CEE 501: Models for Disaster Risk Reduction
Lecture 7: Earthquake Hazard - Part 2

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Learning Objectives

1. **Modeling Earthquake Hazards**

    1.1 Source Model

    1.2 Source to Site Distance Model

    1.3 Ground Motion Model

    1.4 Hazard Curve

    1.5 Uncertainties

2. **Example Calculation**
3. **Useful Tools**

1 Modeling seismic hazard

Probabilistic Seismic Hazard Analysis (PSHA)

Somewhere M there

PDF of Magnitudes given source

All Sources

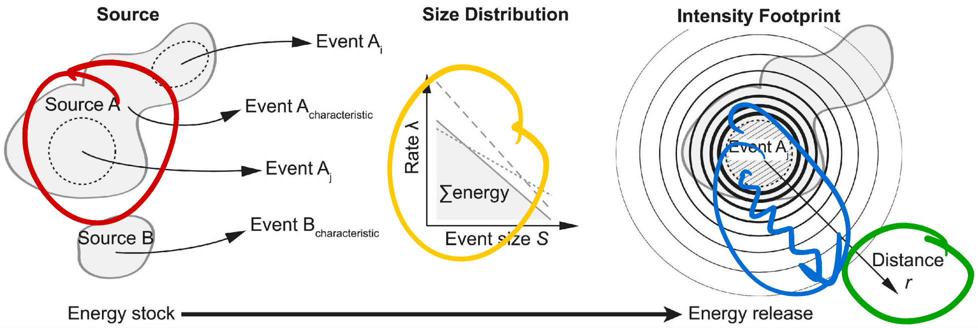
$\lambda = g_\lambda(S, r(x,y))$

$$\lambda(IM > im) = \sum_{S} \lambda(EQ_s) \int_R \int_M P(IM > im|r,m,s)\, f_R(r|m,s)\, f_M(m|s)\, dm\, dr$$

Rate of any EQ occurring on this source

Probability of intensity > im given source, magnitude distance

PDF of source to site distance given source, magnitude (site)

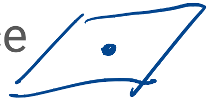


1.1 Earthquake Hazard Model - Source Model

Types of EQ Sources

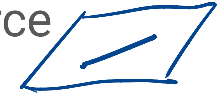
– ways human approximate the natrual geographic formation

Point Source



Volcano, Induced seismicity

Line Source



Strike-slip faults. San Andres. Hayward

Finite Fault Source

Peruvian-Chilean Megathrust

Area Source / Background Source

New Mexico Seismic Zone

Volume Source

Cascadian fault

1.1 Earthquake Hazard Model - Source Model
EQ Source Description

Source Model : Rupture Rate.
Magnitude Distribution

General Category: tectonic setting, type of fault system

Fault Geometry: Geographic location, Length, width, depth, dip, strike

Ground Motion Model :

Fault Mechanics: Slip rate, shear modulus ...

Rupture Characteristics: Slip distribution, rupture area, velocity, stress drop, rise time, directivity.

Fault Interaction: Segmentation, stress interactions.

Historical/Recurrence Data: Recurrence event and interval, paleoseismology, historical records.

1.1 Earthquake Hazard Model - Occurrence Model

Magnitude Frequency Distribution

of event with magnitude > m

Gutenberg–Richter Relationship

$$\log_{10} N = a - bM$$

$$\ln N = \alpha - \beta M$$

Rate of event M>m
$\lambda_m = N(m)/N_{total}$

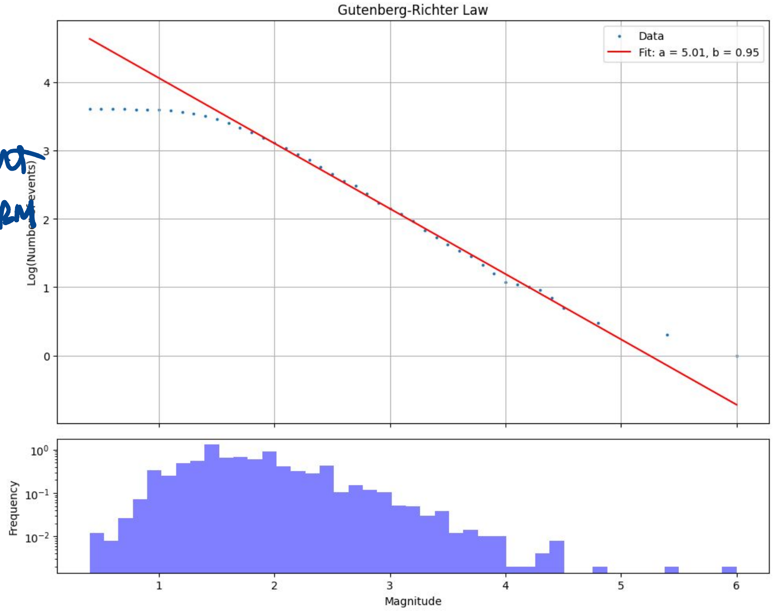
P(M≥m) given event
$= e^{\alpha-\beta M}/e^{\alpha} = e^{-\beta M}$

PDF of Magnitudes

$$f_M(m) = \beta e^{-\beta m}$$

P(M<m) CDF
$= 1 - e^{-\beta M}$

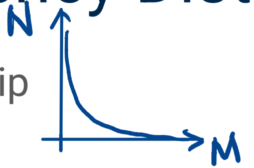
P(M=m) PDF
$= d\,P(M<m)/dm$



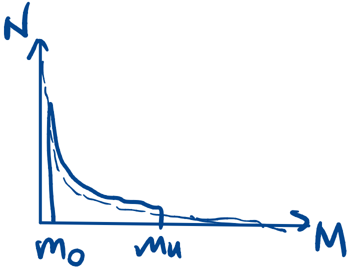
1.1 Earthquake Hazard Model - Occurrence Model

Magnitude Frequency Distribution

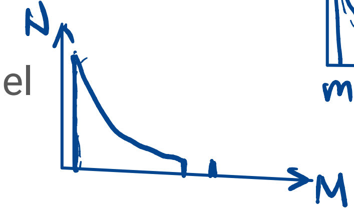
Gutenberg–Richter Relationship



Truncated Gutenberg–Richter Relationship



Characteristic Earthquake Model



$$f_M(m) = \frac{\beta e^{-\beta(m-m_0)}}{1-e^{-\beta(m_u-m_0)}}$$

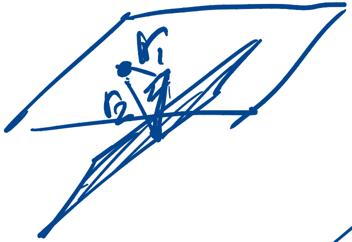
Time Dependent Model

depends on the time since most recent rupture

Statistical Approach + Physical Approach

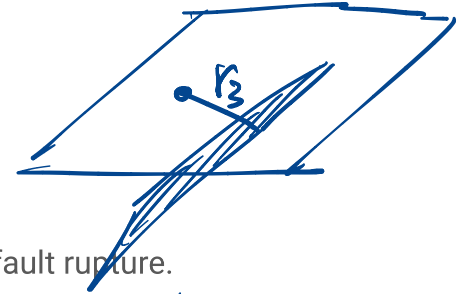
1.2 Earthquake Hazard Model - Distance Model

Source to Site Distance Definition



The horizontal distance between the site and the epicenter.

The straight-line distance from the site to the hypocenter.

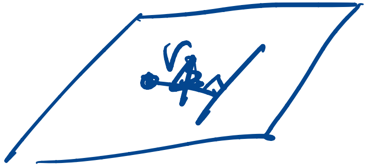


The shortest distance from the site to the ruptured portion of the fault.

The shortest horizontal distance from the site to the surface projection of the fault rupture.

…

Depends on GMPE requirement.

