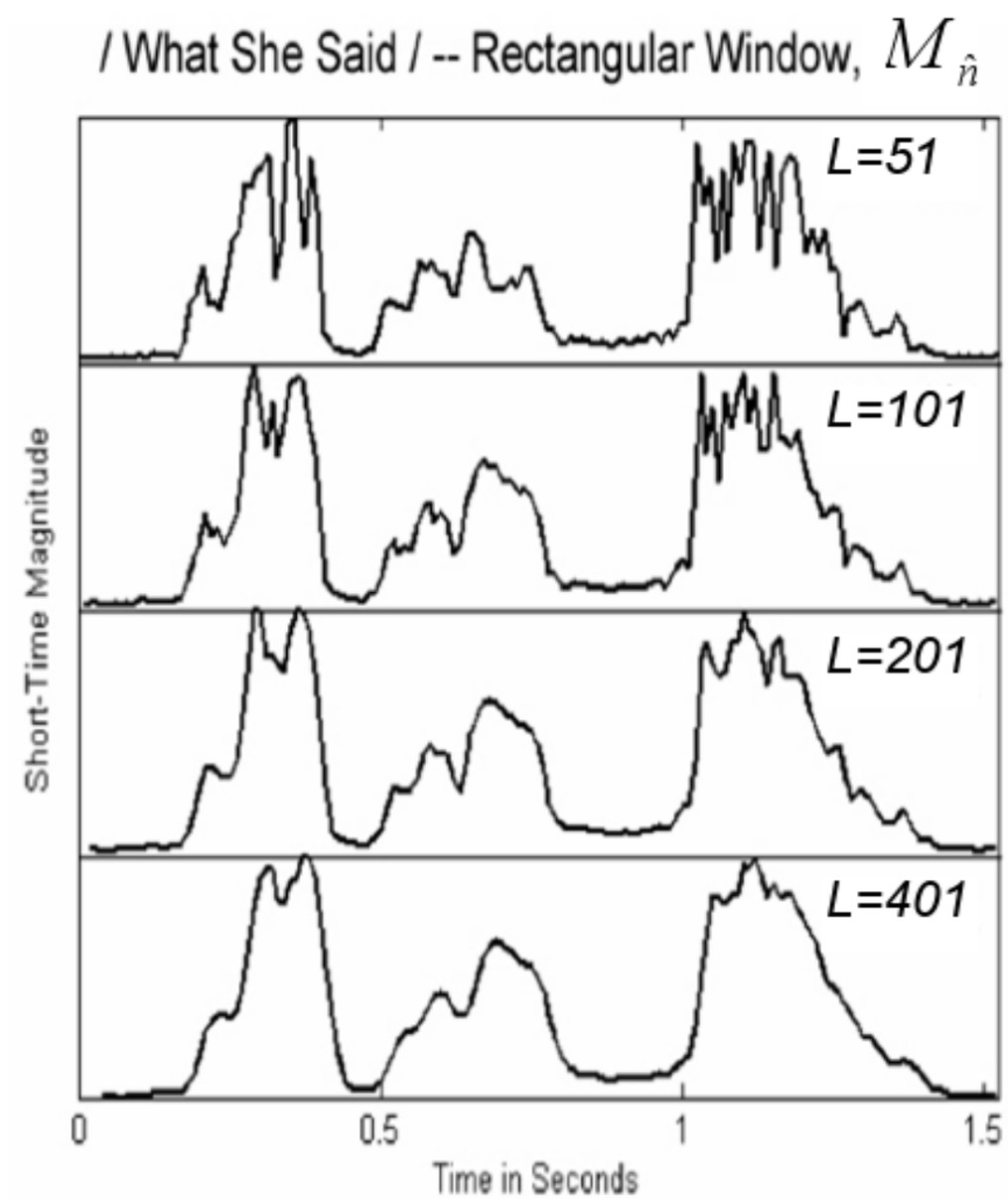




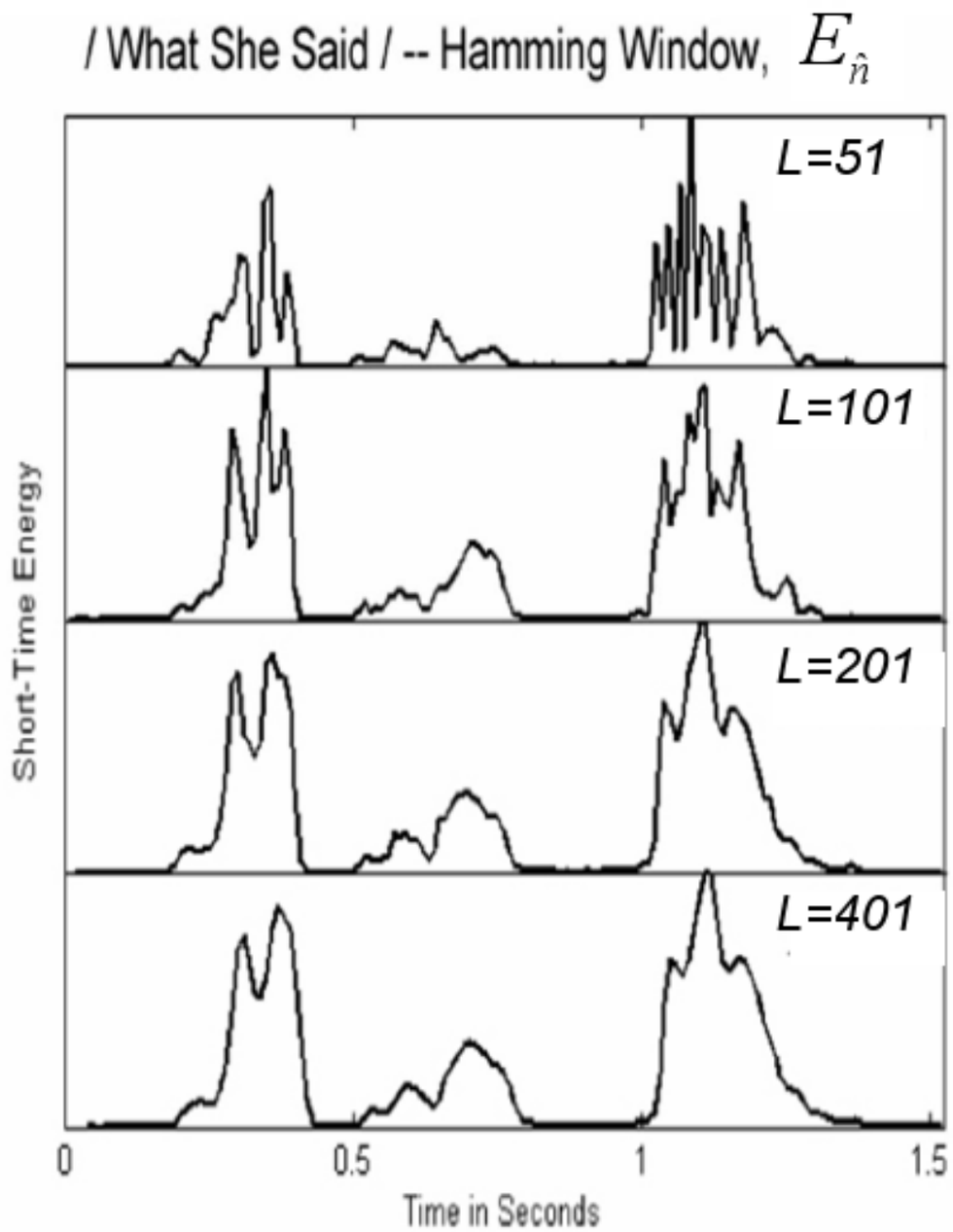
- differences between E_n and M_n noticeable in unvoiced regions
- dynamic range of M_n ~ square root (dynamic range of E_n) => level differences between voiced and unvoiced segments are smaller
- E_n and M_n can be sampled at a rate of 100/sec for window durations of 20 msec or so => efficient representation of signal energy/magnitude

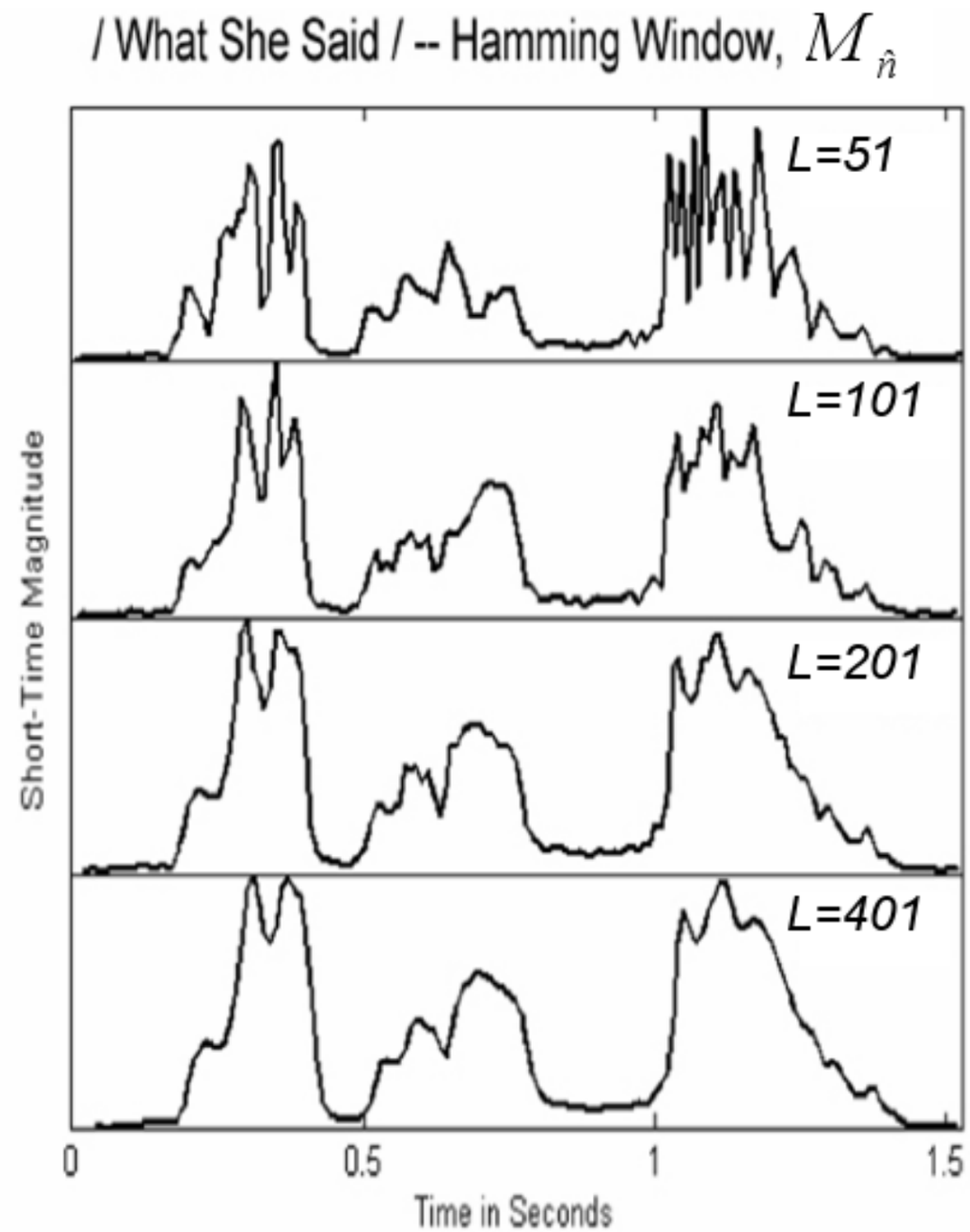
Short Time Energy and Magnitude—Rectangular Window





Short Time Energy and Magnitude—Hamming Window





Other Lowpass Windows

- can replace RW or HW with any lowpass filer
- window should be positive since this guarantees E_n and M_n will be positive
- FIR windows are efficient computationally since they can slide by R samples for efficiency with no loss of information (what should R be?)
- can even use an infinite duration window if its z-transform is a rational function, i.e.,

$$h[n] = a^n, \quad n \geq 0, \quad 0 < a < 1$$
$$= 0 \qquad n < 0$$

$$H(z) = \frac{1}{1 - az^{-1}} \qquad |z| > |a|$$

Other Lowpass Windows

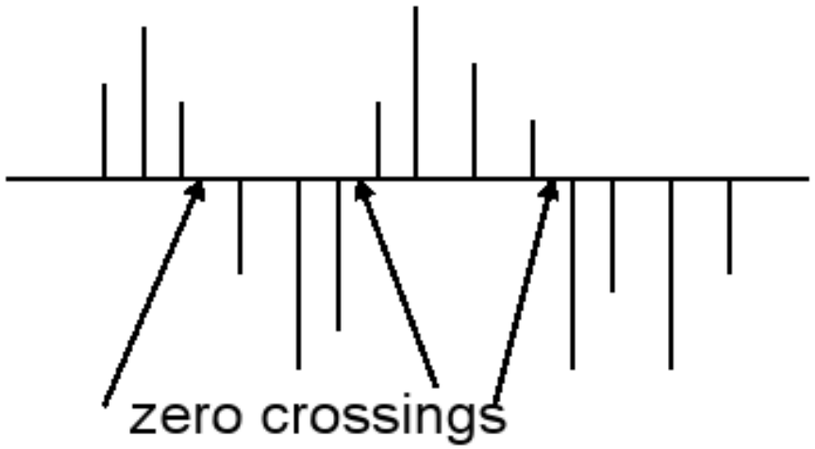
- this simple lowpass filter can be used to implement E_n and M_n recursively as:

$$E_n = aE_{n-1} + (1-a)x^2[n] \text{ – short-time energy}$$

$$M_n = aM_{n-1} + (1-a)\,|\,x[n]\,| \text{ – short-time magnitude}$$

- need to compute E_n or M_n every sample and then down-sample to 100/sec rate
- recursive computation has a non-linear phase, so delay cannot be compensated exactly

Short-Time Average ZC Rate



zero crossing => successive samples have different algebraic signs

- zero crossing rate is a simple measure of the 'frequency content' of a signal—especially true for narrowband signals (e.g., sinusoids)
- sinusoid at frequency F_0 with sampling rate F_S has F_S/F_0 samples per cycle with two zero crossings per cycle, giving an average zero crossing rate of

$z_1=(2)$ crossings/cycle x (F_0 / F_S) cycles/sample

$z_1=2F_0 / F_S$ crossings/sample (i.e., **z_1 proportional to F_0**)

$z_M=M\ (2F_0 /F_S)$ crossings/(M samples)

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Sinusoid Zero Crossing Rates

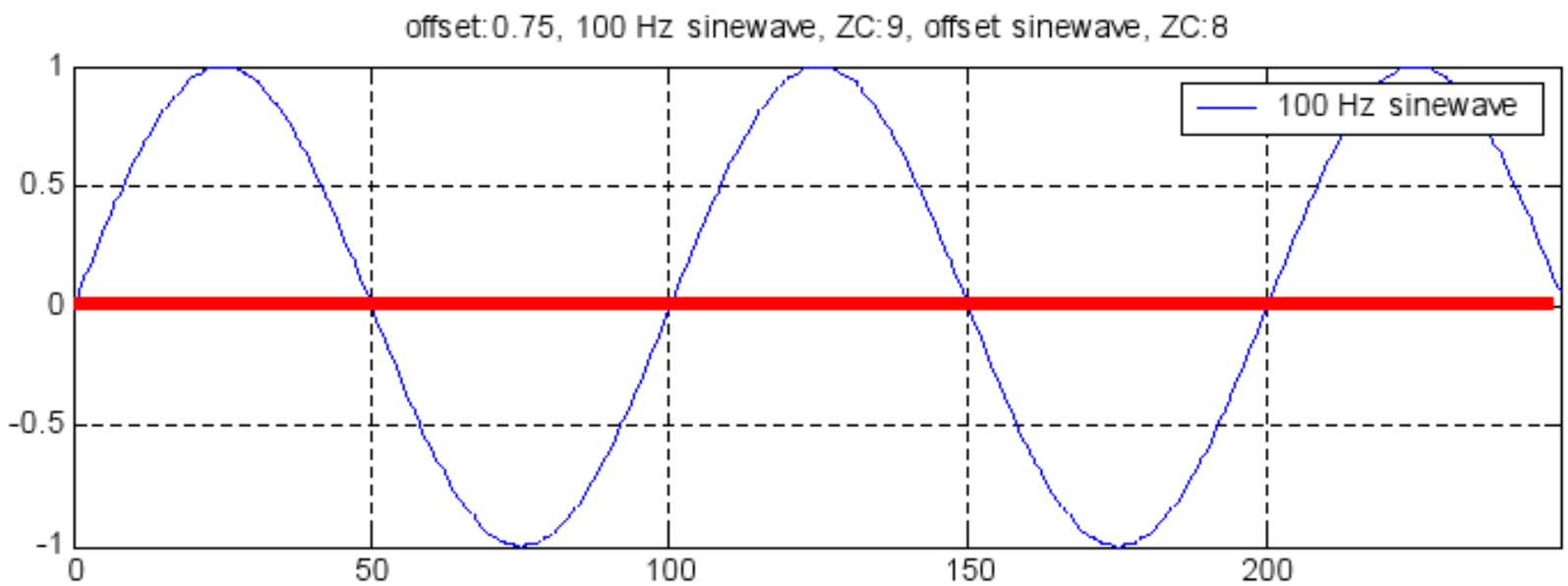
Assume the sampling rate is $F_S = 10{,}000$ Hz

1. $F_0 = 100$ Hz sinusoid has $F_S / F_0 = 10{,}000 / 100 = 100$ samples/cycle; or $z_1 = 2/100$ crossings/sample, or $z_{100} = 2/100 * 100 =$ 2 crossings/10 msec interval

2. $F_0 = 1000$ Hz sinusoid has $F_S / F_0 = 10{,}000 / 1000 = 10$ samples/cycle; or $z_1 = 2/10$ crossings/sample, or $z_{100} = 2/10 * 100 =$ 20 crossings/10 msec interval

