Title:

On the fineness ratio of a nerf gun dart

Extended Essay in Physics

Word count: 3876

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Abstract

This paper will investigate how the fineness ratio of a nerf gun dart affects its drag force while moving through air. The research question for this essay is “How does the fineness ratio of a nerf gun dart affect its drag force while moving through air.” It is important to understand the nature of the frictional regime experienced by fast moving objects through air, as well the role of the co-efficient of drag in aerodynamic motion. These ideas will be reviewed.

This essay will study the fineness ratio of a standard nerf gun dart. Changes to the fineness ratio will be achieved by successive reductions of the dart’s length, whilst keeping the width constant. Experimental issues described in the paper limit the study of fineness ratios greater than 5.5. Other relevant variables are carefully controlled. In situations where the control of certain variables is difficult a large quantity of data is collected so that accuracy can be achieved.

Current research on the fineness ratio is reviewed. Much of this research suggests that there is an optimum fineness ratio, that is a ratio for which the coefficient of drag has a minimum value.

A computer iteration is developed to predict the range of a horizontally fired nerf gun dart for various input drag coefficients. Practical measurements of the range of the darts are carried out. The drag coefficient of the dart is determined by matching the iteration to the practical data. The results suggest that there exists an optimum fineness ratio for which the drag coefficient has a minimum value. This ratio has a value between 4.9 and 5.1. These findings differ in some respect from existing research relating to a solid flat-nosed cylinder.

Abstract word count: 283

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Introduction

The fineness ratio is an important concept in the field of aerodynamics. The width to length ratio of an object plays a significant role in its behavior as it moves through a fluid. The long and slender profile of Concorde has its origins in the physics of the fineness ratio as does the balloon shaped fuselage of the Questair Venture[1] and other craft. The application of the concept of the fineness ratio extends beyond aerodynamics. Many studies have been carried out on the fineness ratio of aquatic animals such as whales, sharks, and dolphins. A Nerf gun dart provides an opportunity to investigate the concept of fineness ratio in a controlled environment where practical measurements are possible. Conclusions drawn in such as study can be extended to other areas.

The research question addressed in this essay is:

"How does the fineness ratio of a nerf gun dart affect the coefficient of drag of the dart as it moves through air?"

The first objective of this essay is to determine the coefficient of drag for a range of fineness ratios of a nerf gun dart. The second objective is to determine whether there exists an optimum fineness ratio for which the drag coefficient has a minimum value. It is necessary to determine the coefficient of drag for different fineness ratios. To do this the theoretical background of fluid motion is reviewed and is used to create a computer iteration which simulates the trajectory of a horizontally fired dart, subject to the input of certain parameters including the drag coefficient. Practical data for the range of darts with different fineness ratio is collected, and parameters such as the nozzle velocity are measured. The coefficient of

[1] "Fineness Ratio." Wikipedia, Wikimedia Foundation, 13 Feb. 2018, en.wikipedia.org/wiki/Fineness_ratio.

drag is determined by matching the prediction of the iteration with those of practical experimentation. Once this is achieved the relationship between the fineness ratio and the coefficient of drag can be studied.

The computer iteration requires the accurate input of certain parameters including the radius, mass, exit velocity, amongst other variables. Care is taken to control these variables. A possibility exists that the nozzle velocity will change with fineness ratio. To this end, the exit velocity is carefully determined for each fineness ratio, by the means of video analysis.

Theory

Overview

When any object moves through a fluid it experiences an opposing drag force. The drag force of an object depends on the nature of the fluid it is moving in, and can be identified as laminar or turbulent as shown in figure 1 bellow.