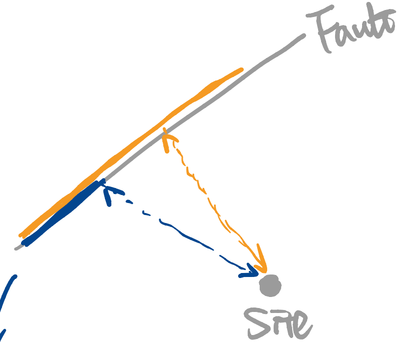


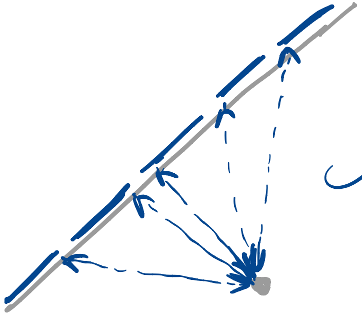
1.2 Earthquake Hazard Model - Distance Model

Source to Site Distance Calculation

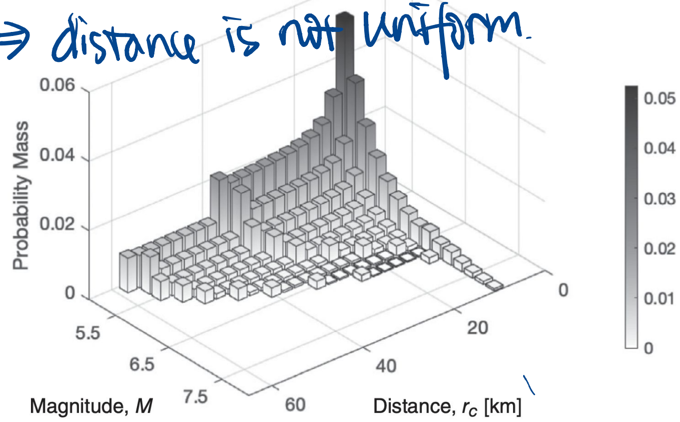
This is a probability distribution given magnitude.

for given rupture length
assume uniform distribution of location
⇒ distance is not uniform.





rupture
length differs depends
on magnitude

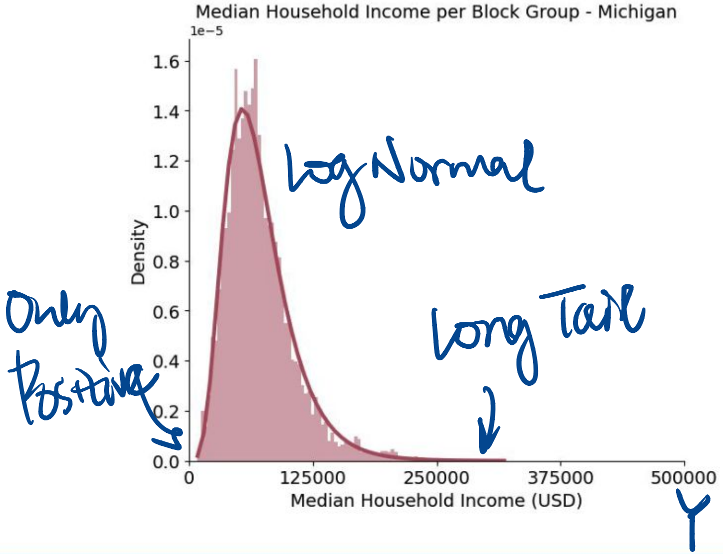


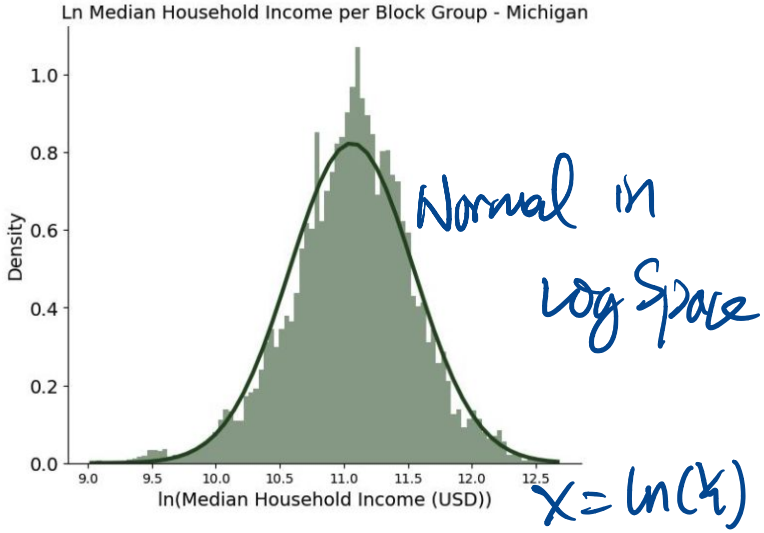
Joint distribution of magnitude and distance accounting for finite rupture dimensions and Gutenberg–Richter event probabilities.

0 Stats Review - Lognormal Distribution

Y follows a lognormal distribution

X = ln(Y) follows a normal distribution





0 Stats Review - Lognormal Distribution

Probability Density Function (PDF) of Lognormal Distribution

$$f(y) = \frac{1}{y\sigma_{\ln Y}\sqrt{2\pi}} \exp\left(-\frac{(\ln y - \mu_{\ln Y})^2}{2\sigma^2_{\ln Y}}\right)$$

Cumulative Distribution Function (CDF) of Lognormal Distribution

$$F(y) = P(Y \le y) = \Phi\left(\frac{\ln y - \mu_{\ln Y}}{\sigma_{\ln Y}}\right)$$

CDF of Standard Normal Distribution

1.3 Earthquake Hazard Model - Ground Motion Model

General Form

SAME

Ground Motion Model
Ground Motion Prediction Equation (GMPE)
Ground Motion Attenuation Function

Decide which Eq and param set to use

Ground motion at site depends on Source Type, Directivity of Rupture, Extend of Rupture / Magnitude, Propagation Path, Local Soil Condition…

Error/Uncertainty

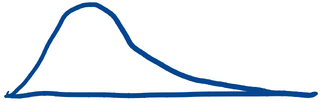
$$Y = f(M, R, Site, \sigma)$$

Intuition M↑. Y↑
R↑ Y↓

Site: Vs30.
760 Rock. 180 Soft Soil

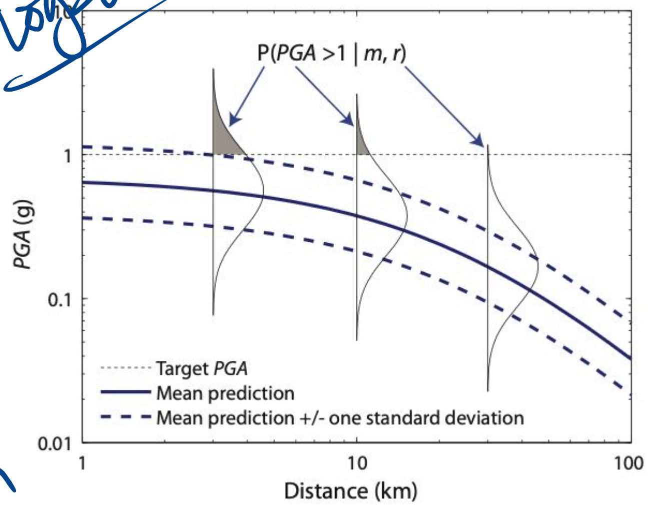
1.3 Earthquake Hazard Model - Ground Motion Model

General Form



Almost always assume IM has a Log Normal Distribution.

Log Scale



$$IM \sim LN(\mu_{\ln IM}, \sigma^2_{\ln IM})$$

$\mu_{\ln Y}$ = Mean of $\ln(Y)$

$\exp(\mu_{\ln Y})$ = Median of Y

In Log Space

$$\ln IM = \underbrace{\mu_{\ln IM}}_{\text{Mean}}(M, R, Site) + \underbrace{\sigma_{\ln IM}}_{\text{Std}}(M, R, Site) \cdot \underbrace{\varepsilon}_{\text{Error } \sim N(0,1)}$$

1.3 Earthquake Hazard Model - Ground Motion Model

A Simple Example of GMM

Boore, Joyner and Fumal (1997) Model

△ Check for application range
for shallow earthquakes in western North America

$$\mu_{\ln SA} = a_0 + a_1(M-6) + a_2(M-6)^2 + a_3 \ln\left(\sqrt{R^2 + a_4^2}\right) + a_5 \ln(V_{s,30})$$

their special definition of distance
R_{bj}

△ Check for particular IM.
SA (T = 0.1s. ξ = 5%)

$$\sigma_{\ln SA} = a_6$$

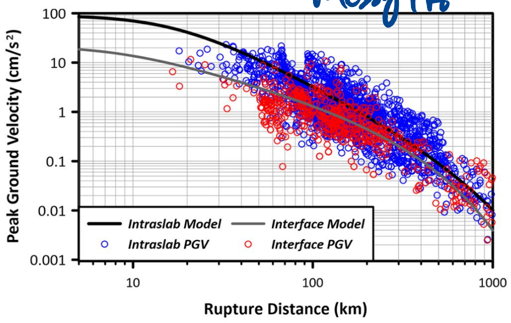
7 sets empirical coefficients
for different SA periods

1.3 Earthquake Hazard Model - Ground Motion Model

A Complex Example of GMPE

Parker, Stewart, Boore, Atkinson, and Hassani (2022) Model

Messy Fit



$$\mu_{\ln Y} = c_0 + F_P + F_M + F_D + F_S$$

Distance

$$F_P = c_1 \ln R + b_4 \mathbf{M} \ln(R/R_{ref}) + a_0 R$$

$$R = \sqrt{R_{rup}^2 + h^2}$$

$$R_{ref} = \sqrt{1 + h^2}$$

$$h = 10^{-0.82 + 0.252\mathbf{M}} \text{ (interface events)}$$

$$h = \begin{cases} 10^{\left(\frac{1.050}{m_c - 4}\right)(\mathbf{M} - m_c) + 1.544} & \mathbf{M} \leq m_c \\ 35km & \mathbf{M} > m_c \end{cases} \text{ (intraslab events)}$$