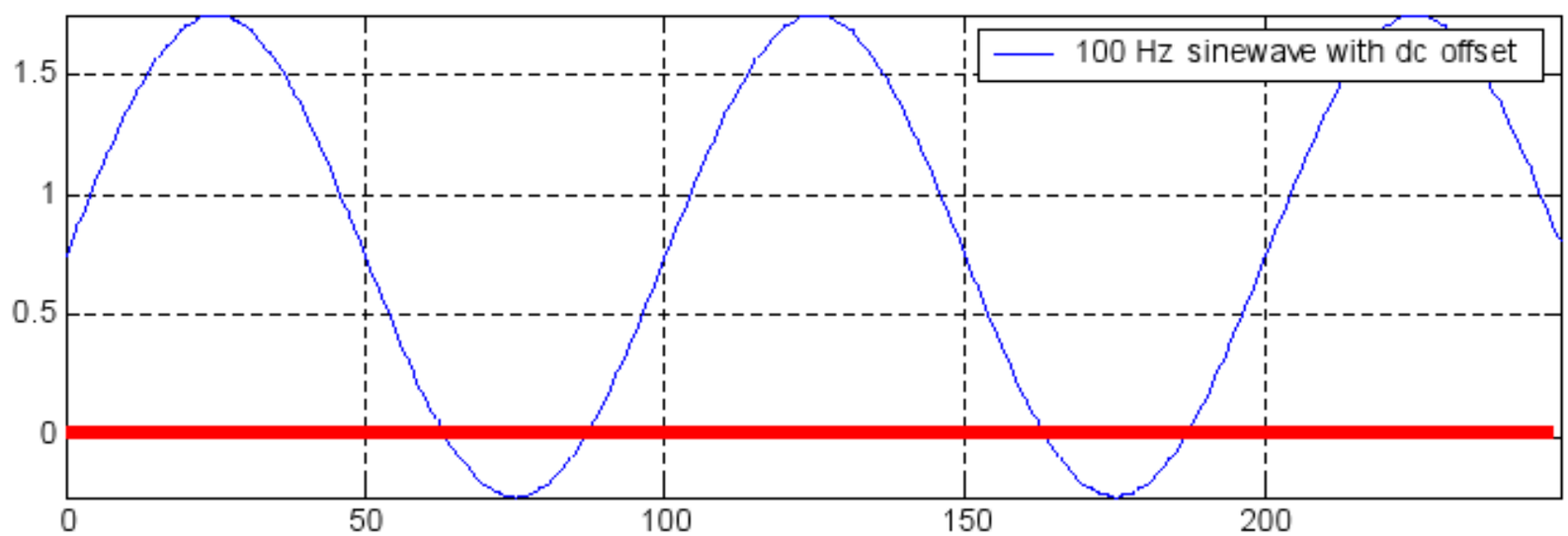
3. $F_0 = 5000$ Hz sinusoid has $F_S / F_0 = 10{,}000 / 5000 = 2$ samples/cycle; or $z_1 = 2/2$ crossings/sample, or $z_{100} = 2/2 * 100 =$ 100 crossings/10 msec interval

Zero Crossing for Sinusoids



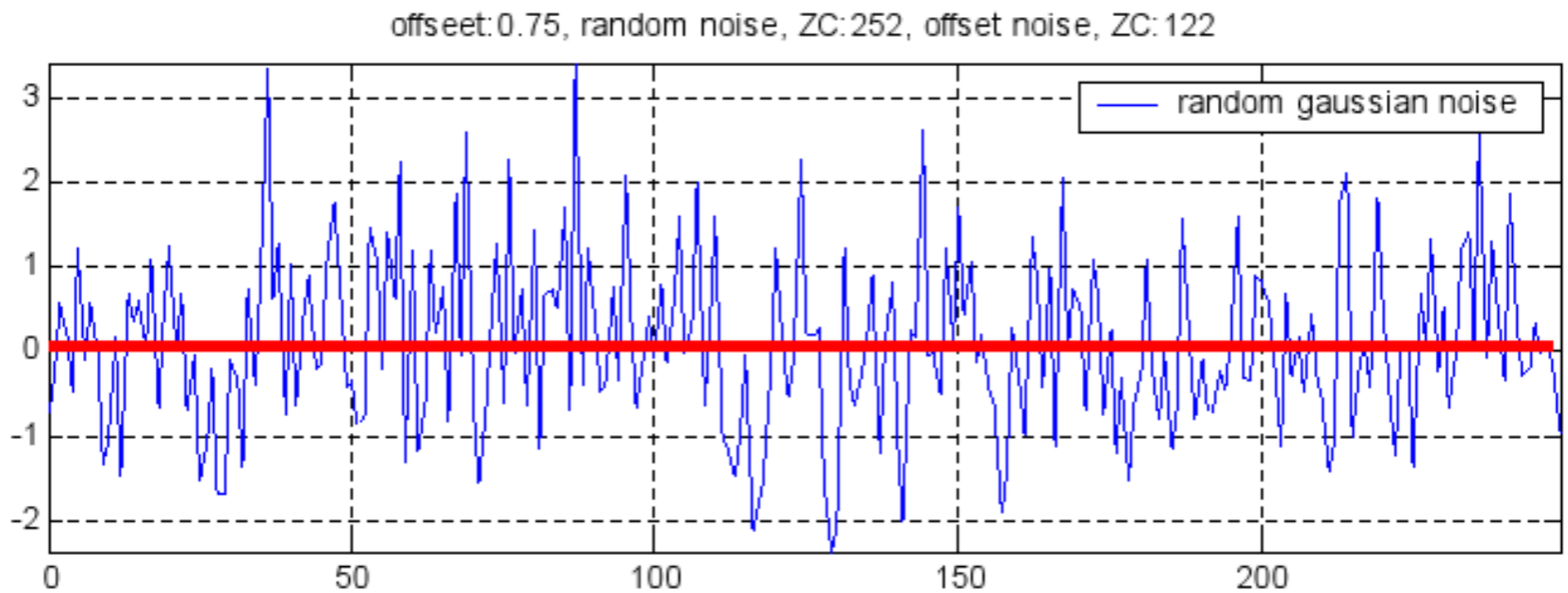
ZC=9

Offset=0.75



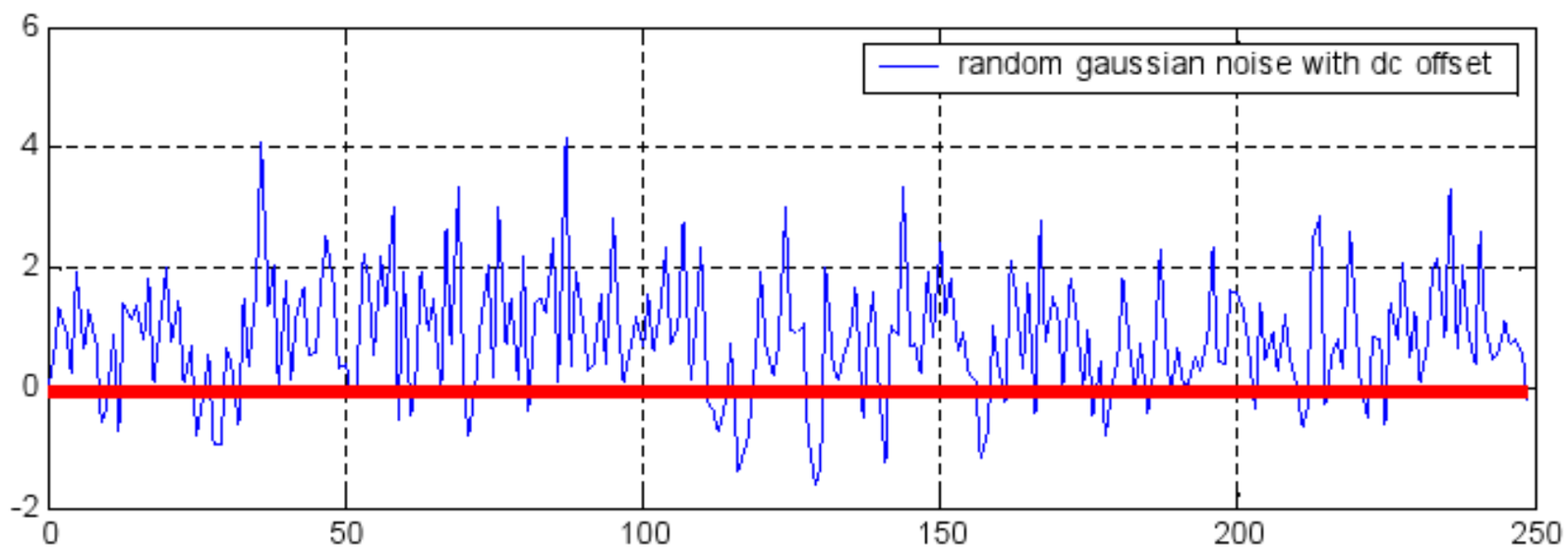
ZC=8

Zero Crossings for Noise



ZC=252

Offset=0.75



ZC=122

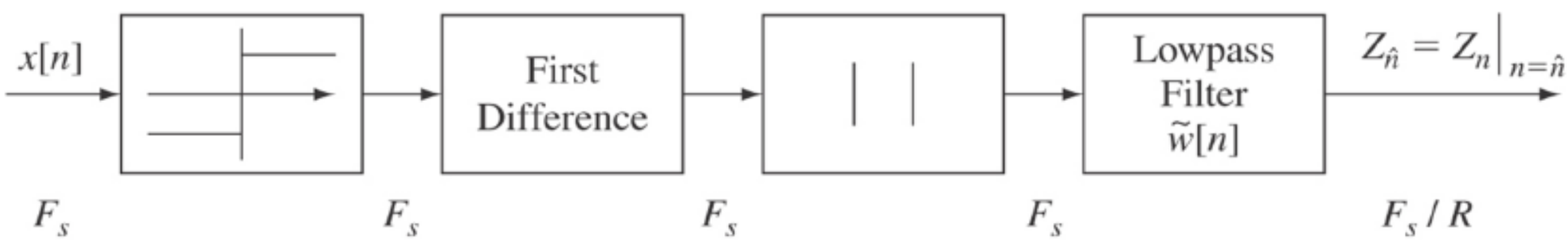
ZC Rate Definitions

$$Z_{\hat{n}} = \frac{1}{2L_{\text{eff}}} \sum_{m=\hat{n}-L+1}^{\hat{n}} |\,\text{sgn}(x[m]) - \text{sgn}(x[m-1])\,|\,\tilde{w}[\hat{n}-m]$$

$$\text{sgn}(x[n]) = 1 \qquad x[n] \geq 0$$
$$= -1 \qquad x[n] < 0$$

- simple rectangular window:

$$\tilde{w}[n] = 1 \qquad 0 \leq n \leq L-1$$
$$= 0 \qquad \text{otherwise}$$
$$L_{\text{eff}} = L$$



Same form for $Z_{\hat{n}}$ as for $E_{\hat{n}}$ or $M_{\hat{n}}$

ZC Normalization

- The formal definition of $Z_{\hat{n}}$ is:

$$Z_{\hat{n}} = z_1 = \frac{1}{2L} \sum_{m=\hat{n}-L+1}^{\hat{n}} |\,\mathrm{sgn}(x[m]) - \mathrm{sgn}(x[m-1])\,|$$

  is interpreted as the number of zero crossings per sample.

- For most practical applications, we need the rate of zero crossings per fixed interval of M samples, which is

$$z_M = z_1 \cdot M = \text{rate of zero crossings per } M \text{ sample interval}$$

  Thus, for an interval of τ sec., corresponding to M samples we get

$$z_M = z_1 \cdot M; \quad M = \tau F_S = \tau / T$$

- $F_S = 10{,}000$ Hz; $T = 100$ μsec; τ=10 msec; $M = 100$ samples
- $F_S = 8{,}000$ Hz; $T = 125$ μsec; τ=10 msec; $M = 80$ samples
- $F_S = 16{,}000$ Hz; $T = 62.5$ μsec; τ=10 msec; $M = 160$ samples

Zero crossings/10 msec interval as a function of sampling rate

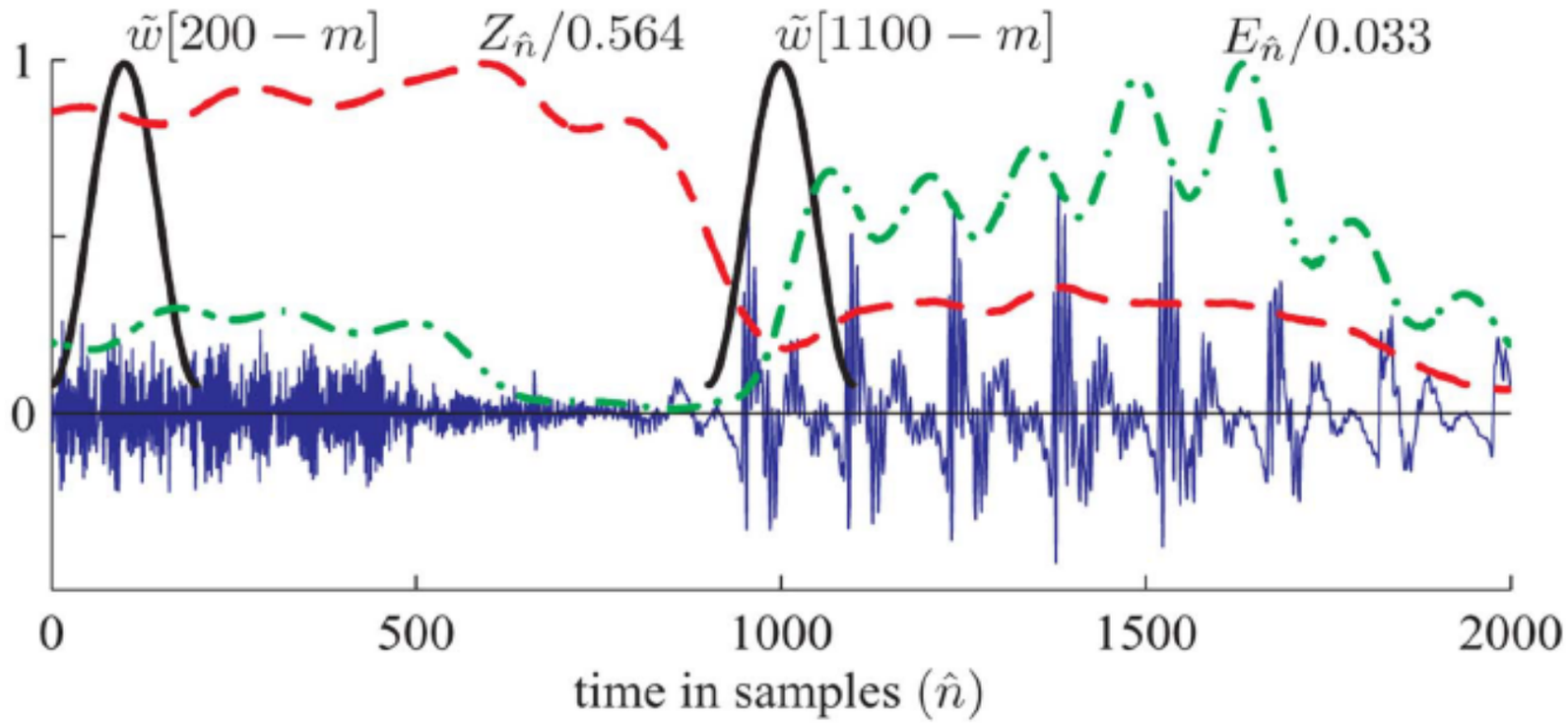
ZC Normalization

- For a 1000 Hz sinewave as input, using a 40 msec window length (L), with various values of sampling rate (F_S), we get the following:

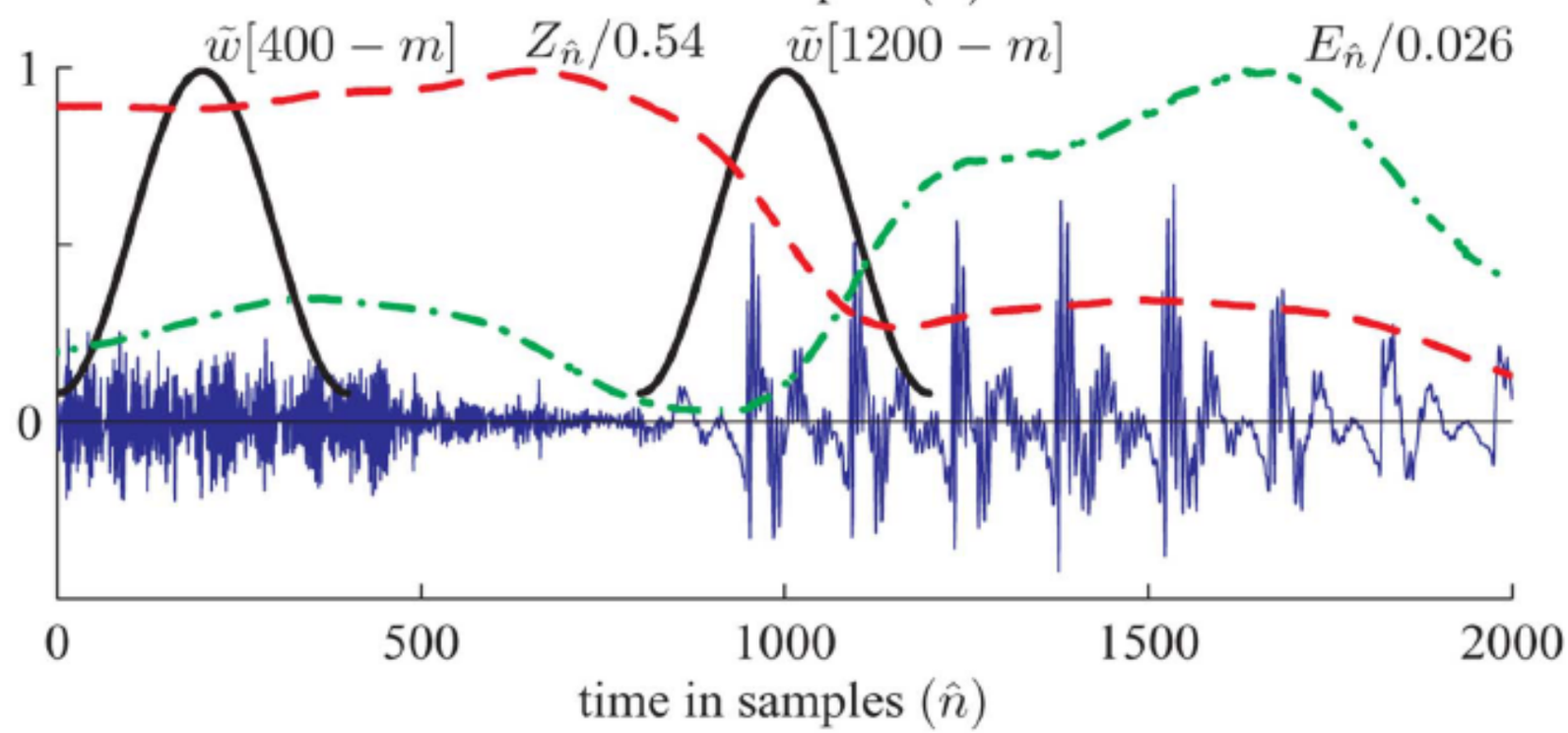
F_S	L	z_1	M	z_M
8000	320	1/4	80	20
10000	400	1/5	100	20
16000	640	1/8	160	20

- Thus we see that the normalized (per interval) zero crossing rate, z_M, is independent of the sampling rate and can be used as a measure of the dominant energy in a band.