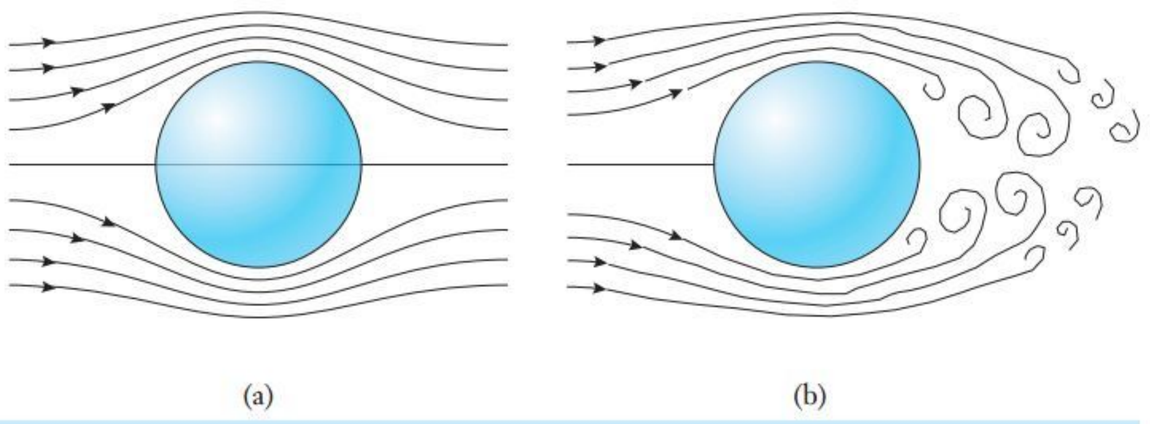

Fig. 1: Laminar flow and turbulent flow[2]

Flow (a) represents laminar flow, where the fluid flows in parallel layers with no exchange of momentum between them[3]. This is the case when smooth spherical objects move through viscous fluids. In such situations, the drag force is modelled by Stokes's law

$$F = 6\pi r\eta v \quad [1]$$

Where F represents the viscous drag force, r represent the radius of the object, η, the viscosity of the fluid, and v the velocity of the object. Stoke's law applies only to spheres, and the concept of fineness ratio cannot be applied in this situation.

[2] "Laminar and Turbulent Flow." Laminar Flow, Wikipedia, 21 Jan. 2018, en.wikipedia.org/wiki/Laminar_flow,

[3] "Laminar Flow." Wikipedia, Wikimedia Foundation, 12 Jan. 2018, en.wikipedia.org/wiki/Laminar_flow.

An alternative to laminar flow is turbulent flow as shown in figure (b). In turbulent flow, there is exchange of momentum between the fluid packets immediately surrounding the moving object. Turbulent flow is influenced by the presence of vortexes. Turbulent flow dominates when the object is moving in high speeds through fluids of low density. Fluid drag in non laminar regimes is governed by the drag equation:

$$F = \frac{1}{2} Cd\, A\, \rho\, V^2 \text{ [2]}$$

Where F represents the turbulent drag force, A represents the cross-sectional area at the widest point, ρ, the fluid density, and V represents the velocity of the object. The drag equation for turbulent regimes differs from Stokes's law in several ways. In the drag equation for turbulent flow the resistive force varies quadratically with the velocity of the object, whereas for laminar flow the relationship is linear. A major difference for the purposes of this essay is the inclusion of the drag coefficient in the equation governing turbulent flow. The value of the coefficient of drag depends, amongst other things, on the geometry and physical dimensions of the object. The coefficient of drag is known to depend on the fineness ratio of the object. This fineness ratio is defined as the ratio of the length to the maximum width of the object[4] and is dimensionless.

Before discussing the effect of the fineness ratio on the coefficient of drag it is necessary to show that the nature of the fluid friction as a nerf gun moves through air is turbulent. To do this use is made of the Reynolds number:

$$Re = \frac{sL}{v}\text{[3]}$$

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4 "Fineness Ratio." Merriam-Webster, Merriam-Webster, 12 Jan. 2018www.merriam-webster.com/dictionary/fineness%20ratio.

Where s is the speed of the fluid with respect to the object, L is the characteristic linear dimension, and v is the kinematic viscosity of the fluid. For values of $Re > 10^{3}$[5] it is expected that the flow is turbulent. For a nerf gun dart moving through air, the order of magnitude of the velocity is 10^{1}. The dart has dimensions in the order of $10^{-2}m$. The magnitude of the dynamic viscosity of air is 10^{5}[6]. When these values are placed in the equation [3] an order of magnitude of 10^{4} is obtained for the Reynolds number. It can be assumed that turbulent flow exists.

This paper will investigate the effect of changing the fineness ratio on the drag coefficient of an object moving in a turbulent regime. The fineness ratio (Fr) is modelled by:

$$Fr = \frac{L}{W} \; [4]$$

Where L represents a characteristic length, and W the maximum cross-sectional diameter. To understand the fineness ratio, it is necessary to understand the two dominant forces of drag acting on an object moving through a viscous fluid.