Magnitudes

$$F_M = \begin{cases} c_4(\mathbf{M} - m_c) + c_5(\mathbf{M} - m_c)^2 & \mathbf{M} \leq m_c \\ c_6(\mathbf{M} - m_c) & \mathbf{M} > m_c \end{cases}$$

$$F_D = \begin{cases} m(d_{b1} - d_{b2}) + d & Z_{hyp} < d_{b1} \\ m(Z_{hyp} - d_{b2}) + d & d_{b1} < Z_{hyp} < d_{b2} \\ d & Z_{hyp} > d_{b2} \end{cases}$$

Hypocenter Depth

Site Soil condition

$$F_S = F_{lin} + F_{nl} + F_b$$

$$F_{lin} = \begin{cases} s_1 \ln\left(\frac{V_{S30}}{V_1}\right) + s_2 \ln\left(\frac{V_1}{V_{ref}}\right) & V_{S30} \leq V_1 \\ s_2 \ln\left(\frac{V_{S30}}{V_{ref}}\right) & V_1 < V_{S30} \leq V_2 \\ s_2 \ln\left(\frac{V_2}{V_{ref}}\right) & V_{S30} > V_2 \end{cases}$$

$$F_b = \begin{cases} e_1 & \delta Z_{2.5} \\ e_3 \delta Z_{2.5} & \frac{e_1}{e_3} < \delta Z_{2.5} < \frac{e_2}{e_3} \\ e_2 & \delta Z_{2.5} \geq \frac{e_2}{e_3} \end{cases}$$

$$\delta Z_{2.5} = \ln(Z_{2.5}) - \ln(\mu_{Z2.5}(V_{S30}))$$

$$F_{nl} = f_1 + f_2 \ln\left(\frac{PGA_r + f_3}{f_3}\right)$$

$$\ln(\mu_{Z2.5}) = \ln(10) \times \theta_1 \left[1 + erf\left(\frac{\log_{10}(V_{S30}) - \log_{10}(\nu_\mu)}{\nu_\sigma \sqrt{2}}\right)\right] + \ln(10) \times \theta_0$$

Uncertainty

$$\sigma = \sqrt{\tau^2 + \phi^2}$$

$$\phi = \sqrt{\phi^2(R_{rup}) + \Delta Var(V_{S30})}$$

$$\phi^2(R_{rup}) = \begin{cases} \phi_1^2 & R_{rup} \leq R_1 \\ \frac{\phi_2^2 - \phi_1^2}{\ln(R_2/R_1)} \ln\left(\frac{R_{rup}}{R_1}\right) + \phi_1^2 & R_1 < R_{rup} < R_2 \\ \phi_2^2 & R_{rup} \geq R_2 \end{cases}$$

$$\Delta Var(V_{S30}) = \begin{cases} \phi_V^2 \left(\frac{\ln\left(\frac{R_2}{max(R_1, min(R_2, R_{rup}))}\right)}{\ln\left(\frac{R_2}{R_1}\right)}\right) & V_{S30} \leq V_1 \\ \phi_V^2 \left(\frac{\ln\left(\frac{V_2}{V_{S30}}\right)}{\ln\left(\frac{V_2}{V_1}\right)}\right)\left(\frac{\ln\left(\frac{R_2}{max(R_1, min(R_2, R_{rup}))}\right)}{\ln\left(\frac{R_2}{R_1}\right)}\right) & V_1 < V_{S30} < V_2 \\ 0 & V_{S30} \geq V_2 \end{cases}$$

1.3 Earthquake Hazard Model - Ground Motion Model

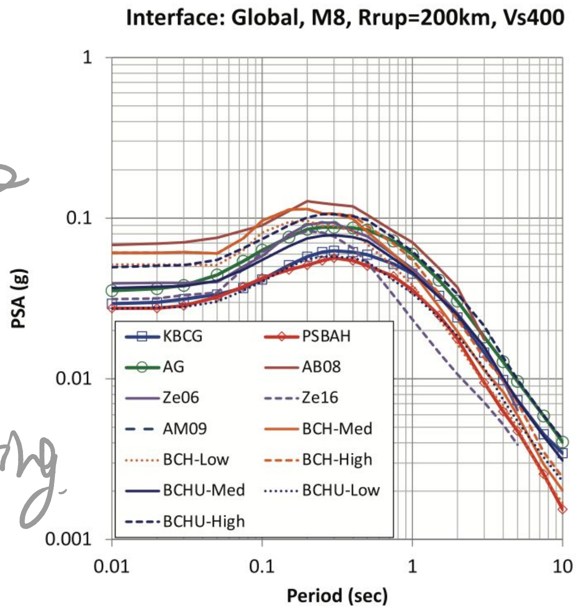
Additional Comments on GMPE

GMPE Construction ① Mixture of physical + empirical

Constructed based on statistical models using ground motion and physical model of earthquake rupture and wave propagation.

GMPE Variability ② More data, narrower range

Many exist, but how much they differ is not clear, mainly due to regional adjustments and dataset variations.



③ More complex equation
More complex parameter fitting.

④ Not much improvement.

1.3 Earthquake Hazard Model - Ground Motion Model

Predicting Using GMPE

f(1)

$$P(IM > im|r, m, s) = \int_{im}^{\infty} \underline{f_{IM}(u)}du$$

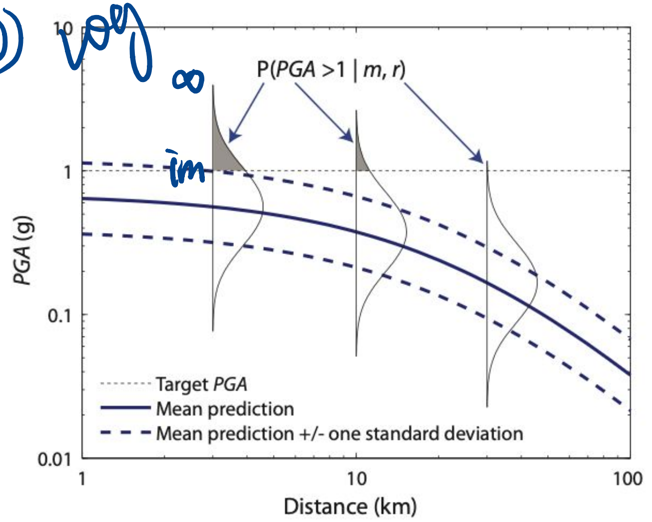
I(x,y) = g_I(s, r(x,y)) log

$$= \int_{im}^{\infty} \frac{1}{\sigma_{\ln IM}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln u - \mu_{\ln IM}}{\sigma_{\ln IM}}\right)^2\right) du$$

$$= 1 - \Phi\left(\frac{\ln im - \mu_{\ln IM}}{\sigma_{\ln IM}}\right)$$

Mean, Std in Logspace

Target IM Level



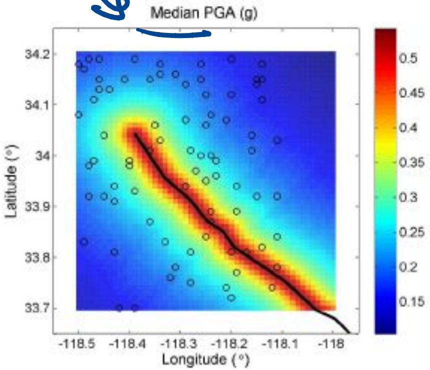
1.3 Earthquake Hazard Model - Ground Motion Model

Intensity Footprint / Ground Model Field (GMF)

$\exp(\mu_{\ln IM})$

$\sigma_{\ln IM} \cdot \varepsilon$ $\sim N(0,1)$

$\exp(\mu_{\ln IM} + \sigma_{\ln IM} \cdot \varepsilon)$



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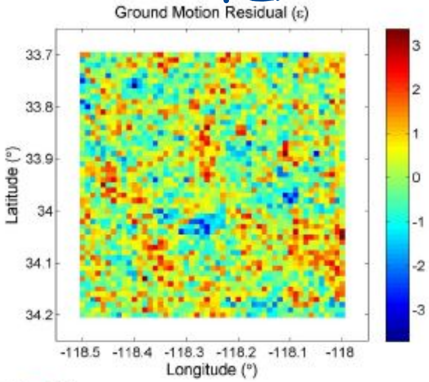

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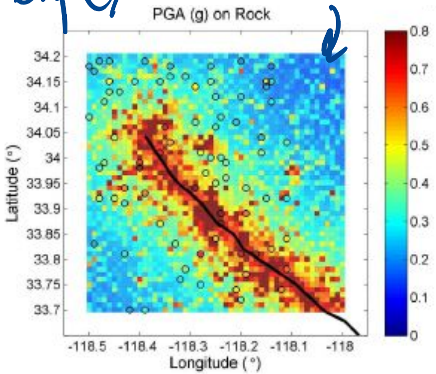


continuous

Run GMPE for all locs.