ZC and Energy Computation



Hamming window with duration *L=201* samples (12.5 msec at *Fs*=16 kHz)



Hamming window with duration *L=401* samples (25 msec at *Fs*=16 kHz)

ZC Rate Distributions

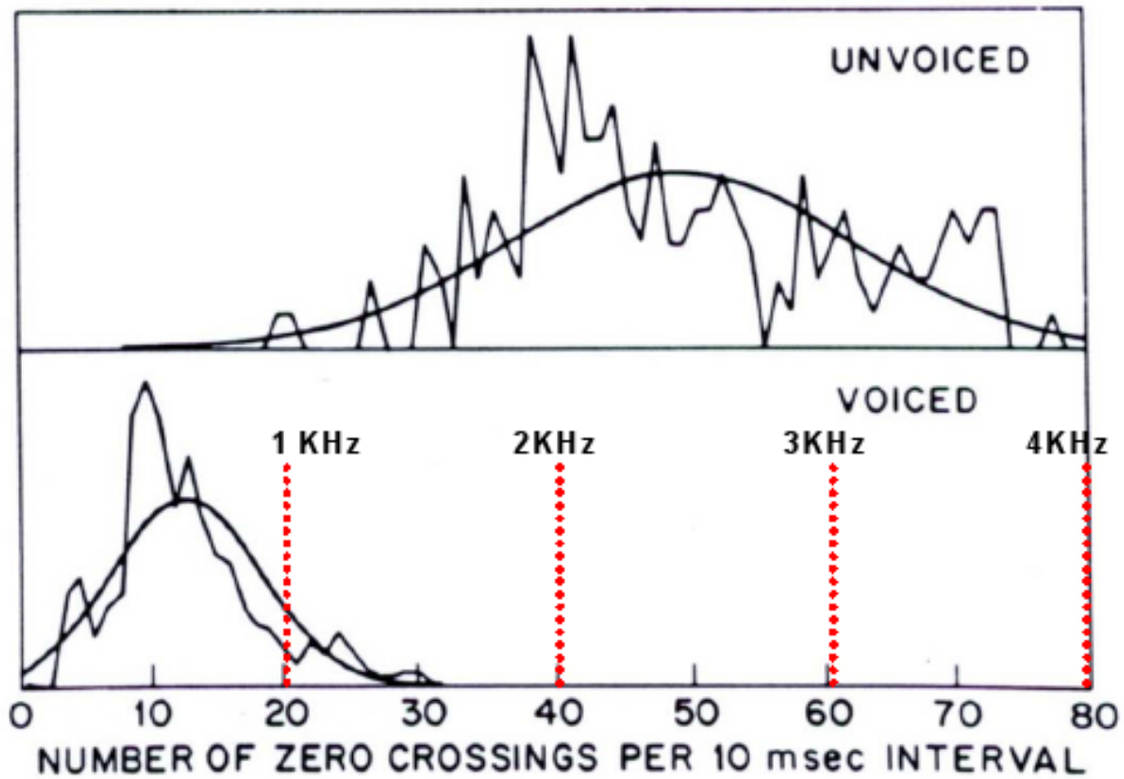


Fig. 4.11 Distribution of zero-crossings for unvoiced and voiced speech.

Unvoiced Speech: the dominant energy component is at about 2.5 kHz

Voiced Speech: the dominant energy component is at about 700 Hz

- for voiced speech, energy is mainly below 1.5 kHz
- for unvoiced speech, energy is mainly above 1.5 kHz
- mean ZC rate for unvoiced speech is 49 per 10 msec interval
- mean ZC rate for voiced speech is 14 per 10 msec interval

ZC Rates for Speech

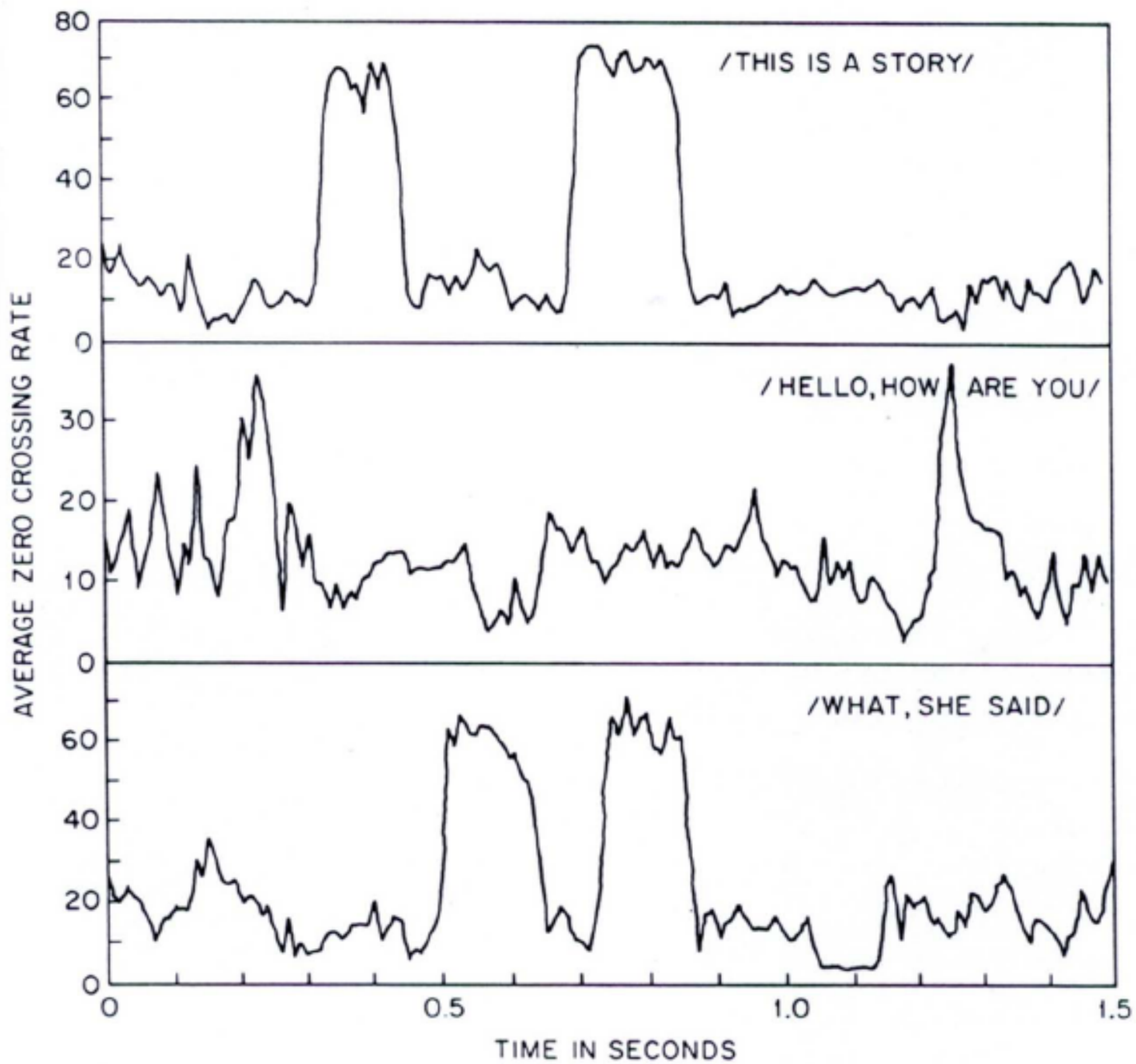
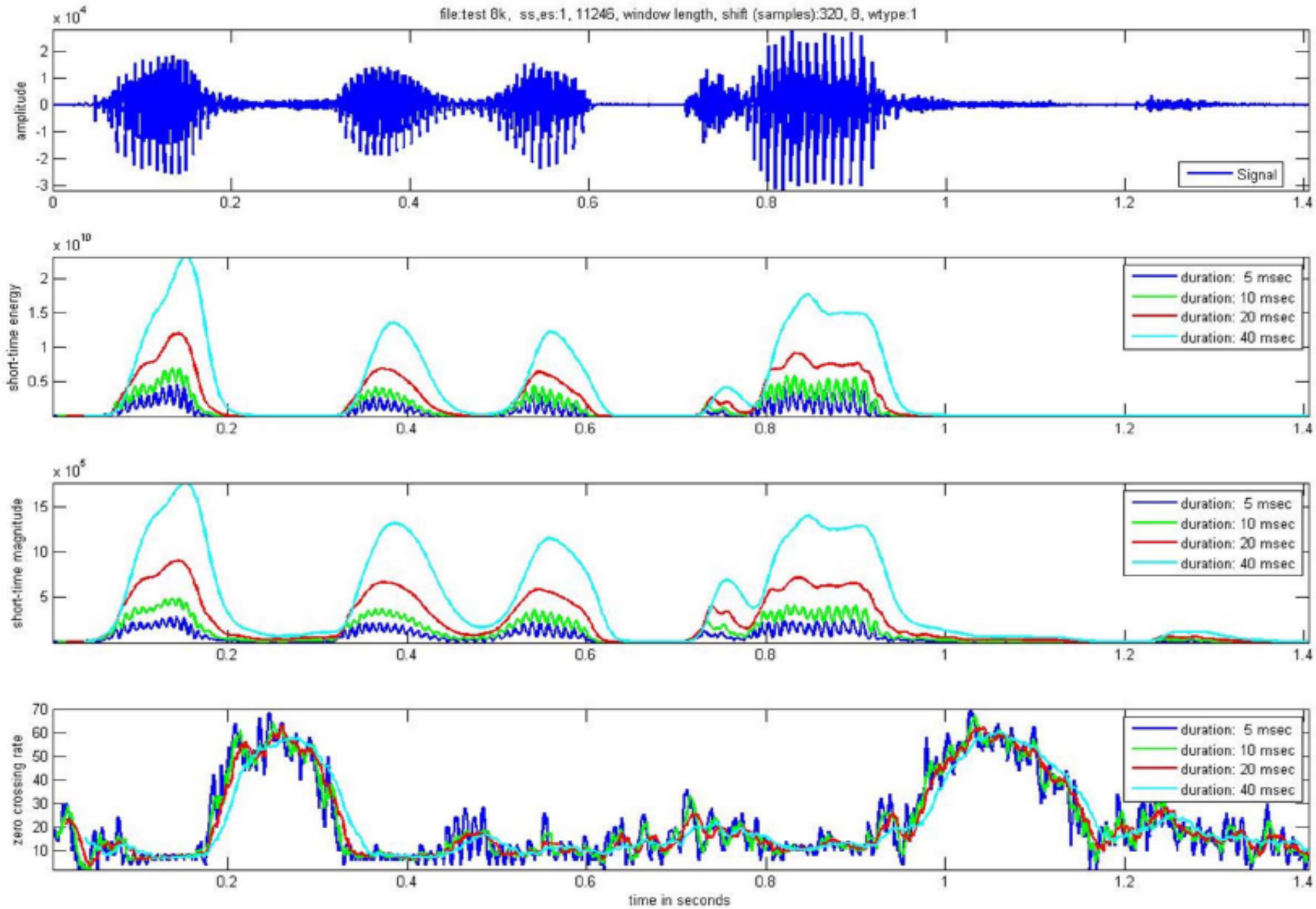


Fig. 4.12 Average zero-crossing rate for three different utterances.

- 15 msec windows
- 100/sec sampling rate on ZC computation

Short-Time Energy, Magnitude, ZC



Issues in ZC Rate Computation

- for zero crossing rate to be accurate, need zero DC in signal => need to remove offsets, hum, noise => use bandpass filter to eliminate DC and hum
- can quantize the signal to 1-bit for computation of ZC rate
- can apply the concept of ZC rate to bandpass filtered speech to give a ‘crude’ spectral estimate in narrow bands of speech (kind of gives an estimate of the strongest frequency in each narrow band of speech)

Summary of Simple Time Domain Measures

$$Q_{\hat{n}} = \sum_{m=-\infty}^{\infty} T(x[m])\tilde{w}[\hat{n}-m]$$

1. Energy:

$$E_{\hat{n}} = \sum_{m=\hat{n}-L+1}^{\hat{n}} x^2[m]\tilde{w}[\hat{n}-m]$$

- can downsample $E_{\hat{n}}$ at rate commensurate with window bandwidth

2. Magnitude:

$$M_{\hat{n}} = \sum_{m=\hat{n}-L+1}^{\hat{n}} \left|x[m]\right|\tilde{w}[\hat{n}-m]$$

3. Zero Crossing Rate:

$$Z_{\hat{n}} = z_1 = \frac{1}{2L}\sum_{m=\hat{n}-L+1}^{\hat{n}} \left|\mathrm{sgn}(x[m]) - \mathrm{sgn}(x[m-1])\right|\tilde{w}[\hat{n}-m]$$

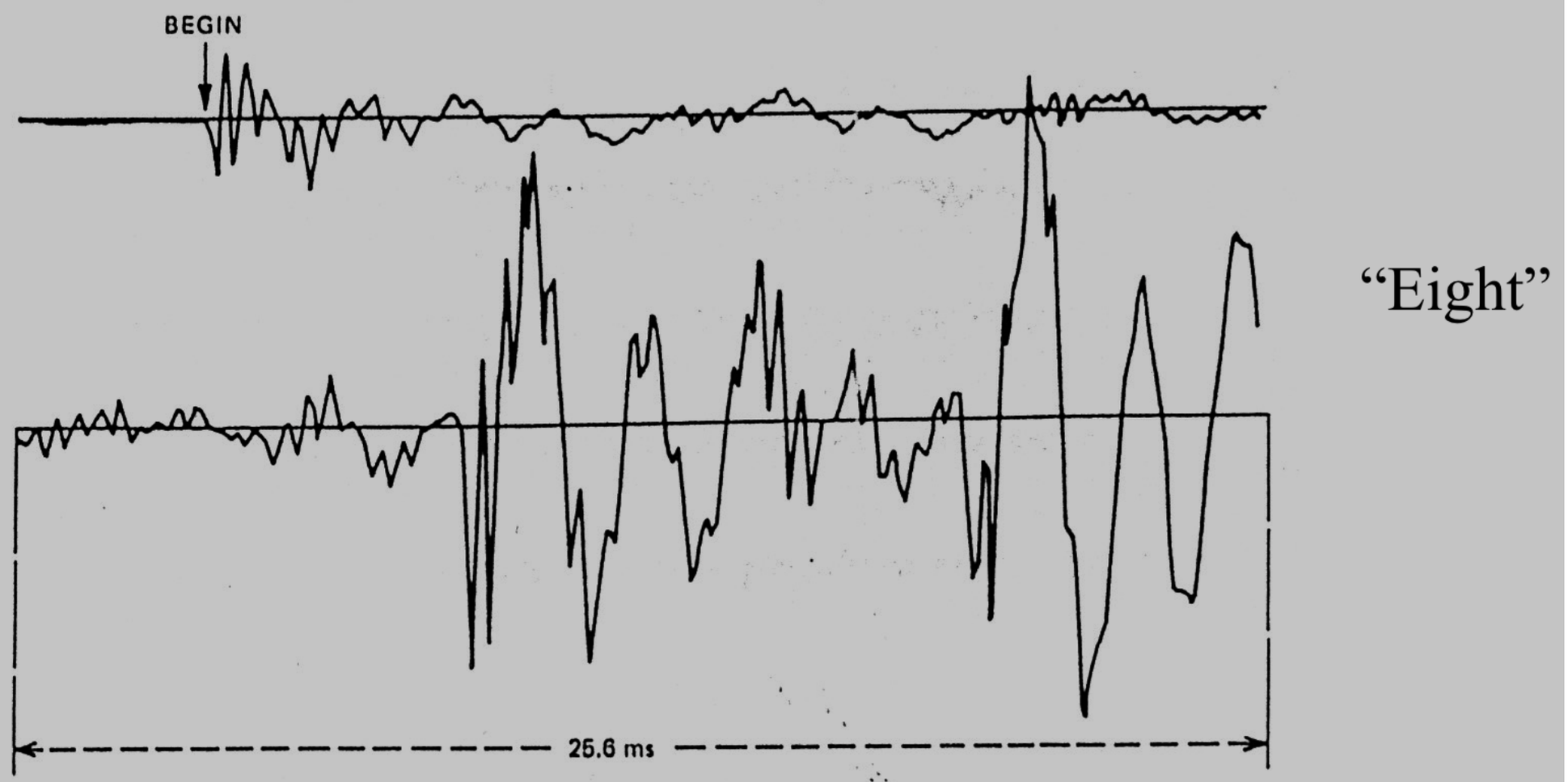
where $\mathrm{sgn}(x[m]) = 1 \quad x[m] \geq 0$

$\qquad\qquad\qquad\quad = -1 \quad x[m] < 0$

Silence Discrimination using ZCR and energy

- Word recognition needs to detect word boundaries in speech
- Recognizing silence can reduce:
  - Processing load
- Easy in sound proof room, with digitized tape