When a nerf gun dart moves through air, viscous forces cause the air in immediate contact with the surface to move at a slower velocity than the air which is located further from the surface. The net effect is that the dart "carries" the air molecules close to its surface, transferring momentum to these molecules. This means that the dart receives a force

[5] IB Physics. IB Physics Data Booklet Revised 2016, IBO, 2016, United Kingdom. https://ibphysics2016.wikispaces.com/file/view/Physics_Data_Booklet_2016.pdf ,

6 "Reynolds Number." Wikipedia, Wikimedia Foundation, 21 Jan. 2018, en.wikipedia.org/wiki/Reynolds_number.

opposing its motion by Newton's third law. This type of drag is known as skin drag[7]. In this paper, increases in the fineness ratio will be produced by increasing the length of the dart while keeping its diameter constant. This will result in a greater surface area, and consequently a greater skin friction. If this were the only source of drag, an increase in the fineness ratio would always lead to an increase in the drag coefficient.

An additional form of drag acting on the nerf gun dart exists. This drag is often referred to as pressure drag. As the dart moves through air, compression occurs at the front of the dart creating an area high pressure[8]. Turbulent flow acts at the end of the dart causing low pressure. Consequently, pressure drag is created and is directed from the leading edge to the trailing edge of the dart. This drag is influenced by the pressure gradient, which is inversely dependent on the distance between the front end and the back end of the dart. Increasing the length of the dart causes a decrease in its pressure gradient, and consequently decreases the pressure drag. For skin drag, increasing the length produced the opposite effect.

It can be seen that the relationship between the fineness ratio and the drag coefficient is complex involving two opposing factors. An increase in the fineness ratio achieved by increasing the length of the dart while keeping the diameter constant will cause an increase in the net skin drag, but a decrease in the net pressure drag. This raises the possibility that there could exist an optimum fineness ratio for which the net drag is minimized.

[7] "SKYbrary Wiki." Friction Drag - SKYbrary Aviation Safety, Skybrary, 27 July 2017, www.skybrary.aero/index.php/Friction_Drag,

[8] "Drag of Blunt Bodies and Streamlined Bodies." Princeton University, The Trustees of Princeton University, 12 Jan. 2018, www.princeton.edu/~asmits/Bicycle_web/blunt.html.

Internet research shows little information on how the coefficient of drag of a nerf gun dart is affected by its fineness ratio. There is research however[9], on the relationship between the drag coefficient and the fineness ratio for cylinders. The results of a study by Syracuse University[10] are shown in figure 2 below. In their paper the university tested both flat and smooth nosed cylinders.

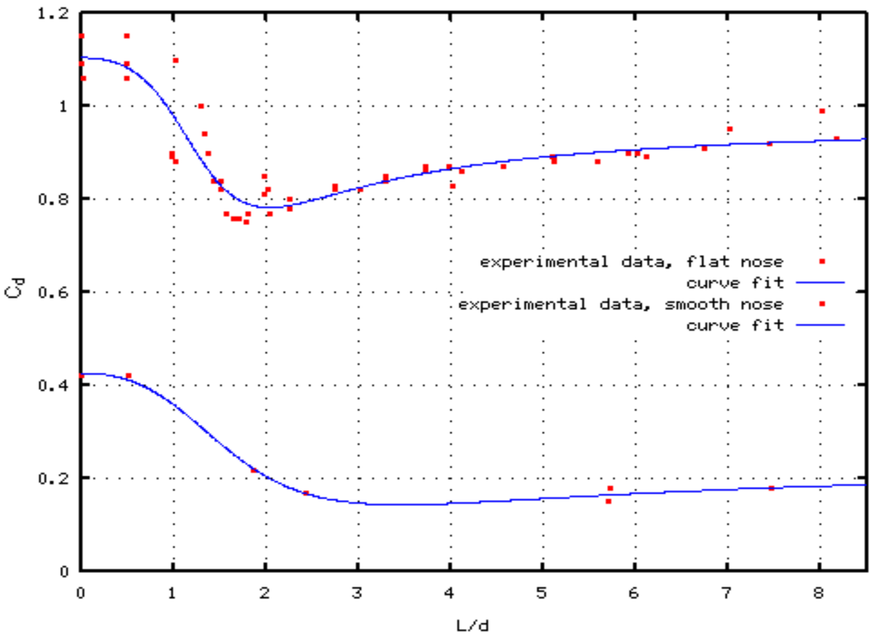

Fig. 2 A graph showing how the drag coefficient for flat and smooth nosed cylinders depends on the fineness ratio. The graph for a flat nosed cylinder shows a distinct minima suggesting an optimum drag coefficient.[11]

The nerf gun darts used in this paper can reasonably be described as flat nosed. The flat nose curve suggests the existence of an optimum fineness ratio where the coefficient of drag is minimized. This is consistent with the two opposing frictional regimes described

