

Introduction to Microeconometrics

Problem set 4: Binary outcome models

1. This problem is based on an example from Mroz's (1987) study of the labor force participation of women, using data from 1976 Panel Study of Income Dynamics.¹ The sample consists of 753 white, married women between the ages of 30 and 60 years. The dependent variable `lfp` equals 1 if a woman is in the labor force, 0 otherwise. Whether women are in the labor force or not, depends on a number of explanatory variables and can be expressed by the following probability

$$p_i = \Pr[y_i = 1 \mid \mathbf{x}_i, \boldsymbol{\beta}] = G(\mathbf{x}_i' \boldsymbol{\beta})$$

- (a) Derive the parameter estimator and the variance-covariance-matrix of a standard logistic model by the maximum likelihood (ML) method.
 - (b) Implement your own ML estimation code for the standard Logit model in Stata using the `ml` command with the `lf` method. Compare your results with the results from the built-in `logit` command in Stata.
 - (c) Calculate marginal effects at the mean, average marginal effects and odds ratios by hand and implement them in Stata.
2. Assume the data-generating process (DGP) is as follows

$$\Pr[y = 1 \mid x] = \Phi(\beta_1 + \beta_2 x)$$

where $\Phi(\cdot)$ is the standard normal cdf, $x \sim \mathcal{N}(0, 1)$, $u \sim \mathcal{N}(0, 1)$, and $(\beta_1, \beta_2) = (0, 1)$. Assume that you only have a small sample (e.g. $N = 40$). For the following problems Wald tests have to be performed with the test statistic:

$$z = W_z = \frac{\hat{\beta}_2 - 1}{\text{se}[\hat{\beta}_2]}$$

- (a) Size of a test: consider a two-sided Wald test of $H_0 : \beta_2 = 1$ against $H_a : \beta_2 \neq 1$. How does the actual size of the Wald test behave in small samples?
- (b) Power of a test: consider the power of the Wald test under $H_a : \beta_2 = 2$. How does the actual power of the Wald test behave in small samples?

¹Thomas A. Mroz (1987): "The Sensitivity of an Empirical Model of Married Women's Hours of Work to Economic and Statistical Assumptions." *Econometrica*, 55(4).