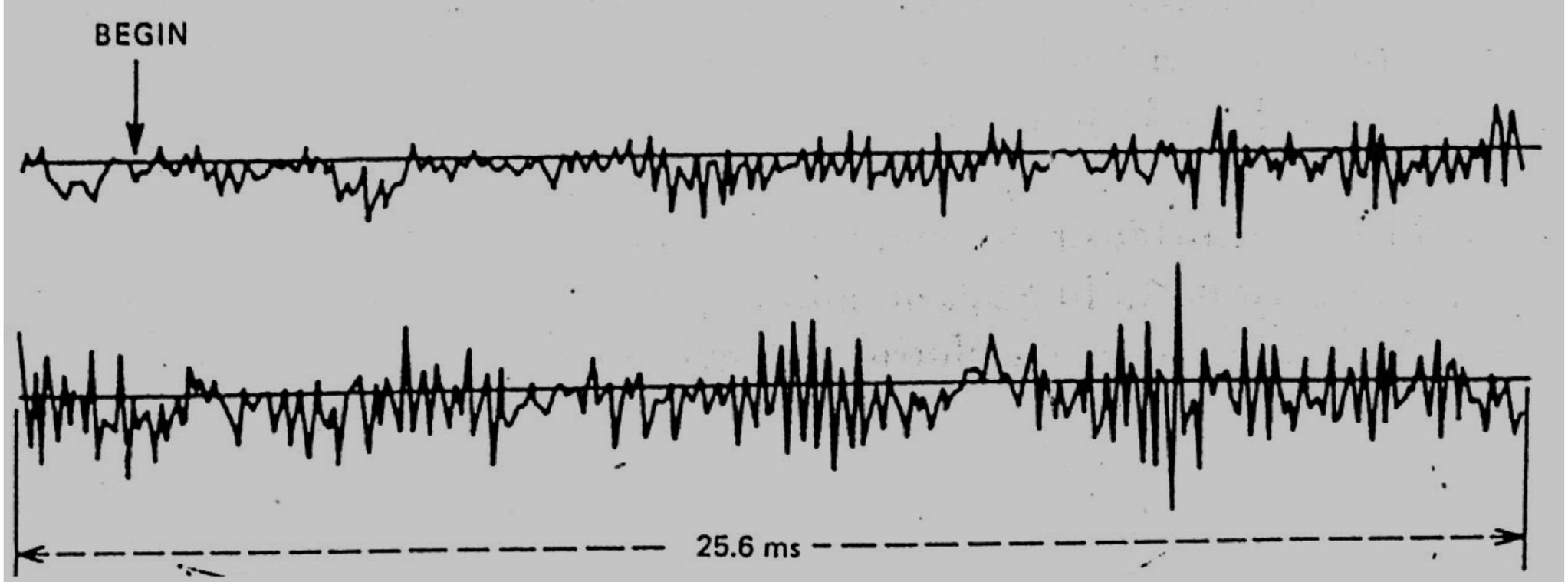
Visual recognition



"Eight"

- Easy
- Note how quiet beginning is (tape)

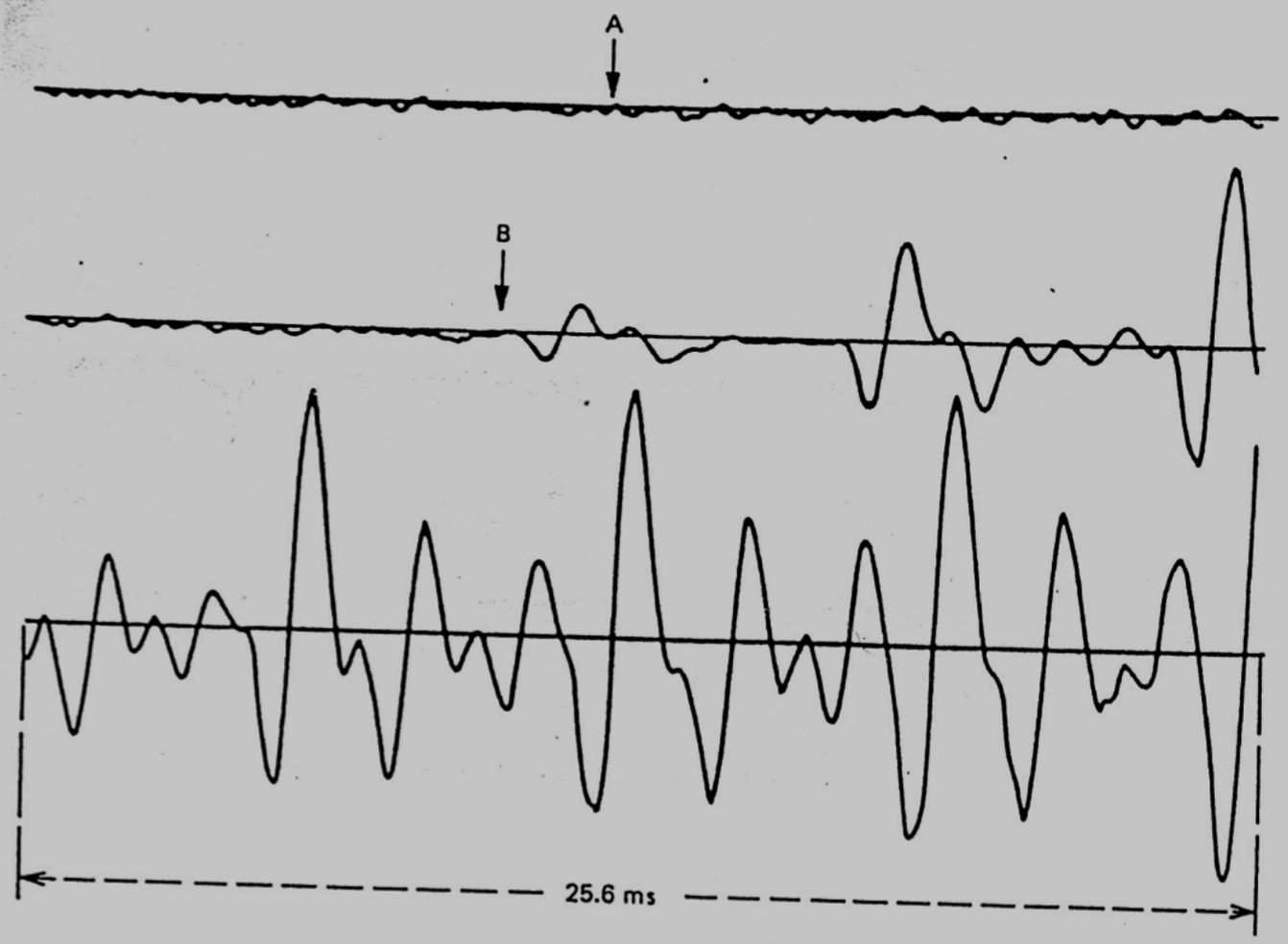
Tougher Visual recognition



"Six"

- "sss" starts crossing the 'zero' line, so can still detect

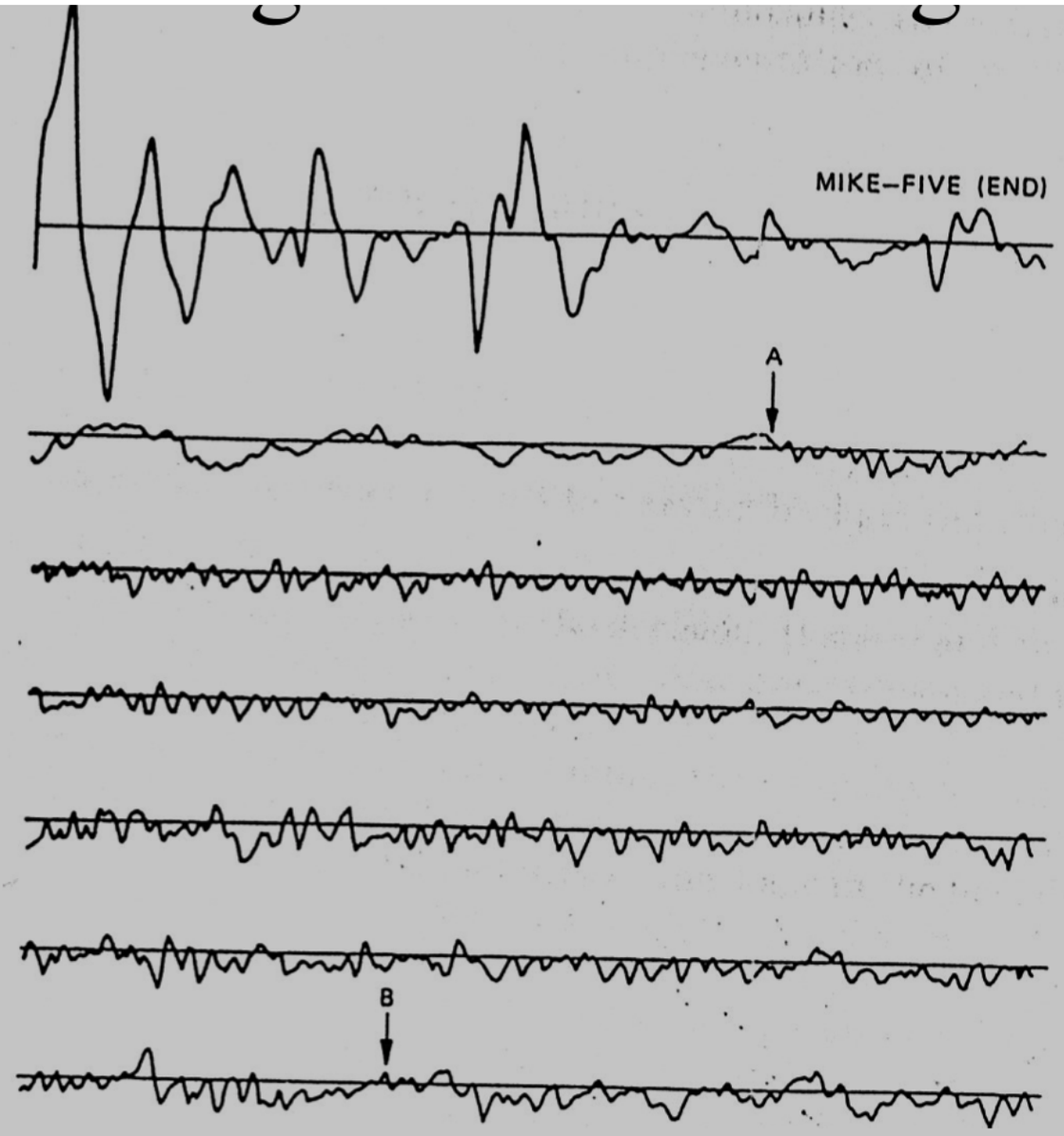
Tougher Visual recognition



"Four"

- Eye picks 'B', but 'A' is real start
  - /f/ is a *weak fricative*

Tougher Visual recognition



"Five"

- Eye picks 'A', but 'B' is real endpoint
  - V becomes *devoiced*

Problem and solution

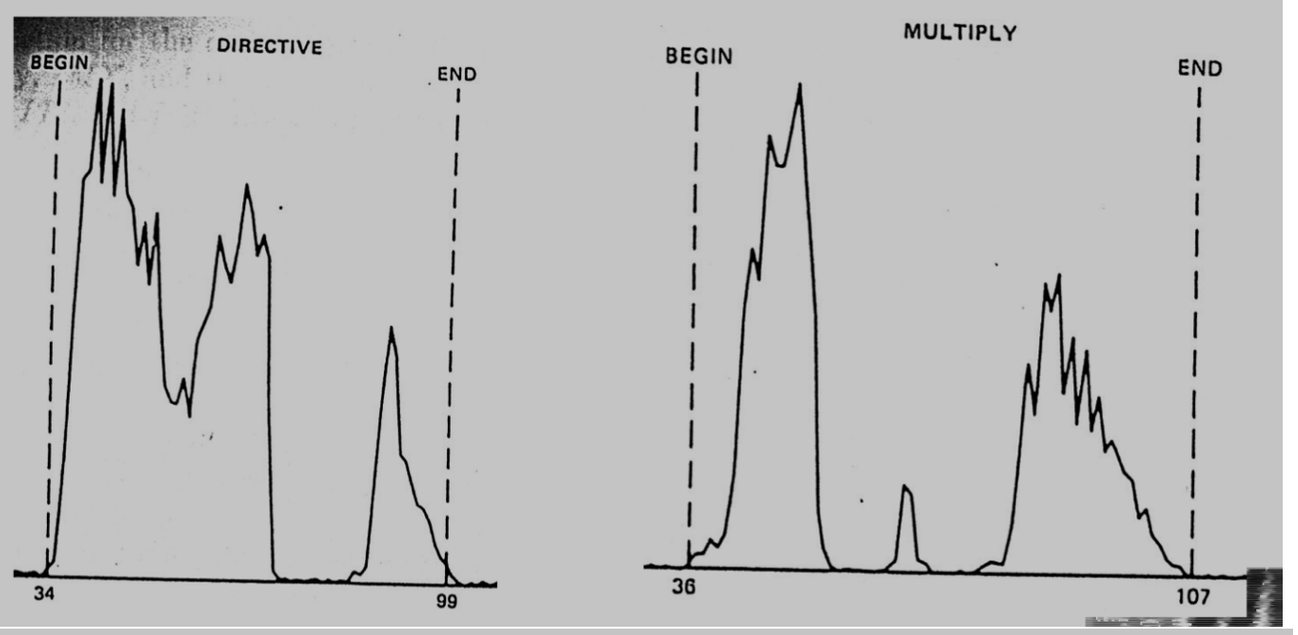
- Noisy computer room with background noise
  - Weak fricatives: /f, th, h/
  - Weak plosive bursts: /p, t, k/
  - Final nasals
  - Voiced fricatives becoming devoiced
  - Trailing off of sounds (ex: binary, three)
- Simple, efficient processing
  - Avoid hardware costs

- Two measurements:
  - Energy
  - Zero crossing rate
- Simple, fast, accurate

Silence Discrimination using ZCR and energy

Energy

- Sum of magnitudes of 10 ms of sound, centered on interval:
  - $E(n) = \sum_{i=-50 \text{ to } 50} |s(n + i)|$



Zero (Level) Crossing Rate

- Number of zero crossings per 10 ms
  - Normal number of cross-overs during silence
  - Increase in cross-overs during speech

Algorithm

- At initialization, record sound for 100ms
  - Assume 'silence'
  - Measure background noise
- Compute average (IZC') and std dev (σ) of zero crossing rate
- Choose Zero-crossing threshold (IZCT)
  - Threshold for unvoiced speech
  - IZCT = min(25 / 10ms, IZC' * 2 σ)
- Compute energy, $E(n)$, for interval
  - Get max, IMX
  - Have silence, IMN

    $I1$ = 0.03 * (IMX – IMN) + IMN

    (3% of peak energy)

    $I2$ = 4 * IMN

    (4x silent energy)

- Get energy thresholds (ITU and ITL)
  - ITL = MIN($I1$, $I2$)
  - ITU = 5 * ITL

Silence Discrimination using ZCR and energy

The Algorithm: Energy Computation

- Search sample for energy greater than ITL
  - Save as start of speech, say N_1
- Search for energy greater than ITU
  - N_1 becomes start of speech
  - If energy falls below ITL, restart
- Search for energy less than ITL
  - Save as end of speech N_2
- Results in conservative estimates
  - Endpoints may be outside

Silence Discrimination using ZCR and energy

The Algorithm: Zero Crossing Computation

- Search back 250 ms
  - Count number of intervals where rate exceeds IZCT
    - If 3+, set starting point, N_1 to first time
    - Else N_1 remains the same
- Do similar search after end