[9] "Drag Coeffecients of Nerf Gun Darts" Btrettel's Nerf Blog, 31 Mar. 2013, 18 Jan. 2018 btrettel.nerfers.com/archives/78,

[10] "Investigating the Drag Coeffecient" Syracuse University http://eng-cs.syr.edu/faculty/higuchi/Papers/JFM08MagSus.pdf,

[11] "Graph of Drag Coefficient versus Fineness Ratio." Btrettels Nerf Blog, 31 Mar. 2013, 18 Jan. 2018 btrettel.nerfers.com/archives/78.

above. Daniel Beaver[12] studied the coefficient of drag for nerf gun darts of 1.25 inches and found a drag coefficient of 0.67. Beaver did not investigate how this value was affected by the fineness ratio. This paper will seek to add to the current research by producing values for drag coefficient of nerf gun darts of various fineness ratios. To do this it is necessary to develop a method to determine the coefficient of drag of a nerf gun dart.

Determining the coefficient of drag of a nerf gun dart

To determine the coefficient of drag, a computer iteration is developed, which uses the drag equation and certain input perimeters such as the nozzle velocity of the nerf gun dart to predict the range of the dart when fired horizontally from a given vertical height. The computer iteration allows for the coefficient of drag to be varied. Practical data is then taken for the range of nerf gun darts of varying fineness ratios. For each range determined experimentally, the drag coefficient in the iteration is changed until a best fit is found.

The iteration utilizes a series of rows, referred to as steps, to calculate the range of the nerf gun dart. Each step represents a small finite time interval. A time interval of 0.01s is used. During this interval, it is assumed that the force and acceleration acting on the dart remain constant. The initial velocity and acceleration of the dart are input in the beginning of the first step. During each step the equations of motion are used to determine the velocity and acceleration at the end of the step. These values are then entered into the beginning of the next step. At the end of each step the distanced moved within that step is determined. The total distance is summed cumulatively as the steps progress. The process is repeated in this

[12] Guitarzan. "Ever Chronographed Your Blasters? - General Nerf." NerfHaven, 18 Mar. 2012, nerfhaven.com/forums/topic/22353-ever-chronographed-your-blasters/#entry313246.

manner over many steps, allowing the trajectory of the dart to be determined over finite time intervals. By changing the value of the drag coefficient used by the iteration, different horizontal ranges are generated. By comparison to practical data the appropriate drag coefficient can be deduced.

The iteration requires an understanding of the vector nature of the drag force under turbulent flow. As discussed earlier the drag force can be written in scalar form as $F = \frac{1}{2} Cd\ A\ \rho\ V^2$. In vector form this equation is written as:

$$F_{(i,j)} = K\left(V_i + V_j\right)|V| \ \ [5]$$

Where K represents $\frac{1}{2} Cd\ A\ \rho$.

This means that two iterations will have to run simultaneously. The first iteration will compute the trajectory in the X direction, whilst the second iteration will compute the trajectory in the Y direction.

Before the iteration can proceed, certain input perimeters must be entered. Figure 3 below shows input data for the nerf gun dart.

	A	B
1	mass	0.0015
2	cd	0.5
3	radius	0.0065
4	density of air	1.18
5	V(iX)	15
6	ΔT	0.01
7		

Fig. 3 The input data for the nerf gun dart. The radius and mass are determined by practical measurement.

Figure 4 below shows a screenshot of the iteration in the horizontal direction.

	1	2	3	4	5	6
A	Vx	Fx	AccelerationX	ΔVx	Vf	Sx
B	15	0.0088056	5.87042625	0.0587043	14.941296	0.1497065
C	14.94129574	0.008737	5.824693269	0.0582469	14.883049	0.2988282
D	14.88304880	0.0086696	5.779745921	0.0577975	14.825251	0.4473697
E	14.82525135	0.0086034	5.735571	0.0573557	14.767896	0.5953354

Fig. 4 The iteration of the horizontal velocity and distance travelled of the nerf gun dart. The first four steps have been shown.

The iteration starts at cell B1. The initial horizontal nozzle velocity is input into this cell. In cell B2 the velocity found in cell B1 is used to determine the horizontal drag force acting on the dart using equation [2]. In cell B3 the horizontal acceleration of the dart is determined by dividing the drag force by the inertia of the nerf gun dart. In cell B4 the acceleration in cell B3 is used find the change in velocity during the time interval. In cell B5 the final velocity is determined by subtracting the initial velocity found in B1 by the change in velocity found in cell B4. In cell B6 the horizontal distance travelled is determined by multiplying the average velocity by the time interval.