error

→ correlated Residual

Ground Motion Correlation

1.4 Earthquake Hazard Model - Hazard Curve

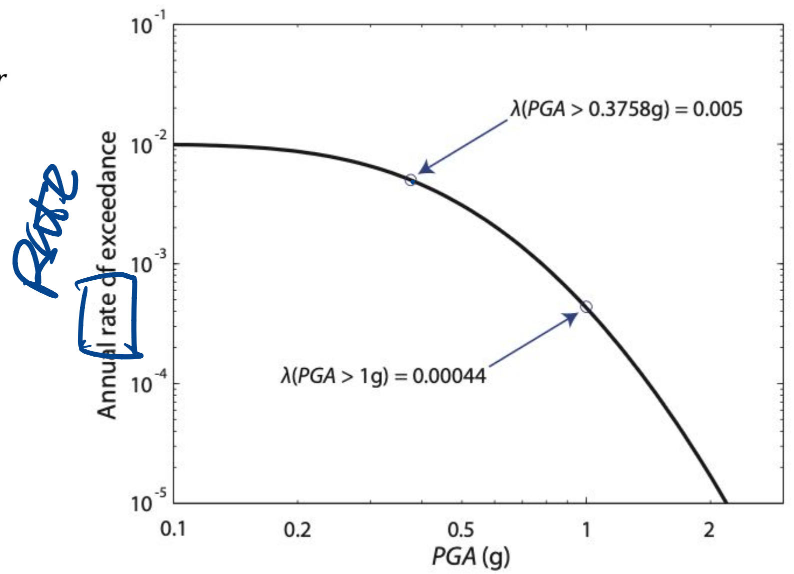
Constructing Hazard Curve

"Exceedance Curve" = Annual Rate of Exceedance

$$\lambda(IM > im) = \sum_{S} \lambda(EQ_s) \int_R \int_M P(IM > im|r,m,s) f_R(r|m,s)\, f_M(m|s)\, dm\, dr$$

Rate:
Num of a event larger than intensity im happen in a given time period

compute a $\lambda(IM>im)$ for each im in relevent range to form a curve.



1.4 Earthquake Hazard Model - Hazard Curve

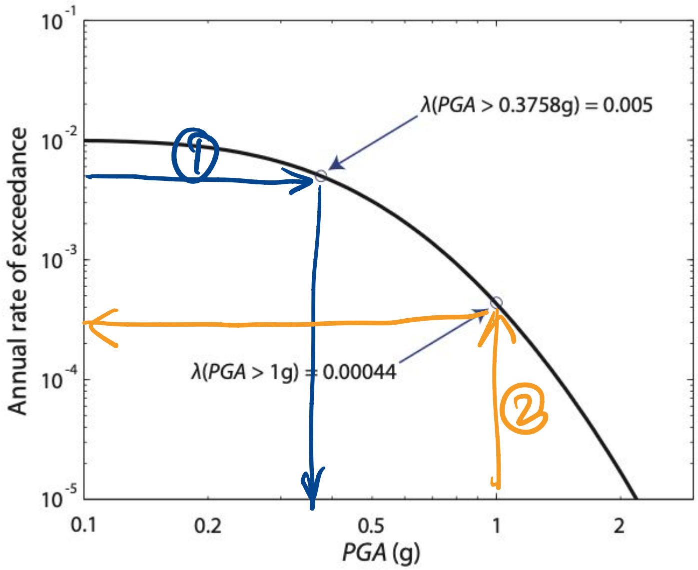
Interpreting Hazard Curve

① Given a rate

$\lambda = 0.005 \rightarrow IM = 0.3758g$

② Given a IM

$IM = 1g \rightarrow \lambda = 0.00044$



1.4 Earthquake Hazard Model - Hazard Curve

Return Period

Return Period - inverse of rate

$$RP = \frac{1}{\lambda}$$

Im > certain level

The mean # of event happen in a given period

Probability of Exceeding

Assume Poisson Distr.

for very small λ

$$P(N \geq 1) = 1 - e^{-\lambda t} \approx \lambda t$$

Probability of Exceeding IM = im in a give time period t.

2% prob 50 years. $\Rightarrow 0.02 = 1 - e^{50\lambda} \Rightarrow \lambda = -\frac{1}{50}\ln(1-0.02)$

$= 0.000405\ldots$

$RP = 1/\lambda$

$= 2475$ yrs.

1.4 Earthquake Hazard Model - Uncertainties
Epistemic and Aleatory Uncertainty

Aleatory Uncertainty - Due to nature's inherent randomness.

No way to predict EQ.   98% CI = 54 to 268

Hayward Fault. 1686. Mean 160. std 65

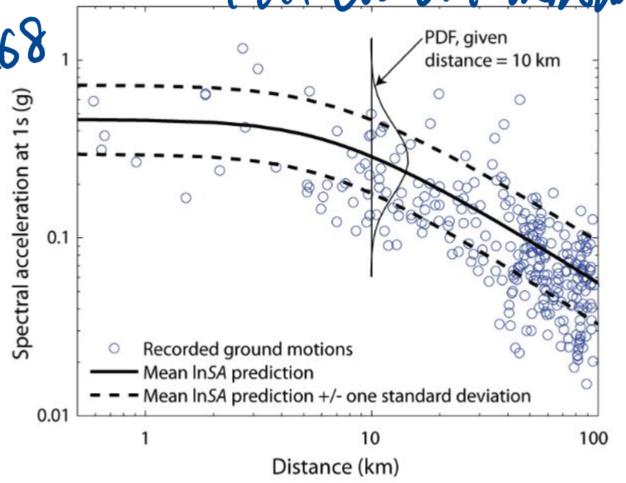
Epistemic Uncertainty - due to human's lack of knowledge

All parameters a, b, r, GMPEs are uncertain

Each component has # of different models, don't know which is better.

Can be reduced, but just so much.

1999 Chi-Chi Taiwan EQ



1.4 Earthquake Hazard Model - Uncertainties

Logic Tree

- Uncertainty branching of possible inputs.

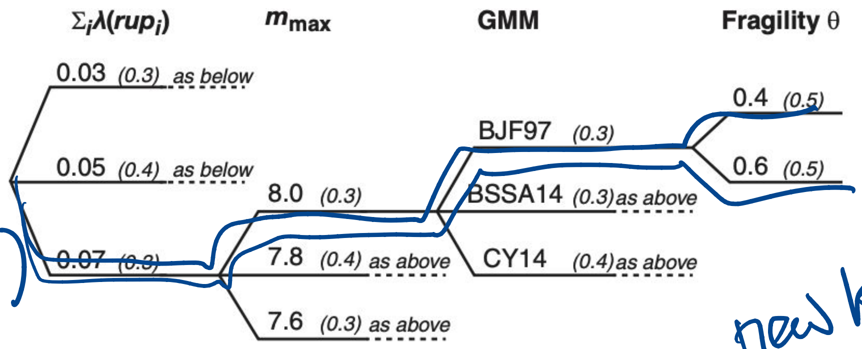
CAN BLEND BOTH

Don't know ⇒ Equal weight

Issue:

Lognormal Distr.

$E[Y] = \exp(\mu_{\ln Y} + \frac{\sigma^2_{\ln Y}}{2})$



new knowledge about what we don't know. Or think too much

as branches increase

$\sigma_{\ln Y} \uparrow$, $E[Y] \uparrow$ Artificial

2 Sample Calculation

Develop a seismic hazard curve for a building at Site S with a 1-second period of vibration. The site has dense soil underground with Vs30 = 400m/s. There are two faults, A and B, near Site S.

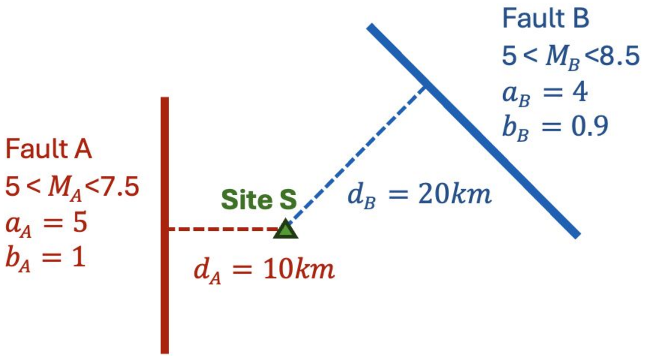
Fault A is a strike-slip fault located 10 km from the site. It has the potential to generate earthquakes with magnitudes ranging from 5 to 7.5. The annual rate of occurrence is modeled using the Gutenberg-Richter relationship with parameters a=5 and b=1.

Fault B is a strike-slip fault located 20 km from the site. It has the potential to generate earthquakes with magnitudes ranging from 5 to 8.5. The annual rate of occurrence is modeled using the Gutenberg-Richter relationship with parameters a=4 and b=0.9.