Silence Discrimination using ZCR and energy

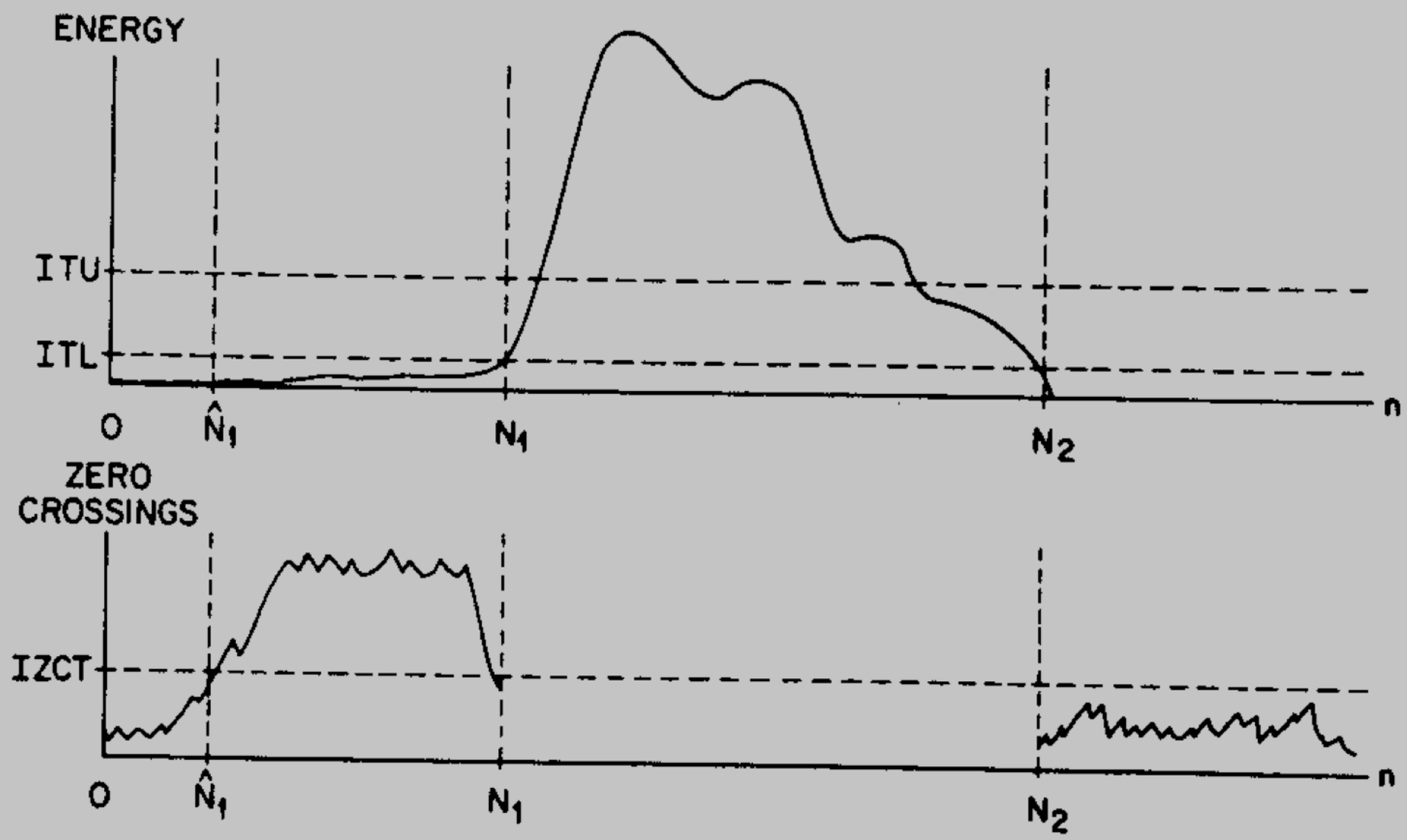
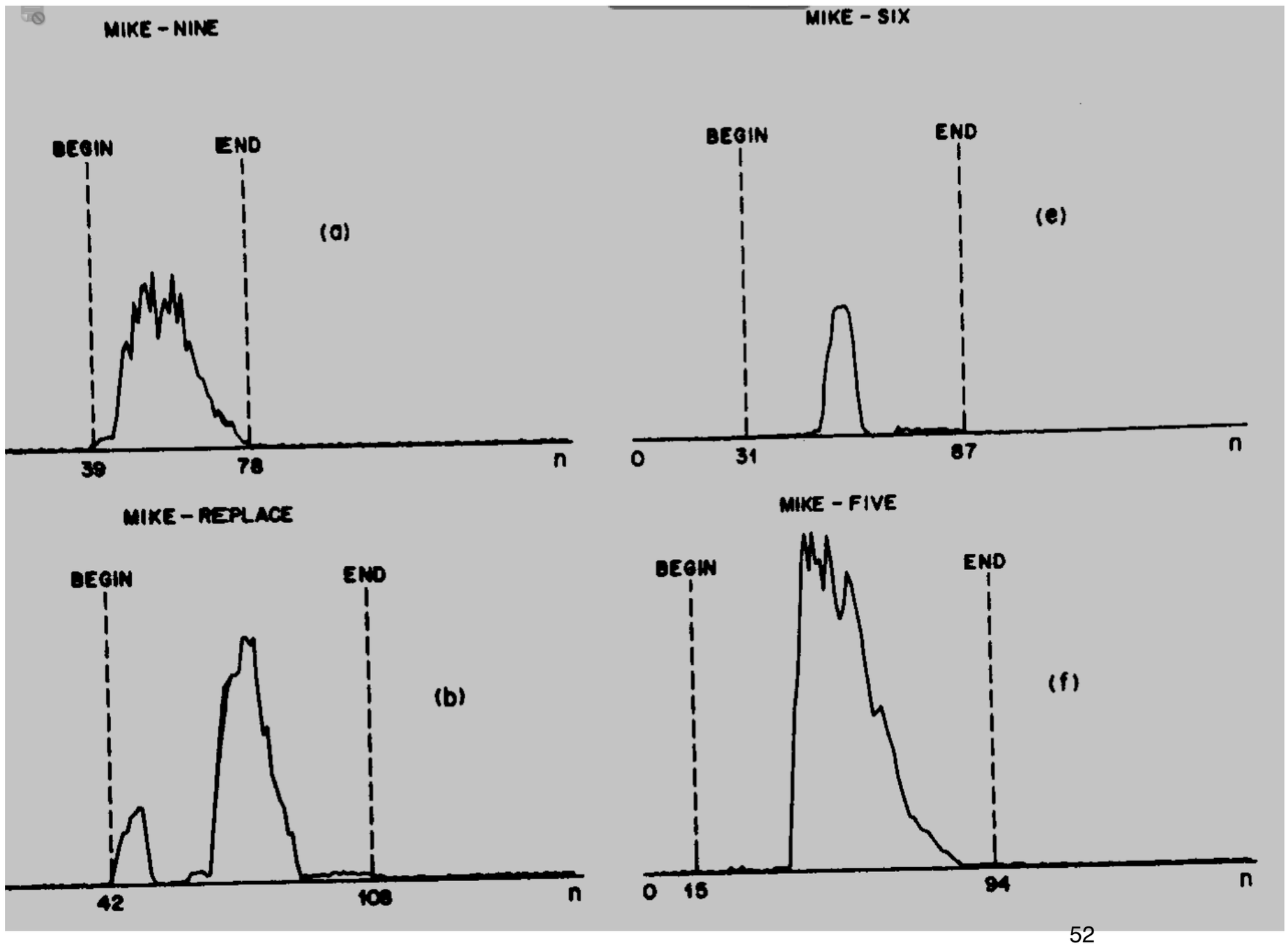


Fig. 4.16 Typical example of average magnitude and zero-crossing measurements for a word with a strong fricative at the beginning. (After Rabiner and Sambur [6].)

Silence Discrimination using ZCR and energy



Silence Discrimination using ZCR and energy

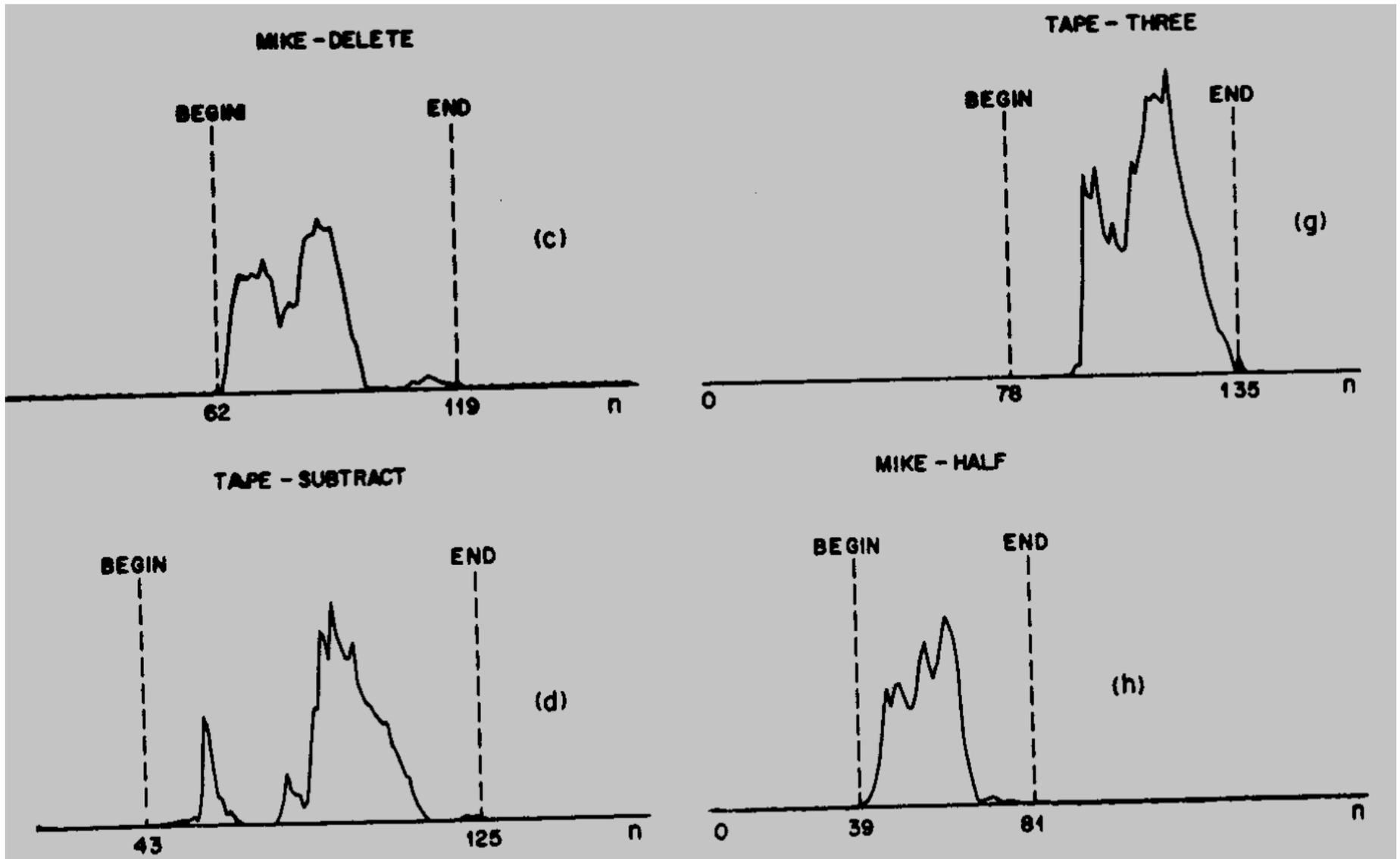


Fig. 4.17 Sequence of average magnitude plots showing how the end-point algorithm performed over a variety of words. (After Rabiner and Sambur [6].)

Short-Time Autocorrelation

-for a deterministic signal, the autocorrelation function is defined as:

$$\Phi[k] = \sum_{m=-\infty}^{\infty} x[m]\,x[m+k]$$

-for a random or periodic signal, the autocorrelation function is:

$$\Phi[k] = \lim_{L\to\infty} \frac{1}{(2L+1)} \sum_{m=-L}^{L} x[m]x[m+k]$$

- if $x[n] = x[n+P]$, then $\Phi[k] = \Phi[k+P]$, => the autocorrelation function preserves periodicity

-properties of $\Phi[k]$:

1. $\Phi[k]$ is even, $\Phi[k] = \Phi[-k]$
2. $\Phi[k]$ is maximum at $k = 0$, $|\Phi[k]| \leq \Phi[0]$, $\forall k$
3. $\Phi[0]$ is the signal energy or power (for random signals)

Periodic Signals

- for a periodic signal we have (at least in theory) $\Phi[P]=\Phi[0]$ so the period of a periodic signal can be estimated as the first non-zero maximum of $\Phi[k]$
  - this means that the autocorrelation function is a good candidate for speech pitch detection algorithms
  - it also means that we need a good way of measuring the short-time autocorrelation function for speech signals

Short-Time Autocorrelation

- a reasonable definition for the short-time autocorrelation is:

$$R_{\hat{n}}[k] = \sum_{m=-\infty}^{\infty} x[m]\,\tilde{w}[\hat{n}-m]\,x[m+k]\,\tilde{w}[\hat{n}-k-m]$$

1. select a segment of speech by windowing
2. compute deterministic autocorrelation of the windowed speech

$R_{\hat{n}}[k] = R_{\hat{n}}[-k]$ - symmetry

$$= \sum_{m=-\infty}^{\infty} x[m]\,x[m-k]\left[\tilde{w}[\hat{n}-m]\tilde{w}[\hat{n}+k-m]\right]$$

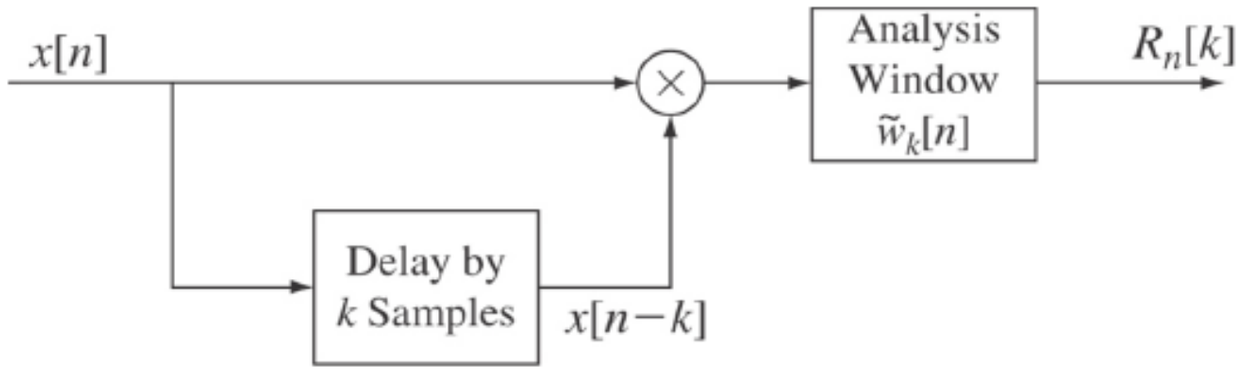
- define filter of the form

$$\tilde{w}_k[\hat{n}] = \tilde{w}[\hat{n}]\,\tilde{w}[\hat{n}+k]$$

- this enables us to write the short-time autocorrelation in the form:

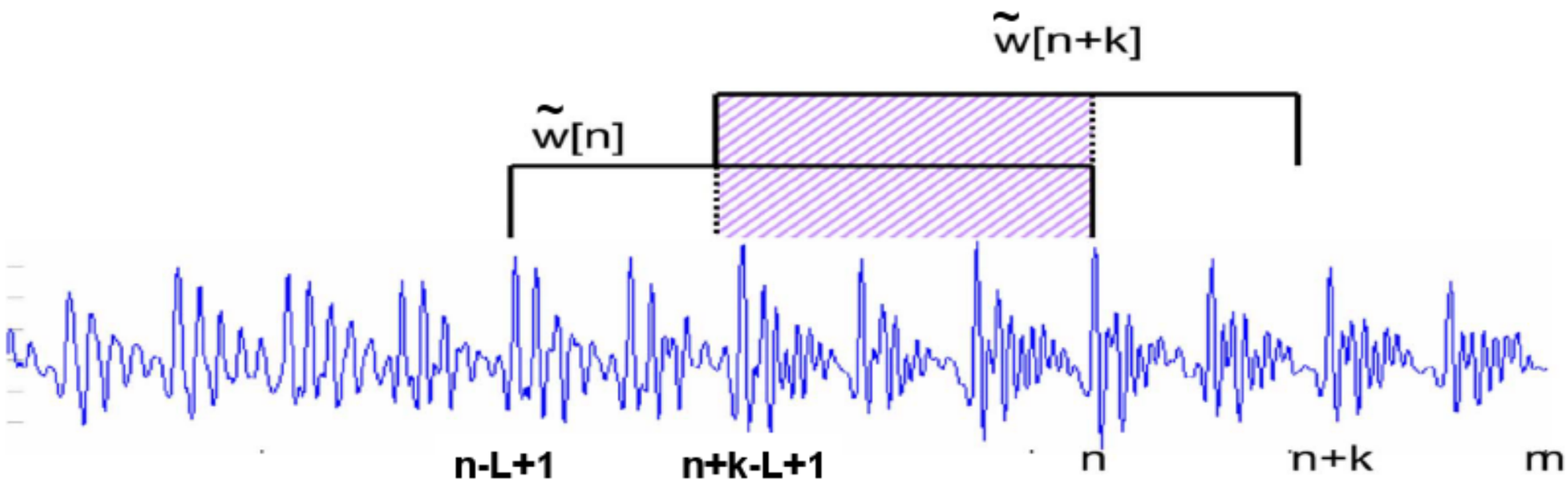
$$R_{\hat{n}}[k] = \sum_{m=-\infty}^{\infty} x[m]\,x[m-k]\,\tilde{w}_k[\hat{n}-m]$$

- the value of $\tilde{w}_{\hat{n}}[k]$ at time $\hat{n}$ for the kth lag is obtained by filtering the sequence $x[\hat{n}]\,x[\hat{n}-k]$ with a filter with impulse response $\tilde{w}_k[\hat{n}]$



Short-Time Autocorrelation

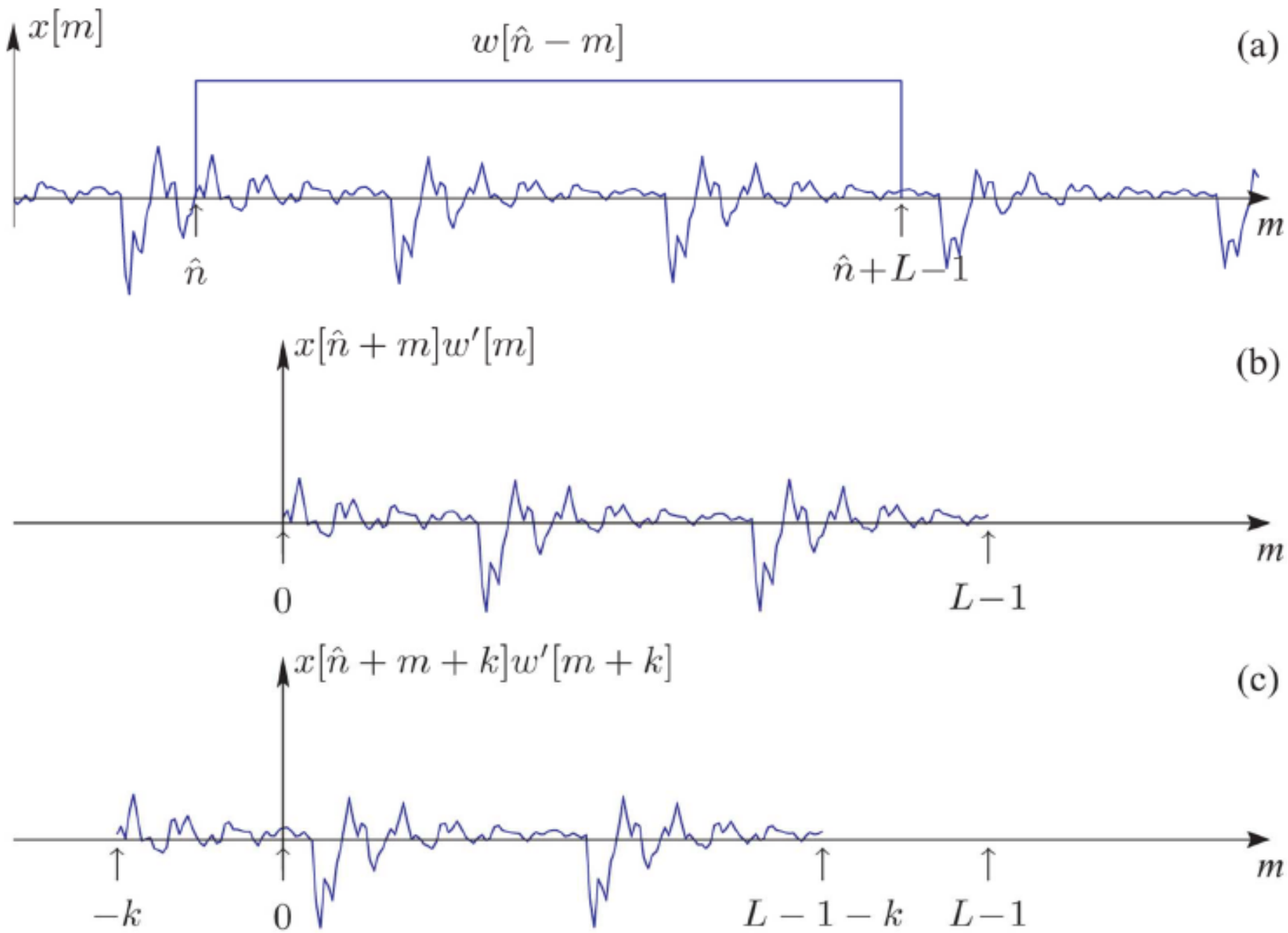
$$R_{\hat{n}}[k] = \sum_{m=-\infty}^{\infty} \left[x[m]\tilde{w}[\hat{n}-m] \right] \left[x[m+k]\tilde{w}[\hat{n}+k-m] \right]$$