Whilst this step is being iterated in the horizontal direction, a simultaneous iteration proceeds to determine the velocity and displacement in the Y direction. Figure 5 shows the iteration in the Y direction:

	7	8	9	10	11	12		13
THEORITICAL DATA								
A	Vy	Fx	AccelerationY	ΔVy	Vyf	Sy		Vr
B	0	0.01476	9.84	0.0984	0.0984	0.000492		14.94162
C	0.0984	0.0147025	9.80163989	0.0980164	0.1964164	0.0019661		14.884345
D	0.1964164	0.0146456	9.76372283	0.0976372	0.2940536	0.0044184		14.828167
E	0.2940536	0.0145894	9.72623697	0.0972624	0.391316	0.0078453		14.773079

Fig. 5 The iteration of the vertical velocity and distance travelled of the nerf gun dart. The first four steps have been shown.

The initial vertical velocity is 0. Cell B8 represents the weight of the object. Cell B9 represents the acceleration due to gravity. In cell B10 the change in vertical velocity is determined during the step. In cell B11 the final vertical velocity is found by adding the change in velocity to the initial velocity. The vertical distance travelled within the step is determined in cell B12 by multiplying the average vertical velocity within the step by the time interval. An additional column exists as B13. In this cell the resultant velocity at the end of the first step is determined by using the equation below:

$$|V| = \sqrt{(V_i)^2 + (V_j)^2} \ [6]$$

A knowledge of the resultant force at the end of the first step is necessary to complete future iterations. The second step of both the vertical and horizontal iteration differs from that of the first, since the velocity of the dart at the beginning of the second step is no longer horizontal. The drag force in the horizontal direction is determined by multiplying its horizontal velocity by the magnitude of the velocity determined at the end of step 1 according to equation [4]. From this moment onwards the iteration continues as described in step 1. For the vertical iteration, the second step differs from the first, in how the net force is determined. The drag force is determined like that of the horizontal direction described above, however the effect of the weight must be accounted for. The net force in cell C8 is determined by subtracting the drag force from the weight. The iteration then proceeds as for step 1. At the end of each step the cumulative distances traversed in both the horizontal and vertical directions are summed.

Experimental Method

Overview

To determine the relationship between the coefficient of drag and the fineness ratio of a nerf gun dart, nine darts of varying fineness ratios were used and their average horizontal range when fired through air from a known vertical height was determined. A total of twenty trials were conducted for each dart. The horizontal ranges were then compared to the ranges calculated by the computer iteration for varying drag coefficients, and a drag coefficient is determined for which the iteration best matched the practical data. A graph of the fineness ratio versus the corresponding drag coefficient can then be plotted

I. Altering the fineness ratio of the darts

Altering the width of the dart was not practical thus the fineness ratio was altered by shortening the length. Dart lengths ranging from 46.39mm $\pm$0.5mm to 67.37m $\pm$0.5mm were used to collect the experimental data. These lengths had corresponding fineness ratios from 3.80$\pm$0.04mm to 5.52$\pm$0.05mm. Darts of length less than 46mm were observed to behave erratically. These darts tended to side-wind as they travelled through air. In addition, video analysis of their nozzle velocity revealed considerable variation for darts of these lengths. This imposed a practical lower limit on the values of fineness ratios investigated.

II. Determining the nozzle velocity of the darts

To determine the horizontal velocity of the dart, video analysis was used. The software employed was Tracker 4.11.0 operating on an OSX platform. The nerf gun darts were filmed at 240fps using an iPhone X. The nerf gun dart was fired horizontally, parallel to a meter ruler for scaling purposes as shown below in Fig.6:

Fig. 6 Nerf gun dart fired from rifle parallel to meter ruler

Five videos were filmed for each dart to determine the average velocity. The software was calibrated with reference to the meter ruler and allows for the position of the dart to be tracked for consecutive frames as shown in Fig.7:

Fig. 7 Sample example of the video analysis. Where the diamond shape represents the position of the dart at a certain frame

The software produces a graph of the horizontal velocity versus time. A screenshot for a particular trial is shown in Fig.8:

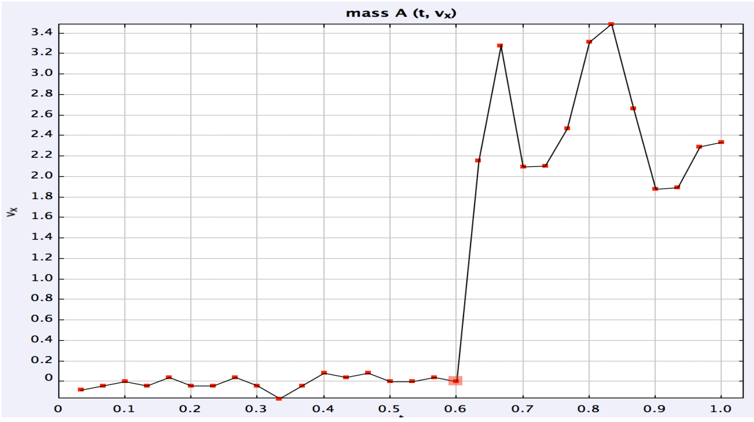

Fig. 8 The graph of velocity versus time of flight for the nerf gun dart. The dart starts moving at t= 0.6

The velocity of the dart was determined by taking an average of the maximum and minimum values in the interval $[0.6s, 1.0s]$.

III. Collecting the horizontal range of the darts

To determine the average horizontal range of the dart, twenty trials were conducted. The dart was fired horizontally from a vertical stand of height 1.23m as shown below in Fig.9:

Fig. 9 Data collection setup for recording the horizontal range of the dart

To determine the distance travelled by the dart, points were marked where it was first observed to contact the floor. The location of each cross was measured using a tape measure. An average range for each dart was determined, alongside its average deviation.

IV. Determining the drag coefficient

Key input parameters, including the nozzle velocity, density of air, and the mass of the dart, are entered into the iteration. The coefficient of drag is changed until the theoretical horizontal and vertical ranges are best matched to those of the experimental results. The predicted range of the iteration is subject to error. The primary source of error in the iterated results lies in the uncertainty associated with the nozzle velocity as an input parameter. This uncertainty in the iterative value for the range needs to be considered when determining the errors associated with the final drag coefficient.

Uncertainties in Raw and Processed Data

Experimental results within the field of fluid motion may be subject to many random errors, thus it is important to account for the uncertainties present. To minimize uncertainties associated with the range, twenty trials were conducted. The precision of each trial was estimated as ± 0.05m reflecting the difficulty in precisely locating the first point of contact. The most significant source of error was the deviation between trials. To this end, the average deviation of the twenty trials is taken to represent the uncertainty of the experimentally determined values.

The determination of the nozzle velocity is also subject to error and is taken as the average deviation between the five trials. In practice the errors associated with the nozzle velocity measurements were in the order of $\pm 0.5ms^{-1}$. The smallest percentage error was 1% associated with a dart length 60.0mm, and the largest was 8% associated with a dart length 47.1mm.

The dart lengths and diameters were measured with a digital Vernier caliper with a resolution of ± 0.01mm. The spongy nature of the dart caused it to compress when using the calibre to determine its length. For this reason, an error of ± 0.5mm was attributed. The diameter of the dart was taken to be the diameter of the plastic cap. This was less comprisable and an error of ± 0.1mm was used. The uncertainty in length also caused an error in the fineness ratio. The error in the fineness ratio is taken as being the sum of the fractional errors on the diameter and length.

Determining the error on the coefficient of drag is less simple, since there exists an error on the practically determined range as well as the theoretical iterated range due to the uncertainty into the inputted values of nozzle velocity. For a given dart, the maximum nozzle velocity was entered into the iteration. The coefficient of drag was adjusted so that two values were found corresponding to the maximum and minimum practically determined range. The process was repeated for the minimum nozzle velocity. By this means, two ranges of values are obtained for the drag coefficient. The intersection of the data is taken as to represent the final range of values for the drag coefficient. The midpoint of this range is plotted on the graph, with the error bars representing the limits of the range.

Experimental Results

Dart	$L \times 10^{-3}$ / m +/- 5×10^{-4}m	F	ΔF
A	67.4	5.52	0.05
B	60.0	4.92	0.05
C	54.0	4.42	0.05
D	50.4	4.13	0.04
E	47.1	3.86	0.04
F	62.9	5.15	0.05
G	62.7	5.14	0.05
H	59.1	4.84	0.05