Use the Boore, Joyner, and Fumal (1997) Ground Motion Model for a strike-slip fault. The equation, substituted with values for coefficient prediction SA(1s) is:

$$\mu_{\ln SA(1s)} = -3.4415 + 1.42M - 0.032M^2 - 0.798\ln\left(\sqrt{R^2 + 8.41}\right) - 0.698\ln(V_{S,30})$$

$$\sigma_{\ln SA(1s)} = 0.52$$



2.1 Sample Calculation - Numerical Integration

$$\lambda(IM > im) = \sum_{S} \lambda(EQ_s) \cancel{\int_R} \int_M P(IM > im|r,m,s) \cancel{f_R(r|m,s)} f_M(m|s)\, dm\, \cancel{dr}$$

Discritized

constant

$$\lambda(IM > im) = \sum_{S} \sum_{M} \lambda(EQ_{m,s})\, P(IM > im|r,m,s)$$

2.1 Sample Calculation - Numerical Integration

Step 1: Discretize the range of magnitude

```
# For Fault A
# Minimum Considered Magnitude
M0 = 5
# Maximum Considered Magnutyde
Mu = 7.5
# Size of Magnitude Bin
del_M = 0.1
# Discretized Range of Magnitudes
Mi = np.arange(M0, Mu+del_M, del_M)
```

```
Mi
```

```
array([5. , 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 6. , 6.1, 6.2,
       6.3, 6.4, 6.5, 6.6, 6.7, 6.8, 6.9, 7. , 7.1, 7.2, 7.3, 7.4, 7.5])
```

$$\lambda(IM > im) = \sum_{S} \sum_{M} \lambda(EQ_{m,s})\, P(IM > im | r, m, s)$$

2.1 Sample Calculation - Numerical Integration

Step 2: Calculate the rate of occurrence for each magnitude bin

```python
# Occurance Model Parameterss
a = 5
b = 1
# Rate of Events with M > Upper Bound of Magnitude Bin
Lambda_u = 10**(a - b * (Mi + delta_M/2))
# Rate of Events with M > Lower Bound of Magnitude Bin
Lambda_0 = 10**(a - b * (Mi - delta_M/2))
# Rate of Events within the Magnitude Bin
Lambda_bin = Lambda_0 - Lambda_u
```

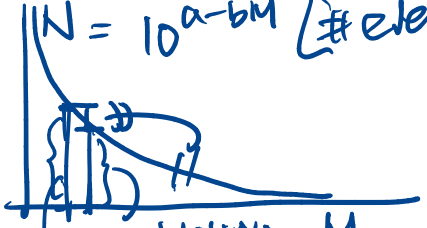
```python
Lambda_bin
```

```
array([2.30263596e-02, 1.82904876e-02, 1.45286507e-02, 1.15405175e-02,
       9.16695887e-03, 7.28157426e-03, 5.78396003e-03, 4.59436276e-03,
       3.64943206e-03, 2.89884692e-03, 2.30263596e-03, 1.82904876e-03,
       1.45286507e-03, 1.15405175e-03, 9.16695887e-04, 7.28157426e-04,
       5.78396003e-04, 4.59436276e-04, 3.64943206e-04, 2.89884692e-04,
       2.30263596e-04, 1.82904876e-04, 1.45286507e-04, 1.15405175e-04,
       9.16695887e-05, 7.28157426e-05])
```

G-R Relationship

$$\log_{10} N = a - bM$$

$N = 10^{a-bM}$ [# event/yr]



$10^{a-b(M+\Delta/2)}$ M

$10^{a-b(M-\Delta/2)}$

$$\lambda(IM > im) = \sum_S \sum_M \lambda(EQ_{m,s}) P(IM > im|r,m,s)$$

2.1 Sample Calculation - Numerical Integration

Now we have a Stochastic Event Table

```
# Create Stochastic Event Table
data = {
    "Event ID": [f"EQ{str(i+1).zfill(3)}" for i in range(len(Mi))],
    "Source": ["Fault A"] * len(Mi),
    "Size (Mw)": [f"[{M-delta_M/2:.2f}, {M+delta_M/2:.2f}]" for M in Mi],
    "Rate (λ)": Lambda_bin
}
df_event = pd.DataFrame(data)

# Display
df_event
```

Book Table 2.1

	Event ID	Source	Size (Mw)	Rate (λ)
0	EQ001	Fault A	[4.95, 5.05]	0.230768
1	EQ002	Fault A	[5.05, 5.15]	0.183305
2	EQ003	Fault A	[5.15, 5.25]	0.145604
3	EQ004	Fault A	[5.25, 5.35]	0.115658
4	EQ005	Fault A	[5.35, 5.45]	0.091870
5	EQ006	Fault A	[5.45, 5.55]	0.072975
6	EQ007	Fault A	[5.55, 5.65]	0.057966
7	EQ008	Fault A	[5.65, 5.75]	0.046044
8	EQ009	Fault A	[5.75, 5.85]	0.036574
9	EQ010	Fault A	[5.85, 5.95]	0.029052
10	EQ011	Fault A	[5.95, 6.05]	0.023077
11	EQ012	Fault A	[6.05, 6.15]	0.018331
12	EQ013	Fault A	[6.15, 6.25]	0.014560
13	EQ014	Fault A	[6.25, 6.35]	0.011566
14	EQ015	Fault A	[6.35, 6.45]	0.009187

2.1 Sample Calculation - Numerical Integration

Step 3: For each magnitude bin, calculate the log-mean and log-std for IM

```
# Function for GMPE
def GMPE_BJF97_SA1(Mag, Dis, Vs30):
    Mu_lnIM = -3.4415 + 1.42*Mag - 0.032*Mag**2 - 0.798*np.log((Dis**2+8.41)**0.5)-0.698*np.log(Vs30)
    Sigma_lnIM = 0.52
    return Mu_lnIM, Sigma_lnIM
```

GMPE

$$\mu_{\ln SA(1s)} = -3.4415 + 1.42M - 0.032M^2 - 0.798\ln\left(\sqrt{R^2+8.41}\right) - 0.698\ln(V_{S,30})$$

$$\sigma_{\ln SA(1s)} = 0.52$$

```
# Distance from Source to Site
Dis = 10
# Site Soil Condition Paramter
Vs30 = 400
# Mean of IM in Log Space
Mu_lnIM = [ GMPE_BJF97_SA1(M, Dis, Vs30)[0] for M in Mi]
# Std of IM in Log Space
Sigma_lnIM = [ GMPE_BJF97_SA1(M, Dis, Vs30)[1] for M in Mi]
```

Magnitude	LogMeanIM	LogStdIM
5.0	-3.193224	0.52
5.1	-3.083544	0.52
5.2	-2.974504	0.52
5.3	-2.866104	0.52
5.4	-2.758344	0.52
5.5	-2.651224	0.52
5.6	-2.544744	0.52
5.7	-2.438904	0.52
5.8	-2.333704	0.52
5.9	-2.229144	0.52
6.0	-2.125224	0.52

$$1 - \Phi\left(\frac{\ln IM - \mu_{\ln IM}}{\sigma_{\ln IM}}\right)$$

$$\lambda(IM > im) = \sum_S \sum_M \lambda(EQ_{m,s}) \underbrace{P(IM > im|r,m,s)}_{= 1 - \Phi\left(\frac{\ln IM - \mu_{\ln IM}}{\sigma_{\ln IM}}\right)}$$

2.1 Sample Calculation - Numerical Integration

Step 4: Discretize the range of intensity

```
# Discretize range of relevant intensity
del_IM = 0.1
Target_IM = np.arange(0.1, 3, del_IM)
```

```
Target_IM
```

```
array([0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. , 1.1, 1.2, 1.3,
       1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2. , 2.1, 2.2, 2.3, 2.4, 2.5, 2.6,
       2.7, 2.8, 2.9])
```

IM levels / threshold

$$\lambda(IM > im) = \sum_S \sum_M \lambda\left(EQ_{m,s}\right) P(IM > im|r, m, s)$$

2.1 Sample Calculation - Numerical Integration

Step 5: Calculate P(IM>im) at each intensity level for each magnitude

```python
# Function Calculate Probability of Exceeding
# Given a Threshold Intensity Level, Logmean and LogStd of the Mag Distr
def Prob_exceed (Target_IM, Mu_lnIM, Sigma_lnIM):
    P_exceed = 1 - scipy.stats.norm.cdf(np.log(Target_IM[i]),
                                        loc=Mu_lnIM,
                                        scale=Sigma_lnIM)
    return P_exceed
```

$$P(IM > im) = 1 - \Phi\left(\frac{\ln im - \mu_{\ln IM}}{\sigma_{\ln IM}}\right)$$

```python
# Calculate Probability of Exceedance for each IM level and each Mag level
df_ex_prob = pd.DataFrame(columns= Target_IM, index = Mi)
for i in range(len(Target_IM)):
    for j in range(len(Mi)):
        p = Prob_exceed(Target_IM,Mu_lnIM[j],Sigma_lnIM[j])
        df_ex_prob.loc[Mi[j], Target_IM[i]] = p
```

df_ex_prob

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5.0	0.043378	0.001161	0.000065	0.000006	0.000001	0.0	0.0	0.0	0.0	0.0
5.1	0.066569	0.002293	0.00015	0.000015	0.000002	0.0	0.0	0.0	0.0	0.0
5.2	0.098152	0.004331	0.000331	0.000038	0.000006	0.000001	0.0	0.0	0.0	0.0
5.3	0.139251	0.007832	0.000696	0.000089	0.000015	0.000003	0.000001	0.0	0.0	0.0
5.4	0.19039	0.013572	0.001399	0.000198	0.000036	0.000008	0.000002	0.000001	0.0	0.0
5.5	0.251282	0.022565	0.002691	0.000424	0.000083	0.000019	0.000005	0.000002	0.0	0.0
5.6	0.320718	0.036036	0.004963	0.000869	0.000185	0.000046	0.000013	0.000004	0.000001	0.0
5.7	0.396602	0.055342	0.008778	0.001705	0.000394	0.000105	0.000031	0.00001	0.000004	0.000001
5.8	0.47614	0.081837	0.014907	0.003207	0.000803	0.000228	0.000072	0.000025	0.000009	0.000004
5.9	0.556157	0.116681	0.024334	0.00579	0.001569	0.000476	0.000159	0.000057	0.000022	0.000009