$\Rightarrow L$ points used to compute $R_{\hat{n}}[0]$;

$\Rightarrow L-k$ points used to compute $R_{\hat{n}}[k]$;

Short-Time Autocorrelation



Examples of Autocorrelations

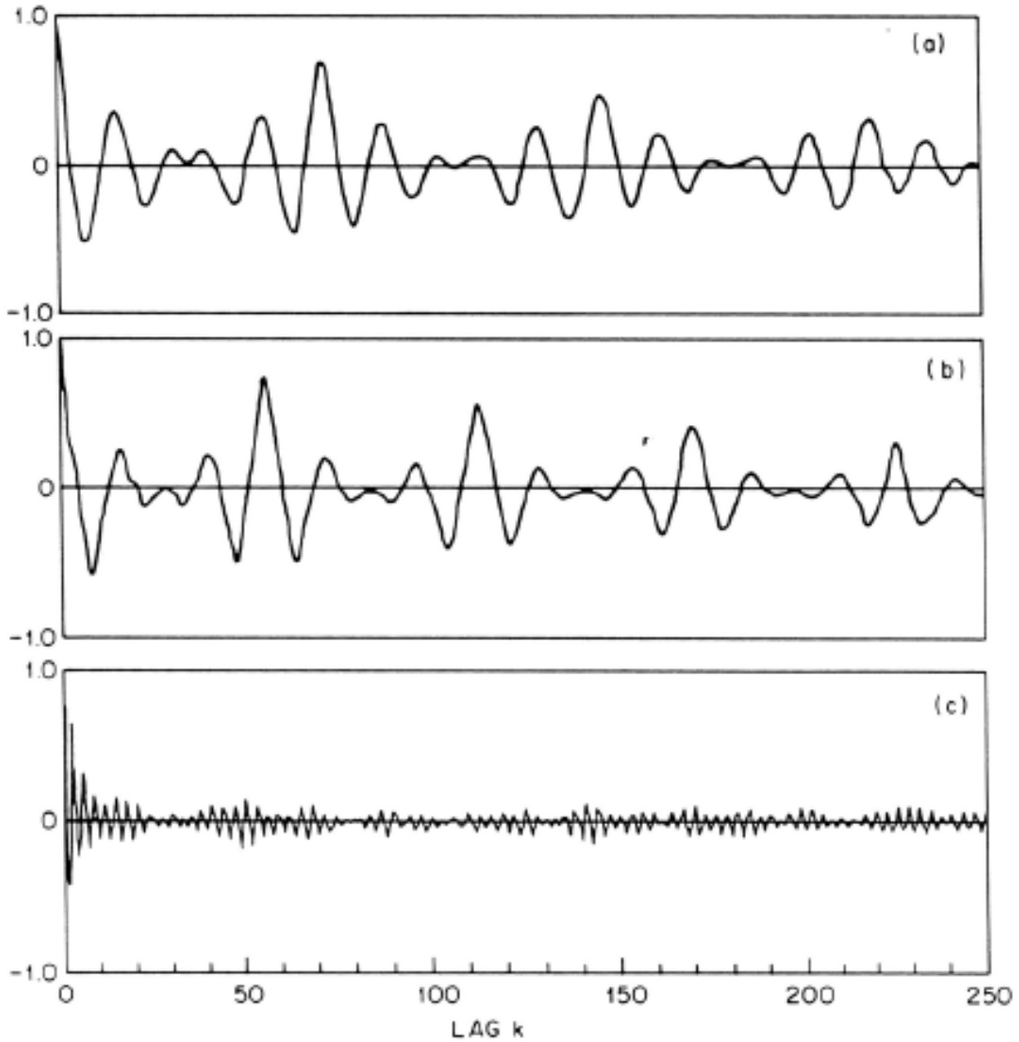


Fig. 4.24 Autocorrelation function for (a) and (b) voiced speech; and (c) unvoiced speech, using a rectangular window with $N = 401$.

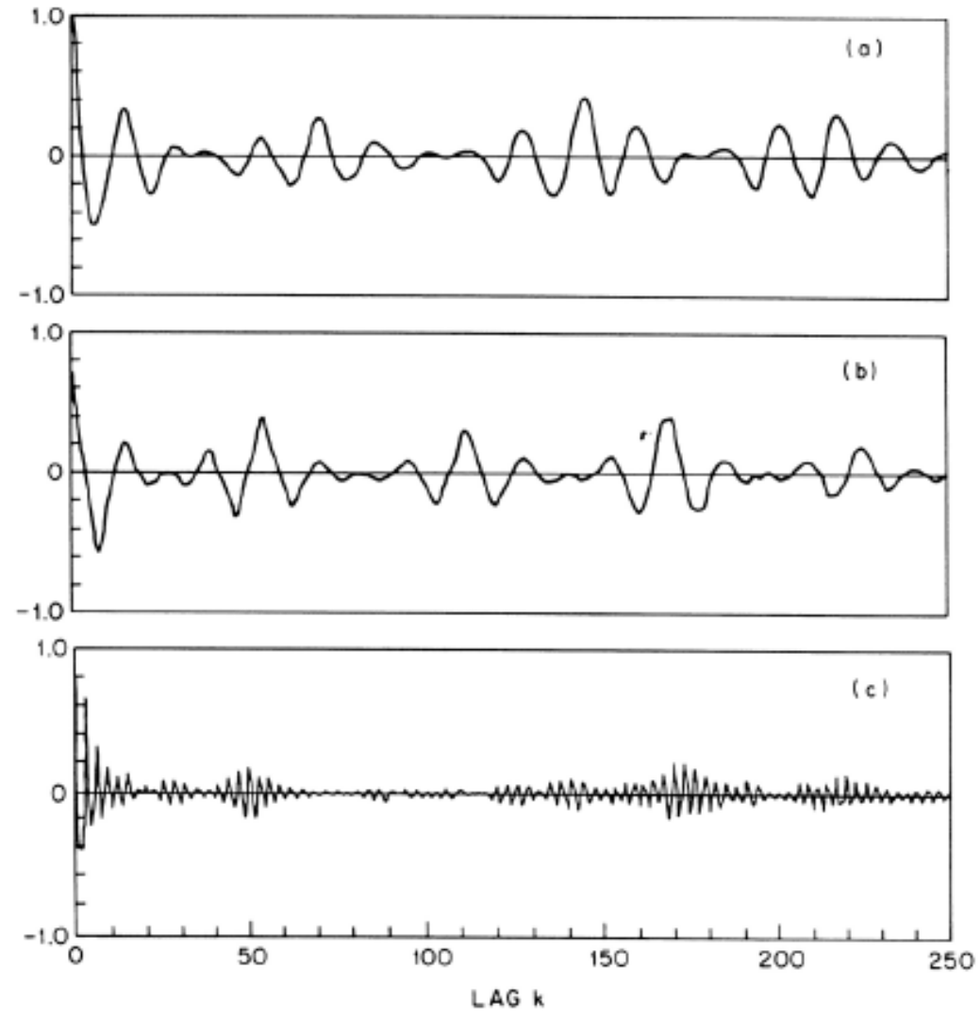
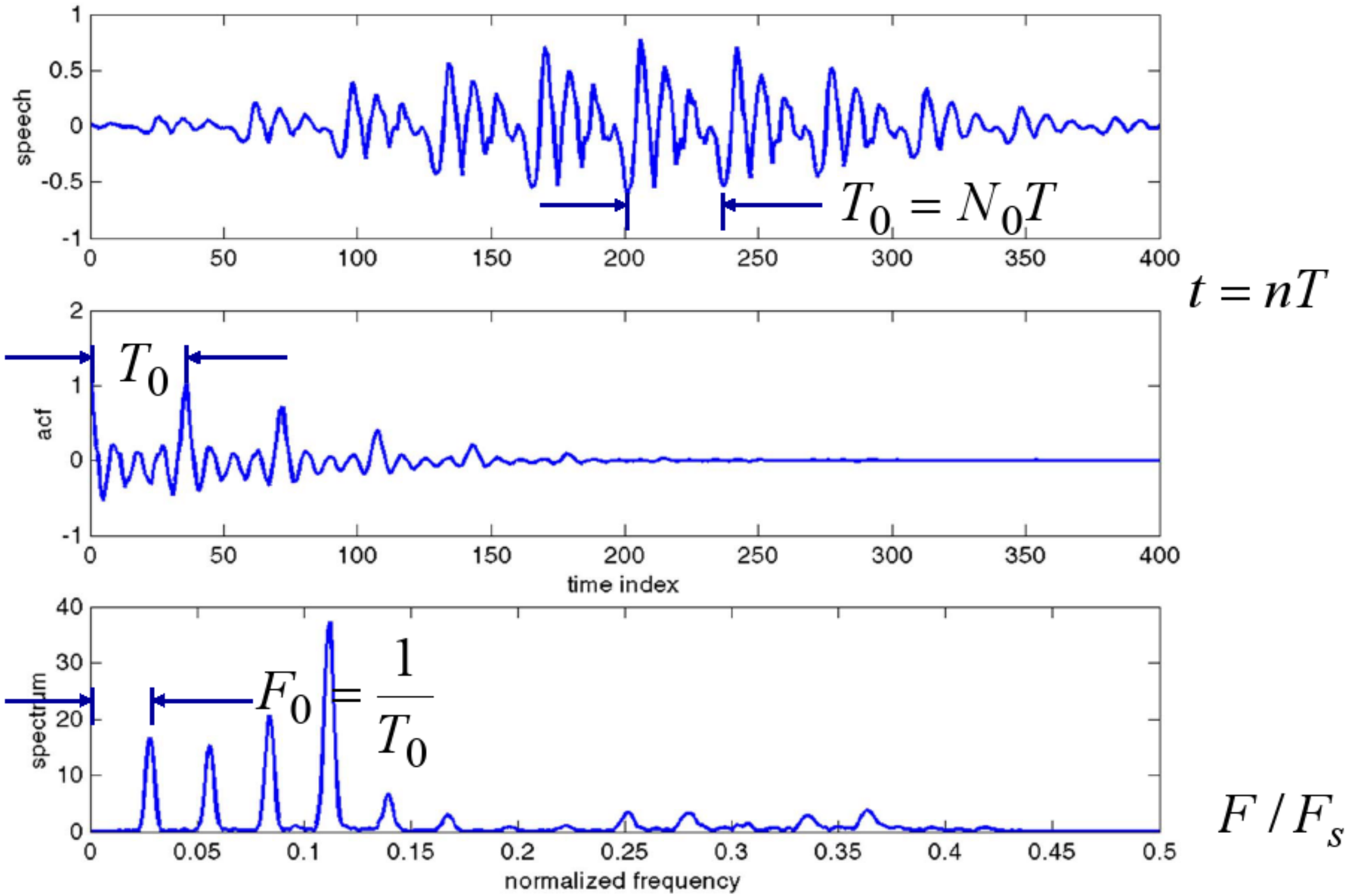


Fig. 4.25 Autocorrelation functions for (a) and (b) voiced speech; and (c) unvoiced speech, using a Hamming window with $N = 401$.

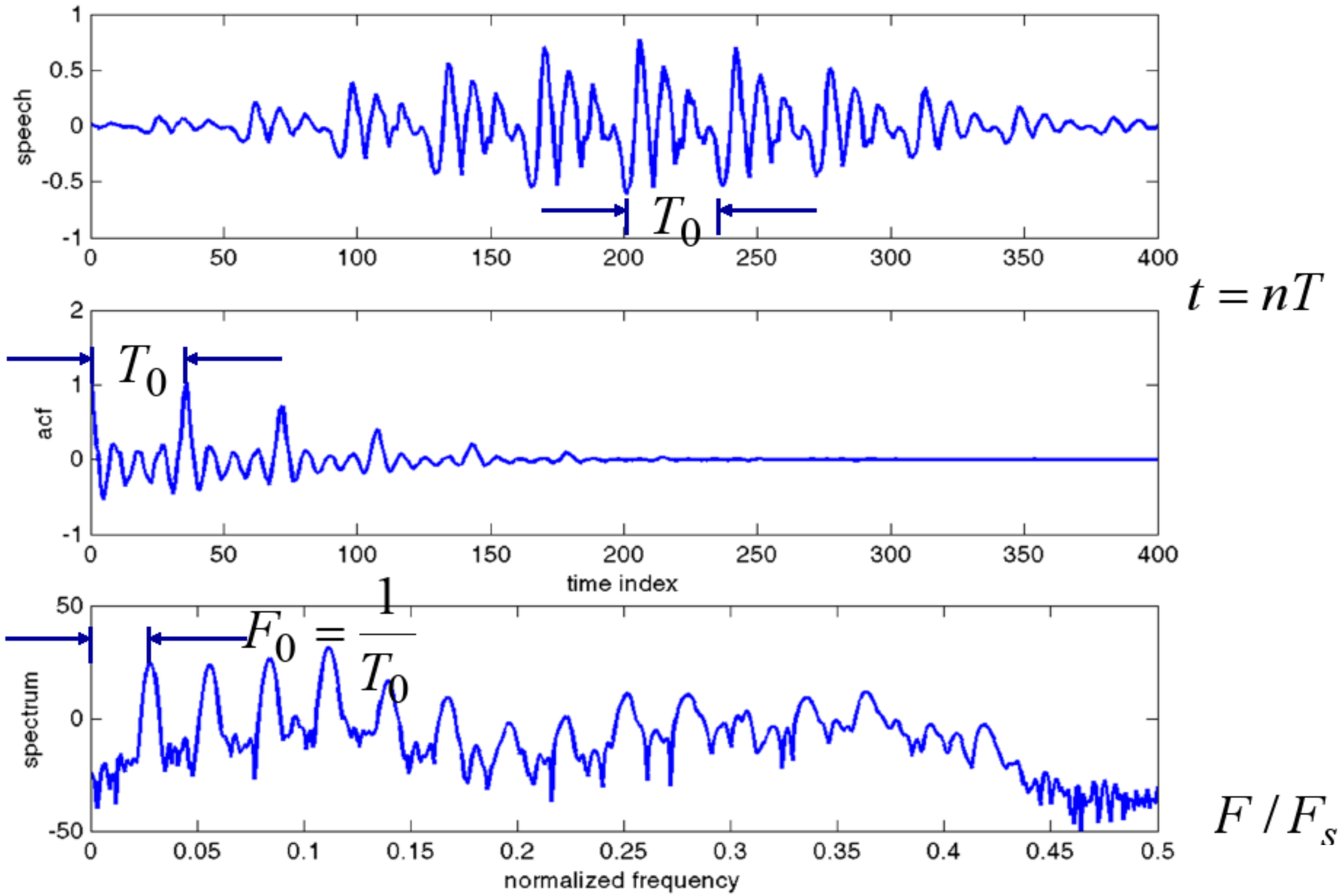
- autocorrelation peaks occur at k=72, 144, ... => 140 Hz pitch
- $\Phi(P)<\Phi(0)$ since windowed speech is not perfectly periodic
- over a 401 sample window (40 msec of signal), pitch period changes occur, so P is not perfectly defined

- **much less clear estimates of periodicity since HW tapers signal so strongly, making it look like a non-periodic signal**
- **no strong peak for unvoiced speech**

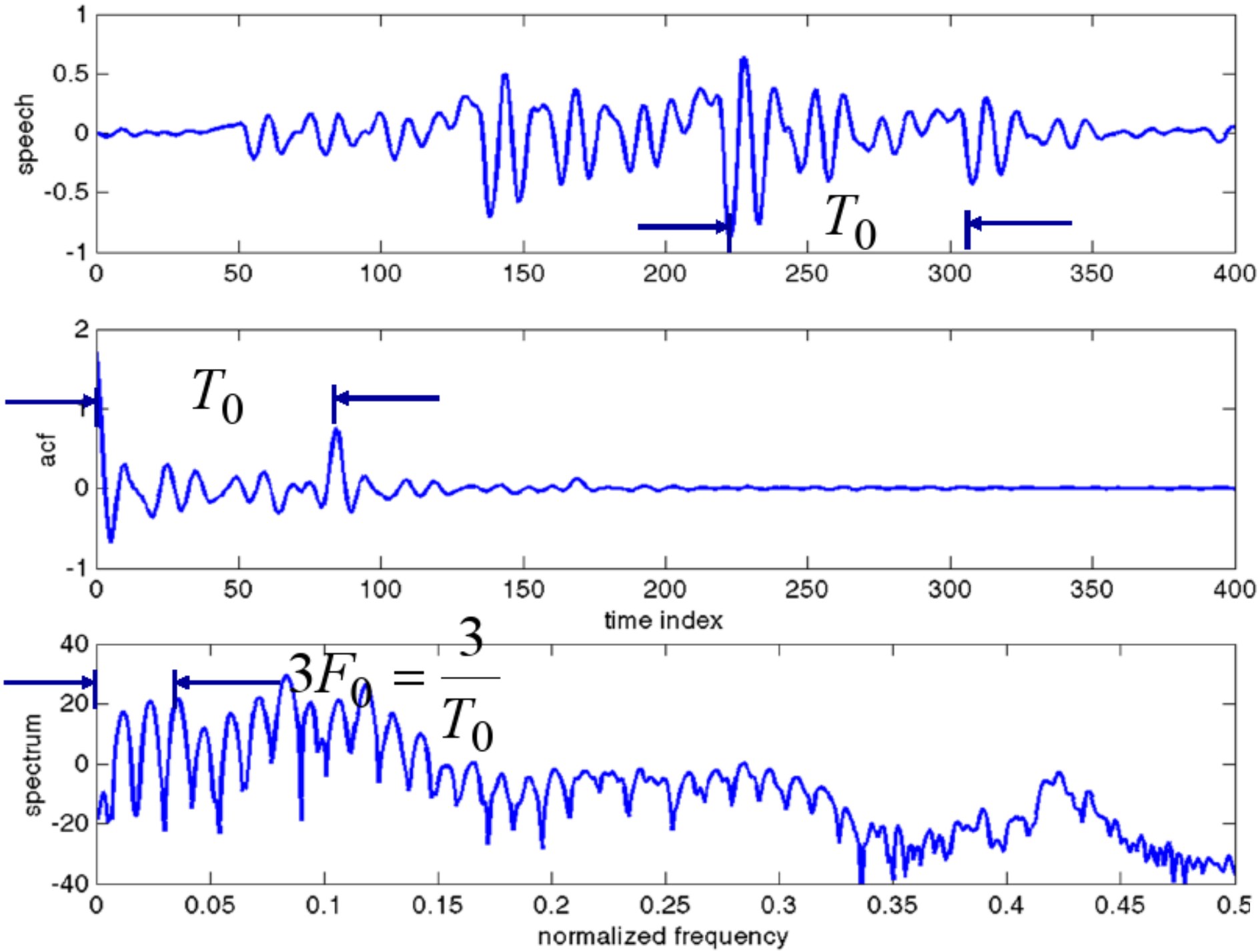
Voiced (female) *L=401* (magnitude)