Fig. 10. Lengths and fineness ratios of dart used

	Range /m +/- 0.05m																			
DART	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14	#15	#16	#17	#18	#19	#20
A	6.10	6.20	6.55	6.90	6.93	6.92	6.98	7.00	7.09	7.09	7.10	7.20	7.38	7.40	7.48	7.55	7.80	7.78	8.00	8.08
B	7.07	7.15	7.40	7.41	7.45	7.43	7.65	7.87	7.95	7.98	8.03	8.12	8.12	8.23	8.28	8.36	8.46	9.05	8.32	8.21
C	6.66	6.62	6.68	6.95	6.94	6.90	7.05	7.05	7.08	7.10	7.25	7.73	7.30	7.45	7.44	7.55	7.62	7.78	8.30	8.01
D	5.80	9.95	6.02	6.07	6.08	6.18	6.19	6.25	6.33	6.41	6.45	6.51	6.68	6.68	6.10	6.83	7.08	0.31	7.44	7.50
E	3.45	3.44	3.65	3.72	3.80	4.02	4.03	4.22	4.38	4.44	4.65	4.58	4.72	4.82	4.97	5.08	5.15	5.35	5.52	5.05
F	7.40	7.40	7.75	7.92	8.00	8.20	8.22	8.25	8.23	8.45	8.52	9.07	9.07	9.15	9.14	9.25	9.28	9.48	9.90	10.01
G	7.40	7.62	7.90	7.95	8.00	8.08	8.22	8.38	8.34	8.48	8.65	8.88	9.02	9.20	9.50	9.60	9.60	10.20	10.30	10.85
H	7.40	7.60	7.58	7.58	7.90	7.90	8.02	8.10	8.05	8.00	8.20	8.35	8.45	8.60	8.70	8.90	9.00	9.30	9.32	9.25

Fig.11. Horizontal range of each nerf gun dart conducted with twenty trials

Dart	Average R /m	ΔR /m
A	7.2	0.4
B	7.9	0.4
C	7.3	0.4
D	6.3	0.8
E	4.5	0.5
F	8.6	0.7
G	8.8	0.8
H	8.3	0.5

Fig.12. The average value and average deviation of the range

Dart	Nozzle Velocity /ms^{-1}						
A	#1	#2	#3	#4	#5	AVG V /ms^{-1}	ΔV /ms^{-1}
B	17.6	16.0	16.0	16.8	17.6	16.8	0.6
C	16.8	16.8	16.8	16.4	16.8	16.7	0.1
D	18.4	19.2	19.2	16.8	20.0	18.7	0.9
E	21.0	20.6	20.4	21.0	21.2	20.8	0.3
F	15.5	16.8	14.4	13.6	12.8	14.6	1.2
G	20.0	18.4	18.8	19.6	19.6	19.3	0.5
H	20.4	20.0	20.0	18.4	19.2	19.6	0.6

Fig.13. Measured values of the nozzle velocity, its average, and average deviation

Dart	Cd	ΔCd	Fr	ΔFr
A	0.8	0.1	5.52	0.05
B	0.49	0.03	4.92	0.05
C	1.1	0.3	4.42	0.05
D	2.9	0.1	4.13	0.04
E	3	0.9	3.86	0.04
F	0.5	0.4	5.15	0.05
G	0.5	0.4	5.14	0.05
H	0.65	0.3	4.84	0.05

Fig.14. The coefficient of drag and its error, and the fineness ratio for each dart