$$\lambda(IM > im) = \sum_{S}\sum_{M}\lambda(EQ_{m,s})P(IM > im|r,m,s)$$

2.1 Sample Calculation - Numerical Integration

Step 6: Calculate λ(IM>im) at each intensity level for each magnitude

```python
# Calculate Rate of Exceedance for each IM level and each event
df_ex_rate = pd.DataFrame(columns=Target_IM, index=Mi)
for i in range(len(Target_IM)):
    for j in range(len(Mi)):
        prob_ex = df_ex_prob.loc[Mi[j], Target_IM[i]]
        rate = Lambda_bin[j]
        rate_ex = prob_ex * rate
        df_ex_rate.loc[Mi[j], Target_IM[i]] = rate_ex
```

$$\lambda(IM > im) = \sum_{S}\sum_{M} \lambda(EQ_{m,s}) P(IM > im|r,m,s)$$

Sum

```
df_ex_rate
```

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
5.0	0.000999	0.000027	0.000002	0.0	0.0	0.0	0.0	0.0
5.1	0.001218	0.000042	0.000003	0.0	0.0	0.0	0.0	0.0
5.2	0.001426	0.000063	0.000005	0.000001	0.0	0.0	0.0	0.0
5.3	0.001607	0.00009	0.000008	0.000001	0.0	0.0	0.0	0.0
5.4	0.001745	0.000124	0.000013	0.000002	0.0	0.0	0.0	0.0
5.5	0.00183	0.000164	0.00002	0.000003	0.000001	0.0	0.0	0.0
5.6	0.001855	0.000208	0.000029	0.000005	0.000001	0.0	0.0	0.0
5.7	0.001822	0.000254	0.00004	0.000008	0.000002	0.0	0.0	0.0
5.8	0.001738	0.000299	0.000054	0.000012	0.000003	0.000001	0.0	0.0
5.9	0.001612	0.000338	0.000071	0.000017	0.000005	0.000001	0.0	0.0
6.0	0.001459	0.00037	0.000088	0.000023	0.000007	0.000002	0.000001	0.0
6.1	0.00129	0.000391	0.000106	0.000031	0.00001	0.000003	0.000001	0.0
6.2	0.001118	0.0004	0.000123	0.000039	0.000013	0.000005	0.000002	0.000001
6.3	0.000952	0.000398	0.000137	0.000048	0.000018	0.000007	0.000003	0.000001

2.1 Sample Calculation - Numerical Integration

Step 7: Sum λ(IM>im) for at intensity level for all magnitudes

```
# Calculate Rate of Exceedance at each IM level for all magnitudes
Rate_Ex = df_ex_rate.sum()
```

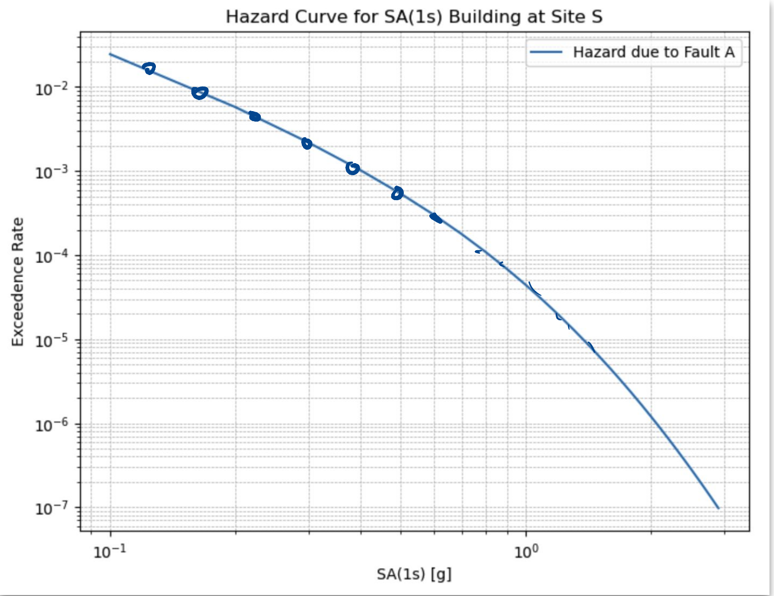
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	...
Total Exc Rate	0.02458	0.005772	0.002185	0.001024	0.000537	0.000302	0.000178	0.000109	0.000068	0.000044	...

$$\lambda(IM > im) = \sum_{S} \sum_{M} \lambda(EQ_{m,s})\, P(IM > im|r, m, s)$$

2.1 Sample Calculation - Numerical Integration

Step 8: Plot exceedance curve due to Fault A

```
# Plot Exceedance Curve

plt.figure(figsize=(8, 6))
plt.loglog(Target_IM, Rate_Ex, label = 'Hazard due to Fault A')

plt.xlabel("SA(1s) [g]")
plt.ylabel("Exceedence Rate")
plt.title("Hazard Curve for SA(1s) Building at Site S")
plt.legend()
plt.grid(True, which='both', linestyle='--', linewidth=0.5)

plt.show()
```



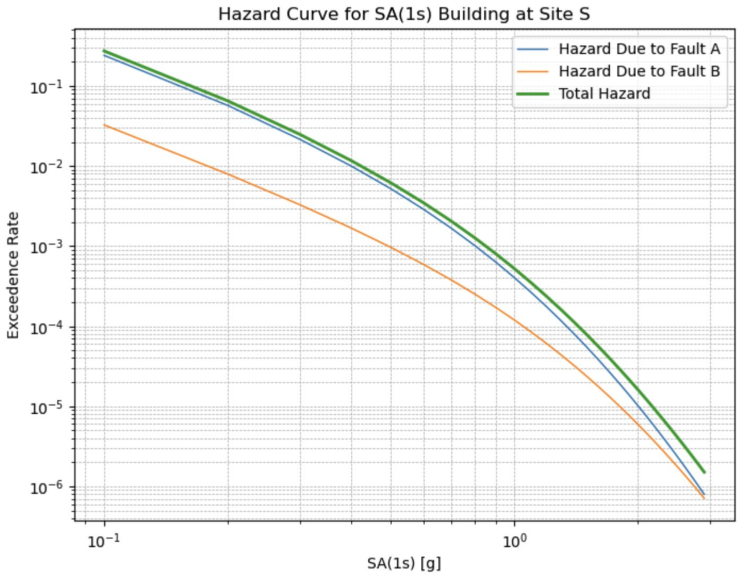
$$\lambda(IM > im) = \sum_S \sum_M \lambda(EQ_{m,s})\, P(IM > im|r, m, s)$$

2.1 Sample Calculation - Numerical Integration

Step 9: Repeat same calculation from Fault B

Step 10: Sum the exceedance rate from Fault A and B

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Exceed Rate Fault A	0.241174	0.057309	0.021567	0.010017	0.005200	0.002891	0.001685	0.001018
Exceed Rate Fault B	0.032629	0.007962	0.003287	0.001683	0.000965	0.000592	0.000380	0.000252
Exceed Rate Total	0.273803	0.065270	0.024855	0.011700	0.006165	0.003483	0.002066	0.001271



$$\lambda(IM > im) = \sum_{S}\sum_{M} \lambda(EQ_{m,s})\, P(IM > im|r,m,s)$$

GEM Insurance

2.2 Sample Calculation - Monte Carlo Simulation

Almost all large scale model.

Capturing Uncertainty:

Logic Tree - Uncertainty about Model to use

Explicitly samples probability distributions, incorporating aleatory and epistemic uncertainties.

Computational Efficiency:

If all parameter has range and PDF

Avoids high-dimensional numerical integration, reducing computational cost.

Handling Complex Ruptures:

Simulates multi-fault and cascading earthquake scenarios more effectively.