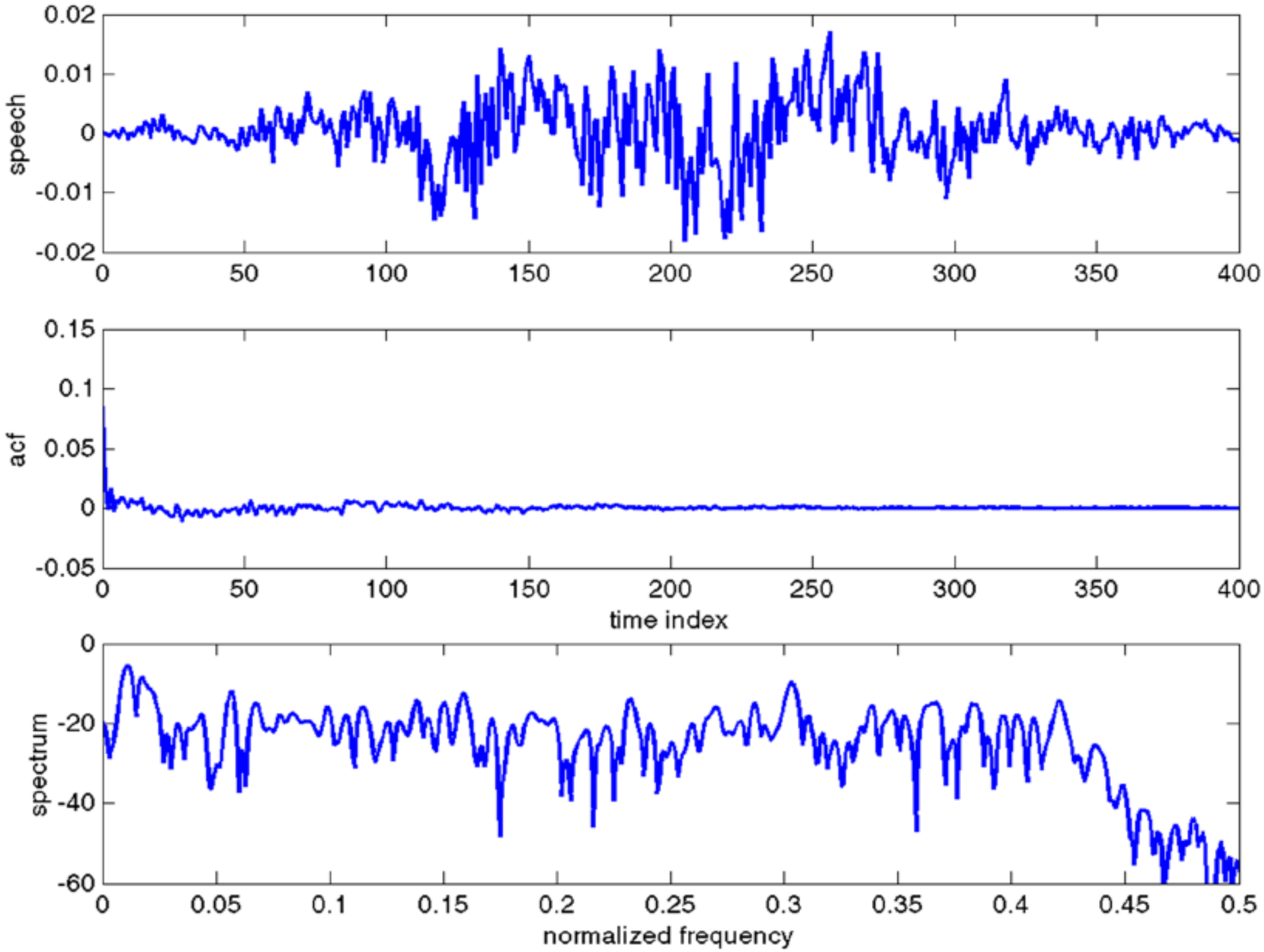
Voiced (female) *L=401* (log mag)



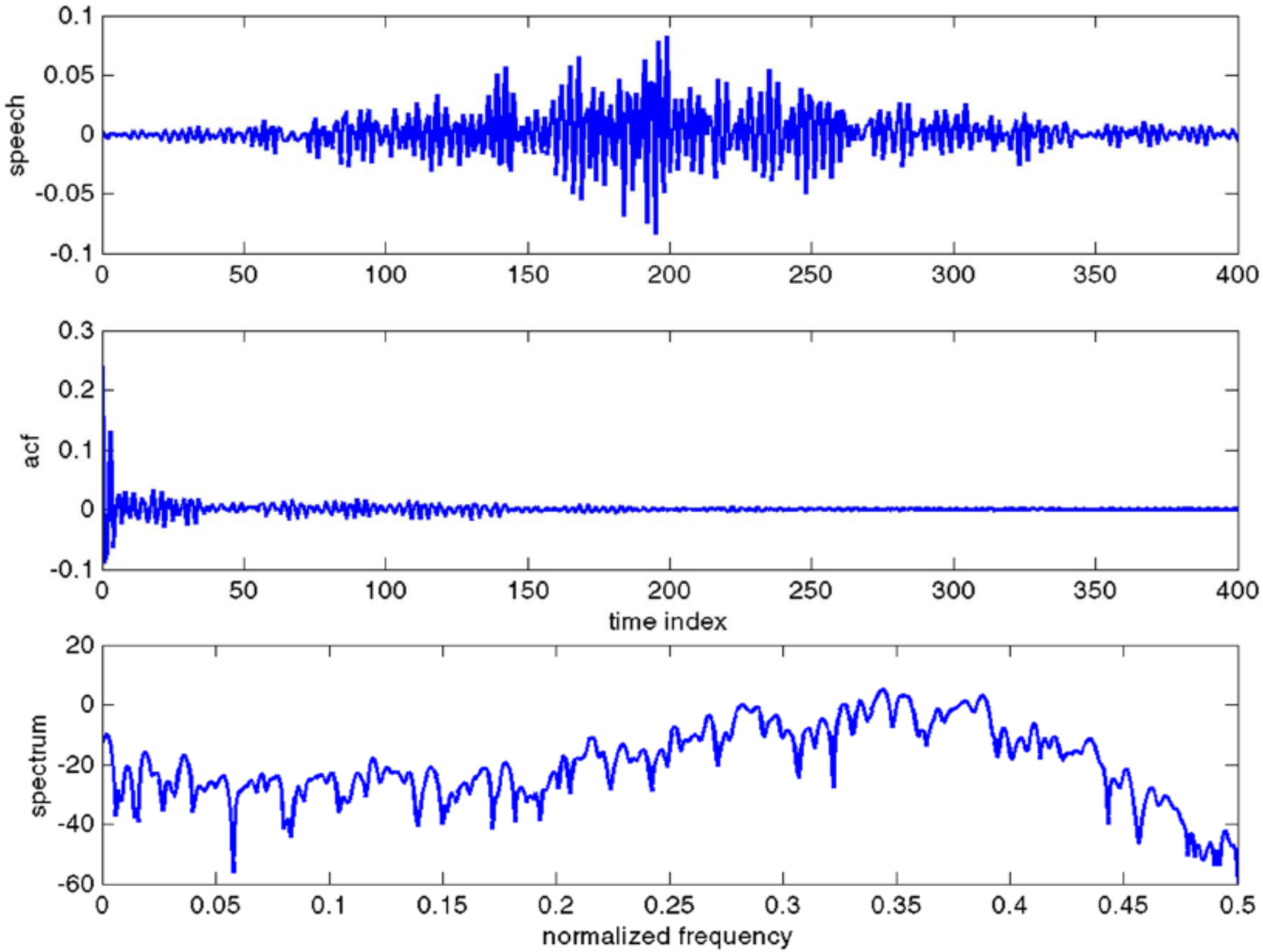
Voiced (male) *L=401*



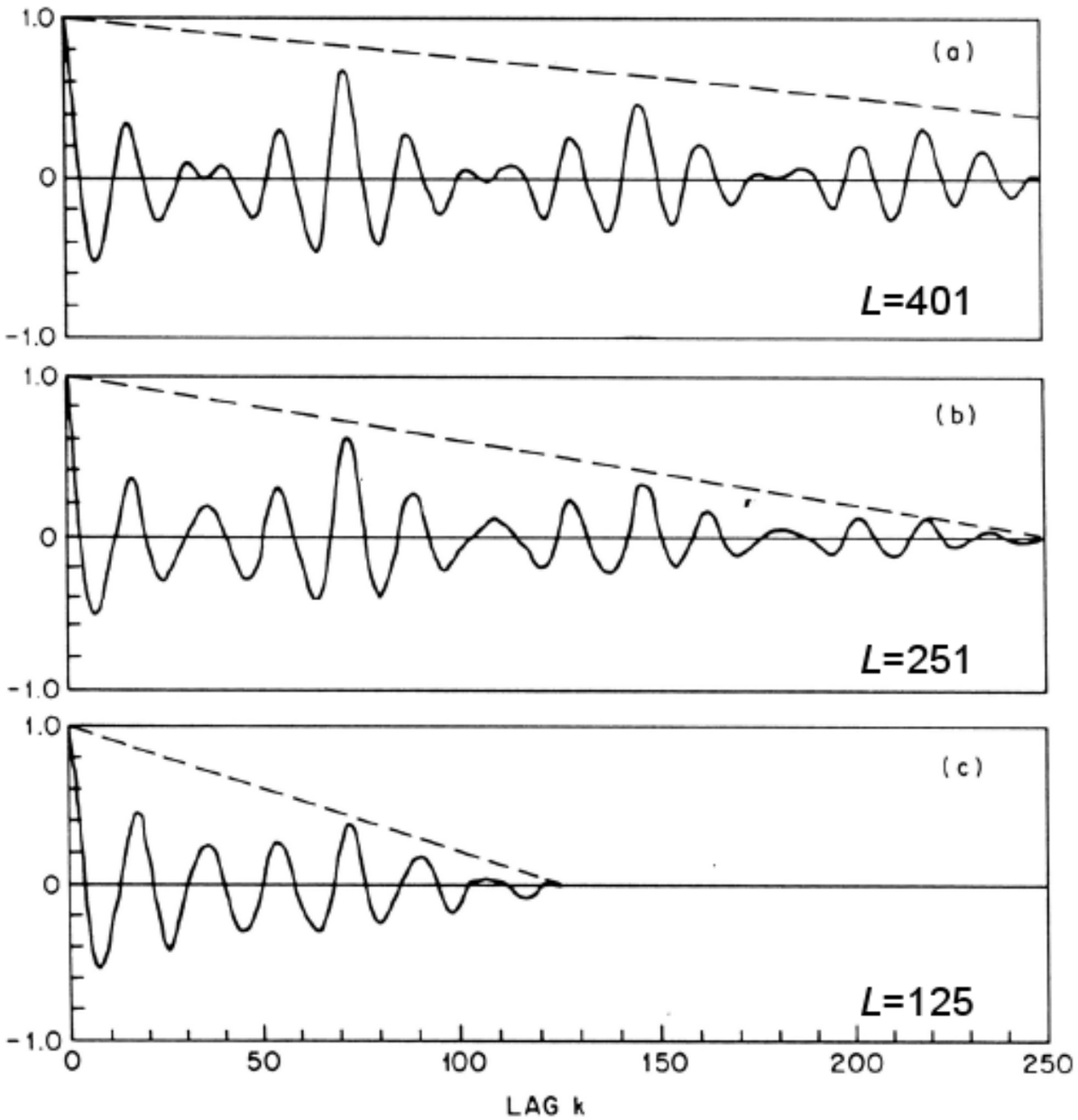
Unvoiced *L=401*



Unvoiced *L=401*



Effects of Window Size



- choice of L, window duration
  - small L so pitch period almost constant in window
  - large L so clear periodicity seen in window
  - as k increases, the number of window points decrease, reducing the accuracy and size of $R_n(k)$ for large k => have a taper of the type $R(k)=1-k/L$, $|k|<L$ shaping of autocorrelation (this is the autocorrelation of size L rectangular window)
- allow L to vary with detected pitch periods (so that at least 2 full periods are included)

Modified Autocorrelation

- want to solve problem of differing number of samples for each different k term in $R_{\hat{n}}[k]$, so modify definition as follows:

$$\hat{R}_{\hat{n}}[k] = \sum_{m=-\infty}^{\infty} x[m]\tilde{w}_1[\hat{n}-m]\,x[m+k]\tilde{w}_2[\hat{n}-m-k]$$

- where $\tilde{w}_1$ is standard L-point window, and $\tilde{w}_2$ is extended window of duration $L+K$ samples, where K is the largest lag of interest
- we can rewrite modified autocorrelation as:

$$\hat{R}_{\hat{n}}[k] = \sum_{m=-\infty}^{\infty} x[\hat{n}+m]\hat{w}_1[m]\,x[\hat{n}+m+k]\hat{w}_2[m+k]$$

- where

$$\hat{w}_1[m] = \tilde{w}_1[-m] \text{ and } \hat{w}_2[m] = \tilde{w}_2[-m]$$

- for rectangular windows we choose the following:

$$\hat{w}_1[m] = 1, \quad 0 \le m \le L-1$$

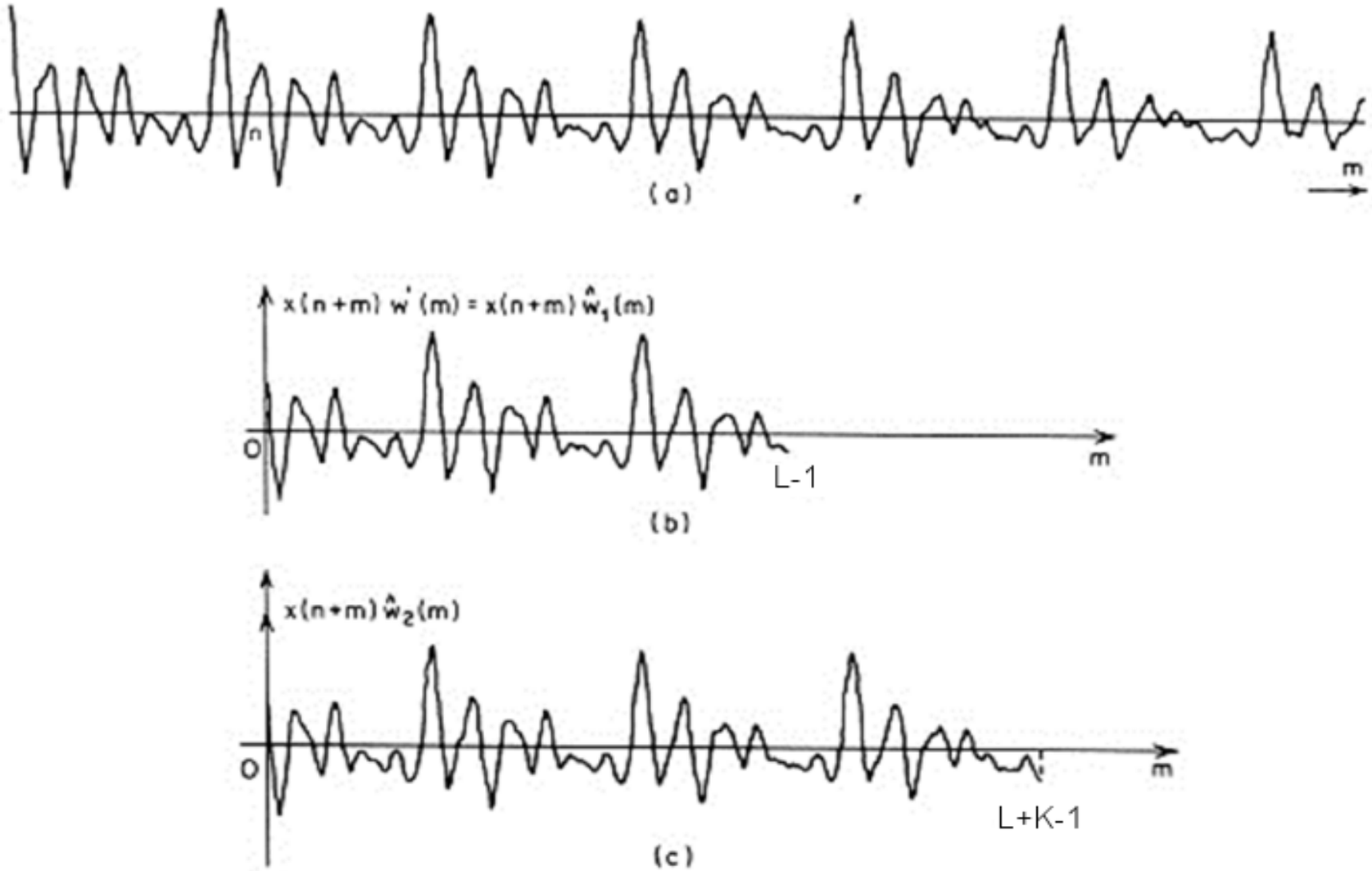
$$\hat{w}_2[m] = 1, \quad 0 \le m \le L-1+K$$

-giving

$$\hat{R}_{\hat{n}}[k] = \sum_{m=0}^{L-1} x[\hat{n}+m]\,x[\hat{n}+m+k], \; 0 \le k \le K$$