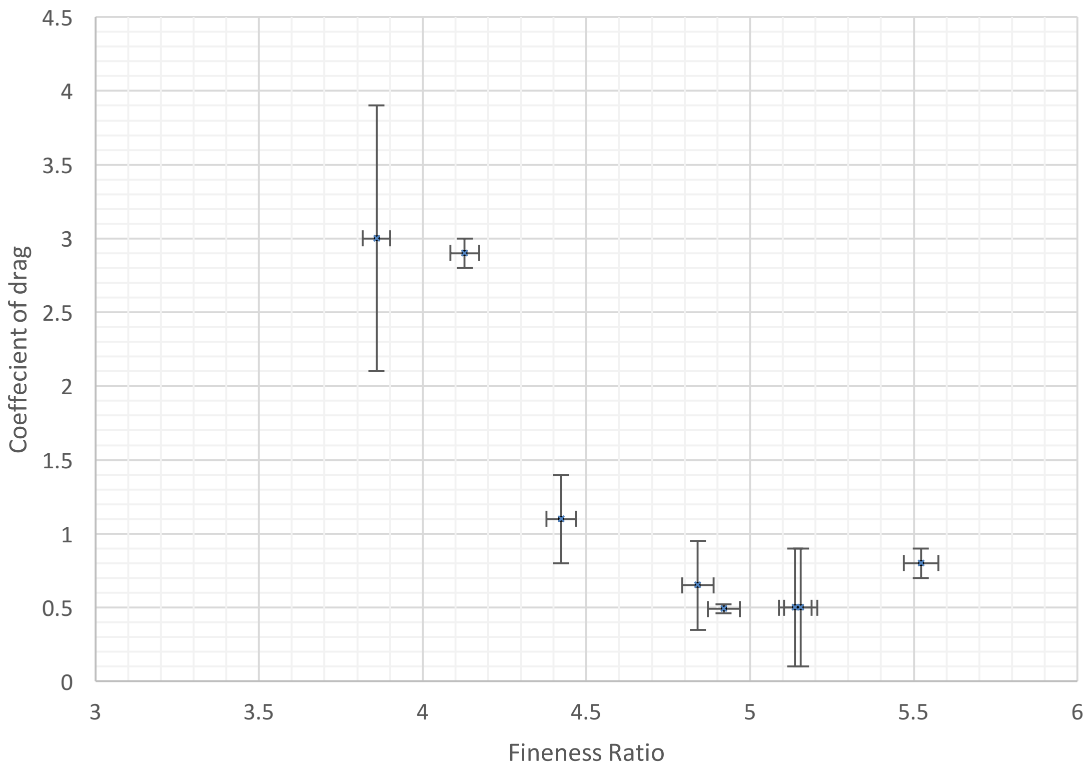

Fig.15. Graph of coefficient of drag versus the fineness ratio

Discussion

This paper set out to investigate the relationship between the coefficient of drag of a nerf gun dart, and its fineness ratio. It was argued that there exists a possibility that there is an optimum fineness ratio for which the coefficient of drag is minimized. This argument was based on a consideration of the different frictional regimes governing the motion of the dart. The argument was supported by research data (figure 2) for flat nosed solid cylinders, which suggests an optimum fineness ratio occurring at an approximate value of 1.8. The experimental results in this paper are represented by figure 15. Figure 15 provides convincing evidence of the existence of an optimum fineness ratio. Due to the uncertainties present in the data, it is difficult to precisely determine the value of the optimum fineness ratio. It seems reasonable to suggest that the optimum lies between the data points (4.9, 0.5) and (5.1, 0.5). In this respect the data differs from that of the researched data for solid nose cones.[13] However, there are a number of similarities between the researched data, and the practical data provided by this study. Both graphs display asymmetry with a positive skew being a key feature. The fall in fineness ratio leading up to the optimum is sharper than the rise in the fineness ratio for values exceeding the optimum. Figure 15 shows a strong negative gradient for the domain [4.1, 4.9]. Comparing the behaviour of the researched data and that of figure 15 is less easy for values of the fineness ratio exceeding the optimum, since the domain is limited by the maximum practical fineness ratio (5.5) corresponding to the un-shortened dart. Nevertheless, the data points seem to support the argument that the graph is positively skewed.

The graph shows large error bars associated with the data point (3.9, 3). This data point corresponds to the shortest fineness ratio investigated. This can be understood since the error on the coefficient of drag was determined through consideration of both the error on the

[13] Syracuse University.

nozzle velocity (theoretical iteration) and on the practically determined range. Shortening the dart caused the exit velocity to vary considerably between shots and the bullet tended to side-wind causing significant deviations in the measured ranges. These errors compound to produce the large error bar observed in figure 15. The data points corresponding to (5.14, 0.5) and (5.15, 0.6) are of interest. These data points correspond to two darts of very similar fineness ratios. The fact that their coefficient of drag are very similar sharing common uncertainty boundaries is reassuring from an experimental viewpoint, and adds confidence to the data points as a whole. In any experiment involving fluid friction, there are many sources of random error. The error bars on most data points are surprisingly small, and this justifies the use twenty trials for each dart.

Conclusion

The experimental results strongly suggest that there exists an optimum fineness ratio for which the drag coefficient is minimum, this minimum has a value of approximately 0.5 and occurs for a fineness ratio that lies between 4.9 and 5.1. The presence of an optimum fineness ratio supports the findings of the University of Syracuse, however the findings of this paper differ in the value of the fineness ratio at which the optimum occurs. A likely reason for this is that the University of Syracuse studied a flat-nosed cylinder rather than the specific case of a nerf gun dart. Due to the methodology used, there was an upper limit on the fineness ratios that could be investigated in this study. Ratios above 5.5 remain uninvestigated. This study used a particular nerf gun dart. There are many different darts available in the market and it is possible that using these darts might allow for a greater domain of fineness ratios to be investigated.

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