One trigger another

Each Event is a single scenario story

2.2 Sample Calculation - Monte Carlo Simulation

Create a Simulated Event Catalog

```
def Func_MC_Event_Sim(M0, Mu, del_M, a, b, T, Source):

    # Magnitude Levels
    Mi = np.arange(M0, Mu, del_M)
    # Rate of Occurance for Each Magnitudes
    Lambda = 10**(a - b * (Mi - del_M/2)) - 10**(a - b * (Mi + del_M/2))

    # Simulation Table
    Events = []

    # For each Magnitude Level
    for i in range(len(Mi)):
        # Magnitude
        m = Mi[i]
        # Generate Events in Each Year
        Ns = np.random.poisson(Lambda[i],T)
        # For each Year
        for i in range(T):
            # Year
            t = i
            # Number of Events for this Magnitude
            n = Ns[i]
            # Store Events in Event Table
            for _ in range(n):
                Events.append([t, Source, m])

    # Create DataFrame
    df_result = pd.DataFrame(Events, columns=["Year", "Source", "Magnitude"])

    return df_result
```

```
# Simulated Events for Each Fault for 1000 Years
Sim_Events_A = Func_MC_Event_Sim(M0=5, Mu=7.5, del_M=0.1, a=5, b=1, T=25000, Source='Fault A')
Sim_Events_B = Func_MC_Event_Sim(M0=5, Mu=8.5, del_M=0.1, a=4, b=0.9, T=25000, Source='Fault B')

# Combine Simulated Event Table
Sim_Events = pd.concat([Sim_Events_A, Sim_Events_B], ignore_index=True)

# Sort by "Year" column in ascending order
Sim_Events = Sim_Events.sort_values(by="Year", ascending=True).reset_index(drop=True)
```

Possion Random Num Gen

Give λ,
Gen Rand # of events / yr
with Poisson Distr

Key Part of Hazard Model

	Year	Source	Magnitude
0	0	Fault B	5.7
1	1	Fault A	5.6
2	1	Fault B	5.0
3	1	Fault A	5.0
4	2	Fault A	5.0
...	...	...	...
36544	24993	Fault A	6.8
36545	24996	Fault B	5.0
36546	24996	Fault A	5.0
36547	24996	Fault A	6.3
36548	24999	Fault B	5.5

2.2 Sample Calculation - Monte Carlo Simulation

Simulate one IM for each event in Catalog

```
# GMPE Function
def GMPE_BJF97_SA1(Mag, Dis, Vs30):
    Mu_lnIM = -3.4415 + 1.42*Mag - 0.032*Mag**2 - 0.798*np.log((Dis**2+8.41)**0.5)-0.698*np.log(Vs30)
    Sigma_lnIM = 0.52
    return Mu_lnIM, Sigma_lnIM
```

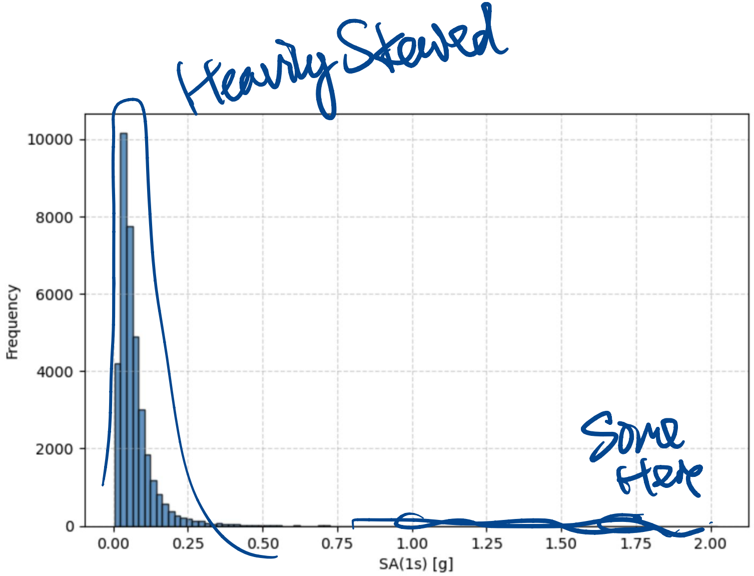
```
# Parameterss
Dis = {'Fault A':10, 'Fault B':20}
#
Sim_IM = []

# For each event, simulate a IM
for _, row in Sim_Events.iterrows():

    # Magnitude and Distance for this Event
    M = row['Magnitude']
    D = Dis[row['Source']]

    # Log Mean and Std of IM for this event
    Mu_lnIM, Sigma_lnIM = GMPE_BJF97_SA1(M, D, 400)

    # Simulate a IM
    im = np.random.lognormal(mean=Mu_lnIM, sigma=Sigma_lnIM)
    Sim_IM.append(im)

# Add to Simulation Event Table
Sim_Events['SA(1s)'] = Sim_IM
```

Random Sampled IM from Lognormal

	Year	Source	Magnitude	SA(1s)
0	0	Fault A	5.0	0.070801
1	1	Fault A	5.3	0.112042
2	2	Fault A	5.1	0.032134
3	4	Fault A	5.9	0.066846
4	4	Fault B	5.4	0.018697
...	...	...	...	...
36258	24995	Fault A	5.3	0.034298
36259	24996	Fault B	5.3	0.041628
36260	24999	Fault A	5.3	0.050429
36261	24999	Fault A	5.0	0.028374
36262	24999	Fault A	5.4	0.135489

2.2 Sample Calculation - Monte Carlo Simulation

Histogram for simulated IMs



	Year	Source	Magnitude	SA(1s)
0	0	Fault A	5.0	0.070801
1	1	Fault A	5.3	0.112042
2	2	Fault A	5.1	0.032134
3	4	Fault A	5.9	0.066846
4	4	Fault B	5.4	0.018697
...	...	...	...	...
36258	24995	Fault A	5.3	0.034298
36259	24996	Fault B	5.3	0.041628
36260	24999	Fault A	5.3	0.050429
36261	24999	Fault A	5.0	0.028374
36262	24999	Fault A	5.4	0.135489

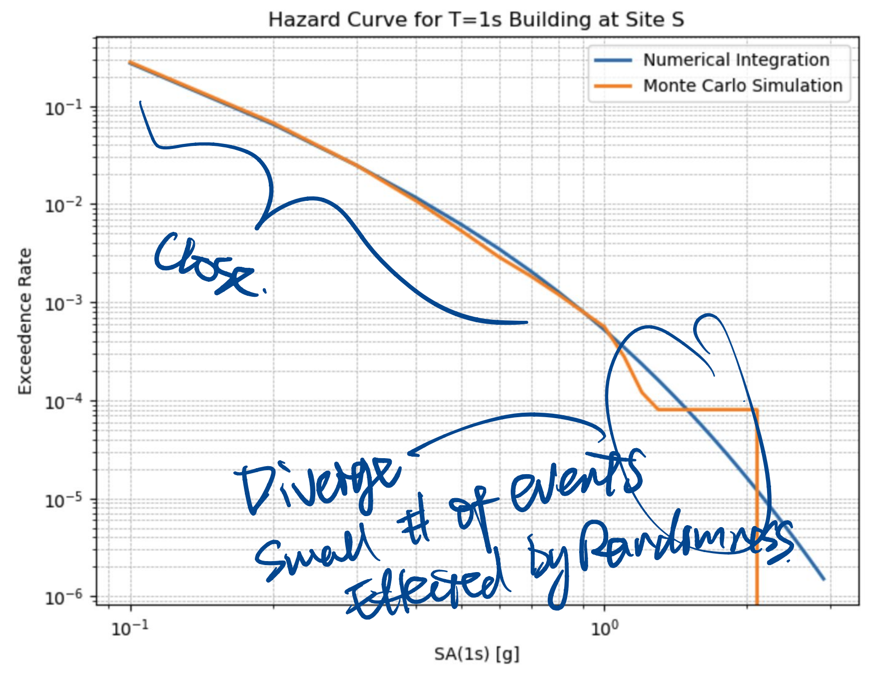
2.2 Sample Calculation - Monte Carlo Simulation

Calculate simulated Exceedance Rate for each IM level

```python
def Sim_Rate_Ex(SimIM, TargetIM, T):

    ex_rates = []

    # For each IM threshold
    for im in TargetIM:

        # Count the number of simulated IM > threshold
        count_ex = np.sum(np.array(SimIM) > im)

        # Calculate Rate = Counte / Years
        rate = count_ex / T

        ex_rates.append(rate)

    df_result = pd.DataFrame({'IM': TargetIM, 'Exceedance_Rate': ex_rates})

    return df_result
```

```python
Rate_Ex_Sim = Sim_Rate_Ex(SimIM = Sim_Events['SA(1s)'],
                          TargetIM = np.arange(0.1,3,0.1),
                          T=25000)
```

Exceedance Rate = count of IM sim / years simulated



Close.

Diverge

Small # of events

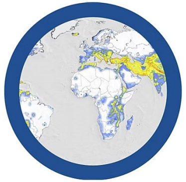
Effected by Randomness

3 Useful Resources for Earthquake Modeling
Global Earthquake Model (GEM)

Global Seismic Hazard Map:

https://www.globalquakemodel.org/product/global-seismic-hazard-map

Flagship Products



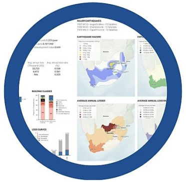
Global Hazard Map

OpenQuake Engine

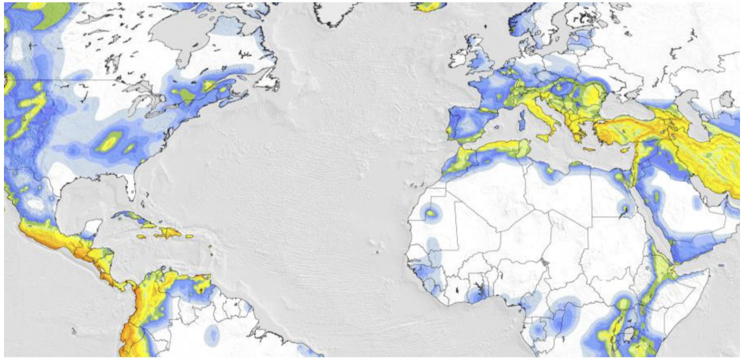
Global Risk Map

Global Exposure Model

Global Vulnerability Model



Risk Profiles



Description

The Global Earthquake Model (GEM) Global Seismic Hazard Map (version 2023.1) depicts the geographic distribution of the Peak Ground Acceleration (PGA) with a 10% probability of being exceeded in 50 years, computed for reference rock conditions (shear wave velocity, Vs30, of 760-800 m/s).

