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Assignment - 3

Section-A

1. What is the use of homogeneous coordinates and matrix representation?

→ To treat all 3 transformations in a consistent way, we use homogeneous coordinates and matrix representation

2. What two dimensional rotation equation in the matrix form?

→ $P' = R * P$

3. What is transformation?

→ Transformation is the process of changing the size, orientation, or position of an object in a scene. It involves using mathematical operations to manipulate the points or vertices in a 2D or 3D space.

4. Define scaling in transformation?

→ A scaling transformation alters the size of an object. In the scaling process we either compress or expand the dimension of the object. Scaling operation can be achieved by multiplying each vertex coordinate (x, y) of the polygon by scaling factor S_x and S_y to produce the transformed coordinates as (x', y').

5. Explain Reflection in transformation?

→ Reflection is a transformation which produces a mirror image of an object. The mirror image can be either about x-axis or y-axis. The object is rotated by 180°.

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Section - B

1. What do you mean by transformation?

→ Transformation refers to the process of altering the geometric properties of an object, such as its position, size orientation or shape. These transformations are essential for creating dynamic and interactive graphics, as they enable us to manipulate objects on the screen in various ways. By employing mathematical operations vertices or points in a 3D or 2D space are manipulated to achieve the desired changes.

Types of transformations:

i) Translation
ii) Scaling
iii) Rotating
iv) Reflection
v) Shearing

2. Why do you need transformations in computer graphics?

→ Transformations in Computer graphics are needed for:

- Moving and positioning objects (translation)
- Resizing objects (scaling)
- Changing orientation (rotation).
- Handling perspective and camera views.
- Combining multiple transformations efficiently.
- Animating objects (motion).
- Converting between coordinate systems.
- Enabling interactive user input.

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3. What is the need of Homogeneous coordinates?

→ Homogeneous coordinates are used in computer graphics to represent geometric transformations, including translation, rotation, scaling and perspective projection, as a single matrix multiplication. This simplifies calculations and allows for efficient chaining of multiple transformations. Additionally they enable the representation of points at infinity, which is useful for certain types of projections and calculations. To combine these three transformations into a single transformation, homogeneous coordinates are used.

Section-C

1. What are basic transformations? Describe each with their matrix representation.

→ The basic transformations are:

i) Rotation
ii) Translation
iii) Scaling
~~iv) Reflection~~
~~v) Shearing~~

(i) Rotation: It is the process of changing the angle of the object. It can be clockwise or anti-clockwise

Matrix form:

$$[A'B'] = [A, B] \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \text{OR} \quad P' = P \cdot R$$

Here $R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is rotation matrix used when angle is positive (anti-clockwise)

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and $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is rotation matrix used when angle is negative (clockwise).

(ii) Translation: The straight line movement of an object from one position to another is called translation.

$x' = x + t_x$

$y' = y + t_y$

t_x, t_y are shift or translation vector.

$P = [x] / [y]$ $P' = [x'] / [y']$ $T = [t_x] / [t_y]$

OR $P' = P + T$

(iii) Scaling: It is used to alter or change the size of objects. The change is done using scaling factors: S_x in x-direction

S_y in y-direction

Matrix form: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$

OR $P' = P \cdot S$

2. What do you mean by reflection and shearing? Explain with example.

→ Reflection: It is a transformation which produces a mirror image of an object. The mirror image can be either about x-axis or y-axis. The object is rotated by 180°.

Example: A triangle ABC is given with

A(3,4)

B(6,4)

C(4,8)

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To find reflected position of triangle i.e to x- axis.

The matrix for reflection about x- axis
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for A point coordinates after reflection:
$$(x,y) = (3,4)\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$(x,y) = (3,-4)$

for B, similarly:

$(x,y) = (6,-4)$

for C,

$(x,y) = (4,-8)$

coordinates of reflected triangle:

$A'(3,-4)$

$B'(6,-4)$

$C'(4,-8)$

Shearing: Shearing transformation slants or distorts an object or coordinate system along either x- axis or y-axis in a 2D plane.

To shear along x- axis, we use the shearing parameter sh_x.

Eq^n will be:

$X' = X + Y Sh_x$

$Y' = X$

Shearing matrix along with x-axis:

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

To shear along y-axis
Eqⁿ will be:

$$y' = x.sh_y + y$$
$$x' = x$$

Shearing matrix along y-axis

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3.(a) Given a triangle with points (1,1), (0,0) and (1,0). Apply shear of 2 units on x axis and 2 units on Y axis. What are the new coordinates of the object

→ Given : A (1,1) , B(0,0) , C (1,0)

$Sh_x = 2$, $Sh_y = 2$

Shearing in X-axis:

for A (1,1)

$$x' = x + y\,Sh_x = 1 + 1(2) = 3$$
$$y' = x = 1$$

~~for~~, A' (3,1)

for B (0,0)

$$x' = 0 + 0(2) = 0$$
$$y' = 0$$
$$B' = (0,0)$$

for C (1,0)

$$x' = 1 + 0(2) = 1$$
$$y' = 0$$
$$C' = (1,0)$$

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New coordinates of triangle after shearing along x-axis
$A'(3,1)$, $B'(0,0)$, $C'(1,0)$

New Shearing along Y-axis
for A (1,1)

$x' = x$
$y' = x.sh_y + y$
$x' = 1$
$y' = 1 + 1(2) = 3$
$A'(1,3)$

for B (0,0)
$x' = x = 0$
$y' = y + x.sh_y = 0 + 0(2) = 0$
$B'(0,0)$

for C (1,0)
$x' = x = 1$
$y' = 0 + 1(2) = 2$
$C'(1,2)$

New coordinates of shearing along y-axis:
~~A' B' C~~
$A'(1,1)$ $B'(0,0)$ $C'(1,2)$

(b) Given a triangle with corner coordinates (0,0), (3,0) and (3,3) Rotate the triangle by 90° in anticlockwise. Find new coordinates of triangle after translating it by 2 units in both x and y.

→ Given coordinates
A (0,0) B (3,0) C (3,3)

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Rotation angle $\theta = 90°$

for $A(0,0)$

$X' = X\cos\theta - Y\sin\theta$

$= 0(\cos 90) - 0(\sin 90)$

$= 0$

$Y' = X\sin\theta + Y\cos\theta$

$= 0(\sin 90) + 0(\cos 90)$

$= 0$

$A' = (0,0)$

for $B(3,0)$

$X' = X\cos\theta - Y\sin\theta = 3(\cos 90) - 0(\sin 90) = (0-0) = 0$

$Y' = X\sin\theta + Y\cos\theta = 3(\sin 90) - 0(\cos 90) = 3 - 0 = 3$

$B' = (0,3)$

for $C(3,3)$

$X' = X\cos\theta - Y\sin\theta = 3(\cos 90) - 3(\sin 90) = 0 - 3 = -3$

$Y' = X\sin\theta + Y\cos\theta = 3(\sin 90) + 3(\cos 90) = 3 + 0 = 3$

$C' = (-3,3)$

Coordinates of new triangle : $A'(0,0)$, $B'(0,3)$, $C'(-3,3)$

Translating it by 2 units in both x and y :

$A'' = (0+2, 0+2) = (2,2)$

$B'' = (0+2, 3+2) = (2,5)$

$C'' = (-3+2, 3+2) = (-1,5)$