Uniform Distribution and Its Types

In probability theory and statistics, a uniform distribution is a probability distribution where all outcomes are equally likely within a given range. This means that if you were to select a random value from this range, any value would be as likely as any other value.

Types:

- Discrete Uniform Distribution
- Continuous Uniform Distribution

Denoted as:

- Discrete Uniform Distribution: $X \sim U(a, b)$
- Continuous Uniform Distribution: $X \sim \mathrm{Unif}(a, b)$

In these notations, 'a' and 'b' represent the lower and upper bounds of the uniform distribution, respectively. For instance, in the context of a dice roll, 'a' is 1 and 'b' is 6, meaning the outcomes range from 1 to 6.

Examples

1. Continuous Uniform Distribution:

Height Example: Imagine you randomly select a person from a group of people whose heights range from 5'6" to 6'0". If every height within this range is equally likely, then the height follows a continuous uniform distribution. This means there's no preference for any specific height within this range.

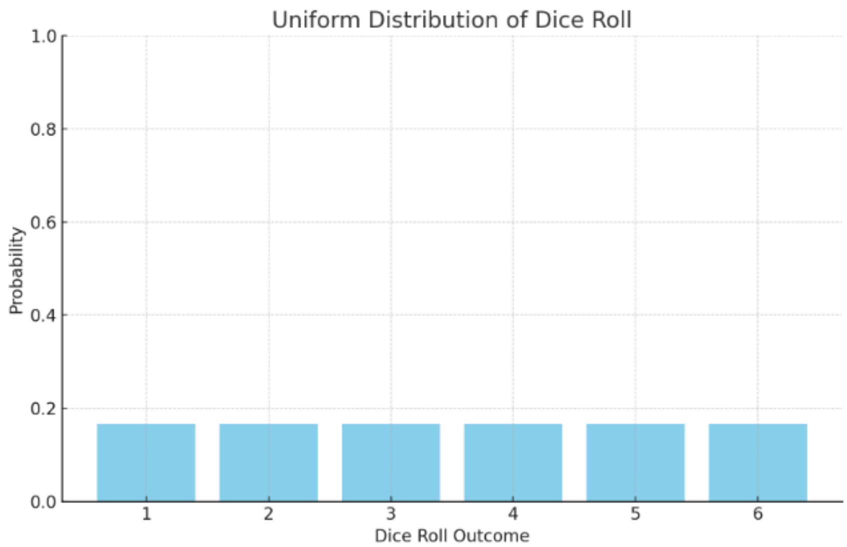
Production Time Example: Suppose a machine takes between 5 to 10 minutes to produce a product. If every minute within this range is equally likely for the production time, then the time follows a continuous uniform distribution.

Travel Distance Example: Consider a car that can travel between 300 to 400 miles on a tank of gas. If every mile within this range is equally likely, then the travel distance follows a continuous uniform distribution.

Apple Weight Example: Imagine you have a basket of apples weighing between 100 and 200 grams. If the weight of any apple within this range is equally likely, then the weight follows a continuous uniform distribution.

2. Discrete Uniform Distribution:

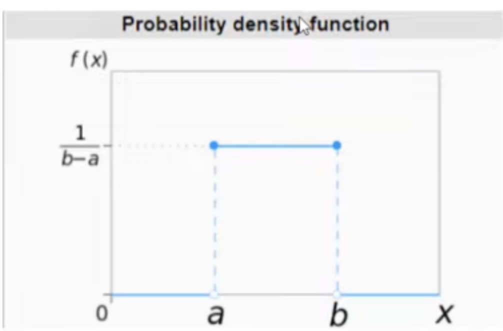
Rolling a Dice: Let's roll a dice. There are 6 possible outcomes (1, 2, 3, 4, 5, 6). Each outcome is equally likely with a probability of (\frac{1}{6}). If you plot the graph, all the heights (probabilities) will be equal and look uniform. Here, 'a' is 1 and 'b' is 6, representing the range of possible outcomes.



Mathematical Terms of Uniform Distribution

Probability Density Function (PDF):
For a continuous uniform distribution between 'a' and 'b', the probability density function (PDF) is given by:

$$f(x) = \{\, 1/(b-a)\ for\ a \leq x \leq b \\ 0\ otherwise \,\}$$



Explanation of $(\frac{1}{b-a})$:
In a continuous uniform distribution, the term $(\frac{1}{b-a})$ represents the height of the probability density function (PDF). This ensures that the total area under the curve is 1, representing 100% probability. For any value of 'x' within the range 'a' to 'b', the height of the PDF is $(\frac{1}{b-a})$

To understand this, recall that the area of a rectangle is given by the formula:

$$[\text{Area} = \text{Length} \times \text{Breadth}]$$
$$\text{Length} = \text{Area}/\text{Breadth}$$

Area = 1 (For the PDF of a continuous uniform distribution, the area under the curve must be 1)
The breadth is ($b - a$) (the range of values.
Therefore, the length (or height) must be:

$$[\text{Length} = \frac{1}{b-a}]$$

Cumulative Distribution Function (CDF):
The cumulative distribution function (CDF) for a continuous uniform distribution between 'a' and 'b' is:

$$F(x) \;=\; \{\, 0\, for \;\; x \;<\; a$$
$$(x \;-\; a)(b \;-\; a)\, for\; a \;\leq x \;\leq b$$
$$1 \;\; x \;>\; b\, \}$$