3 Useful Resources for Earthquake Modeling

United States Geological Survey (USGS)

Earthquake Monitoring & Data

1. USGS Earthquake Map (Latest Earthquakes) – Interactive real-time earthquake map.
2. ANSS (Advanced National Seismic System) – A nationwide network for earthquake monitoring.
3. ShakeMap – Generates real-time ground shaking maps.
4. PAGER (Prompt Assessment of Global Earthquakes for Response) – Provides estimates of earthquake impact on population and economic losses.
5. Did You Feel It? – A community-driven tool where people report earthquake experiences.
6. ComCat (Comprehensive Earthquake Catalog) – A searchable database of earthquake events.

Hazard & Risk Assessment

7. National Seismic Hazard Maps – Maps of earthquake hazard levels across the U.S.
8. USGS ShakeAlert® – An early warning system for earthquakes on the West Coast.
9. USGS Seismic Design Maps – Provides seismic hazard data for engineers and builders.
10. Earthquake Scenarios – Hypothetical earthquake scenarios for preparedness planning.

USGS science for a changing world

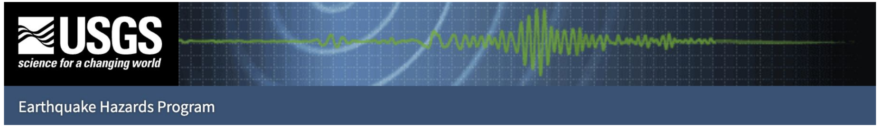
OpenSHA is an open-source, Java-based platform for conducting Seismic Hazard Analysis (SHA)

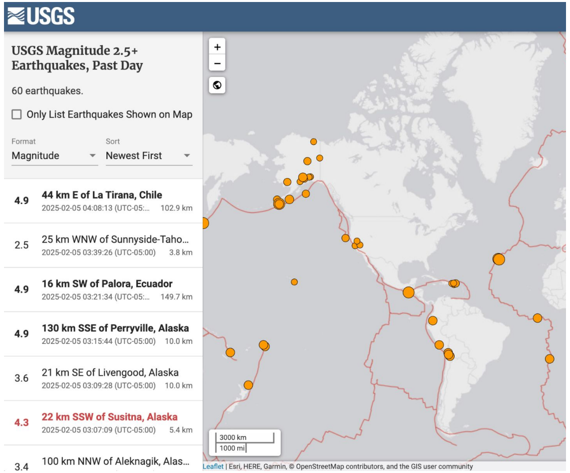
As an object-oriented framework, OpenSHA can accomodate arbitrarily complex (e.g., physics based) earthquake rupture forecasts (ERFs), ground-motion models, and engineering-response models, which narrows the gap between cutting-edge geophysics and state-of-the-art hazard and risk evaluations.

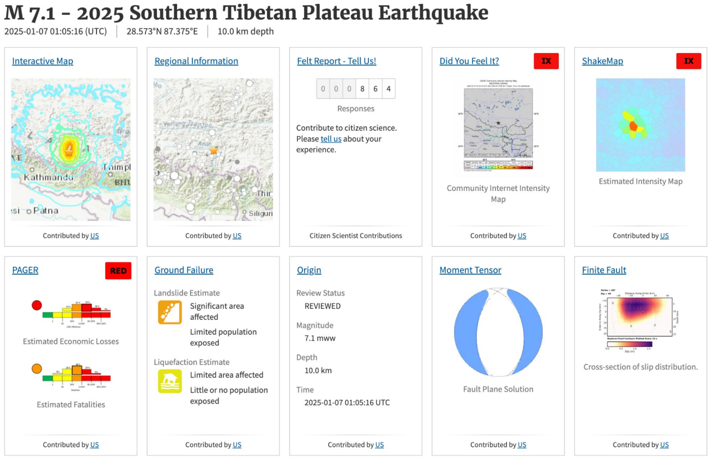
https://opensha.org/

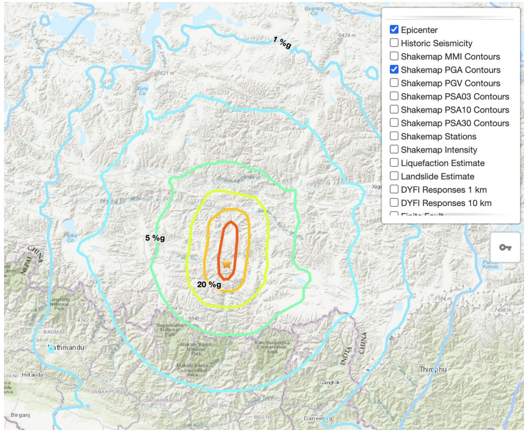
3 Useful Resources for Earthquake Modeling

United States Geological Survey (USGS)









USGS Earthquake Catalog https://earthquake.usgs.gov/earthquakes/search/

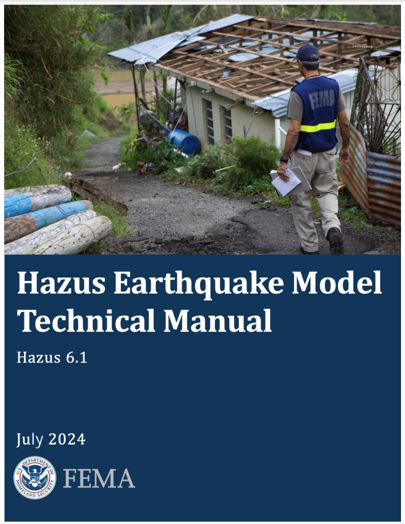
3 Useful Resources for Earthquake Modeling

HAZUS

HAZUS (Hazards U.S.) is a FEMA-developed, GIS-based tool used for modeling and estimating potential losses from natural disasters, including earthquakes, floods, hurricanes, and tsunamis, to support risk assessment and mitigation planning.

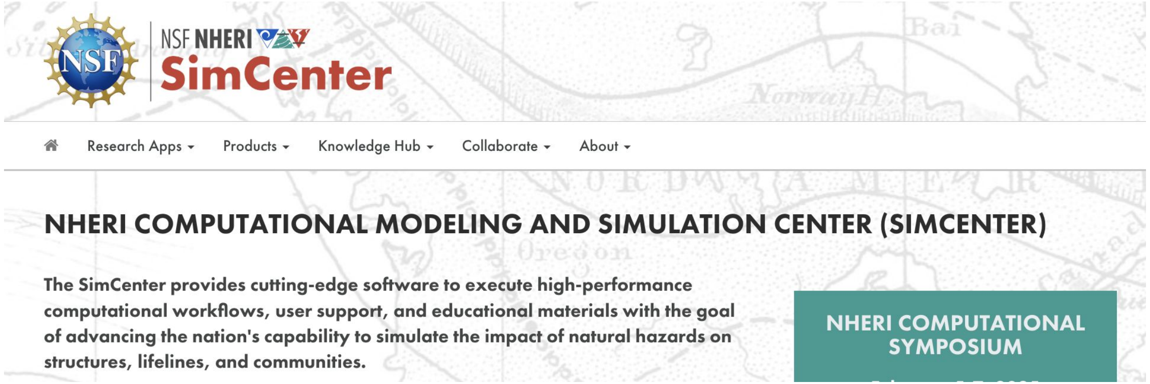
https://www.fema.gov/flood-maps/products-tools/hazus

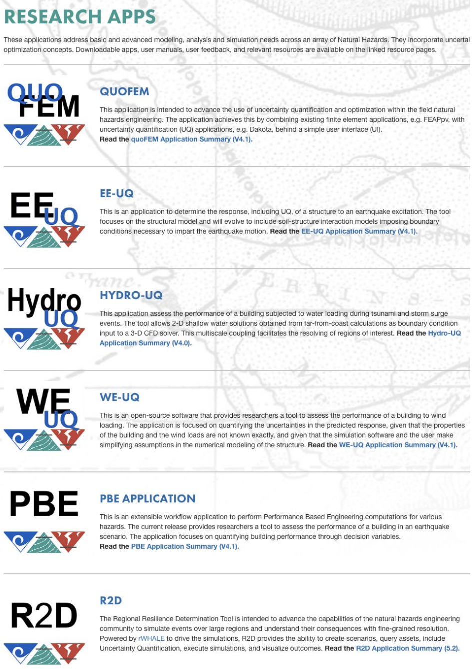
https://www.fema.gov/flood-maps/tools-resources/flood-map-products/hazus/user-technical-manuals



3 Useful Resources for Earthquake Modeling

HAZUS





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