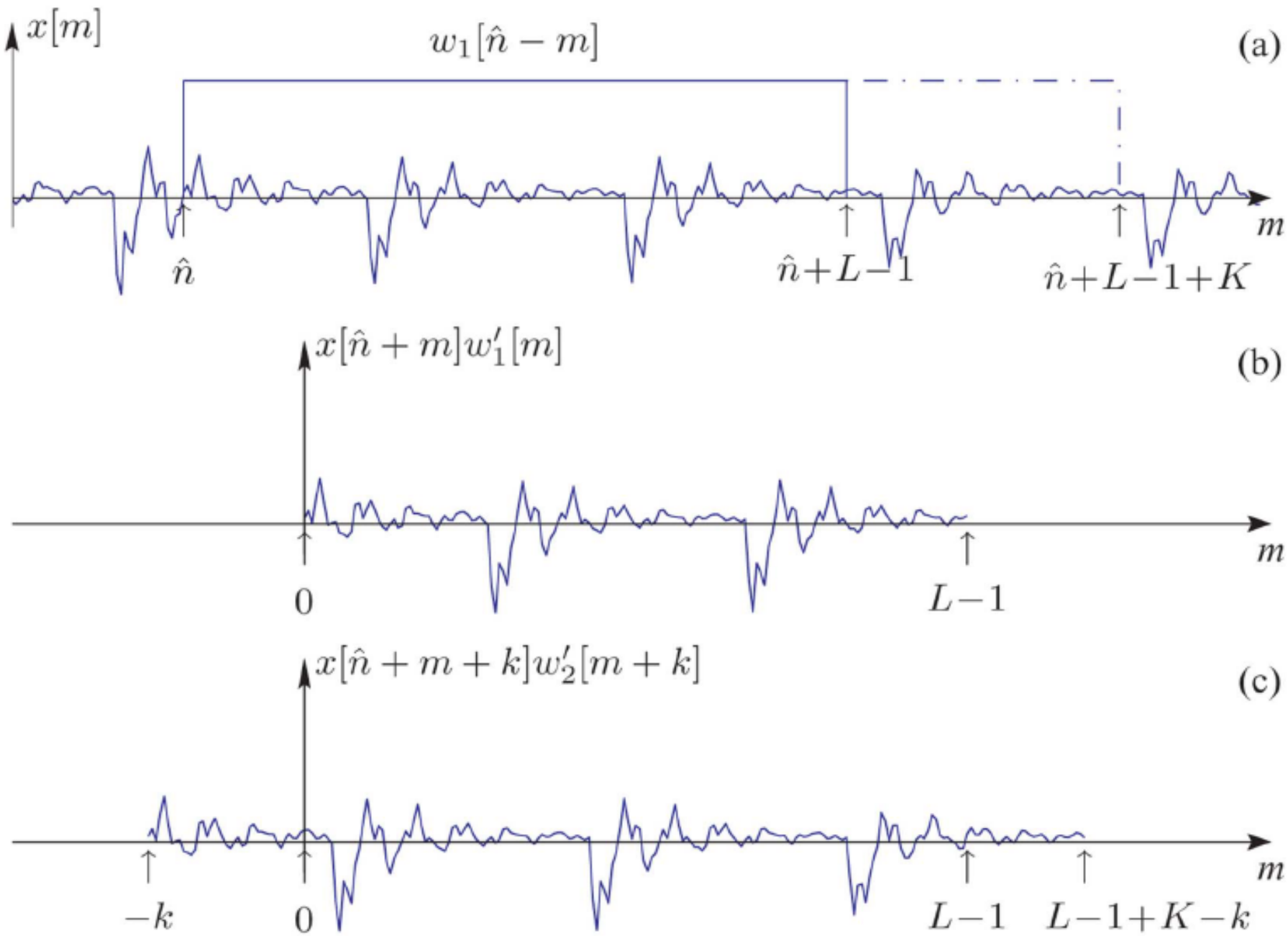
- always use L samples in computation of $\hat{R}_{\hat{n}}[k]\ \forall k$

Examples of Modified Autocorrelation



- $\hat{R}_n[k]$ is a <u>cross-correlation</u>, not an <u>auto-correlation</u>
- $\hat{R}_n[k] \neq \hat{R}_n[-k]$
- $\hat{R}_n[k]$ will have a strong peak at $k = P$ for periodic signals and will not fall off for large k

Examples of Modified Autocorrelation



Pitch period estimation using Autocorrelation function

$$y(n) = C[x(n)]$$

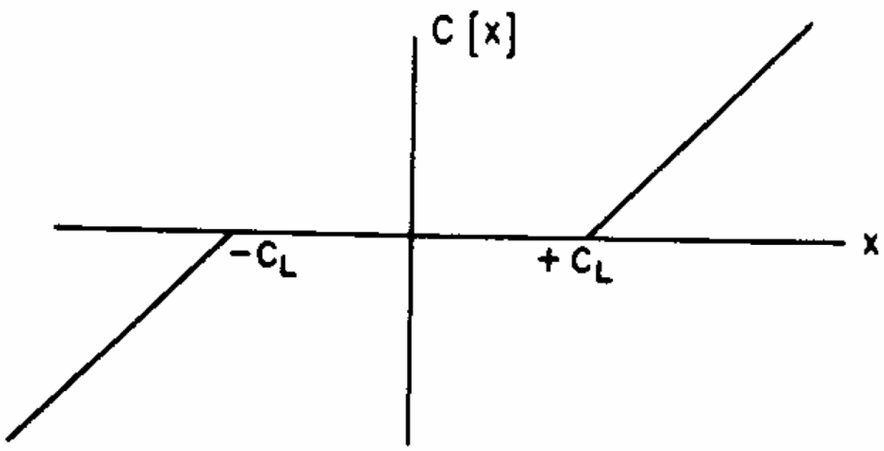


Fig. 4.31 Center clipping function.

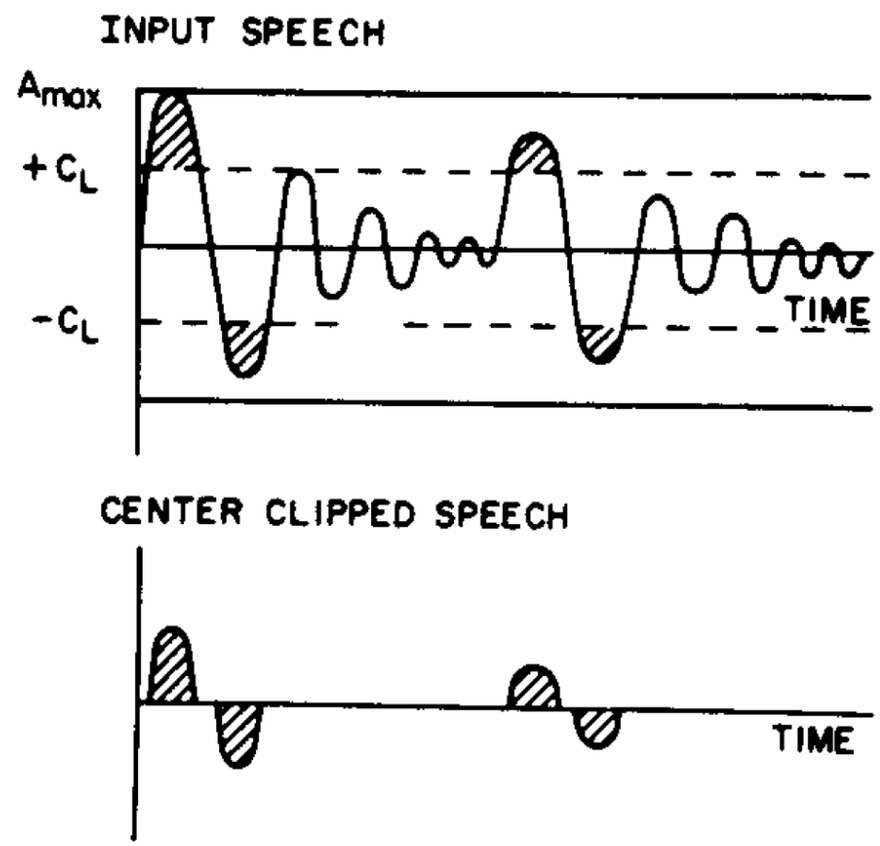


Fig. 4.32 An example showing how center clipping affects a speech waveform. (After Sondhi [17].)

Pitch period estimation using Autocorrelation function

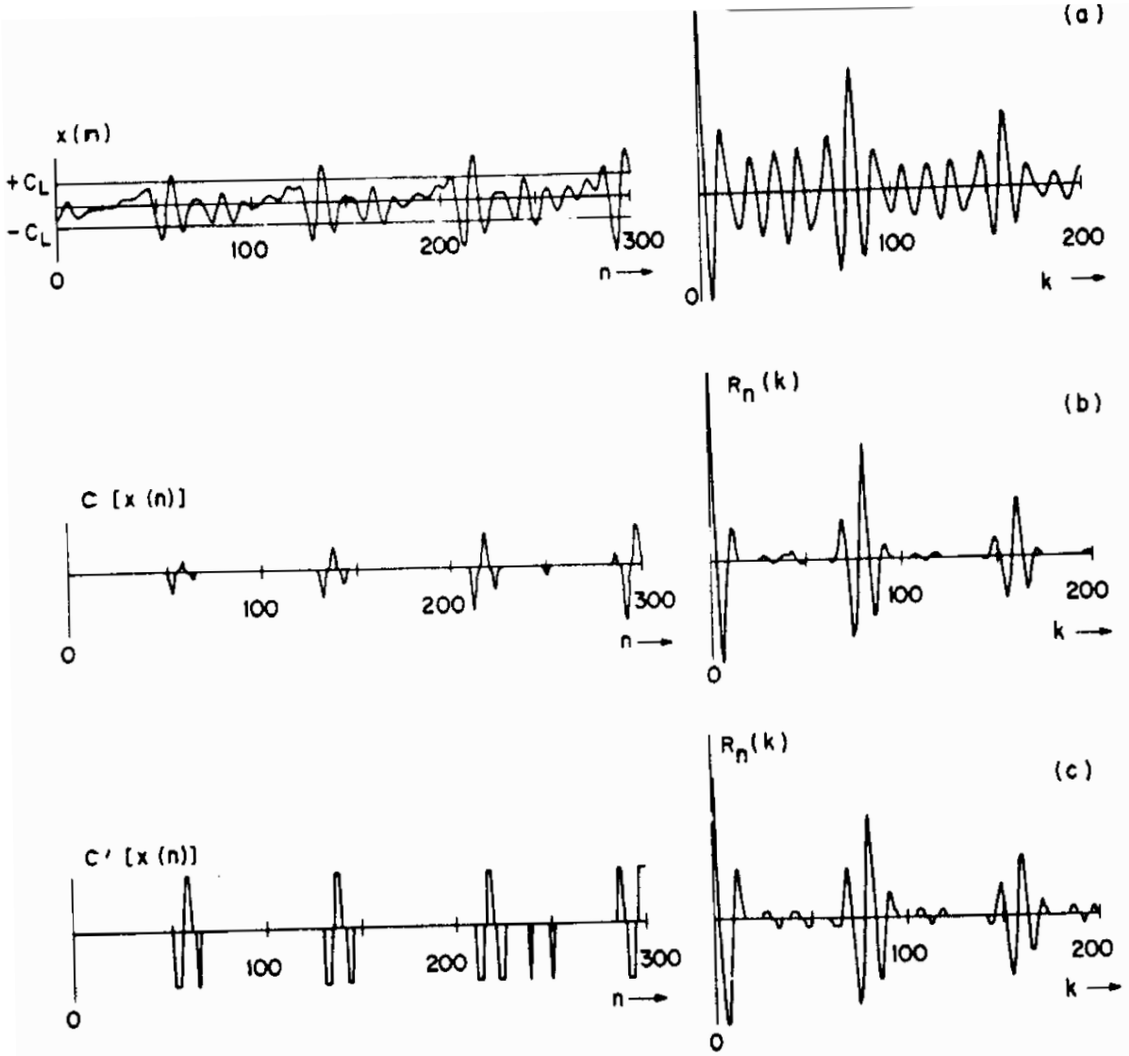


Fig. 4.33 Example of waveforms and correlation function; (a) no clipping; (b) center clipped; (c) 3-level center clipped. (All correlation functions normalized to 1.0.) (After Rabiner [18].)

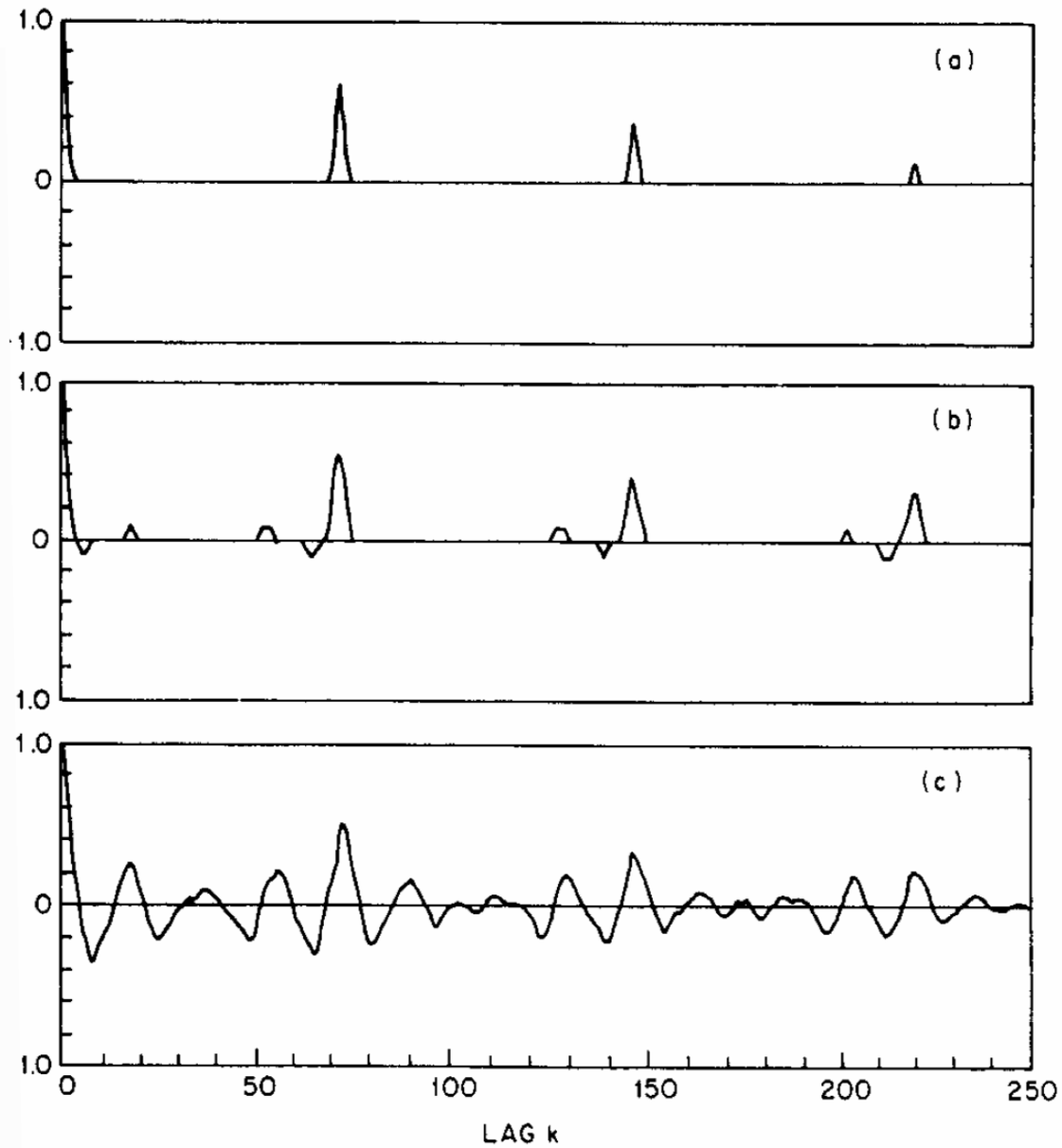


Fig. 4.34 Autocorrelation functions of center clipped speech using $N = 401$; (a) C_L set at 80% of maximum; (b) 64%; (c) 48%. (Speech segment same as for Fig. 4.26a.)

Pitch period estimation using Autocorrelation function

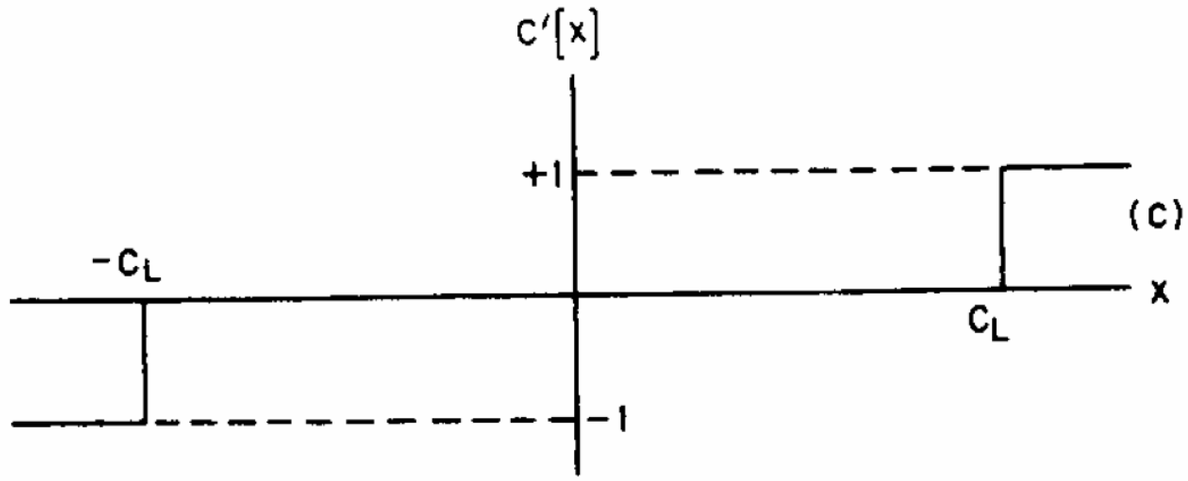


Fig. 4.35 3-level center clipping function.

The computation of the autocorrelation function for a 3-level center clipped signal is particularly simple. If we denote the output of the 3-level center clipper as $y(n)$ then the product terms $y(n+m)y(n+m+k)$ in the autocorrelation function

$$R_n(k) = \sum_{m=0}^{N-k-1} y(n+m)y(n+m+k) \tag{4.41}$$

can have only three different values

$$\begin{aligned} y(n+m)y(n+m+k) &= 0 && \text{if} \quad y(n+m) = 0 \ \ or \ \ y(n+m+k) = 0 \\ &= +1 && \text{if} \quad y(n+m) = y(n+m+k) \\ &= -1 && \text{if} \quad y(n+m) \neq y(n+m+k) \end{aligned} \tag{4.42}$$